Evaluating Empirical Tests of Asset Pricing Models: Alternative Interpretations

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Recent tests of asset pricing models have proceeded in one of two ways. The first method, implemented by Lars Hansen and Kenneth Singleton (1982), begins by estimating parameters using various moment restrictions implied by an economic model, and then tests a set of overidentifying restrictions. In the second, due to Rajnish Mehra and Edward Prescott (1985) and known as calibration, a model is *tested* by seeing if it can exactly reproduce various moments of asset price data for given parameter values. Both of these procedures have shortcomings. The usual practice of testing overidentifying restrictions both involves asking whether a model can match aspects of the data that may be of little economic importance and fails to isolate those characteristics of the data that the model does not capture. Model calibration, on the other hand, does not take account of the statistical properties of both the sample moments trying to be matched and the model implied values to which they are being compared.

This paper describes an alternative strategy for testing asset pricing models that combines features from these two earlier approaches. Following Hansen and Singleton, we use statistical inference techniques to establish formal criteria that a successful model must satisfy. But in the spirit of Mehra and Prescott, we attempt to isolate moments of the data that we feel are of economic importance.

Our method has four natural steps. First we specify an economic model that allows us to characterize the evolution of asset prices. This requires a parameterization of both preferences and the stochastic process driving the endowment in the model. Following our earlier work with Pok-sang Lam (1990a, b), we assume that the utility function is of the constant relative risk aversion class and that the endowment can be accurately characterized by the Markov switching model pioneered by James Hamilton (1989).

In the second step, we choose a set of moments of the asset price data that we use to test the model. Clearly, any simple, parsimoniously parameterized model cannot match more than a small set of properties of the data. So instead of requiring that a model be capable of emulating all of the features of the data, we proceed to choose a small number of moments that are of particular economic interest. Following the literature, we examine the first and second moments of the equity premium and the risk free rate, as well as the variance ratio statistics computed from equity returns first presented by James Poterba and Lawrence Summers (1988).

The third step is estimation. In order to test whether the model is capable of matching the moments specified in the second step, we need to estimate the moments, the parameters of the endowment process and their joint covariance matrix.

In the fourth and final step we test the model. Following Mehra-Prescott and the calibration literature, we do not estimate the preference parameters. Instead we fix the discount factor (β) and the coefficient of relative risk aversion (γ), and then ask whether the moments that are implied by the model are close to those computed from the asset price data.

The methodology is simple and can be applied in any setting where model calibration has previously been used. Since it allows the investigator to choose various moments of the data for study, the method allows us to identify the specific points where the

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model is failing. This can help direct investigations into new and better alternative models.

In our study of asset returns, we find that there exist (β, γ) pairs for which the moments chosen are not rejected at standard levels of statistical significance (i.e., the 5 percent level). But in order to achieve this result, we must allow the coefficient of relative risk aversion to exceed 20. We conclude by commenting on what we believe to be appropriate criteria for choosing γ.

I. The Model

This section summarizes both a model of asset pricing and the solution for returns based on the endowment process introduced by Hamilton. Details can be found in our (1990a) paper with Lam.

Begin by specifying preferences as

$$U(C_t) = (C_t^{1-\gamma} - 1)/(1-\gamma), \tag{1}$$

where $0 < \gamma < \infty$ is the coefficient of relative risk aversion and C_t is the level of per capita real consumption.

Using (1), the standard first-order conditions that describe the evolution of the prices of traded assets can be written as

$$P_t^e C_t^{-\gamma} = \beta E_t C_{t+1}^{-\gamma}(P_{t+1} + D_{t+1}), \tag{2}$$

$$P_t^f = E_t \beta (C_{t+1}/C_t)^{-\gamma}; \tag{3}$$

where P_t^e is the real price of the traded asset, or equity; P_t^f is the real price of the risk-free asset; D_t is the payoff or dividend from owning one unit of equity; β is the discount factor; and E_t is the mathematical expectation conditional on information at time t.

