UNDERSTANDING SPOT AND FORWARD EXCHANGE RATE REGRESSIONS

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SUMMARY

Using the Kalman filter, we obtain maximum likelihood estimates of a permanent–transitory components model for log spot and forward dollar prices of the pound, the franc, and the yen. This simple parametric model is useful in understanding why the forward rate may be an unbiased predictor of the future spot rate even though an increase in the forward premium predicts a dollar appreciation. Our estimates of the expected excess return on short-term dollar-denominated assets are persistent and reasonable in magnitude. They also exhibit sign fluctuations and negative covariance with the estimated expected depreciation. © 1997 John Wiley & Sons, Ltd.

1. INTRODUCTION

In this paper we formulate, estimate, and study simple parametric models of log spot and forward exchange rates that combine permanent and transitory dynamics. The aims of the research are first, to improve our understanding of some puzzling features of the foreign exchange market, and second, to generate plausible estimates of the unobserved expected excess return on short-term US dollar-denominated debt. These expected excess returns are commonly referred to in the international finance literature as deviations from uncovered interest parity, expected profits from forward exchange speculation, or as the forward foreign exchange risk premium.[1]

Our investigation is guided by two stylized facts. First, log spot and forward exchange rates appear to be cointegrated with cointegration vector $(1, -1)$. This in turn implies that the slope coefficient in a regression of the k-period-ahead log spot rate on the log k-period-forward rate is 1 and that the forward premium is I(0).[2] Second, regressions of the future depreciation on the current forward premium yield negative estimates of the slope coefficient. These two facts are puzzling because according to the cointegrating or 'levels' regressions, the forward rate is an

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[1] The equivalence between the deviation from uncovered interest parity and the expected profit from forward foreign exchange speculation follows from the covered interest parity condition. We remain agnostic as to whether this excess return is a risk premium.
[2] We use the standard notation I(d) to denote that a time-series is dth order integrated and requires differencing d times to induce stationarity.

unbiased predictor of the future spot rate while at the same time, the forward premium predicts the future depreciation with the wrong negative sign.

The 'levels' regressions were originally fitted by researchers such as Bilson (1981), Cornell (1977), and Frankel (1981), who were interested in testing the efficient market hypothesis — that the forward rate is the optimal predictor of the future spot rate under risk neutrality.[3] Although these early studies employed standard least-squares procedures, we demonstrate below that the hypothesis that the slope coefficient is 1 cannot be rejected when appropriate cointegrating regression estimation is employed. But when the cointegrating regressions are transformed by subtracting the current log spot rate from both the regressor and the regressand, the resulting slope coefficient in regressions of the future depreciation on the forward premium are typically negative. This anomalous result was first reported in the literature by Cumby and Obstfeld (1984) and Fama (1984). Fama attributes these findings to the presence of a time-varying expected excess currency return that is negatively correlated with and is more volatile than the expected depreciation.[4]

We show that these two features of the data can be accounted for by a simple parametric permanent–transitory components model for spot and forward exchange rates. The two-component specification draws its motivation from Mussa's (1982) sticky-price model in which the exchange rate is represented by a fundamental value and a transient disequilibrium term. We model the fundamental value by a stochastic trend that evolves as a driftless random walk that is common to both spot and forward rates. The temporary part, which measures the short-run disequilibrium of the economy, is represented by a vector ARMA process. The model is estimated by maximum-likelihood using the Kalman filter for monthly observations on bilateral exchange rates between the US dollar and the pound, the French franc, and the yen from 1976:1 to 1992:8. In addition to commonly employed diagnostic tests on residuals, we also gauge the adequacy of the model by its ability to account for various functions of the data that are not explicitly imposed in estimation. The accepted model is then used to generate estimates of the unobserved expected currency return and the expected depreciation.[5] The resulting estimates of the excess return are reasonable in magnitude, persistent, display sign fluctuations, and covary negatively with the estimated expected depreciation.

The remainder of the paper is organized as follows. The next section reviews the stylized facts that form the focus of the paper. Section 3 discusses the permanent–transitory components model that we use. The results of the maximum likelihood estimation are reported in Section 4. Section 5 discusses the simulation methodology we employ to supplement standard diagnostic tests in evaluating the model. Section 6 presents estimates of the expected excess foreign exchange return and the expected depreciation. Finally, some conclusions are contained in Section 7.

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[3] See Hodrick (1987) and Boothe and Longworth (1986), for surveys of this literature. Engel (1984) and Frankel and Razin (1980) point out that it is the real forward rate that is the optimal predictor of the real future spot rate under risk neutrality. Empirical studies have shown that it makes little difference whether real or nominal rates are used. Accordingly, our analysis employs only nominal exchange rates.

[4] Recent attempts to understand these aspects of the data include Hodrick and Srivastava's (1986) demonstration that the negative correlation between the forward premium and the future depreciation is possible in Lucas's (1982) two-country model, Backus, Gregory, and Telmer's (1993) calibrations of that model, Froot and Frankel's (1989) argument for expectational errors, McCallum's (1994) policy reaction model, and Evans and Lewis's (1992) 'peso problem' model.

[5] An alternative strategy would be to employ the simulated method of moments estimator of Duffie and Singleton (1993) in which we would be assured of choosing parameter values that would match, as closely as possible, the sample moments of the data that we are interested in studying. A potential difficulty, however, is that the estimates may be sensitive to the moment conditions used in estimation. We hope to avoid these issues of robustness by employing maximum likelihood estimation.

