INTERNATIONAL ECONOMIC REVIEW
Vol. 00, No. 0, xxxx 2018 DOI: 10.1111/iere.12334

IDENTIFYING EXCHANGE RATE COMMON FACTORS*

BY RYAN GREENAWAY-MCGREVY, NELSON C. MARK, DONGGYU SUL, AND JYH-LIN WU[1]

University of Auckland, New Zealand; University of Notre Dame, U.S.A., and NBER, U.S.A.; University of Texas at Dallas, U.S.A.; National Sun Yat-Sen University, Taiwan

Using recently developed model selection procedures, we determine that exchange rate returns are driven by a two-factor model. We identify them as a dollar factor and a euro factor. Exchange rates are thus driven by global, U.S., and euro-zone stochastic discount factors. The identified factors can also be given a risk-based interpretation. Identification motivates multilateral models for bilateral exchange rates. Out-of-sample forecast accuracy of empirically identified multilateral models dominates the random walk and a bilateral purchasing power parity fundamentals prediction model. Twenty-four-month-ahead forecast accuracy of the multilateral model dominates those of a principal components forecasting model.

1. INTRODUCTION

Exchange rate returns (first differences of log exchange rates) show substantial cross-sectional correlation. In a sample of 27 monthly exchange rate returns from 1999.01 to 2015.12, the average correlation is 0.43 when the U.S. dollar (USD) is the numeraire currency. Similarly, the average correlation is 0.32 when the euro is the numeraire and 0.39 when the Canadian dollar is the numeraire.[2] Recent research has focused on understanding the source of these exchange rate comovements. Engel et al. (2015) assume a factor structure for exchange rates and take a small number (2 or 3) of principal components (PCs) to be the common factors. They find that the PCs remain significant after controlling for macroeconomic fundamental determinants and use them to predict future exchange rate returns. Verdelhan (2018) also assumes a two-factor structure and argues that a dollar exchange rate return and a carry exchange rate return are exchange rate common factors. He gives them a risk-based interpretation by showing that the carry and dollar factors can account for two different cross sections of currency risk premia.

In this article, we obtain factor identification using econometric methods developed by Bai and Ng (2002, 2006) and Parker and Sul (2016). Our analysis identifies a two-factor structure consisting of a dollar factor and a euro factor. The analysis does not find the carry return to be a factor, and identification is robust to the choice of the numeraire currency. The data also support a risk-based interpretation to the factors. Using time-varying dollar and euro factor loadings to sort currency excess returns into portfolios, the average returns are generally increasing in their currency's loadings on the factors. The data also reveal a geographical dimension to the euro factor. European currencies generally load positively on the euro factor whereas all others generally load negatively. Commodity exporting countries tend to load positively on the dollar factor.

*Manuscript received June 2015; revised August 2017.

[1] Some of the work was performed while Mark was a Visiting Fellow at the HKIMR (Hong Kong Institute for Monetary Research) and the Federal Reserve Bank of St. Louis. Research support provided by these institutions is gratefully acknowledged. This is a revision of a paper originally circulated in March 2012 under the title "Exchange Rates as Exchange Rate Common Factors." We have benefitted from comments by seminar participants at Michigan State University and the Bank of Canada. Thoughtful comments and suggestions from two anonymous referees helped us to improve the article. Please address correspondence to: Donggyu Sul, Department of Economics, University of Texas at Dallas, 800 W. Campbell Road, Richardson, TX 75080-3021, U.S.A. E-mail: *d.sul@utdallas.edu*.

[2] This cross-sectional correlation has been recognized in research at least since O'Connell (1988) but has primarily been treated as a nuisance parameter in panel data models (Mark and Sul, 2001; Engel et al., 2007)

The methodology we use is designed to uncover the relationship between the vector of true but unobserved factors and a vector of economic variables put forth as candidates for empirical factors. The first step in the procedure uses an information criterion, proposed by Bai and Ng (2002), to determine the number of common factors k in a panel of exchange rate returns. The second step determines the number of common factors in residuals from regressions of exchange rate returns on unique combinations of k-element groupings of the candidate economic variables. Identification is based on the idea that if this particular group of k variables are empirical factors, then there are no common factors in the residuals. If one or more common factors are found in the residual panel, this particular set of variables is rejected as the empirical factors.

The candidate list of economic variables is potentially large. Searching over all possibilities is not feasible. We therefore limit empirical factor candidates to exchange rate returns. This is not unreasonable because exchange rate returns, being the difference between countries' (possibly unobservable) log stochastic discount factors (SDFs), may contain information that is difficult to observe in other macroeconomic fundamentals.

What is the value added of empirical factor identification? One is that it guides us toward an economic interpretation of the source of exchange rate comovements (as opposed to the descriptive PCs analysis). Drawing on the SDF approach to the exchange rate, as in Lustig et al. (2011) and Verdelhan (2018), implies that comovements of exchange rate returns and log SDFs across countries are heavily influenced, if not dominated, by the dynamics of the log SDF of the U.S. and the euro zone. We mount a limited exploration into a risk-based interpretation of the dollar and euro factors.

A second value to the identification is that it can be exploited to improve the performance of empirical exchange rate models. Our dollar and euro factor identification suggests a multilateral model of bilateral exchange rates that contrasts with typical bilateral formulations. That is, bilateral exchange rates in conventional models are determined by variables from the pair of countries associated with the bilateral exchange rate.[3] Instead of fixating on the details of every bilateral country pair, knowing the determinants of the dollar and the euro allows one to understand a substantial proportion of the variation in any bilateral exchange rate. To assess empirical model performance of the multilateral model, we employ an out-of-sample forecasting methodology that has been a standard procedure for model assessment since Meese and Rogoff (1983). We reserve the period from 2004.01 to 2015.12 for out-of-sample forecast evaluation and generate 1, 12, and 24-month-ahead forecasts based on 60-month rolling regressions.

In the forecasting analysis, we compare our multilateral "dollar–euro" model with alternative models considered in the literature. The first is the bilateral purchasing-power parity (PPP)-based fundamentals model (Bi-PPP). We use this as a comparison model because Engel et al. (2007) find that it gives the best forecast accuracy among several bilateral fundamentals-based formulations considered in the literature. We find that prediction accuracy from our dollar–euro model dominates those from the PPP-based model as well as those from the driftless random walk.

The empirical exchange rate literature finds that sample size matters for forecast accuracy. Rapach and Wohar (2001) and Lothian and Taylor (1996) report significant predictive power when working with long historical time-series data. To obtain more observations within the post-Bretton Woods floating regime, a first generation of papers (Mark and Sul, 2001; Rapach and Wohar, 2004; Groen, 2005) expanded observations cross sectionally with the use of panel data methods. The panel aspect of our data expands observations by exploiting the cross section.

Improved forecast performance over the random walk and the bilateral PPP-based model does not fully answer the question of whether identification has predictive value in empirical

[3] Berg and Mark (2015) is an exception. They argue that bilateral exchange rates are driven in part by third-country (rest of world) shocks.

modeling since the factor structure can also be estimated by PCs and used to forecast. Engel et al. (2015) found that quarterly forecasts from a two-principal components model were significantly more accurate than random walk predictions over the 1999–2007 period. When we compare the dollar–euro factor forecasts to the two-principal components model, we find, on balance, that the dollar–euro model has lower mean-square prediction error (MSPE) at the longer (24-month) horizons.

The article is organized as follows: The next section presents the common factor structure that we assume and the identification methodology that we use. Our data set is described in Section 3. Empirical factor identification results are presented in Section 4. A limited exploration into geographical aspects of the factors and a possible risk-based interpretation of the factors is undertaken in Section 5. Forecasting results are presented in Section 6, and Section 7 concludes.

2. COMMON FACTORS IN EXCHANGE RATE VARIATION

This section develops the factor structure for exchange rate returns that guides our empirical work. To fix notation, let f_t be the k-dimensional vector of the true but unobserved common (global) factors and f_t^p be an m-dimensional vector of economic variables that are candidates for empirical identification as true common factors. Note that m is potentially very large. The goal is to identify a unique set of k elements from f_t^p that describe the evolution of f_t. We present ideas developed for the nominal exchange rate. The parallel development for real exchange rates is straightforward and omitted.

Let there be $N+1$ currencies. The USD is currency "0," and the euro is currency "1." Nominal exchange rates s_{it} are stated as logarithms of the price of the USD in country i currency. s_{it} increases when the dollar appreciates. If, within a country, markets are complete or if markets are incomplete but the law-of-one price holds and there is no arbitrage, the country will have a unique SDF. Let n_{it} be the log nominal SDF for country $i = 0, \ldots, N$. In the SDF approach to exchange rates, the exchange rate return is the difference between the log SDFs,

$$\Delta s_{it} = n_{it} - n_{0t}. \tag{1}$$

Because Δs_{it} varies (quite a bit) over time, we know that SDFs evolve differently across countries. A representation of the log SDF that is consistent with such cross-country heterogeneity is the factor structure,

$$n_{it} = \delta_i' f_t + n_{it}^o, \tag{2}$$

where δ_i is a k-element vector of factor loadings and n_{it}^o is the idiosyncratic component of the country i log SDF. The latent factors may be correlated with each other $\text{cov}(f_{it}, f_{jt}) \neq 0$, for $i \neq j$, whereas the idiosyncratic components are uncorrelated across countries, $\text{cov}(n_{it}^o, n_{jt}^o) = 0$. Heterogeneous responses to factor movements across countries are necessary for exchange rate returns Δs_{it} to vary over time. If there were no cross-country differences in factor loadings δ_i, the exchange rate return would be driven only by idiosyncratic components of the log SDF and would then be cross-sectionally uncorrelated. Because the factors f_t drive common movements in every country's log SDF, they are global in nature. Lustig et al. (2011) and Verdelhan (2018) also decompose the log SDF into a common global component and a country-specific idiosyncratic component. We take Equations (1) and (2) to represent the *truth*.

Substituting (2) into (1) gives the factor representation for exchange rate returns

$$\Delta s_{it} = (\delta_i' - \delta_0') f_t + n_{it}^o - n_{0t}^o. \tag{3}$$

Notice from (3) that the idiosyncratic part of the numeraire country's log SDF, n_{0t}^o, appears for all i and is also a common source of exchange rate comovement. Our interest is in the identification of f_t, not n^o_{0t}. To attenuate the numeraire effect of n^o_{0t} in exchange rate comovements, we transform observations into deviations from the cross-sectional mean

(4)
$$\frac{1}{N}\sum_{i=1}^{N}\Delta s_{it} = \Delta\bar{s}^{\$}_t = \left(\bar{\delta}' - \delta'_0\right) f_t - n^o_{0t},$$

where $\bar{\delta}' = (\frac{1}{N}\sum_{i=1}^{N}\delta_{i,1}, \ldots, \frac{1}{N}\sum_{i=1}^{N}\delta_{i,k})$ is the cross-sectional average of factor loadings and $\tilde{\delta}'_i = (\delta'_i - \bar{\delta}')$ is the deviation from the mean loadings. In deviations from the cross-sectional mean form, $\Delta\tilde{s}_{it} = \Delta s_{it} - \Delta\bar{s}^{\$}_t$, the n^o_{0t} component is removed and f_t is rendered the only common factor component of the exchange rate return,

(5)
$$\Delta\tilde{s}_{it} = \tilde{\delta}'_i f_t + \tilde{n}^o_{it},$$

where $\tilde{n}^o_{it} \to n^o_{it}$ as $N \to \infty$. Hence, the underlying factor structure in deviations from the mean form is numeraire invariant when N is large, but in any finite sample, changing the numeraire currency results in some variation in the $\tilde{\delta}_i$ factor loadings.[4]

2.1. *Identification Method.* The common factor representation has successfully been used as the statistical foundation for modeling comovements across exchange rates, but because the factors are not identified, the economic interpretation for the underlying mechanism is not obvious. To address this issue, Bai and Ng (2006) and Parker and Sul (2016) develop econometric methods to identify the unobserved common factors with observed economic variables. In this section, we draw on these methods to identify the common factors for exchange rate returns. The procedure involves two steps. The first step identifies the number of common global factors k present in the data. The second step evaluates restrictions imposed on candidate empirical factors by the factor representation to identify those economic variables that closely mimic the k true latent factors.

The panel data are N exchange rate returns over T time periods in deviations from the mean form $\Delta\tilde{s}_{it}$. The number k of common factors is identified using Bai and Ng's (2002) IC_2 information criterion on standardized observations.[5] Let $C_{NT} = \min(N, T)$, and λ_i be the ith largest eigenvalue of the sample covariance matrix. The information criterion is

(6)
$$IC_2 = \ln\left(\sum_{i=k+1}^{C_{NT}}\lambda_i\right) + k\left(\frac{N+T}{NT}\right)\ln C_{NT},$$

and the number of common factors in the panel is the value of k that minimizes (6).

For concreteness and to foreshadow our findings, assume step 1 determined exchange rates $\Delta\tilde{s}_{it}$ are driven by $k = 2$ common factors. In step 2, viewing Equation (5) as the true factor representation, we test the null hypothesis that a unique pair of economic variables (f^p_{jt}, f^p_{st}) span the same space as the two true common factors (f_{1t}, f_{2t}),

(7)
$$f_{1t} = a_{11}f^p_{jt} + a_{12}f^p_{st} + \epsilon_{1t},$$

(8)
$$f_{2t} = a_{21}f^p_{jt} + a_{22}f^p_{st} + \epsilon_{2t},$$

[4] If the United States is the numeraire country, $\bar{\delta}$ is the average of all other (not the United States) country factor loadings. If instead, Canada is used as the numeraire, Canada's factor loadings are replaced by the United States's δ in computing the average, $\bar{\delta}$. The effect of swapping numeraires on $\tilde{\delta}_i$ vanishes when N is large.

