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SOME EVIDENCE IN FAVOR OF A MONETARY RATIONAL EXPECTATIONS EXCHANGE RATE MODEL WITH IMPERFECT CAPITAL SUBSTITUTABILITY*

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We develop and test a monetary rational expectations model of the Swiss/U.S. exchange rate. Two salient features of the model are the assumption that domestic and foreign currency denominated assets are imperfect substitutes, and that purchasing power parity need not hold. We fail to reject overidentifying restrictions imposed on the model by the rational expectations hypothesis. Our point estimates, especially for the income elasticity of the demand for money, are plausible. Finally, the model outperforms the random walk model established as a benchmark by Meese and Rogoff.

1. INTRODUCTION

Despite the apparent successes of early tests of monetary rational expectations models of exchange rate behavior, their reputations have been tarnished as more data and alternative econometric techniques have become available (see Driskill and Sheffrin 1981, and Meese and Rogoff 1983). Professional response to this event has taken an interesting form: some early proponents of the usefulness of rational expectations have now eschewed them, instead clinging to other features of the monetary models (e.g., Bilson 1981).

In this paper, we take another tack, and maintain the rational expectations hypothesis while amending the monetary model to incorporate imperfect capital substitutability and current-account effects. We then make use of implications of the rational expectations hypothesis to test the model. Our primary finding is that this model is generally consistent with the data, providing some evidence in favor of the joint hypothesis of rational expectations and our monetary model. For purposes of comparison with other exchange-rate studies, we also look at out-of-sample prediction, comparing it to the Meese-Rogoff benchmark of a random walk.

Our theoretical model is part of a line of work that emphasizes the interplay between risk-aversion, rational speculators and current account flow-market phenomena. Our model thus belongs to the generic class of inventory-speculation models beginning with Muth (1961) and extending, in the foreign exchange literature, through Black (1972), and Driskill and McCafferty (1980). To keep the analysis tractable, we have been forced to take a partial equilibrium approach, whereby money, real income, and price levels are treated as exogenous to the foreign exchange market. Even with our partial equilibrium approach, though, we are left with an ambitious econometric project. To mitigate the effects of our

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[1] We thank Pok-sang Lam for useful discussions. The comments of two anonymous referees led to an improvement in the paper.

partial-equilibrium simplification, we specifically choose the Swiss franc/U.S. dollar exchange rate for our empirical work: it seems quite plausible that the Swiss/U.S. exchange rate has small effects on other Swiss/U.S. macrovariables.

Our choice of the Swiss/U.S. exchange rate highlights additional methodological considerations. As argued elsewhere (Driskill and Sheffrin 1981, p. 1072, and Driskill 1981) most other commonly used bilateral exchange rates offer special problems that make it inappropriate to confront them with simple empirical monetary models. In brief, these problems arise either from managed floating vis-á-vis the dollar, or from the explicit linking of currency values first as in the "Snake" arrangements and subsequently in the EMS. Our desire is to find an appropriate testing ground to explore whether these theoretical notions have some empirical validity. In this manner, we hope to identify those components of a monetary model that can be used as building blocks in models more appropriate for other exchange rate investigations.[2]

The Swiss franc, which is not part of any currency area, seems suitable for the following reasons. First, while there has been some exchange market intervention, indirect evidence indicates that it has been mostly short term operations. The evidence is that, first, the Swiss authorities have allowed substantial movements in the real exchange rate. Second, high level Swiss National Bank officials have stated that exchange rate considerations have always been subordinate to price-level or inflation concerns. Third, we find that the quarterly data used in our study does not provide particularly strong evidence that the exchange rate Granger causes the relative money supply, relative income level, and relative price level.

Our empirical work has intellectual linkages in several directions. In the exchange rate literature, one link is to Driskill (1981) who estimated an exchange rate reduced form equation for the Swiss franc/U.S. dollar rate. His study, though, was not based on a rational expectations model and provided no structural estimates. Our study improves on Driskill's in these two dimensions because it is based on a rational expectations model, and does provide structural estimates. Another link is to McNelis and Condor (1982) who also investiage the Swiss/U.S. exchange rate, but again do not appeal to rational expectations or uncover structural parameters. Their emphasis is on the empirical gains achieved by estimating time-varying parameters. A third link is to Papell (1986), who also estimated a structural exchange rate model under rational expectations. In contrast to our work, Papell did not test hypotheses, partly because his time-series model of the exogenous forcing variables required estimation of a large number of parameters. Our view is that a central message of this research is that solutions for the endogenous variables incorporate rational agent's optimal forecasts of future realizations of the exogenous forcing variables. Our approach lets us test this hypothesis.