To obtain a closed-form solution to the functional equations (2) and (3), we must specify the stochastic process governing $\{C_t, D_t\}$. In our earlier paper with Lam (1990b), we show that both consumption and dividend growth have significant negative skewness and leptokurtosis that is well captured by Hamilton's Markov switching model. We assume that the log of C_t and D_t, c_t and d_t, are given by a bivariate random walk with a two-state Markov drift. The model can be written as

$$\begin{pmatrix} c_t \\ d_t \end{pmatrix} = \begin{pmatrix} c_{t-1} \\ d_{t-1} \end{pmatrix} + \begin{pmatrix} \alpha_0^c \\ \alpha_0^d \end{pmatrix} + \begin{pmatrix} \alpha_1^c \\ \alpha_1^d \end{pmatrix} S_{t-1} + \begin{pmatrix} \varepsilon_t^c \\ \varepsilon_t^d \end{pmatrix}, \tag{4}$$

where $(\varepsilon_t^c \varepsilon_t^d)'$ is jointly i.i.d. normal with mean zero and covariance matrix Σ, and S_t is a Markov random variable that takes on values of 0 or 1 with transition probabilities $\Pr[S_t = 1 | S_{t-1} = 1] = p$ and $\Pr[S_t = 0 | S_{t-1} = 0] = q$. The model requires estimation of the parameter vector $\mathbf{B} = (\alpha_0^c, \alpha_0^d, \alpha_1^c, \alpha_1^d, p, q, \Sigma)$.

The model can now be solved for the prices of the risky and the riskless assets as functions of the process parameters $\mathbf{B}$, the state S_t, and the preference parameters β and γ. The result can be written as

$$P_t^e = \lambda(\mathbf{B}, \beta, \gamma; S_t) D_t; \tag{5}$$

$$P_t^f = \theta(\mathbf{B}, \beta, \gamma; S_t). \tag{6}$$

The various moments of the equity return, the equity premium and the risk-free rate implied by the model can be computed directly from (5) and (6). In general, these can be written as $M(\mathbf{B}; \beta, \gamma)$. In the next section we choose to study the first and second moments of the equity premium and the risk-free rate, as well as the variance ratios computed from equity returns. The variance ratios are simple linear functions of the autocorrelations of the returns data and are all equal to one if returns are truly serially independent.

II. Estimation and Testing

In order to evaluate the performance of the model described in Section I, and to perform the desired test, we need to estimate the process parameters $\mathbf{B}$, and the vector of moments of interest, call these μ. Using annual data from 1889 to 1987, we have computed estimates of both the process pa-

TABLE 1:—SELECTED SAMPLE MOMENTS OF ASSET RETURNS

	Mean	St. Dev.
Risk-Free Rate (R^f, σ_f)	1.15	5.87
Risk Premium (R^p, σ_p)	6.03	19.15
Correlation (ρ_{fp})	−0.02	
	Variance Ratios	
Two-Year (VR_2)	0.99	
Three-Year (VR_3)	0.83	
Five-Year (VR_5)	0.82	
Ten-Year (VR_{10})	0.71	

rameters from data on total per capita consumption (including durables) and dividends ($\hat{\mathbf{B}}$), and a set of sample moments from asset return data ($\hat{\mu}$), along with an estimate of the covariance matrix of $(\hat{\mathbf{B}}\hat{\mu})'$. Possible sample moments to be considered are reported in Table 1. (While the table only reports the variance ratios at two-, three-, five- and ten-year horizons, all the tests reported below include statistics at all horizons from two to ten years.)

Table 1 reflects a number of well-known aspects of the returns data. While the risk-free rate averaged approximately 1 percent, the equity premium exceeded 6 percent. In addition, the standard deviation of the equity premium is more than three times that of the risk-free rate. Finally, the variance ratio statistics lie below 1.0, implying a small amount of negative serial correlation in equity returns.

Model performance is evaluated by examining the difference between the model's implied values for the moments of interest, $M(\hat{\mathbf{B}};\beta,\gamma)$, and the estimates from the asset price data, $\hat{\mu}$. We wish to find out if $[M(\hat{\mathbf{B}};\beta,\gamma)-\hat{\mu}]$ is small. The estimate of this difference is plagued by uncertainty that arises from the fact that both the moments of the data, μ, and the process parameters, $\mathbf{B}$, must be estimated. Taking account of both sources of uncertainty, we can compute the sampling distribution of $[M(\hat{\mathbf{B}};\beta,\gamma)-\hat{\mu}]$ directly from the estimate of the covariance matrix of the vector $(\hat{\mathbf{B}}\hat{\mu})'$; call this $\hat{\Gamma}$.