[5] Bai and Ng (2002), Hallin and Liska (2007), Onatski (2009, 2010), and Ahn and Horenstein (2013) propose alternative methods to determine the number of common factors. We employ Bai and Ng's (2002) IC_2 because Parker and Sul (2016) show that it has good robustness properties.

where for $j = 1, 2$, $\text{var}(\epsilon_{jt}) \to 0$ as $T \to \infty$. Asymptotically, the economic variables give an exact identification of the factors in the sense that the error terms are $O_p(1/\sqrt{T})$. It is also possible that some of the a_{js} coefficients are zero. If, for example, $a_{12} = a_{21} = 0$, the latent factors are uniquely identified. This implies that the residuals Δs_{it}^o, from regressions of $\Delta\tilde{s}_{it}$ on (f_{jt}^p, f_{st}^p),

$$\Delta\tilde{s}_{it} = a_i + b_{i1}f_{jt}^p + b_{i2}f_{st}^p + \Delta s_{it}^o \tag{9}$$

have no common factors. We are guided by the following two results, established by Parker and Sul (2016):

(1) If there are no (zero) common factors in the panel of residuals Δs_{it}^o, then (f_{jt}^p, f_{st}^p) are the true common factors.
(2) If there are one or more common factors in the panel of residuals Δs_{it}^o, then either (f_{jt}^p or f_{st}^p) or both (f_{jt}^p, f_{st}^p) are not the true common factors.

Hence, we examine whether pairs of economic variables are approximately the true factors by regressing $\Delta\tilde{s}_{it}$ on all combinations of two candidates f_{st}^p and f_{jt}^p and then using the IC_2 information criteria (6) to determine the number of common factors in the regression residuals. If there are no common factors in the panel of residuals, then f_{st}^p and f_{jt}^p are identified as empirical factors.

3. DATA

Observations are split into two data sets. The first, which we refer to as the euro-epoch data, consists of exchange rates and interest rates of $N = 27$ countries from 1999.01 to 2015.12. Currency selection was based on data availability and whether or not countries allowed their exchange rate to float. Factor identification is more precise when N is large and when exchange rates are flexible. Little or no information is contributed by adding exchange rates that are pegged. Currencies included in the sample were consistently classified as either "floating" or "managed floating without a predetermined path" in the International Monetary Fund (IMF) *Annual Report on Exchange Arrangements and Exchange Restrictions*.[6]

The euro-epoch data emphasize the important role played by the euro in international finance and reflects a trend among emerging market economies to allow their exchange rates to float. The euro-epoch data consist of the currencies of Australia (AUS), Brazil (BRA), Canada (CAN), Chile (CHI), Columbia (COL), the Czech Republic (CZE), the euro (EUR), Hungary (HUN), Iceland (ICE), India (IND), Israel (ISR), Japan (JPN), Korea (KOR), Mexico (MEX), Norway (NOR), New Zealand (NZL), the Philippines (PHI), Poland (POL), Romania (ROM), Singapore (SIN), South Africa (RSA), Sweden (SWE), Switzerland (SWI), Taiwan (TWN), Thailand (THA), Turkey (TUR), the United Kingdom (GBR), and the United States (USA).[7]

As seen in Table 1, the euro has consistently been the second most important currency (behind the U.S. dollar) in terms of foreign exchange market turnover. An attractive feature of the euro-epoch data is it does not extend across different regimes or institutional structures.

[6] The IMF report does not cover Taiwan since it is not part of the IMF. We include it in the sample however since the central bank of Taiwan states it uses a managed floating regime. In any case, the standard deviation of monthly returns of the USD/New Taiwan dollar is 1.48% between 1999.01 and 2015.12, which is of similar order of magnitude as that of the Singapore dollar 1.81%, which has consistently been classified as a "managed float with no predetermined path" by the IMF.

[7] Country abbreviations follow International Olympic Committee three-letter country codes (except Taiwan, which we designate as TWN).

TABLE 1
TOP TEN CURRENCIES RANKED BY GLOBAL FOREIGN EXCHANGE MARKET VOLUME

	Percentage Shares of Average Daily Volume						
	1998	2001	2004	2007	2010	2013	Average
U.S. dollar	86.8	89.9	88	85.6	84.9	87	87.0
Euro	...	37.9	37.4	37	39.1	33.4	37.0
Yen	21.7	23.5	20.8	17.2	19	23	20.9
Pound	11	13	16.5	14.9	12.9	11.8	13.4
Swiss franc	7.1	6	6	6.8	6.3	5.2	6.2
Australian dollar	3	4.3	6	6.6	7.6	8.6	6.0
Canadian dollar	3.5	4.5	4.2	4.3	5.3	4.6	4.4
Swedish krona	0.3	2.5	2.2	2.7	2.2	1.8	2.0
Norwegian krone	0.2	1.5	1.4	2.1	1.3	1.4	1.3
Other	65.4	14.7	15.7	20.1	19	21.8	20.0
Total	200	200	200	200	200	200	200

The second data set is from the pre-euro epoch and is of more historical interest, spanning time from 1983.10 to 1998.12. The pre-euro currencies are from AUS, CAN, GBR, Germany (GER), ICE, ISR, JPN, KOR, NOR, NZL, PHI, RSA, SIN, SWE, SWI, and USA. Many of the European currencies are excluded because they were effectively pegged to the deutsche mark during the European Monetary System. Similarly, we exclude emerging market currencies as they were generally pegged to the USD during that time.

Exchange rates are end-of-month point-sampled and obtained from IHS Global insight. We also use implied interest-rate differentials through the forward premium to construct the carry factor return.[8] Further details on the data used in the construction of the carry factor can be found in the Appendix.

4. EMPIRICAL FACTOR IDENTIFICATION

A large number of macro and financial variables potentially have influence on bilateral exchange rates. What economic variables should we include in the vector f_t^p ? To narrow down the set of candidates, our search for common factors is restricted to exchange rate returns. One of the returns we consider is the "carry," studied by Verdelhan (2018). In his examination of nominal exchange rate returns with the USD as the numeraire currency, he concludes that exchange rates have a two-factor representation. The first is a dollar factor, which is the average of the cross section of USD exchange rate returns. Henceforth, we denote the dollar factor by $\Delta\bar{s}_t^{\$}$. Verdelhan's second factor is the "carry factor," which is the cross-rate currency return on a portfolio of high interest rate countries relative to a portfolio of low interest rate countries. He calls this exchange rate return the carry because a (portfolio) carry trade is formed by taking a short position in the low interest rate portfolio and using the proceeds to take a long position in the high interest rate portfolio. Verdelhan (2018) gives a risk-based interpretation to the factors. The dollar risk is interpreted as a global macro-level risk and the carry as representing volatility and uncertainty risk. On account of his findings, we also consider the carry as a factor candidate.

The carry return is constructed as follows: For each time period t, sort the countries by their interest rate and divide, alternatively, into quintiles, quartiles, and tertiles from low to high. Let N_{Ht} be the number of countries in the highest quantile and N_{Lt} be the number in the lowest

[8] By covered interest parity, the forward premium is equal to the interest differential. We follow the literature (e.g., Verdelhan, 2018) which routinely uses the forward premium to measure the interest differential.

quantile.[9] The nominal carry exchange rate return $\Delta\bar{s}_t^c$ is the cross-exchange rate return between P_{Ht} and P_{Lt} currencies

$$\Delta\bar{s}_t^c \equiv \frac{1}{N_{Ht}} \sum_{j \in P_{Ht}} \Delta s_{jt} - \frac{1}{N_{Lt}} \sum_{i \in P_{Lt}} \Delta s_{it}. \tag{10}$$

The carry return constructed this way rebalances the portfolios each period depending on the rank ordering of interest rates. We refer to this as the conditional carry return. We do this using the average interest rate of all countries, and, with the average interest rate, only developed countries are included in the construction of the carry factor.[10] We also consider an unconditional carry return, where the portfolios are sorted once and for all in 1998.12 based on the average interest rates for developed countries from 1990.01 to 1998.12. Additional details on the construction of the carry factor can be found in the Appendix.

The other variables in our candidate list f_t^p are the cross-sectional averages of alternative numeraire exchange rates. These are alternative country i versions of the dollar factor. If $s_{it} - s_{1,t}$ is the log currency i price of the euro, the euro factor candidate, $\Delta\bar{s}_t^{€} = N^{-1} \sum_{i=1}^{N} \Delta s_{it} - \Delta s_{1,t}$, is the cross-sectional average of individual bilateral exchange rate returns with the euro as the numeraire. In the euro-epoch data set, there are 27 such factor candidates.

4.1. *Empirical Identification in the Euro-Epoch Sample.* The IC_2 employed on the euro-epoch sample of standardized and unstandardized exchange rate returns in deviation from mean form, $\{\Delta\tilde{s}_{it}\}$. Taking the minimum of the two determines there to be $k = 2$ common factors. Using other methods, Verdelhan (2018) and Engel et al. (2015) also determine that there are two common factors in exchange rates.

Given that there are two factors, we run the Parker–Sul identification on all possible pairs of factor candidates. There are 27 numeraire factor candidates plus three carry candidates, which vary by portfolio sizes (sorted into quintiles, quartiles, or tertiles). To test if the dollar and the euro are factors, take residuals from the regression $\Delta\tilde{s}_{it} = \alpha_i + \tilde{\delta}_{i1}\Delta\bar{s}_t^{\$} + \tilde{\delta}_{i2}\Delta\bar{s}_t^{€} + \Delta s_{it}^o$ and use IC_2 to determine the number of common factors in the panel $\{\Delta s_{it}^o\}$. Do this for all pairs of candidates. To check robustness over time, we also run the procedure on 47 recursively backdated samples. The sample always ends on 2015.12. The first sample runs from 2002.12 to 2015.12, the second from 2002.11 to 2015.12, and so on through the last sample, which runs from 1999.02 to 2015.12. We always find the dollar, $\Delta\bar{s}_t^{\$}$, to be a factor.

Table 2 reports the proportion of samples that finds a variable to be a common factor along with the dollar factor. As there are a great number of results, the table reports only a subset of the essentials. Look at the first row labeled USA. These are results using the USD as the numeraire. Conditional on the dollar factor, the table reports the proportion of samples in which the candidate is also detected as a factor. “EUR,” “JPN,” and “SWI” stand for the cross-sectional averages of the depreciation rates with the numeraires of the euro, yen, and Swiss franc. The entry 1 under the EUR column indicates that a dollar and a euro factor have been found in all 47 samples. The 0 entry under the JPN column says, conditional on the dollar, the yen is never determined to be a factor. Similarly, the Swiss franc is never found to be a factor. Moving further across the row, we form the carry return sorting over all countries in the sample alternatively into quintiles, quartiles, and tertiles (see Equation (10)). Carry factors are constructed by deleting the currency being analyzed from the carry portfolios and are standardized. (Results with nondeletion are exactly the same.) Conditional on the dollar, none of the carry candidates are determined to be factors in any sample.

[9] The carry trade takes an USD short position in the P_L portfolio and uses the proceeds to take a corresponding USD long position in the P_H portfolio. This return is accessible to investors in any country.

[10] The set of developed countries are the G-10 currencies (AUS, CAN, GBR, GER, JPN, NOR, NZL, SWE, SWI, and USA).

TABLE 2
FREQUENCY OF COMMON FACTOR DETECTION IN POST-EURO RESIDUAL PANEL CONDITIONAL ON DOLLAR FACTOR

	Candidate Factors											
				Conditional Carry			Conditional Carry			Unconditional Carry		
	Selected Currencies			All Countries			Developed Countries			Developed Countries		
Numeraire	EUR	JPN	SWI	Quint.	Quart.	Tert.	Quint.	Quart.	Tert.	Quint.	Quart.	Tert.
USA	1	0	0	0	0	0	0	0	0	0	0	0
AUS	1	0	0	0	0	0	0	0	0	0	0	0
BRA	1	0	0	0	0	0	0	0	0	0	0	0
CAN	1	0	0	0	0	0	0	0	0	0	0	0
CHI	1	0	0	0	0	0	0	0	0	0	0	0
COL	1	0	0	0	0	0	0	0	0	0	0	0
CZE	1	0	0	0	0	0	0	0	0	0	0	0
EUR	1	0	0	0	0	0	0	0	0	0	0	0
GBR	1	0	0	0	0	0	0	0	0	0	0	0
HUN	1	0	0	0	0	0	0	0	0	0	0	0
ICE	1	0	0	0	0	0	0	0	0	0	0	0
IND	1	0	0	0	0	0	0	0	0	0	0	0
ISR	1	0	0	0	0	0	0	0	0	0	0	0
JPN	1	0	0	0	0	0	0	0	0	0	0	0
KOR	1	0	0	0	0	0	0	0	0	0	0	0
MEX	1	0	0	0	0	0	0	0	0	0	0	0
NOR	1	0	0	0	0	0	0	0	0	0	0	0
NZL	1	0	0	0	0	0	0	0	0	0	0	0
PHI	1	0	0	0	0	0	0	0	0	0	0	0
POL	1	0	0	0	0	0	0	0	0	0	0	0
ROM	1	0	0	0	0	0	0	0	0	0	0	0
RSA	1	0	0	0	0	0	0	0	0	0	0	0
SIN	1	0	0	0	0	0	0	0	0	0	0	0
SWE	1	0	0	0	0	0	0	0	0	0	0	0
SWI	1	0	0	0	0	0	0	0	0	0	0	0
THA	1	0	0	0	0	0	0	0	0	0	0	0
TUR	1	0	0	0	0	0	0	0	0	0	0	0
TWN	1	0	0	0	0	0	0	0	0	0	0	0

NOTES: For each numeraire currency, the identification procedure is applied to 47 recursively backdated samples. Every sample ends 2015.12. The first sample begins 2002.12. The last sample begins 1999.02. Table shows frequency with which a common factor is detected in the residual panel out of 47 trials. Developed countries are the G-10.