This leads us to a final linkage from our work to the literature on testing and estimating rational expectations models by making use of the overidentifying

[2] At the suggestion of a referee, we investigated these conjectures by estimating our model for both the dollar/DM and the dollar/yen exchange rates. In both cases, the model did poorly. For example, the point estimates of some of the structural parameters came out the wrong sign.

restrictions imposed by the rational expectations assumption. The theoretical underpinnings of this work are associated with Wallis (1981) and Hansen and Sargent (1980). Implementation of this work includes work by Driskill and Sheffrin (1981), Eckstein (1984), Hoffman and Schlagenhoff (1983), and Woo (1985). The Eckstein and Hansen and Sargent papers are relevant, in that they estimate models with an inherent speculative structure broadly analogous to our own. Hoffman and Schlagenhoff assert that they fail to reject the most stark rational expectations monetary model, one that assumes short-run purchasing power parity and uncovered interest parity. They do not take up the Meese/Rogoff challenge of out-of-sample prediction, and work by Anderson (1986) shows that their estimates are highly sensitive to their particular sample period. Woo estimates and tests a model that assumes purchasing-power parity, uncovered interest parity, but lagged money demand adjustment. This provides a theoretical rationale for incorporating the lagged exchange rate in the reduced form derived from the structural model. Woo fails to reject the model under rational expectations, and also investigates the out-of-sample prediction properties. In one respect, our model is closely related to Woo's: our assumption of imperfect goods and capital substitutability provides a theoretical rationale for incorporating the lagged exchange rate in the observable reduced form. Our structural model, though, includes quite different cross-equation restrictions on the observable reduced form than does Woo's. Our econometric implementation also differs from Woo's: we induce stationarity by differencing, whereas he detrends with a polynomial in time. As Granger and Newbold (1974) and Nelson and Kang (1981) have emphasized, the use of residuals from deterministic trends as data can lead to both spurious cyclical phenomena and spurious correlations. We also test the differenced data for stationarity; Woo does not. Thus, there are potential problems in Woo's estimation procedure, and special problems in interpreting his forecasting exercises.

The remainder of the paper is organized as follows. The specification of the model that we estimate and test is contained in the next section. Section 3 reports the estimation results and the out of sample prediction experiments that we perform. Some concluding remarks are reserved for Section 4.

2. THE MODEL

Our model may be termed "monetary" in as much as relative money supplies play a prominent role in exchange-rate determination. The feature that distinguishes it from most other monetary models is its assumption about stock/flow interations under conditions of less-than-perfect international capital substitutability. As a result, we do not require purchasing power parity to hold, which as an empirical relationship, has fared poorly during the American float.[3] In addition, we focus primarily on the foreign exchange and money markets, assuming prices, real incomes, and money supplies are exogenous to the exchange rate. We provide Granger causality tests that support this assumption.

The basic building blocks of the model are a money market equilibrium

[3] Mark (1990) finds statistical evidence that purchasing power parity fails to hold even in the long run.

condition, a foreign exchange market equilibrium condition, and a specification of the stochastic processes governing the behavior of the exogenous forcing variables, in our case, relative money supplies, income levels, and price levels. Most variables will be expressed as logarithms of *relative* variables, that is, as the log of the ratio of Swiss to U.S. variables.

Money Market Equilibrium. Real money demand takes the well-known form of

$$(m_t - p_t)^d = -\lambda^{-1} r_t + \hat{\pi} y_t + \hat{\varepsilon}_t, \tag{1}$$

where m, p, and y are logs of the ratio of domestic to foreign money supplies, price levels, and real income levels, respectively, and $\hat{\varepsilon}$ is a random variable, the difference of which follows a random walk.[4] The interest rate differential is r_t, which means that λ^{-1} is the interest semi-elasticity of the demand for money. The money demand income elasticity is $\hat{\pi}$. Assuming demand equals the exogenous supply, equation (1) can be rewritten

$$r_t = -\lambda m_t + \lambda p_t + \pi y_t + \varepsilon_t, \tag{2}$$

where $\pi = \lambda \hat{\pi}$, and $\varepsilon = \lambda \hat{\varepsilon}$.

Foreign Exchange Market Equilibrium. The major components of the foreign exchange building block are specifications of trade balance behavior, capital flow behavior, and a market equilibrium condition.

The trade balance, measured in foreign currency units, is expressed as follows:

$$T_t = \alpha(e_t - p_t) - \psi y_t + u_t, \tag{3}$$

where $\alpha > 0$, $\psi > 0$, and u_t is a random variable assumed to follow a random walk. This equation simply says that the trade balance depends on relative prices and income. Assuming $\alpha > 0$ amounts to an assumption that the Marshall-Lerner condition is satisfied.

The net stock demand for foreign assets is assumed to be proportional to the expected relative rate of return:

$$F_t = \eta[E_t e_{t+1} - e_t - r_t], \qquad \eta > 0, \tag{4}$$

where, $E_t(x_{t+k})$ is the expectation of x_{t+k} conditioned on information available at time t. This, of course, is a very simple specification of asset demand, but captures, we think, the critical feature of speculative behavior in international capital markets. Perfect capital substitutability would correspond to $\eta \to \infty$.