The resulting test statistic is given by

$$(7)\quad K=\left[M(\hat{\mathbf{B}};\beta,\gamma)-\hat{\mu}\right]'\hat{\Gamma}^{-1}\times\left[M(\hat{\mathbf{B}};\beta,\gamma)-\hat{\mu}\right],$$

TABLE 2:—MODEL EVALUATION USING VARIOUS SAMPLE MOMENTS

	Preference Parameters		
β	1.135	1.095	0.945
γ	14.00	21.75	30.00
	Moments Implied by the Model, $M(\hat{\mathbf{B}},\beta,\gamma)$		
	First and Second Moments		
R^f	1.00	1.86	5.63
R^p	1.77	2.78	4.00
σ_f	9.86	13.71	17.38
σ_p	11.26	21.88	47.61
ρ_{fp}	0.02	0.02	0.01
	Variance Ratio Statistics		
VR_2	0.89	0.71	0.74
VR_3	0.83	0.56	0.61
VR_5	0.78	0.42	0.50
VR_{10}	0.74	0.31	0.41
	Marginal Significance Level for Tests		
$K(R^f,R^p)$	0.54	0.24	0.59
$K(R^f,R^p,\sigma_p,\sigma_f,\rho_{fp})$	0.00	0.55	0.24
$K(VR_2,\dots,VR_{10})$	0.12	0.84	0.20

which is distributed as *chi*-squared with degrees of freedom equal to the number of moments being considered, under the null hypothesis that $M(\mathbf{B};\beta,\gamma)=\mu$. (See our paper with Lam, 1990a, for details.) When K is small, as measured by a high p-value from the cumulative *chi*-squared distribution, we can conclude that the model's implied moments are close to the data.

Implementation of this procedure requires the choice of a (β,γ) pair. Table 2 presents calculations for three possibilities. The general result is that the model comes fairly close to the sample moments when considering either the variance ratio statistics, or the means of the equity premium and the risk free rate. But the second moments of the premium and the risk-free rate are slightly more difficult to match. In fact, the point estimates for the standard deviations, σ_f and σ_p, exceed the sample values by apparently large amounts. But because the standard errors about these standard deviations are quite large, the marginal significance levels of the tests including them remain high.

All of the examples in Table 2 involve values of γ that may seem large.[1] It is im-

[1] In addition, we have included cases in which $\beta>1.0$. Both Narayana Kocherlakota (forthcoming) and our

portant to note that there exist no (β, γ) pairs with γ less than 15.0, for which even the simplest mean test does not reject the model at the 5 percent significance level. But if γ is allowed to exceed 20, then the model performs very well.

III. Discussion

Traditionally, researchers have restricted their attention to values of γ that are less than 10.0. But the methodology we employ to evaluate the intertemporal capital asset pricing model finds that the coefficient of relative risk aversion must lie outside of this range in order for the model to perform well. Given this evidence, we have two choices. We can either abandon this simple formulation of the model because it requires values of γ that are unacceptably large, or we can change the acceptable range for γ.

Our view is that it is useful to consider values for the coefficient of relative risk aversion that are larger than have been considered in the past. The purpose of modeling is, after all, to provide an organizing framework that helps us to make sense of the data. To this end, a simple model is always preferred to a more complex one. There is mounting evidence that the basic model presented in this paper provides a consistent picture when γ is large, but not when it is small. (See also Shmuel Kandel and Robert Stambaugh, 1988.) In short, the data seem to be telling us that γ is larger than previously thought.

It is interesting to note that introspection can also reveal values of γ that are larger than previously agreed upon. Inspired by a question first put forth by Kandel and Stambaugh, we recently asked members of the faculty of the Department of Economics at Ohio State and the attendees of the session at the AEA annual meeting where this paper was presented to make an estimate of their income and then to answer the following question: *Based on the flip of a fair coin, I will either pay you \$1000 or you will have to pay me \$1000. How much would you be willing to pay me to avoid this bet?* We obtained a total of 22 responses to this question from which we constructed estimates of the coefficient of relative risk aversion (using the constant relative risk aversion utility function given in equation (1)). The responses implied γ's ranging from 0 to 77, with an average of 12.7 and a standard deviation of 15.7. Although the size of the sample is small, this evidence suggests that concentrating on γ greater than 10 is not unreasonable.

1990a paper with Lam discuss why it is sensible to consider discount factors in excess of 1.0.

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