Since the observations are deviations from the cross-sectional mean, identification is asymptotically (as $N \to \infty$) robust to numeraire choice. In any finite sample, this may not be true. The other rows in the table run the identification procedure using alternative currencies as the numeraire.

The overwhelming evidence finds a dollar and a euro factor. No evidence is found for the yen or the Swiss franc to be a factor, nor for any of the candidate carry factors. Having found the dollar and the euro to be factors, when either the dollar or the euro is the numeraire, it does not matter if exchange rate returns are expressed as deviations from the mean or not. Say the dollar is the numeraire. The factor structure for deviations from the mean is $\Delta\tilde{s}_{it} = \tilde{\delta}_{i1}\Delta\bar{s}_t^{\$} + \tilde{\delta}_{i2}\Delta\bar{s}_t^{€} + \epsilon_{it}$. If we do not take deviations from the mean, it is still the two-factor structure, $\Delta s_{it} = (\tilde{\delta}_{i1} + 1)\Delta\bar{s}_t^{\$} + \tilde{\delta}_{i2}\Delta\bar{s}_t^{€} + \epsilon_{it}$. This is true also when the euro is the numeraire. Now suppose currency j is the numeraire. The exchange rate panel consists of $\Delta\tilde{s}_{it}^j = \Delta s_{it}^j - \Delta\bar{s}_t^j$, where $s_{it}^j = s_{it} - s_{jt}$ is the price of currency j in terms of currency i. The structure is a dollar and euro factor structure for deviations from the mean, $\Delta\tilde{s}_{it}^j = \delta_{i1}\Delta\bar{s}_t^{\$} + \delta_{i2}\Delta\bar{s}_t^{€} + \epsilon_{it}$, but for the not demeaned return, $\Delta s_{it}^j = \delta_{i1}\Delta\bar{s}_t^{\$} + \delta_{i2}\Delta\bar{s}_t^{€} + \Delta\bar{s}_{it}^j + \epsilon_{it}$. That is, $\Delta\bar{s}_t^j$ is also a common factor.

TABLE 3

FREQUENCY OF COMMON FACTOR DETECTION IN PRE-EURO RESIDUAL PANEL CONDITIONAL ON DOLLAR FACTOR

	Candidate Factors								
				Conditional Carry			Conditional Carry		
	Selected Currencies			All Countries			Developed Countries		
Numeraire	GER	JPN	SWI	Quint.	Quart.	Tert.	Quint.	Quart.	Tert.
USA	1	0	0	0	0	0	0	0	0
AUS	1	0	0	0	0	0	0	0	0
CAN	1	0	0	0	0	0	0	0	0
GBR	0.769	0	0	0	0	0	0	0	0
GER	1	0	0	0	0	0	0	0	0
ICE	0.962	0	0	0	0	0	0	0	0
ISR	1	0	0	0	0	0	0	0	0
JAP	1	0	0	0	0	0	0	0	0
KOR	1	0	0	0	0	0	0	0	0
NOR	1	0	0	0	0	0	0	0	0
NZL	1	0	0	0	0	0	0	0	0
PHI	1	0	0	0	0	0	0	0	0
RSA	1	0	0	0	0	0	0	0	0
SIN	1	0	0	0	0	0	0	0	0
SWE	0.615	0	0	0	0	0	0	0	0
SWI	1	0	0	0	0	0	0	0	0
TWN	1	0	0	0	0	0	0	0	0

NOTES: For each numeraire currency, the identification procedure is applied to 26 recursively backdated samples. Every sample ends 1998.12. The first sample begins 1985.12 The last sample begins 1983.11. Table shows frequency with which a common factor is detected in the residual panel out of 26 trials. Developed countries are the G-10.

4.2. *Empirical Identification in the Pre-Euro-Epoch Sample.* The last observation in the pre-euro sample is 1998.12. The first sample runs from 1985.12 and the last sample begins in 1983.11, so that identification is also performed on 26 recursively backdated samples. The cross section is smaller because currencies of emerging market economies in the euro-epoch sample either were not convertible or were pegged. We do not attempt to combine the euro and pre-euro epoch samples because the disappearance and emergence of currencies over time introduces blocks of zeros in the cross-moment matrix from which eigenvalues are computed for the IC_2, which makes the procedure unreliable.[11]

Results for the pre-euro epoch sample are displayed in Table 3. Our findings are similar to those from the euro-epoch sample. The cross section of dollar and deutsche mark exchange rate returns are found to be factors in the vast majority of the samples.

4.3. *Empirical Identification with Verdelhan's Method.* Consider the regression of currency i's depreciation on the nominal interest differential with the U.S. $r_{it} - r_{\$t}$, the dollar factor $\Delta\bar{s}_t^{\$}$, the carry factor $\Delta\bar{s}_t^{c}$, and the dollar and carry factors interacted with the interest differential

$$\begin{aligned}\Delta s_{it+1} &= a + \beta_{i1}(r_{it} - r_{\$t}) + \beta_{i2}\Delta\bar{s}_{t+1}^{\$} \\ &\quad + \beta_{i3}\Delta\bar{s}_{t+1}^{\$}(r_{it} - r_{\$t}) + \beta_{i4}\Delta\bar{s}_{it+1}^{c} + \beta_{i5}\Delta\bar{s}_{it+1}^{c}(r_{it} - r_{\$t}) + \epsilon_{it+1}.\end{aligned} \tag{11}$$

[11] If X is the panel of residuals, the number of factors identification requires calculation of Trace(XX'). We do not combine pre- and post-euro epoch countries because the available currencies would be added and disappear at points in time. The presence of blocks of zeros in XX' creates a problem for the identification procedure.

Verdelhan (2018) identifies the dollar and carry returns to be factors by obtaining significant t-ratios on β_{i2}, β_{i4}, and β_{i5}.[12] The regression controls for the effect of the interest differential through uncovered interest parity. Verdelhan calls the interaction term the "conditional carry" factor, which tries to capture the idea that the comovement between the carry factor and country i exchange rate return is higher in times when the interest differential is bigger.

We estimate (11) with our data. The carry factor $\Delta\bar{s}^{c}_{it}$ is constructed by sorting all countries by interest rates into quintiles, and the carry factor used in the regression omits currency i from the construction of the carry. For example, if $i =$ CAN, CAN is removed from the quintile portfolio it falls into before we construct the carry. Whether a currency is pegged or floats does not introduce complications to this regression methodology here, so we combine the euro and pre-euro samples. We also include, in the pre-euro sample, the currencies of France (FRA), Germany (GER), Greece (GRE), Italy (ITA), and the Netherlands (NET). For each currency, we use as many observations as available, beginning 1983.10. Observations for European currencies in the euro-zone end in 1998.12, whereas observations for the euro begin in 1999.01. The carry factor is generated by sorting countries into quintiles on the basis of their interest rates. The t-ratio on the interest differential is never significant. The t-ratio on the dollar factor coefficient is always highly significant, which is not surprising and not reported. t-Ratios for the key coefficients of interest (β_{i3}, β_{i4}, and β_{i5}) are shown on the left side of Table 4. Our estimates of Equation (11), as in Verdelhan (2018), shows the regression has high explanatory power. The $\bar{R}^2$ values range from 0.21 (TWN) to 0.91 (NET). β_{i4} for the carry is significant at the 5% level for 11 of 33 exchange rates. The carry interacted with the interest differential β_{i5} is significant for five exchange rates.

Now, what happens if we add the euro factor as a regressor to Equation (11)? The right side of Table 4 shows t-ratios for β_{i3}, β_{i4}, β_{i5}, and β_{i6} from

$$\begin{aligned}\Delta s_{it+1} = {} & a + \beta_{i1}\,(r_{it} - r_{\$t}) + \beta_{i2}\Delta\bar{s}^{\$}_{t+1} \\ & + \beta_{i3}\Delta\bar{s}^{\$}_{t+1}\,(r_{it} - r_{\$,t}) + \beta_{i4}\Delta\bar{s}^{c}_{it+1} + \beta_{i5}\Delta\bar{s}^{c}_{it+1}\,(r_{it} - r_{\$t}) + \beta_{i6}\Delta\bar{s}^{€}_{t+1} + \epsilon_{it+1}. \qquad (12)\end{aligned}$$

Here, we see the euro factor is significant for 28 of the 33 exchange rates. The interaction terms (β_{i5}) continue to be significant for six exchange rates, but the carry (β_{i4}) is now significant for only eight exchange rates. The adjusted R^2 values all increase.

Table 5 reports the t-ratios on the coefficients of interest estimated on the euro-epoch sample. These results tell a similar story. The carry (β_{i4}) is significant for 16 of 27 exchange rates in (11) and for 11 exchange rates when the equation is augmented by the euro factor. The euro factor is significant in 23 of 27 exchange rates. Adding the euro factor increases the $\bar{R}^2$.

To summarize this section, our evidence shows that exchange rate returns are driven by a two-factor structure. We identified a dollar factor and a euro factor. The carry return is not identified to be an exchange rate common factor using the Parker–Sul method. Verdelhan's regression method is less definitive. It provides strong evidence that the euro currency return is an exchange rate common factor and only weak evidence that the carry factor is an exchange rate common factor.We note that Aloosh and Bekaert (2017), employing cluster analysis, also identify two currency factors—one associated with "dollar" currencies and the other associated with "European" currencies—and that their two factors also drive out the carry factor. The similarity in the adjusted R^2 values in Tables 4 and 5 says the euro factor and carry factors share common information, but the lower significance of the carry in the Parker–Sul and in the Verdelhan methodologies leads to the conclusion that exchange rate dynamics are more directly linked and driven by the euro factor.

[12] Verdelhan (2018) did not include $\Delta\bar{s}^{\$}_{t+1}(r_{it} - r_{\$,t})$ in his regressions.

Table 4
FACTOR IDENTIFICATION BY VERDELHAN'S METHOD OVER THE FULL SAMPLE

	Equation (11)				Equation (12)				
	$t_{\beta_{i3}}$	$t_{\beta_{i4}}$	$t_{\beta_{i5}}$	$\bar{R}^2$	$t_{\beta_{i3}}$	$t_{\beta_{i4}}$	$t_{\beta_{i5}}$	$t_{\beta_{i6}}$	$\bar{R}^2$
AUS	**−2.714**	0.459	1.092	0.783	**−2.814**	0.037	1.402	**2.825**	0.792
BRA	−1.572	−0.195	1.215	0.527	−0.890	−0.711	1.273	**6.019**	0.610
CAN	**−5.575**	**5.160**	−1.129	0.373	**−4.912**	**4.077**	−0.994	**5.595**	0.412
CHI	−0.608	0.585	−0.058	0.457	−0.631	−0.164	0.102	**3.069**	0.481
COL	**2.561**	1.580	−1.676	0.476	**2.835**	0.929	**−2.079**	**6.359**	0.569
CZE	**2.052**	**−2.011**	**2.433**	0.661	**2.225**	0.097	1.328	**−13.07**	0.814
DEN	−1.774	−1.394	1.794	0.904	**−3.976**	−1.933	**6.067**	**−21.25**	0.964
GER	0.636	−1.486	0.176	0.691	−0.069	−0.007	0.132	**−205.0**	0.998
FRA	0.643	−0.396	0.160	0.890	−0.316	−0.478	**2.661**	**−19.59**	0.958
GBR	1.167	−0.133	−0.832	0.478	1.125	0.447	−1.275	**−2.831**	0.494
GRE	0.349	**−2.994**	**3.829**	0.737	1.341	**−2.946**	**3.433**	**−2.414**	0.768
HUN	0.684	−0.467	−0.157	0.710	0.358	1.423	−1.500	**−8.448**	0.784
ICE	−1.435	−0.756	0.995	0.335	−1.473	−0.888	1.053	0.974	0.339
IND	**2.419**	−0.228	0.547	0.419	**2.314**	−0.895	0.847	**4.092**	0.464
ISR	−0.193	**−2.423**	1.061	0.406	−0.206	**−2.224**	1.050	−0.592	0.407
ITA	**2.701**	1.058	−0.687	0.772	**2.690**	1.057	−0.573	−1.302	0.775
JPN	−1.471	**−2.193**	0.087	0.213	−1.619	−1.866	0.098	−1.691	0.220
KOR	**−3.693**	0.022	0.345	0.204	**−2.819**	−0.938	0.702	**5.445**	0.259
MEX	**−3.707**	**2.384**	−0.277	0.366	**−4.075**	0.684	1.325	**8.164**	0.494
NET	0.036	−1.005	−0.760	0.906	−0.706	−0.834	0.274	**−69.46**	0.986
NOR	1.261	−1.059	0.863	0.854	0.967	−0.945	1.219	**−3.590**	0.865
NZL	−1.457	**2.826**	−0.777	0.399	−1.069	**2.271**	−0.809	**4.591**	0.426
PHI	**2.202**	**3.099**	**−6.752**	0.405	**2.468**	**2.349**	−6.192	**2.991**	0.431
POL	−1.626	**−2.107**	**2.348**	0.714	**−2.012**	−1.603	**2.176**	**−2.266**	0.722
ROM	0.824	−1.570	1.806	0.707	1.047	0.235	1.091	**−7.313**	0.798
RSA	−1.905	0.385	1.009	0.383	−1.620	−0.816	1.174	**6.081**	0.430
SIN	1.448	−0.565	−0.520	0.738	1.328	−0.811	−0.390	1.130	0.740
SPA	1.372	1.911	0.007	0.862	−0.277	**2.064**	0.549	**−15.74**	0.980
SWE	1.359	−0.950	1.355	0.738	**2.196**	−0.020	0.918	**−6.567**	0.767
SWI	**2.519**	**−6.264**	**2.418**	0.690	**2.274**	**−5.505**	**2.321**	**−16.52**	0.804
THA	−1.107	**−2.857**	1.284	0.358	−1.837	**−3.495**	1.466	**3.711**	0.398
TUR	0.631	1.817	−1.244	0.398	1.672	1.336	−1.579	**7.213**	0.494
TWN	0.022	1.110	1.589	0.210	0.152	0.661	1.759	**3.324**	0.241

Notes: Bold indicates significance at the 5% level. We use any observations available from 1983.10–2015.12. Carry factor formed by sorting countries into quintiles. R^2 and t-ratios on coefficients in Equations (11) and (12).