Market equilibrium is characterized by the equality of net capital exports with the trade balance surplus, T_t, plus autonomous flows, $\bar{A}$:

$$\Delta F_t = T_t + \bar{A}. \tag{5}$$

[4] To justify the assumption that $\hat{\varepsilon}$ follows a random walk, we appeal to the notion that changes in transaction technology, for example the development of automatic tellers, seem likely to be nonstationary.

$\bar{A}$ is assumed constant, and for expositional ease will be set equal to zero. Alternatively, and more realistically, we can treat $\bar{A}$ as following a random walk. Both assumptions are consistent with our estimation results.

The Observable Reduced Form. To facillitate the exposition, we adopt the following vector notation for the exogenous variables and the model's parameters:

$$X_t \equiv \begin{bmatrix} p_t \\ y_t \\ m_t \end{bmatrix}, \qquad a \equiv \begin{bmatrix} \eta\lambda - \alpha \\ \eta\pi - \psi \\ -\eta\lambda \end{bmatrix}, \qquad b \equiv \begin{bmatrix} -\eta\lambda \\ -\eta\pi \\ -\eta\lambda \end{bmatrix},$$

and $\tilde{X}_t = a'X_t + b'X_{t-1}$. In the Appendix, we provide the derivation of the semi-reduced form of our exchange rate equation, which we write as

$$\text{(6)} \quad e_t = \mu e_{t-1} - \frac{1}{(1-\mu)\eta + \alpha} \left[E_t \left\{ \sum_{j=0}^{\infty} (\mu)^j \tilde{X}_{t+j} \right\} - E_{t-1} \left\{ \sum_{j=0}^{\infty} (\mu)^j \tilde{X}_{t+j} \right\} \right] + w_t ,$$

where $\mu \equiv 1 + (1/2)[(\alpha/\eta) - \{(\alpha/\eta)(4 + (\alpha/\eta))\}^{1/2}]$, and w_t is a disturbance term that follows a random walk. Two features of equation (6) deserve comment. First, the exchange rate is seen to depend on current predictions of all future values of the exogenous variables. This forward-looking aspect of the solution is typical of the "asset-market" approach to exchange rate determination. The actual prediction rules agents use in forming equation (6) depends on the stochastic processes governing the exogenous forcing variables. Second, a *lagged* value of the exchange rate appears in the solution. This is a feature of the stock/flow interaction, arising from imperfect asset substitutability.

To close the model, we assume that first-differences of the exogenous variables follow a stationary, kth-order vector autoregression (VAR)

$$\text{(7)} \qquad \Delta X_t = \sum_{j=1}^{k} B_j \Delta X_{t-j} + \nu_t .$$

The B_j ($j = 1, 2, \ldots, k$) are (3×3) matrices of constant coefficients with the r, s'th element denoted by $b_{rs,j}$, and $\{\nu_t\}$ is a (3×1) vector sequence of serially uncorrelated innovations. To simplify the notation, we can represent equation (7) as the following first-order VAR,

$$\text{(8)} \qquad \Delta Z_t = A \Delta Z_{t-1} + \delta_t$$

where

$$Z_t' = (p_t, \ldots, p_{t-k+1}, y_t, \ldots, y_{t-k+1}, m_t, \ldots, m_{t-k+1}),$$

$$\delta_t = (\nu_{1t}, 0, \ldots, 0, \underset{\substack{\uparrow \\ (k+1)\text{th element}}}{\nu_{2t}}, 0, \ldots, 0, \underset{\substack{\uparrow \\ (2k+1)\text{th element}}}{\nu_{3t}}, 0, \ldots, 0), \qquad \text{and}$$

$$A = \begin{bmatrix}
b_{11,1} & \cdot & \cdot & \cdot & b_{11,k} & b_{12,1} & \cdot & \cdot & \cdot & b_{12,k} & b_{13,1} & \cdot & \cdot & \cdot & b_{13,k} \\
1 & 0 & \cdot & \cdot & 0 & 0 & \cdot & \cdot & \cdot & 0 & 0 & \cdot & \cdot & \cdot & 0 \\
0 & 1 & \cdot & \cdot & 0 & 0 & \cdot & \cdot & \cdot & 0 & 0 & \cdot & \cdot & \cdot & 0 \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
0 & \cdot & \cdot & 1 & 0 & 0 & \cdot & \cdot & \cdot & 0 & 0 & \cdot & \cdot & \cdot & 0 \\
b_{21,1} & \cdot & \cdot & \cdot & b_{21,k} & b_{22,1} & \cdot & \cdot & \cdot & b_{22,k} & b_{23,1} & \cdot & \cdot & \cdot & b_{23,k} \\
0 & \cdot & \cdot & \cdot & 0 & 1 & \cdot & \cdot & \cdot & 0 & 0 & \cdot & \cdot & \cdot & 0 \\
0 & \cdot & \cdot & \cdot & 0 & 0 & 1 & \cdot & \cdot & 0 & 0 & \cdot & \cdot & \cdot & 0 \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
0 & \cdot & \cdot & \cdot & 0 & 0 & \cdot & \cdot & \cdot & 1 & 0 & 0 & \cdot & \cdot & 0 \\
b_{31,1} & \cdot & \cdot & \cdot & b_{31,k} & b_{32,1} & \cdot & \cdot & \cdot & b_{32,k} & b_{33,1} & \cdot & \cdot & \cdot & b_{33,k} \\
0 & \cdot & \cdot & \cdot & 0 & 0 & \cdot & \cdot & 0 & 1 & \cdot & \cdot & \cdot & \cdot & 0 \\
0 & \cdot & \cdot & \cdot & 0 & 0 & \cdot & \cdot & \cdot & 0 & 1 & 0 & \cdot & \cdot & 0 \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
0 & \cdot & \cdot & \cdot & 0 & 0 & \cdot & \cdot & \cdot & 0 & \cdot & \cdot & 0 & 1 & 0
\end{bmatrix}$$

The j-step ahead linear-least squares predictor of Z_t is

$$E_t(Z_{t+j}) = Z_t + A[I - A^j](I - A)^{-1}\Delta Z_t . \tag{9}$$

Let G be a $(3 \times 3k)$ matrix of zeroes except for the elements g_{11}, $g_{2,k+1}$, and $g_{3,2k+1}$, which are equal to one. Then $X_t = GZ_t$, and using equation (9), the j-step ahead linear least squares predictor of X_t is conveniently given by

$$E_t(X_{t+j}) = GZ_t + GA[I_{3k} - A^j](I_{3k} - A)^{-1}\Delta Z_t . \tag{10}$$

Now, substitute equation (10) into equation (6) and take first differences to obtain the following solution for the exchange rate:

$$\Delta e_t = \mu \Delta e_{t-1} + [q'G(A + CA + 2C) + r'G]\Delta Z_{t-1} - q'GC\Delta Z_{t-2} + u_t , \tag{11}$$

where

$$q' \equiv -(1/\alpha)(a' + \mu b'), \tag{12}$$

$$r' \equiv -(1/\alpha)[(1 - 2\mu)b' - \mu a'], \tag{13}$$

$$C \equiv \mu A(I_{3k} - \mu A)^{-1}, \tag{14}$$

$$u_t \equiv q'G(I_{3k} + C)\delta_t + \Delta w_t . \tag{15}$$

Equations (8), (11), and (12) through (15) form the system of equations that we estimate jointly below. The identifiable parameters of the model are λ, π, (ψ/α), and (α/η). The parameters α, η, and ψ are not individually identified because only the ratio enters into the solution of the model.

3. EMPIRICAL IMPLEMENTATION

Our sample of quarterly observations runs from 1976:III, the time of the Rambouillet agreement, to 1987:IV.[5] The following data were used: Real GDP's for output measures, the GDP deflator for commodity prices, M1 and M3 for money supplies, and the end of quarter Swiss franc/U.S. dollar exchange rate. All data except the exchange rate were seasonally adjusted at the source. M3 for the U.S. was obtained from the CITIBASE tape. Swiss M1 plus quasi money corresponds to the M3 concept. The Swiss quasi-money series reported by the *International Financial Statistics* contains a change in treatment of time and savings deposits in 1982:II. We were unable to reconstruct an equivalent measure for the earlier time periods, and used a dummy variable for that time period to construct the M3 series. All other observations were taken from the *IFS* tape. All growth rates were converted to percent.

We chose not to use data on the trade balance or capital flows because of the well-known errors in the data for bilateral trade and capital flows. To get some idea of the magnitude of these errors, we constructed the Swiss-U.S. balance of trade for 1973:I to 1983:IV from both: (a) Swiss imports (c.i.f.) from the U.S. and Swiss exports (f.o.b.) to the U.S.; and (b) U.S. imports from Switzerland (c.i.f) and U.S. exports to Switzerland (f.o.b.). The two time series had enormous disparities in both sign and magnitude. For example, the two series for the four quarters of 1974, measured in millions of dollars, were as follows: Swiss-U.S. trade: −50, −41, 9, −8; U.S.-Swiss trade: 107, 110, 17, −6. This data came from OECD *Foreign Trade Monthly Series*, IMF *Direction of Trade*, and U.S. Department of Commerce, Bureau of Census, *Highlights of U.S. Export and Import Trade*.

The asymptotic distribution theory that we use to draw inference requires the use of stationary time series. Although testing the hypothesis that the observations are homogeneously distributed would require an infinite number of observations, we test for a particular form of nonstationarity; that is, the presence of unit roots in the autoregressive polynomial. The discussion here is brief, as the methodology we follow is well known (Fuller 1976, Dickey and Fuller 1979, Dickey, Bell, and Miller 1985).

All observations were first subjected to a logarithmic transformation to conform to the theory stated in relative form. For each series, differences were first regressed on one lagged level and two lags of the differences. Under the null hypothesis that a unit root is present, the coefficient on the lagged level is zero. The studentized coefficient (the t-statistic calculated by a standard regression package) is used to test this hypothesis. While it does not follow a student t distribution under the null, its distribution has been tabulated (Fuller 1976).