5. CHARACTERISTICS OF THE IDENTIFIED FACTORS

Researchers frequently assume the PCs are the factors. Figure 1 plots the cumulated dollar factor and the cumulated first PC. Figure 2 compares the cumulated euro factor with the cumulated second PC. Although there are similarities between our identified factors and the PCs, they are not the same. PCs are constructed under the identifying assumption that they are orthogonal to each other. The factor representation allows the factors to be correlated with each other. The correlation between $\Delta\bar{s}_t^{\$}$ and the first PC is 0.996, between $\Delta\bar{s}_t^{€}$ and the second PC is 0.8, and the correlation between the dollar and the euro factors is −0.267. Generalized strength in the dollar is associated with generalized weakening of the euro.

To give some context for our identification, the implied relationship between the latent factors and the dollar and euro empirical factors is

$$f_{1,t} = a_{1,1}\Delta\bar{s}_t^{\$} + a_{1,2}\Delta\bar{s}_t^{€} + \epsilon_{1,t}, \tag{13}$$

$$f_{2t} = a_{21}\Delta\bar{s}_t^{\$} + a_{22}\Delta\bar{s}_t^{€} + \epsilon_{2t}. \tag{14}$$

TABLE 5
FACTOR IDENTIFICATION BY VERDELHAN'S METHOD OVER THE EURO-EPOCH SAMPLE

	Equation (11)				Equation (12)				
	$t_{\beta_{i3}}$	$t_{\beta_{i4}}$	$t_{\beta_{i5}}$	$\bar{R}^2$	$t_{\beta_{i3}}$	$t_{\beta_{i4}}$	$t_{\beta_{i5}}$	$t_{\beta_{i6}}$	$\bar{R}^2$
AUS	**−2.421**	0.738	0.837	0.783	**−2.229**	−0.109	0.811	**2.204**	0.788
BRA	−1.225	−0.700	**1.980**	0.539	−0.619	−1.022	1.315	**5.355**	0.605
CAN	−1.445	**2.882**	−0.558	0.514	−1.123	1.355	−0.661	**2.326**	0.523
CHI	−0.656	1.602	−0.122	0.475	−0.611	0.724	−0.260	**2.260**	0.487
COL	1.901	**2.079**	−0.338	0.488	1.645	−0.116	−0.071	**5.273**	0.556
CZE	**2.725**	**−6.080**	0.200	0.699	**2.732**	−0.836	0.666	**−11.86**	0.819
EUR	1.165	**−7.457**	−1.019	0.760	**−2.758**	**3.784**	1.800	inf	1.000
GBR	1.293	−1.393	**−2.040**	0.521	0.996	−0.516	−1.918	−1.903	0.530
HUN	0.038	**−2.523**	1.206	0.726	−0.359	−0.658	1.246	**−7.314**	0.785
ICE	**−3.613**	**−3.181**	**3.667**	0.367	**−3.622**	**−3.257**	**3.673**	0.649	0.368
IND	**2.791**	0.625	0.793	0.435	**2.729**	−0.503	0.902	**3.322**	0.465
ISR	**−2.131**	**−2.482**	1.110	0.334	−1.778	**−2.877**	1.127	1.490	0.340
JPN	1.233	**−3.876**	−0.361	0.203	1.362	**−4.563**	−0.528	**2.416**	0.228
KOR	**−4.950**	−1.476	**5.517**	0.647	**−4.929**	**−3.333**	**5.704**	**3.701**	0.677
MEX	**−2.183**	**2.471**	−0.507	0.427	−1.934	0.049	0.514	**6.800**	0.523
NOR	0.012	**−3.550**	1.277	0.667	−0.807	−0.170	1.271	**−6.884**	0.714
NZL	0.264	1.073	−1.382	0.630	0.426	0.746	−1.465	1.195	0.632
PHI	−0.785	0.941	−1.345	0.288	−0.369	−0.057	−1.231	**2.716**	0.317
POL	−1.388	**−2.466**	**2.242**	0.718	−1.687	−1.389	**2.112**	**−1.964**	0.724
ROM	**2.145**	−1.806	0.426	0.716	**2.561**	1.117	0.035	**−7.380**	0.798
RSA	−1.874	0.977	0.352	0.476	−1.132	−0.553	0.025	**3.541**	0.506
SIN	**2.472**	**−2.995**	1.105	0.708	**2.849**	**−5.358**	0.419	**5.774**	0.734
SWE	0.283	**−10.04**	1.374	0.795	−0.200	**−5.250**	1.805	**−8.232**	0.828
SWI	0.808	**−6.280**	1.453	0.672	1.116	**−4.675**	1.089	**−8.610**	0.737
THA	−0.862	−1.493	0.089	0.343	−1.748	**−3.501**	0.175	**4.724**	0.409
TUR	0.295	**4.303**	−0.141	0.446	1.098	**2.132**	−0.687	**5.270**	0.498
TWN	0.853	**−2.519**	1.744	0.512	1.268	**−4.515**	1.859	**4.820**	0.557

NOTES: Bold indicates significance at the 5% level. We use observations from the euro-epoch sample. Carry factor formed by sorting countries into quintiles. R^2 and t-ratios on coefficients in Equations (11) and (12).

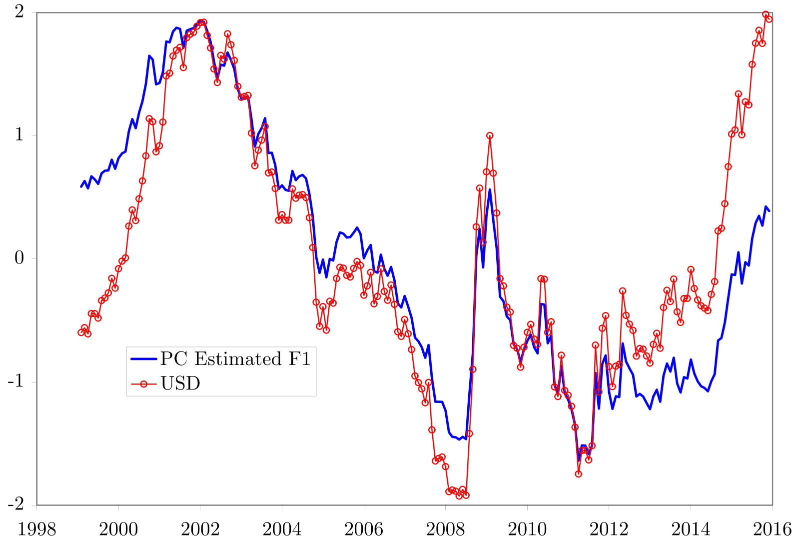

FIGURE 1

DOLLAR FACTOR AND PRINCIPAL COMPONENTS [COLOR FIGURE CAN BE VIEWED AT WILEYONLINELIBRARY.COM]

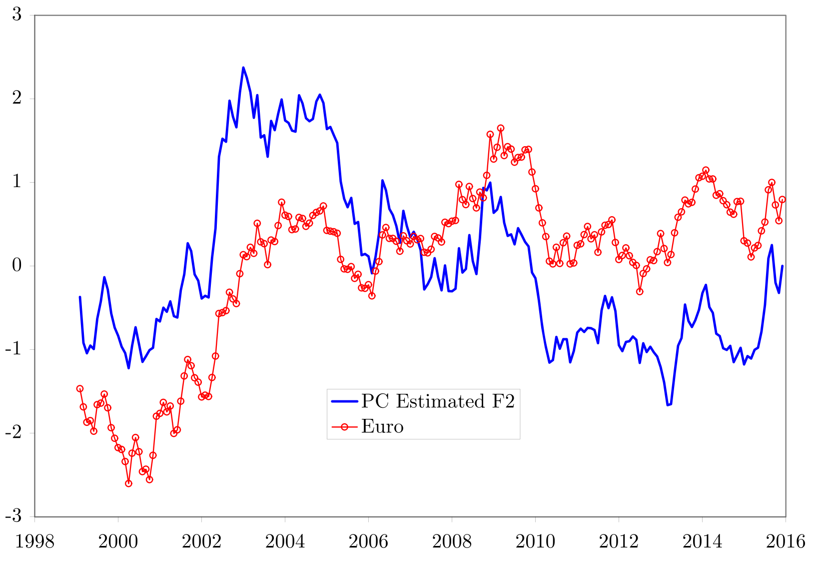

FIGURE 2

EURO FACTOR AND PRINCIPAL COMPONENTS [COLOR FIGURE CAN BE VIEWED AT WILEYONLINELIBRARY.COM]

As before, let USA be country 0 and let the euro zone be country 1. Note that $\Delta\bar{s}_t^{\text{€}} = \Delta\bar{s}_t^{\$} - \Delta s_{1,t}$. Recall from (2) that country i's log SDF has a two-factor structure, which when employed in Equations (13) and (14) gives[13]

$$
\begin{aligned}
f_{1t} &= a_{11}\Delta\bar{s}_t^{\$} + a_{12}\left(\Delta\bar{s}_t^{\$} - \Delta s_{1,t}\right) + \epsilon_{1t},\\
&= (a_{11}+a_{12})\underbrace{[\bar{n}_t - n_{0t}]}_{\Delta\bar{s}_t^{\$}} - a_{12}\underbrace{[\bar{n}_t - n_{1t}]}_{\Delta s_{1t}} + \epsilon_{1t} + O_p(N^{-1}),\\
f_{2t} &= a_{21}\Delta\bar{s}_t^{\$} + a_{22}\left(\Delta\bar{s}_t^{\$} - \Delta s_{1t}\right) + \epsilon_{2t},\\
&= (a_{21}+a_{22})\underbrace{[\bar{n}_t - n_{0t}]}_{\Delta\bar{s}_t^{\$}} - a_{22}\underbrace{[\bar{n}_t - n_{1t}]}_{\Delta s_{1t}} + \epsilon_{2t} + O_p(N^{-1}).
\end{aligned}
$$

Recalling the linear factor representation for the nominal SDF $n_{it} = \delta_{i1}f_{1t} + \delta_{i2}f_{2t} + n_{it}^0$ after some algebra yields

$$
n_{it} = b_{i1}\bar{n}_t - b_{i2}n_{0t} - b_{i3}n_{1t} + \delta_{i1}\epsilon_{1t} + \delta_{i2}\epsilon_{2t} + n_{it}^0, \tag{15}
$$

where

$$
\begin{aligned}
b_{i1} &= \delta_{i1}\left(a_{11}+a_{12}\right) + \delta_{i2}\left(a_{21}+a_{22}\right),\\
b_{i2} &= \delta_{i1}a_{11} + \delta_{i2}a_{21},\\
b_{i3} &= \delta_{i1}a_{12} + \delta_{i2}a_{22}.
\end{aligned}
$$

[13] Note that $\bar{n}_t = \frac{1}{N}\sum_{i=1}^N n_{it}$ and $\Delta\bar{s}_t^{\text{€}} = \frac{1}{N}\sum_{i\notin 1}^N n_{it} - n_{1t} = \bar{n}_t^1 - n_{1t}$. But the difference between $\bar{n}_t$ and $\bar{n}_t^1$ goes to zero as $N \to \infty$. This is because $\bar{n}_t - \bar{n}_t^1 = \frac{1}{N}(n_{1t} + \cdots + n_{Nt}) - \frac{1}{N}(n_{0t} + n_{2t} + \cdots + n_{Nt}) = \frac{1}{N}(n_{1t} + n_{0t}) = O_p(N^{-1})$ since both n_{1t} and n_{0t} are $O_p(1)$.