The first column of Table 1 reports the studentized coefficient on the lagged level. From Table 8.5.2 in Fuller (1976), the 10 percent critical value is (for 50 observations) 2.6 and the 5 percent critical value is 2.93. These results are fairly suggestive that there may be at least one unit root present in all series except relative price levels. For example, the studentized coefficient on m (the log of Swiss

[5] Hansen and Hodrick (1983) argue that the date of the Rambouillet agreement is a reasonable point to begin the sample because the agreement served to legitimize the regime of floating exchange rates.

TABLE 1
STUDENTIZED COEFFICIENTS FOR ϕ IN THE REGRESSION.
$\Delta z_t^d = -\phi\Delta^{d-1}z_{t-1} + \beta_1\Delta^d z_{t-1} + \beta_2\Delta^d z_{t-2} + \varepsilon_t$

z	$d = 1$[1]	$d = 2$[1]	$d = 1$[2]
log relative money supply			
(M1)	0.33	3.27	2.40
(M3)	1.76	3.69	2.54
log relative price levels	3.39	1.85	1.56
log relative income	1.97	4.95	2.46
log exchange rate	0.88	2.96	0.97

[1] Critical values at 5 and 10 percent for 50 observations are 2.93 and 2.60 respectively.
[2] z is the residual from a quadratic trend. 10 percent critical value for 50 observations on a linear trend is 3.18.

money relative to U.S. money) is 1.76 for M3 and 0.33 for M1, the coefficient on p (log Swiss price level relative to the U.S. price level) is 3.39, the coefficient on y (log Swiss output relative to U.S. output) is 1.97, and the coefficient on e is 0.88. The null hypothesis that a unit root is present cannot be rejected at the 5 percent level, nor at the 10 percent level with the exception of relative price levels.

Column 2 of Table 1 reports the studentized coefficient on the coeffcient of a lagged difference in the regression of the second difference on a lagged difference and the lags of second differences. Except for relative price levels, the hypothesis that a second unit root is present is strongly rejected. The studentized coefficients on M1, M3, price levels, income levels, and the exchange rate are 3.27, 3.69, 1.85, 4.95, and 2.96, respectively. Thus, we conclude that first differencing of all the observations will produce behavior roughly consistent with stationarity. The only caveat is that inflation differentials may not be stationary. Our model, though, requires all series to have the same order of differencing, so we cannot second difference just one series.

We should note that other researchers have chosen alternative transformations of the data to induce stationarity. For example, Woo assumes that the residuals from a quadratic trend are stationary. The validity of this assumption can be investigated empirically as well. The tests described above can be performed on the residuals from a trend. The distribution for the studentized coefficient shifts to the right, however. Fuller (1976) reports the empirical distribution when the observations are deviations from a linear trend. The 10 percent critical value is now 3.18.

In column 3, the studentized coefficient on a lagged level is reported from a regression of a difference on a lagged level and two lagged differences, where the observations are residuals from a *quadratic* trend. As is seen, none of these are close to the 10 percent critical value of 3.18 tabulated when the observations are residuals from a linear trend. The studentized coefficients for M1, M3, price levels, income levels, and the exchange rate are 2.40, 2.54, 1.56, 2.46, and 0.97, respectively. In all likelihood, the distribution for the quadratic case would lie to the right of the distribution for the linear case, which suggests that the residuals from a quadratic trend are likely to be nonstationary. Fitting the model with this data is likely to yield large t-ratios, but this might only be an artifact of the detrending procedure (see Granger and Newbold 1974). Furthermore, one is put in the uncomfortable position of arguing that the purely deterministic part of the time series is a *quadratic* in time, which is clearly unrealistic.

We now turn to maximum likelihood estimation of the model. Let β denote the vector of parameters that contains all the elements of the matrix A as well as η/α, ψ/α, λ, and π. Let $\varepsilon_t \equiv (\nu_t', u_t)'$ be the (4×1) dimensional residual vector from our system of equations. For a sample of size T, the log likelihood function is, $L(\beta, \Omega) = -2T \ln(2\pi) - F(\beta, \Omega)$, where

$$F(\beta, \Omega) = \frac{1}{2} \sum_{t=1}^{T} \varepsilon_t(\beta)'\Omega^{-1}\varepsilon_t(\beta) + \frac{T}{2} \ln |\Omega|, \tag{16}$$

and $\Omega = \mathrm{E}[\varepsilon_t \varepsilon_t']$. The likelihood function can be maximized by the following iterated nonlinear seemingly unrelated regression strategy. First, notice that for any fixed value of β,

$$\hat{\Omega}(\beta) = \frac{1}{T} \sum_{t=1}^{T} \varepsilon_t(\beta)\varepsilon_t(\beta)'$$

is the minimizer of $F(\beta, \Omega)$. Now begin with an initial guess value of $\beta = \beta_o$ to compute $\hat{\Omega}(\beta_o)$ and choose β to minimize $F[\beta, \hat{\Omega}(\beta_o)]$. Denote the minimizer of $F[\beta, \hat{\Omega}(\beta_o)]$ by β_1. Next, repeat the minimization with the updated objective function, $F[\beta, \hat{\Omega}(\beta_1)]$. This procedure of minimizing the sequence of updated F functions continues until convergence and produces the maximum likelihood estimator of β because the information matrix is block diagonal. Our convergence criterion was to stop on iteration i if

$$|F[\beta_i, \hat{\Omega}(\beta_{i-1})] - F[\beta_{i-1}, \hat{\Omega}(\beta_{i-2})]| < 0.001.$$

A consistent estimator of the asymptotic covariance matrix is obtained by evaluating

$$\frac{1}{T} \sum_{t=1}^{T} \frac{\partial \varepsilon_t(\hat{\beta})}{\partial \beta} \Omega^{-1} \frac{\partial \varepsilon_t(\hat{\beta})'}{\partial \beta}$$

at the final, maximum likelihood estimates, $\hat{\beta}$. The programs for the computations were written in the GAUSS programming language and the numerical optimization was performed using a Davidon-Fletcher-Powell algorithm.

To proceed, we need to settle on the lag length, k, for the VAR of the forcing process. The first difference specification with quarterly data suggests that four lags may be necessary to adequately account for the serial correlation in the data. An unrestricted fourth-order VAR, however, requires estimating 36 parameters (excluding constants) for the forcing process alone. Because the unrestricted fourth-order VAR is so heavily parameterized, it is difficult to estimate even the VAR coefficients with much precision. To cut back on the parameterization of the model, we maintain the fourth-order specification ($k = 4$) but exclude those explanatory variables whose t-ratios, in the unrestricted VAR, were less than one. A reasonable

TABLE 2
AKAIKE'S INFORMATION CRITERION FOR ALTERNATIVE VAR SPECIFICATIONS FOR THE FORCING PROCESS.

Model	Exclusion restrictions	Money = M1: No. of parameters[1]	Money = M1: AIC	Money = M3: No. of parameters[1]	Money = M3: AIC
VAR (4)	no	39	639.41	39	608.89
VAR (3)	no	30	635.67	30	602.01
VAR (2)	no	21	647.48	21	620.67
VAR (4)	yes	20	610.39	21	584.07
VAR (3)	yes	17	619.75	19	584.61
VAR (2)	yes	15	638.96	11	604.18

[1]Includes constant terms.

case for our "t-ratio" rule can be made by considering Akaike's (1974) information criterion (AIC) which is defined as,

$$\text{AIC} = -2 \log \text{likelihood} + 2(\text{number of parameters}).$$

Akaike instructs us to select the model that minimizes AIC.

Table 2 reports AIC values for second through fourth order unrestricted VAR's and restricted VAR's parameterized according to our selection rule. Clearly, 2 lags are insufficient to account for the serial correlation in the data, as can be seen from the AIC values for the unrestricted models, while the unrestricted fourth order model is overparameterized. Whether we measure the money by M1 or M3, however, the restricted fourth order VAR minimizes the AIC, for the class of models considered.

We next investigate the appropriateness of the exogeneity specification by testing the hypothesis that Δe does not Granger cause Δm, Δy, and Δp. Here, the unconstrained system is taken to be a fourth-order vector autoregression. Using M1, the Lagrange multiplier (LM) test with 12 degrees of freedom yields an LM statistic of 17.00 (p-value = 0.1495). A likelihood ratio test yields an LR statistic of 21.49 (p-value = 0.044). Using M3, the Lagrange multiplier test yields an LM statistic of 14.54 (p-value = 0.267) while the likelihood ratio test yields an LR statistic of 15.86 (p-value = 0.198). There doesn't appear to be any evidence that Δe Granger causes Δp, Δm, and Δy when money is measured by M3. There is marginal evidence that Δe Granger causes Δp, Δm, and Δy using M1, but the evidence is far from conclusive. We therefore maintain the exogeneity assumption while bearing in mind that the results for M1 may need to be interpreted with caution as the exogeneity assumption is more tentative in this case.

In Table 3, we report the estimation results for our model where money is measured by M1. Estimates of the matrix A are not reported to economize on space, but are available upon request.[6] The parameters are not estimated with a

[6] We tried a total of 16 different starting values in estimating the model. We initially assigned a value of either 1 or 10 to the four structural variables, η/α, ψ/α, λ, and π. There are 16 combinations of these

TABLE 3
EMPIRICAL RESULTS FOR THE MONETARY MODEL WITH M1.