TABLE 6
IDENTIFIED FACTOR STRUCTURE DURING EURO EPOCH

		$\Delta\tilde{s}_{it} = \tilde{\delta}_{i1}\Delta\bar{s}_t^{\$} + \tilde{\delta}_{i2}\Delta\bar{s}_t^{€} + \Delta s_{it}^o$				
		Dollar		Euro		
		$\tilde{\delta}_{i1}$	t-Ratio	$\tilde{\delta}_{i2}$	t-Ratio	R^2
Western Europe	GBR	−0.278	−4.950	−0.252	−3.766	0.137
	ICE	0.028	0.237	−0.173	−1.211	0.015
	NOR	0.147	2.926	−0.474	−7.895	0.235
	SWE	0.213	5.407	−0.555	−11.834	0.408
	SWI	−0.137	−2.800	−0.741	−12.658	0.370
Emerging Europe	CZE	0.161	3.181	−0.88	−14.547	0.537
	HUN	0.519	7.827	−0.658	−8.312	0.458
	POL	0.532	6.540	−0.212	−2.184	0.262
	ROM	0.111	1.214	−0.461	−4.228	0.156
Common wealth	AUS	0.501	9.446	0.195	3.091	0.288
	CAN	−0.111	−2.302	0.238	4.158	0.084
	NZL	0.410	6.167	0.051	0.649	0.124
	RSA	0.537	4.365	0.492	3.350	0.131
Mid East	ISR	−0.408	−6.487	0.039	0.519	0.186
	TUR	0.383	2.631	0.678	3.903	0.114
Asia	IND	−0.343	−6.724	0.266	4.369	0.263
	JPN	−0.827	−8.782	−0.16	−1.421	0.311
	KOR	0.086	1.281	0.346	4.330	0.077
	PHI	−0.519	−9.372	0.176	2.675	0.384
	SIN	−0.387	−13.726	0.043	1.266	0.469
	THA	−0.503	−10.701	0.151	2.690	0.404
	TWN	−0.571	−17.283	0.07	1.772	0.621
Latin America	BRA	0.548	4.582	1.071	7.504	0.225
	CHI	−0.068	−0.777	0.373	3.576	0.077
	COL	0.151	1.754	0.718	6.979	0.199
	MEX	−0.141	−2.106	0.623	7.779	0.278

NOTES: Estimated over the euro-epoch sample.

Every country's log SDF is seen to be connected to the global log SDF $\bar{n}_t$, the U.S. log SDF n_{0t}, and the euro-zone log SDF n_{1t}. Upon substitution of (15) into (1), exchange rate returns are seen to be governed by the United States, euro, and a global ($\bar{n}_t$) log SDF. That is,

$$\Delta s_{it} \to b_{i1}\bar{n}_t - b_{i2}n_{0t} - b_{i3}n_{1t} \text{ as } N, T \to \infty.$$

5.1. *Geographical Patterns.* Table 6 shows estimates of the identified factor structure. These are regressions of Equation (9) with the dollar factor for $f_{1t}^p = \Delta\bar{s}_t^{\$}$ and the euro factor for $f_{2t}^p = \Delta\bar{s}_t^{€}$. We estimate by regressing the deviations from the mean $\Delta\tilde{s}_{it}$ so the results are numeraire invariant. Results are broken down by geographical classification. Estimation is for the euro-epoch data set.

In regressions of $\Delta\tilde{s}_{it}$, explanatory power of the identified two-factor model is high, with R^2 ranging from 0.02 (ICE) to 0.62 (TWN). The dollar factor loadings are generally positive for European and commonwealth countries (not Canada), which says, conditional on the euro, a rise in the USD is associated with a decline in these currencies. Conditional on the euro, dollar gains tend to be associated with gains in Asian currencies, which load negatively on the dollar factor. Except for Mexico and Canada, who load negatively on the dollar factor so that their currencies risk with the dollar (and who share a border with the United States), those that load positively on the dollar tend to be commodity currencies

The euro factor loads negatively on European exchange rates and positively on all others (except JPN). The negative loadings says when the euro gains, European currencies also gain. Non-European currencies fall relative to the dollar when the euro gains. There is a distinct geographical pattern in the factor loadings.[14] There is also a shred of evidence that countries that share risk better with the euro zone load negatively on the euro factor. Regressing the euro-factor loadings on the R^2 from regressing a country's consumption growth rate on euro-zone consumption growth gives a slope of -1.064 (t-ratio -1.816) and $R^2 = 0.121$. A positive loading says when the euro gains, that currency loses and is associated with lower consumption correlation with the euro zone.[15]

5.2. *A Risk-Based Interpretation.* The connection between exchange rates and SDFs and the role of SDFs in pricing assets suggests there may be a risk-based interpretation to the factor structure. We pursue this interpretation along the lines developed in Verdelhan (2018).

The operation goes as follows: At date t, estimate the factor structure on a width k backward looking window of observations

$$\Delta \tilde{s}_{it} = a_{it_0} + \delta_{i1,t_0} \Delta \bar{s}_t^{\$} + \delta_{i2,t_0} \Delta \bar{s}_t^{€} + \epsilon_{it}, \qquad \text{for } t = t_0 - k + 1, \ldots, t_0. \tag{16}$$

Currency i is omitted in construction of both factors. Next, sort the time-varying factor loadings $\hat{\delta}_{i1,t_0}$ and $\hat{\delta}_{i2,t_0}$ from smallest to largest and form four portfolios of currency excess returns grouped by the ranking on dollar exposure ($\hat{\delta}_{i1,t_0}$) and four portfolios grouped by ranking on euro exposure ($\hat{\delta}_{i2,t_0}$). The investor takes a long position in the dollar portfolios if the average G-10 currency interest differential $(\frac{1}{N}\sum_i r_{it}) - r_{\$,t}$ at time t_0 is positive and short if the differential is negative. Similarly, the investor takes a long position in the euro-beta sorted portfolios if the average G-10 currency interest differential with respect to the euro area $(\frac{1}{N}\sum_i r_{it}) - r_{€,t}$ is positive.[16] Note that each currency appears in both a dollar "beta-sorted" portfolio and a euro "beta-sorted" portfolio.[17]

The dollar and euro beta-sorted returns, which serve as test asset returns, are

$$\bar{r}_{j,t+1}^{\$} = \left[\frac{1}{N_{P_{\$j,t}}} \sum_{i \in P_{\$j,t}} (r_{it} + \Delta s_{it+1}) - r_{\$,t} \right] \cdot I\left(\frac{1}{N} \sum_i r_{it} - r_{\$,t} \right),$$

$$\bar{r}_{j,t+1}^{€} = \left[\frac{1}{N_{P_{€j,t}}} \sum_{i \in P_{€j,t}} \left(r_{it} + \Delta s_{it+1}^{€}\right) - r_{€,t} \right] \cdot I\left(\frac{1}{N} \sum_i r_{it} - r_{€,t} \right),$$

where the indicator function $I(\cdot) = 1$ if the argument is positive and is -1 if the argument is negative. $N_{P_{\$j,t}}$ ($N_{P_{€j,t}}$) is the number of currencies in the dollar (euro) beta-sorted portfolio j at time t, and $s_{it}^{€}$ is the log currency i price of the euro.

The aggregate portfolio excess returns, $RE_{t_0+1}^{\$} = \sum_{j=1}^{4} \bar{r}_{j,t+1}^{\$}$ and $RE_{t_0+1}^{€} = \sum_{j=1}^{4} \bar{r}_{j,t+1}^{€}$ are interpreted as the risk factors. We construct these conditional returns for each $t_0 = k, \ldots, T-1$ and use them to estimate a two-factor beta-risk model.[18] Stack the test asset returns in the

[14] Lustig and Richmond (2017) undertake a systematic investigation of the relationship between dollar exposure and geography.

[15] Annual consumption data are from Penn World Tables version 8.1 (Feenstra et al., 2015).

[16] We are applying the Lustig et al. (2014) investment strategy for the dollar to the dollar and the euro.

[17] Because the portfolios and the portfolio returns depend on interest rate differentials, the dollar and euro portfolios are constructed using the same data set that we used to make the carry returns. Details on these data are contained in the Appendix.

[18] Because the returns are conditional on interest differential realizations, the literature refers to them as "conditional" returns.

TABLE 7
RISK-BASED INTERPRETATION OF DOLLAR AND EURO FACTORS

A. Return Characteristics

	Portfolios Sorted by Dollar Loadings				Portfolios Sorted by Euro Loadings			
	P_1	P_2	P_3	P_4	P_1	P_2	P_3	P_4
	Excess Return							
Mean	−2.868	0.865	4.004	3.702	1.468	5.605	3.353	3.683
Std. dev.	28.015	37.150	32.546	37.214	17.028	39.031	26.190	30.711
	Depreciation Rate							
Mean	0.329	−1.275	−1.594	−0.356	0.224	1.148	0.189	1.121
Std. dev.	14.974	26.178	32.434	36.426	12.553	23.197	26.410	30.203
	Interest Differential							
Mean	−2.539	−0.410	2.410	3.346	1.692	6.753	3.542	4.804
Std. dev.	21.665	24.148	2.945	6.058	11.317	29.484	5.198	4.117

Aggregate Conditional Portfolio Excess Return

	Sorted by Dollar Load	Sorted by Euro Load
Mean	1.426	3.527
Std. dev.	27.223	21.645
Sharpe	0.052	0.163

B. Two-Factor Beta-Risk Model

First-Stage Betas

	β_1	β_2	β_3	β_4	β_5	β_6	β_7	β_8
Dollar Risk	0.604	1.078	1.082	1.235	−0.002	−0.105	−0.005	0.112
Euro Risk	0.044	−0.160	0.048	0.068	0.284	1.429	1.085	1.203

Second Stage

$\lambda_{\$}$	t-Ratio	$\lambda_{€}$	t-Ratio	R^2	χ^2_7	p-Value
1.858	0.947	3.548	2.196	0.761	4.393	0.734

NOTES: Estimated over the euro-epoch sample.

vector $y_t = (\bar{r}^{\$}_{1,t}, \ldots, \bar{r}^{\$}_{4,t}, \bar{r}^{€}_{1,t}, \ldots, \bar{r}^{€}_{4,t})'$ and the risk factors in the vector $x_t = (RE^{\$}_t, RE^{€}_t)'$. Using the two-stage method, the first stage runs the time-series regression of the return differential on the portfolio excess return.

$$y_{it} = a_i + x_t'\beta_i + \epsilon_{it} \tag{17}$$

for $t = t_0, t_0 + 1, \ldots, T$, $i = 1, \ldots, 8$, and $\beta_i = (\beta_{i\$}, \beta_{i€})'$ is the two-dimensional vector of betas on the dollar and euro risk factors. As in Verdelhan (2018), the second stage runs the cross-sectional regression of the average returns on the betas without a constant

$$\bar{y}_i = \lambda'\hat{\beta}_i + \alpha_i, \tag{18}$$

where $\bar{y}_i$ is the time-series average of y_{it} and α_i is the "pricing error." We compute standard errors by GMM to account for the fact that the betas in stage 2 are generated regressors.

Table 7 reports the results. The beta-risk model is estimated on observations from 1999.01 to 2015.12. The initial rolling factor loadings $(\delta_{i1,t_0}, \delta_{i2,t_0})$ are estimated on observations from

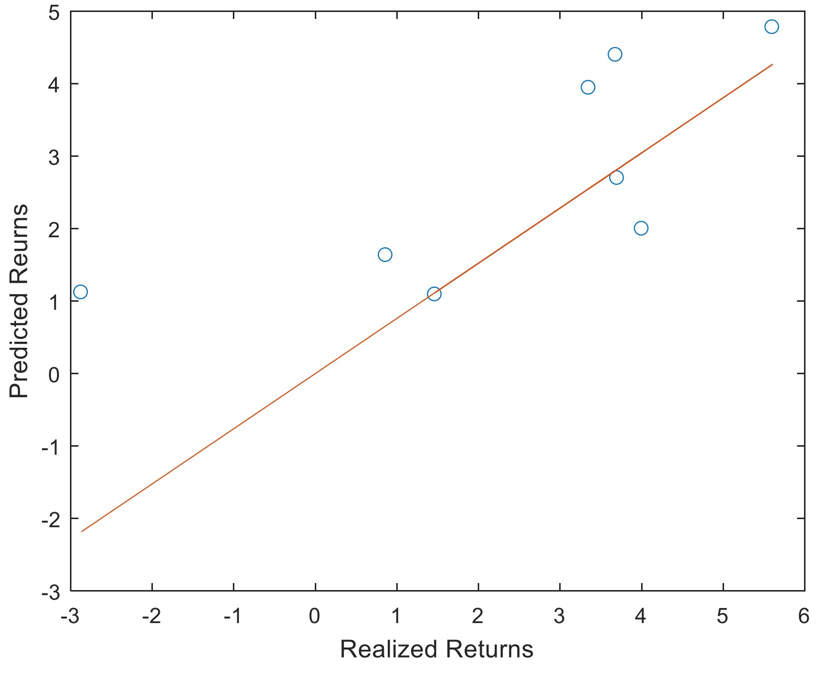

FIGURE 3

ACTUAL AND PREDICTED AVERAGE EXCESS RETURNS [COLOR FIGURE CAN BE VIEWED AT WILEYONLINELIBRARY.COM]

1994.01 through 1999.01. The deutsche mark is used in place of the euro for 1994.01 through to 1998.12 in the rolling regressions.

Returns are stated in percent per annum. Some support for a risk-based interpretation of the factors is provided by the mean conditional excess currency returns. The mean returns are generally (but not monotonically) increasing in exposure to the dollar factor and to the euro factor. Interestingly, the conditional excess returns are driven more heavily by interest differentials than by exchange rate depreciation.

The dollar risk premium estimate $\lambda_\$$ is 1.8% (p-value = 0.34) whereas the euro risk premium estimate is 3.5% (p-value = 0.03). The test for randomness in the pricing errors is insignificant, and the second stage R^2 is a respectable 0.76.[19] Figure 3 plots the actual and predicted excess returns.

To summarize, the empirical factor identification is useful in that it helps to give an economic interpretation for cross-currency comovements of exchange rates. The data reveal both geographical and risk-based dimensions to the dollar and euro factors. In the next section, we show that the identification can also work to improve empirical exchange rate models in terms of their ability to forecast.

6. MULTILATERAL EMPIRICAL EXCHANGE RATE MODELING

This section conducts an out-of-sample forecasting exercise with the factor models. Although Inoue and Kilian (2004) point out that in-sample tests are more powerful than out-of-sample tests in testing the predictability of exchange rates, ever since Meese and Rogoff (1983), it has been customary practice to evaluate empirical exchange rate models by their out-of-sample forecast accuracy. Our dollar–euro factor identification motivates a particular *multilateral*

[19] R^2 statistics are calculated using the sum of squared dependent variables (not demeaned) in the denominator to ensure that they are positive.

forecasting model for bilateral exchange rates. We generate forecasts for nominal exchange rate returns at 1-, 12-, and 24-month horizons. The USD is the numeraire. Forecast ability for any pair of exchange rates implies forecast ability for the associated cross rate.[20]

Exchange rates are an asset price. As in other asset-pricing research, exchange rate forecasting aims to exploit information contained in the deviation of the exchange rate from a fundamental value, which is thought to be a measure of central tendency. The strategy shares much with studies of stock prices, where variables such as the dividend–price ratio or book value relative to market value of firms predict future equity returns. For stock prices, a certain multiple of dividends (or book value) plays the role of the central tendency for price.

The identification of the dollar and euro factors lead us to forecast h-period-ahead exchange-rate returns with the empirical model

$$s_{it+h} - s_{it} = \alpha_i + \beta_{i1}\bar{s}_t^{\$} + \beta_{i2}\bar{s}_t^{€} + \beta_{i3}\bar{s}_t^{i} + \beta_{i4}s_{it} + \epsilon_{it+h}. \tag{19}$$

The systematic part of the regression plays the role of an error-correction term. The derivation of Equation (19) is given in the Appendix. The model includes the dollar and euro factors but also includes a currency i factor, $\bar{s}_t^i$, the cross-sectional average of exchange rates with currency i as the numeraire. The Appendix shows how $\bar{s}_t^i$ contains idiosyncratic information that can be exploited. By including it as conditioning information, the forecasts also become numeraire invariant.[21] Forecasts are generated by rolling regression using a 60-month lag window.

For comparison, we also generate forecasts from three other models discussed in the recent literature. One is a dollar and carry factor model, where $\bar{s}_t^{€}$ in (19) is replaced by the carry counterpart $\bar{s}_t^c$, constructed by sorting countries by interest rates into quintiles. A second model is drawn from Engel et al. (2015), who dispense with empirical identification of factors and use PCs as factors $\hat{F}_{j,t}^{pc}$, $j = 1, 2$ for forecasting[22]

$$s_{it+h} - s_{it} = \alpha_i + \beta_{i1}\hat{F}_{1t}^{pc} + \beta_{i2}\hat{F}_{2t}^{pc} + \beta_{i3}s_{it} + \epsilon_{it+h}. \tag{20}$$

The PCs are estimated for every t and each horizon, h.

The third is the bilateral PPP fundamentals model. In this model, the fundamental value of s_{it} is the PPP $p_{it} - p_{0t}$, where p_{it} is the log price level of country i. The model allows s_{it} to deviate from its PPP over the short and medium term but assumes that they share a common trend, so the real exchange rate is stationary and mean reverting. The PPP-based fundamentals model is thus an error correction without the short-run dynamics

$$s_{it+h} - s_{it} = \alpha_i + \beta_i\left(p_{it} - p_{0t} - s_{it}\right) + \epsilon_{it+h}. \tag{21}$$

[20] Drawing motivation from the present value model of exchange rates, Chen et al. (2010) and Sarno and Schmeling (2013) find evidence that today's exchange rate predicts future fundamentals. The importance of cross-sectional information has been recognized since Bilson (1981), who used seemingly unrelated regression to estimate his exchange rate equation. Frankel and Rose (1996) initiated a literature on the panel data analysis of PPP, which is surveyed by Caporale and Cerrato (2006). Cerra and Saxena (2010) employed a panel data set with a large number (98) of countries in a study of the monetary model of exchange rates.

[21] Empirical factors are standardized by the variance of their depreciation rates to avoid exact multicollinearity. Since s_{it} can be perfectly correlated with $\delta_{i1}\bar{s}_t^{\$} + \delta_{i2}\bar{s}_t^{€} + \phi_i\bar{s}_t^i$, without standardizing, the slope coefficients are not estimable in some cases. For example, $\bar{s}_t^{\$}$ in (19) is equal to $N^{-1}\sum_{i=1}^N s_{it}/\sqrt{V(\Delta s_{it})}$ where $V(\Delta s_{it}) = t^{-1}\sum_{\ell=1}^t(\Delta s_{i\ell} - t^{-1}\sum_{\ell=1}^t \Delta s_{i\ell})^2$.

[22] Engel et al. (2015) considered one-, two-, and three-factor models. The forecasting ability of the two- and three-factor models were nearly identical and dominated that of the one-factor model. Using quarterly data beginning in 1973, Engel et al. (2015) find that predictions of the factor-based forecasts significantly dominate random walk forecasts in mean-square error when forecasting from 1999 to 2007. We note that Engel et al. (2015) used the "restricted" version of the forecasting, which includes an extra round of estimation. They forecasted by recursively estimating both the PCs and factor loadings, which were inputted into the forecasting model $s_{it+h} - s_{it} = \alpha_i + \beta_i\hat{s}_{it}^o + \epsilon_{it+h}$, where $\hat{s}_{it}^o = s_{it} - \hat{\delta}_{i1}\hat{F}_{1t} - \hat{\delta}_{i2}\hat{F}_{2t}$. Here, we use PCs in the "unrestricted" forecasting model. This eliminates the estimation of factor loadings, which gives more accurate forecasts than the restricted forecasts.

Table 8
FORECASTING AT ONE-MONTH HORIZON

	Random Walk	Dollar–Euro		Dollar–Carry		Principal Components		Bi-PPP	
	MSPE	U	t_{CW}	U	t_{CW}	U	t_{CW}	U	t_{CW}
AUS	**1.415**	1.236	0.306	1.099	2.573	1.042	1.436	1.004	1.163
BRA	**2.009**	1.267	0.443	1.062	2.286	1.190	0.641	1.021	0.764
CAN	0.829	1.192	−0.649	**0.965**	3.282	1.110	−0.527	1.045	−0.909
CHI	1.186	1.169	−1.302	1.027	1.813	**1.000**	1.358	1.036	−1.172
COL	**1.571**	1.210	−1.347	1.159	0.872	1.127	−0.453	1.032	−0.655
CZE	**1.405**	1.189	−0.416	1.105	1.578	1.153	0.629	1.021	0.223
EUR	**0.910**	1.162	−1.201	1.173	−0.301	1.117	−1.068	1.016	−0.030
GBR	**0.637**	1.193	0.074	1.140	0.966	1.148	0.167	1.018	0.253
HUN	**2.125**	1.174	−0.818	1.157	0.058	1.066	0.956	1.033	−0.922
ICE	**1.924**	1.310	−0.352	1.367	−0.153	1.267	−0.623	1.051	−0.558
IND	0.649	1.155	−0.267	1.046	2.345	1.094	−0.716	**0.997**	1.074
ISR	**0.585**	1.230	−0.002	1.363	0.652	1.259	0.372	1.023	0.163
JPN	**0.721**	1.146	0.300	1.144	0.689	1.143	−0.667	1.056	−0.577
KOR	**1.200**	1.329	−0.496	1.288	−0.012	1.220	−0.863	1.033	−1.499
MEX	**0.859**	1.090	0.871	1.124	2.392	1.183	−0.523	1.025	−0.137
NOR	**1.202**	1.198	−1.494	1.079	0.966	1.078	−0.025	1.025	−0.268
NZL	1.566	1.176	−0.757	1.157	0.709	1.138	−1.434	**0.991**	1.386
PHI	**0.285**	1.336	0.576	1.049	1.733	1.087	0.395	1.056	−0.420
POL	**1.975**	1.260	−0.775	1.173	0.824	1.187	0.269	1.004	1.109
ROM	**1.341**	1.151	−0.728	1.052	1.242	1.078	−0.461	1.045	−0.936
RSA	2.242	1.081	0.522	**0.988**	3.801	1.148	−0.339	0.998	1.594
SIN	**0.291**	1.166	−0.384	1.158	0.580	1.147	−0.445	1.057	−0.355
SWE	**1.197**	1.294	−1.508	1.169	0.102	1.111	−0.213	1.011	1.150
SWI	**0.966**	1.152	−0.790	1.229	0.525	1.176	−1.429	1.022	0.752
THA	**0.265**	1.143	0.527	1.086	0.869	1.058	0.742	1.006	1.370
TUR	**1.638**	1.256	0.685	1.078	1.930	1.190	0.396	1.040	0.478
TWN	**0.207**	1.195	0.282	1.106	1.485	1.020	1.280	1.026	−0.635

Notes: U is the MSPE of the model in question divided by the MSPE of the random walk. t_{CW} is the t-ratio for the Clark–West (2007) statistic and is significant at the 10% level if it exceeds 1.28.

If the nominal exchange rate is not weakly exogenous, the exchange rate s_{it} moves toward the PPP value $p_{it} - p_{0t}$ over time and $\beta_i > 0$. This is a bilateral model in the sense that the fundamentals $p_{it} - p_{0t}$ depend only on variables from the associated bilateral pair of countries. Exchange rate models are typically formulated in bilateral terms. Examples include monetary-based models (Mark, 1995) and Taylor Rule models augmented with the real exchange rate (Molodtsova and Papell, 2009; Molodtsova et al., 2008, 2011). We include the PPP model because Engel et al. (2007) find that it gives the most favorable results among the fundamentals models they consider.

Forecasts are generated at 1-, 12-, and 24-month horizons and for each month from 2004.1 through 2015.12. The initial rolling sample is 1999.1–2003.12 for different forecast horizons. After estimating model parameters under different horizons, the one-month forecast of 2004.1 is generated using the data at 2003.12, whereas the 24-month forecast of 2004.1 is generated using the data at 2002.1. That is, we generate the same number of forecasts for each forecasting horizon. Forecast accuracy of the alternative models is compared to predictions of the driftless random walk. Theil's U statistic, the ratio of MSPE from the model to those from the random walk, is used to assess the relative accuracy of point forecasts. To evaluate whether forecasts are statistically significantly more accurate than the random walk, we use the Clark and West (2007) test of forecast accuracy. Because the regression-based models (19) nest the random walk, their forecasts will have greater bias since there are more parameters to be estimated with the same amount of data. The Clark–West statistic makes an adjustment to the MSPE to account for the greater bias in the model.

TABLE 9
FORECASTING AT 12-MONTH HORIZON

	Random Walk	Dollar–Euro		Dollar–Carry		Principal Components		Bi-PPP	
	MSPE	U	t_{CW}	U	t_{CW}	U	t_{CW}	U	t_{CW}
AUS	20.676	0.455	3.507	0.497	3.372	**0.356**	3.727	1.000	1.893
BRA	33.094	**0.447**	5.131	0.469	5.289	0.469	5.081	0.987	1.372
CAN	9.649	0.525	3.894	0.490	4.038	**0.424**	4.121	0.900	2.234
CHI	14.120	0.506	3.920	0.540	3.698	**0.488**	4.329	1.215	−0.410
COL	21.226	0.546	3.994	0.555	4.066	**0.480**	3.950	1.033	0.784
CZE	16.581	0.485	3.890	0.516	4.394	**0.322**	5.517	0.822	2.886
EUR	10.410	0.484	4.990	0.508	5.219	**0.464**	5.220	0.986	1.984
GBR	10.009	**0.399**	2.303	0.524	2.339	0.497	2.175	1.110	1.093
HUN	18.811	0.457	4.542	0.545	4.140	**0.453**	4.226	0.947	2.181
ICE	37.972	**0.517**	2.395	0.609	2.315	0.764	1.829	1.080	1.497
IND	9.367	**0.346**	4.557	0.360	4.424	0.385	4.361	0.836	2.856
ISR	7.573	0.478	4.098	0.495	4.070	**0.443**	4.454	0.900	2.090
JPN	11.380	**0.457**	5.585	0.466	4.888	0.791	3.796	0.866	2.682
KOR	14.317	0.366	2.242	**0.365**	2.242	0.464	2.305	1.160	−0.033
MEX	11.209	**0.374**	2.778	0.503	2.379	0.682	2.552	1.005	1.125
NOR	16.354	0.443	4.137	0.480	3.818	**0.323**	3.993	0.912	2.146
NZL	20.062	0.420	3.292	0.445	3.092	**0.385**	3.259	1.094	1.398
PHI	4.877	**0.599**	3.451	0.723	3.224	0.794	2.960	1.177	0.512
POL	24.925	0.459	3.329	**0.443**	3.247	0.457	3.095	0.933	1.419
ROM	16.389	**0.309**	3.967	0.379	3.685	0.354	3.628	1.188	0.631
RSA	22.181	0.402	4.169	**0.380**	4.544	0.445	4.069	1.141	0.550
SIN	3.540	0.394	3.892	0.365	4.950	**0.347**	4.363	0.811	2.704
SWE	16.125	0.460	3.839	0.518	3.438	**0.312**	3.710	1.007	1.796
SWI	8.502	0.554	3.094	**0.512**	3.558	0.552	3.160	0.950	2.185
THA	4.041	0.551	5.241	0.617	4.820	**0.531**	5.035	0.834	2.358
TUR	20.188	**0.566**	4.703	0.584	4.623	0.651	4.418	1.243	1.281
TWN	2.206	0.479	4.361	0.501	4.371	**0.449**	3.628	1.124	0.299

NOTES: U is the MSPE of the model in question divided by the MSPE of the random walk. t_{CW} is the t-ratio for the Clark–West (2007) statistic and is significant at the 10% level if it exceeds 1.28.