Log likelihood	No. of parameters	Structural Parameter Estimates η/α (s.e.)	Ψ/α (s.e.)	λ (s.e.)	π (s.e.)
−397.643	23	0.1844 (0.124)	0.3551 (1.305)	10.8164 (6.318)	1.676 (6.000)

Test of Cross-equation Restrictions

Likelihood ratio statistic:	16.101
Degrees of freedom:	11
P-value:	0.137

Tests for First-Order Residual Serial Correlation

Alternative specification	Lagrange-multiplier statistic	Degrees of freedom	p-value
VAR (1)	6.354	16	0.984
AR (1):joint	2.779	4	0.595
AR (1):(p-eqn.)	0.007	1	0.933
AR (1):(y-eqn.)	0.007	1	0.978
AR (1):(m-eqn.)	0.297	1	0.586
AR (1):(e-eqn.)	0.795	1	0.386

Root-Mean Square Errors of Out-of-Sample Exchange Rate Levels Forecasts Forecast Horizon

Forecast dates	Monetary Model 1	2	3	4	Random Walk with Drift 1	2	3	4
85:I-86:IV	6.770	11.634	16.043	21.723	7.828	13.596	18.370	23.964
84:I-85:IV	7.211	12.699	18.208	22.746	7.596	13.638	19.460	24.344

great deal of precision, but this is not surprising given that we are estimating 23 parameters from only 46 quarterly observations. The point estimates seem reasonable. Note in particular that the ratio (π/λ) is the income elasticity of the demand for money, a parameter about which we have some confidence that a "reasonable" value is that it be less than one. Our estimate of (π/λ) is 0.155.

We performed a number of diagnostic tests on the model. First, we test the cross-equation restrictions imposed by equations (12) through (15). These restrictions are rather complicated and their validity can most conveniently be investigated with a likelihood ratio test. In the constrained model, the 15 explanatory variables in the exchange rate equation are $(\Delta e_{t-1}, \Delta p_{t-1}, \dots, \Delta p_{t-4}, \Delta y_{t-1}, \dots, \Delta y_{t-5}, \Delta m_{t-1}, \dots, \Delta m_{t-5})$. We estimate an unconstrained version of the model by including these same 15 explanatory variables in the exchange rate

assignments. In each case, the starting values for the coefficients in the matrix A were assigned the values from least squares estimates.

equation but we relax the restrictions (12) through (15).[7] The likelihood ratio test yields an LR statistic of 16.101. With 11 degrees of freedom, the cross-equation restrictions cannot be rejected at better than the 13 percent level.

We also examine the properties of the residuals by performing Lagrange multiplier tests for the presence of first-order serial correlation. This is appropriate as a first pass test for autocorrelation, since the presence of higher order serial correlation should cause the test of no autocorrelation against the alternative of first-order autocorrelation to reject. We tested the null hypothesis of no serial correlation against the following alternatives: First, that the (4×1) vector $\{\varepsilon_t\}$ follows a VAR (1), second, that $\{\varepsilon_{j,t}\}$, $(j = 1, 2, 3, 4)$ jointly follow univariate AR (1)'s, and third, that the $\{\varepsilon_{j,t}\}$ individually follow univariate AR (1)'s. As can be seen from the table, the null hypothesis that there is no serial correlation in the residuals cannot be rejected at standard significance levels.

Finally, we examine the out-of-sample forecasts of our model. We follow what has become the standard methodology in the literature and compare the forecasts of our model against the random walk specification. The choice of the random walk model as a benchmark for comparison is motivated by the findings of Meese and Rogoff (1983) who found that the random walk specification outperformed a number of popular exchange rate models during their sample period.[8] Our purpose here is only to use the model's forecasting performance to supplement standard statistical analysis in evaluating the model. We do not engage in a comprehensive forecasting exercise that compares the forecasts of several competing exchange rate models.

Recall that our data set extends through 1987:IV. To evaluate the out of sample forecasting performance of the model, our first forecasting experiment estimates the model's parameters using data through 1985:I, and a sequence of forecasts are made, from one quarter to four quarters ahead. As has become standard in such exercises, realized values of the exogenous variables are used in these forecasts. Our exchange rate equation contains a lagged change in the exchange rate, however. For the two through four quarter ahead forecasts, the previous quarter's *predicted* exchange rate is used as the conditioning variable and not the actual, realized exchange rate. The estimation period is then updated one period and the process is repeated. The last sequence of forecasts is obtained by estimating the model through 1986:IV. This procedure generates 8 k-step ahead forecasts $(k = 1, \dots, 4)$. Based on the results of Meese and Rogoff (1983), we take the random walk model (with drift) as the appropriate benchmark.[9] These forecasts are of the log-level of the exchange rate, and forecast performance is judged by the root mean squared error criteria. For all four forecast horizons, the constrained model outperforms the random walk model.

One reason that the model outperforms the random walk may be due to this specific forecasting period which does not include any turning points of the

[7] The exclusion restrictions in the exogenous forcing process are maintained in the unconstrained model, since they are not restrictions imposed by the theory, and because our preliminary results suggested that imposing those restrictions was appropriate.

[8] See also, Diebold and Nason (1990) who find that nonparametric predictors offer little improvement over the random walk model in forecasting exchange rates.

[9] Meese and Rogoff don't allow for drift, but we do since it out performs the no-drift model.