To summarize, we compare the multilateral dollar–euro factor exchange rate model to the dollar–carry model, a two-principal-components model, and the bilateral PPP fundamentals model,

$$\text{Dollar–Euro:} \quad s_{it+h} - s_{it} = \alpha_i + \beta_{i1}s_{it} + \beta_{i2}\bar{s}_t^{\$} + \beta_{i3}\bar{s}_t^{\text{€}} + \beta_{i4}\bar{s}_t^{i} + \epsilon_{it+h},$$

$$\text{Dollar–Carry:} \quad s_{it+h} - s_{it} = \alpha_i + \beta_{i1}s_{it} + \beta_{i2}\bar{s}_t^{\$} + \beta_{i3}\bar{s}_{it}^{c} + \beta_{i4}\bar{s}_t^{i} + \epsilon_{it+h},$$

$$\text{PC:} \quad s_{it+h} - s_{it} = \alpha_i + \beta_{i1}\hat{F}_{1t}^{pc} + \beta_{i2}\hat{F}_{2t}^{pc} + \beta_{i3}s_{it} + \epsilon_{it+h},$$

$$\text{Bi-PPP:} \quad s_{it+h} - s_{it} = \alpha_i + \beta_i(s_{it} - (p_{it} - p_{0t})) + \epsilon_{it+h}.$$

MSPEs of the random walk and Theil's U for competing models for one-month-ahead forecasts are shown in Table 8. Bolded entries indicate the model with the lowest MSPE. For these one-month-ahead forecasts, Bi-PPP is almost as good as the random walk and does better than the three-factor models. But the bottom line is that none of the models can beat the random walk forecasts at the one-month horizon.

Forecasting results at the 12-month horizon are shown in Table 9. Here, the Bi-PPP model deteriorates badly and never dominates. The three-factor models perform significantly better than the random walk (CW > 1.28 is significant at the 10% level and CW > 1.65 is significant at the 5% level). Although there are some large differences (see Theil's U for MEX, PHI)

TABLE 10
FORECASTING AT 24-FOUR MONTH HORIZON

	Random Walk	Dollar–Euro		Dollar–Carry		Principal Components		Bi-PPP	
	MSPE	U	t_{CW}	U	t_{CW}	U	t_{CW}	U	t_{CW}
AUS	36.635	**0.227**	4.368	0.240	4.588	0.291	4.328	0.955	3.272
BRA	58.283	**0.208**	6.585	0.210	7.028	0.256	6.439	0.915	2.622
CAN	17.880	0.230	5.304	**0.206**	5.644	0.223	5.899	0.712	3.780
CHI	23.530	0.566	5.467	**0.290**	6.004	0.721	5.840	1.264	0.809
COL	30.189	**0.318**	5.184	0.411	5.420	0.460	4.928	0.969	2.084
CZE	30.290	0.201	5.023	0.215	5.215	**0.174**	4.979	0.571	4.628
EUR	17.548	**0.228**	3.404	0.233	3.452	0.239	3.877	0.970	2.685
GBR	17.536	**0.191**	4.094	0.223	4.133	0.313	3.941	1.100	2.241
HUN	28.735	**0.270**	4.813	0.281	4.949	0.316	4.235	0.913	3.135
ICE	75.657	0.226	2.439	**0.195**	2.508	0.288	2.392	0.926	2.137
IND	15.526	**0.244**	4.715	0.294	4.347	0.313	4.624	0.770	3.214
ISR	9.681	**0.349**	4.010	0.354	3.949	0.442	3.411	1.070	1.373
JPN	30.351	**0.093**	6.275	0.107	6.272	0.239	6.837	0.339	5.699
KOR	24.046	**0.246**	3.723	0.257	3.556	0.335	3.833	1.178	1.053
MEX	15.515	**0.275**	4.186	0.332	4.201	0.396	3.879	1.021	1.838
NOR	22.219	0.341	5.320	0.365	5.477	**0.329**	4.879	0.858	3.131
NZL	32.739	0.312	4.002	**0.305**	3.922	0.377	3.538	1.027	2.396
PHI	9.555	**0.182**	4.508	0.241	4.407	0.397	4.930	1.080	2.034
POL	32.642	0.358	5.175	0.402	5.302	**0.312**	5.006	0.829	2.459
ROM	25.046	**0.297**	4.452	0.343	4.722	0.380	4.394	1.355	1.626
RSA	60.925	**0.150**	4.634	0.164	4.810	0.199	4.140	1.162	1.315
SIN	7.226	**0.141**	4.780	0.164	5.430	0.207	4.456	0.630	4.862
SWE	25.852	0.228	4.746	**0.220**	4.867	0.351	4.791	1.106	1.752
SWI	15.920	0.197	4.491	0.224	4.727	**0.149**	4.978	0.855	3.625
THA	8.333	0.180	4.265	**0.168**	4.386	0.349	3.572	0.758	3.699
TUR	30.431	0.308	5.138	**0.300**	5.284	0.363	5.321	1.606	2.172
TWN	3.482	0.275	3.999	**0.208**	4.683	0.260	3.822	1.211	1.029

NOTES: U is the MSPE of the model in question divided by the MSPE of the random walk. t_{CW} is the t-ratio for the Clark–West (2007) statistic and is significant at the 10% level if it exceeds 1.28.

where the dollar–euro model performs much better, for the most part, the accuracy is similar across the three-factor models.

Table 10 shows forecasting results at the 24-month horizon. Here, the Bi-PPP model is about as accurate as the random walk, and the factor models are much more accurate. Again, differences across the factor models are not large, but the dollar–euro model has the most accurate point forecasts, as indicated with the lowest Theil's U for 15 exchange rates. The dollar–carry model is most accurate for eight exchange rates, and PCs is most accurate for four exchange rates.

As mentioned earlier, the alternative factor candidates share a good deal of common information. This is why forecasting performance across the three-factor models is similar. It is possible to forecast well even with a model that is inconsistently estimated. This is the case with the dollar–carry model if the carry is not a common factor. Hence, the forecasting exercise should not be viewed as a method to determine which candidate is the true common factor.

6.1. *Daily Forecasting.* The exchange rate conditioning information is observed daily. Here, we show how the dollar–euro model is able to forecast at daily horizons. Here, we consider forecasting with daily exchange rates for the dollar–euro model and the PC model. The daily sample, obtained from IHS Global Insight, extends from January 1, 2013 to March 25, 2016 which gives 844 time-series observations for 25 currencies.[23] Forecasts generated by 60-day rolling regression and the first date forecasted was March 25, 2013.

[23] Daily observations are not available for ICE. From September 2011 to January 2015, SWI pegged to the euro and was omitted.

TABLE 11
FORECASTING AT DAILY HORIZONS

		One-Day Ahead					Four-Weeks Ahead			
	Random Walk	Dollar–Euro		Principal Components		Random Walk	Dollar–Euro		Principal Components	
	MSPE	U	t_{CW}	U	t_{CW}	MSPE	U	t_{CW}	U	t_{CW}
AUS	**0.041**	1.118	−1.818	1.141	−1.009	0.871	**0.256**	9.344	0.259	9.125
BRA	**0.087**	1.108	−0.364	1.143	−1.441	2.366	0.245	7.504	**0.244**	7.659
CAN	**0.021**	1.107	0.654	1.142	0.771	0.555	**0.236**	8.083	0.329	7.366
CHI	**0.030**	1.115	−0.532	1.123	−1.266	0.710	0.278	10.140	**0.276**	10.189
COL	**0.057**	1.107	0.463	1.099	0.330	2.020	**0.226**	7.921	0.274	7.653
CZE	**0.037**	1.128	−1.095	1.129	−0.436	0.718	0.327	7.194	**0.312**	7.553
EUR	**0.029**	1.094	−1.179	1.154	−0.223	0.555	0.319	7.309	**0.251**	7.220
GBR	**0.020**	1.114	−1.137	1.125	−1.607	0.383	**0.253**	8.800	0.290	8.395
HUN	**0.045**	1.129	−1.056	1.117	0.912	0.712	0.319	7.632	**0.307**	6.939
IND	**0.026**	1.129	0.758	1.182	0.027	0.604	0.308	5.289	**0.304**	5.251
ISR	**0.017**	1.132	−1.213	1.087	0.341	0.321	**0.264**	8.987	0.308	8.910
JPN	**0.032**	1.115	0.004	1.064	2.442	0.705	**0.265**	6.961	0.303	6.808
KOR	**0.023**	1.111	−0.296	1.118	0.386	0.457	**0.284**	10.068	0.310	8.871
MEX	**0.034**	1.124	−0.597	1.128	−0.015	0.757	**0.264**	7.679	0.321	7.430
NOR	**0.050**	1.147	−0.409	1.156	−0.487	0.949	**0.249**	7.767	0.300	7.605
NZL	**0.048**	1.141	−0.931	1.100	0.851	0.945	**0.247**	9.487	0.276	9.708
PHI	**0.009**	1.071	2.378	1.125	−0.279	0.205	**0.303**	6.700	0.332	6.941
POL	**0.040**	1.149	−1.234	1.137	0.159	0.745	0.335	8.125	**0.317**	7.877
ROM	**0.033**	1.123	−1.211	1.131	0.741	0.609	0.304	7.662	**0.278**	7.167
RSA	**0.073**	1.185	−1.379	1.182	−1.602	1.344	0.299	8.707	**0.290**	8.007
SIN	**0.011**	1.125	−1.223	1.118	−0.308	0.200	**0.292**	11.391	0.298	10.933
SWE	**0.037**	1.124	0.411	1.132	1.330	0.615	**0.285**	8.544	0.297	8.112
THA	**0.009**	1.077	2.358	1.107	1.416	0.263	0.304	8.002	**0.271**	8.182
TUR	**0.046**	1.164	−1.105	1.168	0.050	1.122	**0.261**	9.402	0.307	8.614
TWN	**0.009**	1.089	1.116	1.124	0.582	0.150	0.265	7.810	**0.255**	7.268

NOTES: U is the MSPE of the model in question divided by the MSPE of the random walk. t_{CW} is the t-ratio for the Clark–West (2007) statistic and is significant at the 10% level if it exceeds 1.28. In some cases, regressors were perfectly collinear even after standardizing the observations. Hence, local currency factor is omitted in the forecasting regression and observations are not standardized.

Table 11 shows forecasting results at the one-day-ahead and four-week-ahead horizons. As with the monthly data, the random walk dominates one-step (one-day)-ahead forecasts in terms of MSPE, but the dollar–euro and PC models are more accurate at longer horizons. Compared to the random walk, both models are able to forecast daily exchange rates at the four-week horizon. At four weeks, the dollar–euro model dominates PCs in MSPE for 14 of 25 exchange rates. Forecasts are statistically significant with strong positive Clark–West rejections for all currencies.

7. CONCLUSION

This article studies the source of comovements across exchange rates. We identified a dollar factor and a euro factor as the pair of common empirical factors driving a panel of exchange rates. The carry return is not identified as a factor. Drawing on the SDF approach to the exchange rate, our identification can be interpreted as evidence that a global, a U.S., and a euro-zone SDF exhibit dominance in exchange rate movements. More generally, these represent global factors that have relevance for understanding asset prices in the international context. A limited exploration finds support for a risk-based interpretation of the factors. The data also reveal a geographical aspect in the way currencies load on the euro factor and a separate pattern of loading on the dollar factor by commodity currencies.

Our identification suggests empirical exchange rate modeling should incorporate multilateral dollar–euro factors. In out-of-sample forecasting, the multilateral model outperforms the random walk and the bilateral PPP fundamentals models. Forecast performance was in line with the pure statistical (PCs) factor forecasting model in terms of mean-square forecast error. The alternative multilateral model consisting of a dollar and carry factor generates similarly accurate forecasts.

The point of the forecasting analysis was not to find the best forecasting model but to demonstrate the value of identification. Instead of looking at bilateral determinants on a case-by-case basis, one implication of our identification is that empirical researchers might focus on understanding the determinants the dollar and euro factors in order to understand most of the variation in any bilateral exchange rate.

Our findings suggest future directions for research. First, macromodeling should recognize the potential importance of multicountry models for exchange rate determination. In empirical modeling, one should pay special attention to the role of the United States and the euro zones on bilateral exchange rates. Consideration of multilateral factors can potentially solve the Obstfeld and Rogoff (2000) exchange rate disconnect puzzle. New directions for international asset pricing might emphasize a heightened role for global, U.S., and euro SDFs.

APPENDIX

A.1. *Data Sources for the Carry, Dollar, and Euro Portfolios.* Interest rate differentials used for the construction of the portfolios and portfolio excess returns are based on the forward premium (log forward minus the log spot rate). End of period spot and one-month forward national currency unit per USD exchange rates were sourced from Datastream. Each spot-forward pair is selected from the same underlying data source: either Barclay's Bank (BB), WM/Reuters (WMR), Thomson Reuters (TR), or the *Taiwan Economic Journal* (TEJ). Currencies were included in the construction of the various portfolio returns over month $t-1$ to t if they had a forward rate at time $t-1$ and spot rate data at time $t-1$ and t: If these data were lacking, then the currency was excluded. We excluded Turkey from the construction of the portfolios between February and November 2001, when the quoted one-month-forward rate is fixed, whereas the spot continues to vary.

The data coverage and source for each currency are as follows: Australian dollar, December 1984–December 2015, BB; Austrian schilling, December 1996–December 1998, WMR; Belgian franc, Febraury 1985–December 1998, WMR; Brazilian real, March 2004–December 2015, WMR; Canadian dollar, December 1984–December 2015, BB; Chilean peso, March 2004–December 2015, WMR; Colombian peso, March 2004–December 2015, WMR; Czech koruna, December 1998–December 2015 WMR; Danish krona, December 1984–December 1998, BB; euro, December 1998–December 2015; Finnish markka, December 1996–December 1998, WMR; French franc, October 1983–December 1998, BB; German mark, October 1983–December 1998, BB; Greek drachma, December 1996–December 1998, WMR; Hungarian forint, December 1998–December 2015, WMR; Icelandic krona, March 2004–December 2015, WMR; Indian rupee, December 1998–December 2015, WMR; Irish pound, December 1996–December 1998, WMR; Israeli shekel, March 2004–December 2015, WMR; Italian lira, March 1984–December 2015, BB; Japanese yen, October 1983–December 2015; Korean won, February 2002–December 2015, WMR; Mexican peso, December 1998–December 2015, WMR; Dutch guilder, February 1985–December 1998, TR; Norwegian krone, December 1984–December 2015, BB; New Zealand dollar, December 1984–December 2015, BB; Philippine peso, December 1996–December 2015, WMR; Polish zloty, December 1998–December 2015, WMR; Portuguese escudo, December 1996–December 1998, WMR; Romanian leu, December 1998–December 2015; South African rand, October 1983–December 2015, BB; Singapore dollar, December 1984–December 2015, BB; Spanish peseta, December 1996–December 1998, WMR; Swedish krona, December 1984–December 2015, BB; Swiss franc, October 1983–December 2015, BB; Taiwan dollar, January 1992–December 2015, TEJ; Thai baht, December

1998–December 2015, TR; Turkish lira, December 1998–February 2001, December 2001–December 2015, WMR; UK pound, October 1983–2015, BB.

A.2. *Derivation of the Forecasting Regression.* In Section 2, we subtracted the cross-sectional averages before estimating the number of the common factors to avoid the impact of the choice of the numeraire. When other currency except for the USD and euro becomes the numeraire, the exchange rates panel must have three common factors: The USD, euro, and the numeraire currency factors. Meanwhile, when either the USD or the euro becomes the numeraire, the exchange rates panel have only two factors. The forecasting regression should not be dependent on the choice of the numeraire. For example, the forecasting regression with NZD/USD must have the same explanatory variables with the forecasting regression with USD/NZD. To take account of this difference, we need to include all three factors in the forecasting regressions always. Rewrite Equation (9) in the level

$$s_{it} = a_i + b^*_{i1}\bar{s}^{\$}_t + b_{i2}\bar{s}^{€}_t + s^o_{it}, \tag{A.1}$$

where $b^*_{i1} = b_{i1} - 1$. Next, we approximate s^o_{it} as the cross-sectional average of the depreciation rates with the ith numeraire currency. Note that

$$s^i_{jt} = s_{jt} - s_{it} = a_j + b^*_{1j}\bar{s}^{\$}_t + b_{2j}\bar{s}^{€}_t + s^o_{jt} - s^o_{it}.$$

Hence, the cross-sectional average of s^i_{jt} becomes

$$N^{-1}\sum\nolimits_{j\neq i}^{N} s^i_{jt} = \bar{a} + \bar{b}^*_1\bar{s}_t + \bar{b}_2\bar{s}^{€}_t + N^{-1}\sum\nolimits_{j\neq i}^{N} s^o_{jt} - s^o_{it}.$$

Then the idiosyncratic component, s^o_{it}, can be written as

$$s^o_{it} = N^{-1}\sum\nolimits_{j\neq i}^{N} s^i_{jt} - \bar{a} - \bar{b}^*_1\bar{s}^{\$}_t - \bar{b}_2\bar{s}^{€}_t + O_p(N^{-1/2}). \tag{A.2}$$

Plugging Equation (A.2) into (A.1) leads to

$$s_{it} = a^+_i + b^+_{i1}\bar{s}^{\$}_t + b^+_{i2}\bar{s}^{€}_t + \bar{s}^i_t + \upsilon_{it},$$

where υ_{it} is the approximation error $b^+_{i1} = b^*_{i1} - \bar{b}^*_1$, $b^+_{i2} = b_2 - \bar{b}_2$, and $a^+_i = a_i - \bar{a}$. We assume that this approximation error, υ_{it}, is stationary. Then there exists the following restrictive error correction model (ECM):

$$s_{it+1} - s_{it} = \alpha_i + \lambda_i\left(s_{it} - b^+_{i1}\bar{s}^{\$}_t - b^+_{i2}\bar{s}^{€}_t - \bar{s}^i_t\right) + \epsilon_{it+1}.$$

To provide more flexibility, we consider the following unrestricted version of the ECM for the h-period-ahead forecasts:

$$s_{it+h} - s_{it} = \alpha_i + \beta_{i1}s_{it} + \beta_{i2}\bar{s}^{\$}_t + \beta_{i3}\bar{s}^{€}_t + \beta_{i4}\bar{s}^i_t + \epsilon_{i,t+h}. \tag{A.3}$$

For some exchange rates, ε_{it} is nearly zero due to near exact multicollinearity among explanatory variables. To avoid this problem, empirical factors are standardized by the variance of their depreciation rates. Since s_{it} can be perfectly correlated with $\delta_{i1}\bar{s}^{\$}_t + \delta_{i2}\bar{s}^{€}_t + \phi_i\bar{s}^i_t$, without standardizing, the slope coefficients are not estimable in some cases. For example, $\bar{s}^{\$}_t$ in (19) is equal to $N^{-1}\sum_{i=1}^{N} s_{it}/\sqrt{V(\Delta s_{it})}$ where$V(\Delta s_{it}) = t^{-1}\sum_{\ell=1}^{t}(\Delta s_{i\ell} - t^{-1}\sum_{\ell=1}^{t}\Delta s_{i\ell})^2$. Also note that when $i =$ euro, $\bar{s}^{€}_t = \bar{s}^i_t$. Hence, we did not include the local currency factor in the case of euro/USD.

A.3. *Clark–West Test.* Interpreting MSPE as an estimator of the true (or population) MSPE of the model, Clark and West (2007) argue that this leads to greater bias in the MSPE of larger models than smaller models due to the fact that the larger model has more parameters to be estimated with the same amount of data. Clark and West (2007) therefore propose an adjusted MSPE to account for this bias. This adjustment is particularly appropriate when using out-of-sample loss as a basis for model evaluation (as it is in the current application). To test whether model a has a lower MSPE than model b we employ Clark and West's (2007) test of equal MSPEs from nested models.

The Clark and West test of the null hypothesis that $\ddot{U}_h^{(a,b)} < 1$ is based on testing whether the mean of

$$J_{ish}^{(a,b)} = \left(\hat{s}_{is+h}^{a} - s_{is+h}\right)^2 - \left(\hat{s}_{is+h}^{a} - s_{is+h}^{b}\right)^2 - P^{-1}\sum_{s=1}^{P}\left(\hat{s}_{is+h}^{b} - s_{is+h}\right)^2$$

is less than zero. Clark and West (2007) show that

$$P^{-1}\sum_{s=1}^{P} J_{ish}^{(a,b)} \Big/ \sqrt{V\left(P^{-1}\sum_{s=1}^{P} J_{ish}^{(a,b)}\right)} \overset{a}{\sim} N(0,1)$$

under the null hypothesis that $\ddot{U}_h^{(a,b)} = 1$. To estimate $V(J_{i,h}^{(a,b)})$ they suggest using the Newey–West estimator. We use the estimator with the truncation lag set to be $h-1$ since the forecast errors overlap $h-1$ periods.

REFERENCES

AHN, S. C., AND A. R. HORENSTEIN, "Eigenvalue Ratio Test for the Number of Factors," *Econometrica* 81 (2013), 1203–27.

ALOOSH, A., AND G. BEKAERT, "Currency Factors," mimeo, Columbia University, 2017.

BAI, J., AND S. NG, "Determining the Number of Factors in Approximate Factor Models," *Econometrica* 70 (2002), 181–221.

———, AND ———, "Evaluating Latent and Observed Factors in Macroeconomics and Finance," *Journal of Econometrics* 131 (2006), 507–37.

BERG, K. A., AND N. C. MARK, "Third-Country Effects on the Exchange Rate," *Journal of International Economics* 96 (2015), 227–43.

BILSON, J. F. O., "Profitability and Stability in International Currency Markets," NBER Working Papers 0664, National Bureau of Economic Research, Inc., 1981.

CAPORALE, G. M., AND M. CERRATO, "Panel Data Tests of PPP: A Critical Overview," *Applied Financial Economics* 16 (2006), 73–91.

CERRA, V., AND S. C. SAXENA, "The Monetary Model Strikes Back: Evidence from the World," *Journal of International Economics* 81 (2010), 184–96.

CHEN, Y.-C., K. S. ROGOFF, AND B. ROSSI, "Can Exchange Rates Forecast Commodity Prices?" *Quarterly Journal of Economics* 60 (2010), 133–60.

CLARK, T. E., AND K. D. WEST, "Approximately Normal Tests for Equal Predictive Accuracy in Nested Models," *Journal of Econometrics* 138 (2007), 291–311.

ENGEL, C., N. C. MARK, AND K. D. WEST, "Exchange Rate Models Aren't as Bad as You Think," *NBER Macroeconomics Annual* 22 (2007), 381–441.

———, ———, AND ———, "Factor Model Forecasts of Exchange Rates," *Econometric Reviews* 34 (2015), 32–55.

FEENSTRA, R. C., R. INKLAAR, AND M. P. TIMMER, "The Next Generation of the Penn World Table," *American Economic Review* 105 (2015), 3150–82.

FRANKEL, J. A., AND A. K. ROSE, "A Panel Project on Purchasing Power Parity: Mean Reversion within and between Countries," *Journal of International Economics* 40 (1996), 209–24.

GROEN, J. J. J., "Exchange Rate Predictability and Monetary Fundamentals in a Small Multi-Country Panel," *Journal of Money, Credit, and Banking* 37 (2005), 495–516.

HALLIN, M., AND R. LISKA, "The Generalized Dynamic Factor Model: Determining the Number of Factors," *Journal of the American Statistical Association* 102 (2007), 603–17.

INOUE, A., AND L. KILIAN, "In-Sample or Out-of Sample Tests of Predictability: Which One Should We Use?" *Econometric Reviews* 23 (2004), 371–402.

LOTHIAN, J. R., AND M. P. TAYLOR, "Real Exchange Rate Behavior: The Recent Float from the Perspective of the Past Two Centuries," *Journal of Political Economy* 104 (1996), 488–509.

LUSTIG, H., AND R. J. RICHMOND, "Gravity in FX R^2: Understanding the Factor Structure in Exchange Rates," mimeo, Stanford University, 2017.

———, N. ROUSSANOV, AND A. VERDELHAN, "Common Risk Factors in Currency Markets," *Review of Financial Studies* 24 (2011), 3731–77.

———, ———, AND ———, "Countercyclical Currency Risk Premia," *Journal of Financial Economics* 111 (2014), 527–53.

MARK, N. C., "Exchange Rates and Fundamentals: Evidence on Long-Horizon Predictability," *American Economic Review* 85 (1995), 201–18.

———, AND D. SUL, "Nominal Exchange Rates and Monetary Fundamentals: Evidence from a Small Post-Bretton Woods Panel," *Journal of International Economics* 53 (2001), 29–52.

MEESE, R. A., AND K. ROGOFF, "Empirical Exchange Rate Models of Seventies: Do They Fit Out-of-Sample?" *Journal of International Economics* 14 (1983), 3–24.

MOLODTSOVA, T., AND D. H. PAPELL, "Out-of-Sample Exchange Rate Predictability with Taylor Rule Fundamentals," *Journal of International Economics* 77 (2009), 67–180.

———, A. NIKOLSKO-RZHEVSKY, AND D. H. PAPELL, "Taylor Rules with Real-Time Data: A Tale of Two Countries and One Exchange Rate," *Journal of Monetary Economics* 55 (2008), 563–79.

———, ———, AND ———, "Taylor Rules and the Euro," *Journal of Money, Credit, and Banking* 43 (2011), 535–52.

OBSTFELD, M., AND K. ROGOFF, "The Six Major Puzzles in International Macroeconomics: Is There a Common Cause?" *NBER Macroeconomics Annual* 15 (2000), 339–412.

O'CONNELL, P. G. J., "The Overvaluation of Purchasing Power Parity," *Journal of International Economics* 44 (1988), 1–19.

ONATSKI, A., "Testing Hypotheses about the Number of Factors in Large Factor Models," *Econometrica* 77 (2009), 1447–79.

———, "Determining the Number of Factors from Empirical Distribution of Eigenvalues," *Review of Economics and Statistics* 92 (2010), 1004–16.

PARKER, J., AND D. SUL, "Identification of Unknown Common Factors: Leaders and Followers," *Journal of Business and Economics Statistics* 34 (2016), 227–39.

RAPACH, D. E., AND M. E. WOHAR, "Testing the Monetary Model of Exchange Rate Determination: New Evidence from a Century of Data," *Journal of International Economics* 58 (2001), 359–85.

———, AND ———, "Testing the Monetary Model of Exchange Rate Determination: A Closer Look at Panels," *Journal of International Money and Finance* 23 (2004), 867–95.

SARNO, L., AND M. SCHMELING, "Which Fundamentals Drive Exchange Rates? A Cross-Sectional Perspective," mimeo, City University London, 2013.

VERDELHAN, A., "The Share of Systematic Variation in Bilateral Exchange Rates," *Journal of Finance* 73 (2018), 375–418.