TABLE 4
EMPIRICAL RESULTS FOR THE MONETARY MODEL WITH M3.

Log likelihood	No. of parameters	Structural Parameter Estimates η/α (s.e.)	Ψ/α (s.e.)	λ (s.e.)	π (s.e.)
−382.277	26	0.1153 (0.214)	0.1329 (1.476)	1.6754 (3.613)	1.3496 (9.618)

Test of Cross-equation Restrictions

Likelihood ratio statistic:	26.511
Degrees of freedom:	11
P-value:	0.005

Tests for First-Order Residual Serial Correlation

Alternative specification	Lagrange-multiplier statistic	Degrees of freedom	*p*-value
VAR (1)	8.988	16	0.913
AR (1):joint	2.730	4	0.604
AR (1):(p-eqn.)	0.034	1	0.853
AR (1):(y-eqn.)	0.143	1	0.706
AR (1):(m-eqn.)	0.473	1	0.491
AR (1):(e-eqn.)	2.126	1	0.145

Root-Mean Square Errors of Out-of-Sample Exchange Rate Levels Forecasts Forecast Horizon

Forecast dates	Monetary Model 1	2	3	4	Random Walk with Drift 1	2	3	4
85:I-86:IV	8.218	15.069	20.737	28.046	7.828	13.596	18.370	23.964
84:I-85:IV	7.552	14.273	21.039	26.965	7.596	13.638	19.460	24.344

exchange rate. The Swiss franc steadily gained relative to the dollar from 1985:I through 1987:IV. To explore this possibility, we repeat the forecasting exercise by beginning in 1984:I and making the last sequence of forecasts at 1985:IV. This puts the turning point for the Swiss franc/dollar rate, which occured in 1985:I, at the midpoint of the forecasting period. The model continues to out perform the random walk model.

Table 4 reports the estimation results for our model using M3 as the monetary variable. This specification requires the estimation of 26 parameters. As before, the point estimates seem reasonable and the implied elasticity of money demand (π/λ) is 0.805.

The model does not hold up as well under diagnostic testing with M3, however. The test of the cross-equation restrictions reject the model at better than the 1 percent level. Although there is little evidence that the residuals are serially correlated, this model does not forecast as well as the random walk.

4. CONCLUSIONS

Our findings suggest that a monetary, rational-expectations model of exchange-rate determination is broadly consistent with recent Swiss/U.S. data. One interpretation of this result is that the post-mortems being conducted on exchange-rate models of the '70's are, if not premature, at least restricted in applicability to the starkly simple models incorporating perfect capital substitutability and mobility, and purchasing power parity. In addition, our failure to reject the joint hypothesis of our structural model and the rational expectations assumption provides some evidence that the empirical demise of these earlier models is not due so much to the assumption of rational expectations as to their particular array of simplifying assumptions.

Of course, our results are, at the moment, more suggestive than definitive. Fruitful extensions may be found in a number of directions. First, the relaxation of the partial-equilibrium assumptions of the model should prove useful. Second, extensions incorporating aspects of managed floating should create a model with empirical implications for exchange rates other than the Swiss-U.S. rate. Third, the introduction of fiscal effects might be important.

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APPENDIX

To derive equation (6) in the text, first combine equations (2) through (5) to obtain the basic difference equation of our model:

$$\text{(A.1)} \quad \eta E_t e_{t+1} - (\eta + \alpha) e_t - \eta E_{t-1} e_t + \eta e_{t-1} = a'X_t + b'X_{t-1} + \eta \Delta \varepsilon_t + u_t,$$

where a' and b' and X are as described in the text. Taking expectations of both sides of equation (A.1) at $t - 1$, we have

$$\text{(A.2)} \quad (1 - \theta F + F^2) E_{t-1} e_{t-1} = (1/\eta)[a'E_{t-1}X_t + b'E_{t-1}X_{t-1}],$$

where F is the forward shift operator and $\theta \equiv 2 + (\alpha/\eta)$. Factoring $(1 - \theta F + F^2)$ as

$$\text{(A.3)} \quad (1 - \mu_1 F)(1 - \mu_2 F),$$

we can solve for $E_{t-1} e_t$ as

$$\text{(A.4)} \quad E_{t-1} e_t = \mu e_{t-1} - \mu E_{t-1} \left\{ (a'/\eta) \sum_{j=0}^{\infty} (\mu)^j X_{t+j} + (b'/\eta) \sum_{j=0}^{\infty} (\mu)^j X_{t-1+j} \right\},$$

where $\mu = \mu_1 = (1/2)(\theta - [\theta^2 - 4]^{1/2})$, $\theta = \mu_1 + \mu_2$, and $\mu_1 = 1/\mu_2$. Updating (A.4) one time period we have an analogous expression for $E_t e_{t+1}$. Substituting these expressions for $E_{t-1} e_t$ and $E_t e_{t+1}$ into the basic difference equation (A.1) yields equation (6) of the text.

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