2. REVIEW OF SPOT AND FORWARD RATE BEHAVIOUR

To fix the notation that we use, let s_t and $f_{k,t}$ denote the spot and k-period-forward dollar price of foreign currency in logarithmic form. The observations are multiplied by 100 so that returns are expressed in per cent. Also, let Δ_k be the k-period difference operator ($\Delta_k s_{t+k} \equiv s_{t+k} - s_t$ with $\Delta_1 \equiv \Delta$), $r_{k,t} \equiv f_{k,t} - E_t s_{t+k}$ be the expected excess return from forward exchange speculation, and $f_{k,t}^p \equiv f_{k,t} - s_t$ be the k-period-forward premium. Following Hansen and Hodrick (1983), our sample begins on 1976:1 after the Rambouillet Conference and extends to 1992:8. These data are taken from the Harris Bank *Weekly Review*, and are drawn from those Fridays occurring nearest the end of the calendar month.

This section documents the presence of the following two stylized facts in our data set. First, spot and forward exchange rates appear to be cointegrated with cointegration vector $(1, -1)$, and second, the slope coefficient in regressions of the future depreciation on the forward premium are negative and significantly less than 1.

2.1. Spot and Forward Rate Cointegration

Since it is generally accepted in the literature that both spot and forward exchange rates are I(1), we dispense with unit-root tests for these data and begin by performing augmented Dickey–Fuller (1979) (ADF) and Phillips–Perron (1988) (PP) tests on the OLS residuals from regressing the k-period-ahead spot rate on the k-period-forward rate,

$$s_{t+k} = \alpha_0 + \beta_0 f_{k,t} + u_{k,t} \tag{1}$$

for $k = 1, 3$.[6] The lag length for the ADF test is chosen optimally following Campbell and Perron (1991), while for the PP test it is fixed at 6. The results of these tests are reported in Table I. Using Engle and Yoo's (1987) 10%, 5%, and 1% critical values of $-3\cdot02$, $-3\cdot37$, and $-4\cdot00$, respectively, the null hypothesis that these spot and forward rates are not cointegrated is easily rejected at conventional significance levels at both monthly and quarterly horizons for all three currencies.

Research on foreign-exchange market efficiency was originally pursued by estimating β_0 by OLS and testing the hypothesis that $\beta_0 = 1$ with standard OLS t-ratios (e.g. Bilson, 1981; Cornell, 1977; Frankel, 1981, and others). As in these early studies, our estimated slope coefficients in Table I are close to 1, suggesting that the forward rate may be an unbiased predictor of the future

Table I. Cointegration tests

Currency	$k = 1$			$k = 3$		
	$\hat{\beta}_0$	$\tau(\text{ADF})$	$\tau(\text{PP})$	$\hat{\beta}_0$	$\tau(\text{ADF})$	$\tau(\text{PP})$
Pound	0·975	−12·594	−12·880	0·911	−3·670	−5·275
Franc	0·987	−7·963	−13·693	0·953	−3·342	−4·949
Yen	0·994	−5·325	−12·547	0·975	−3·224	−5·163

$\hat{\beta}_0$ is the OLS slope-coefficient estimate from the regression, $s_{t+k} = \alpha + \beta_0 f_{k,t} + u_{k,t}$ for $k = 1, 3$. $\tau(\text{ADF})$ and $\tau(\text{PP})$ are Studentized coefficients for the augmented Dickey–Fuller and the Phillips–Perron tests, respectively, that $\{u_{k,t}\}$ has a unit root.

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[6] See, for example, Baillie and Bollerslev (1989), Liu and Maddala (1992), Mark (1990), or Clarida and Taylor (1993).

Table II. Cointegrating regressions

Currency	DOLS			DGLS		
	$\hat{\beta}_0$	$t(\hat{\beta}_0)$	m.s.l.	$\hat{\beta}_0$	$t(\hat{\beta}_0)$	m.s.l.
$k = 1$						
Pound	0·997	−1·533	0·125	0·997	−0·908	0·364
Franc	0·999	−0·828	0·408	0·998	−0·719	0·472
Yen	1·003	1·947	0·052	1·001	0·676	0·500
$k = 3$						
Pound	0·992	−1·455	0·146	0·988	−1·315	0·189
Franc	0·993	−1·439	0·150	0·992	−1·200	0·230
Yen	1·010	2·186	0·029	1·003	0·519	0·604

Estimates of cointegrating regression coefficient, $s_{t+k} = \alpha_0 + \beta_0 f_{k,t} + u_{k,t}$ using Stock and Watson's method with six leads and lags. $t(\beta_0)$ is the asymptotic t-statistics for the test of the hypothesis $\beta_0 = 1$. Marginal significance levels (m.s.l.) are for a two-tailed test and are computed from the t-ratio's asymptotic standard normal distribution.

spot rate. But as is well known, when $\{s_{t+k}\}$ and $\{f_{k,t}\}$ are cointegrated, OLS suffers from a second-order asymptotic bias, and its t-ratio is not asymptotically standard normal. At issue is whether the unbiasedness hypothesis survives when the correct distribution theory is used to conduct inference.

To answer this question, we estimate β_0 with Stock and Watson's (1993) dynamic OLS (DOLS) and dynamic GLS (DGLS) cointegration vector estimators. The results are reported in Table II. As can be seen, the point estimates are qualitatively close to 1 and the unbiasedness hypothesis generally cannot be rejected at the 5% level. The lone exception comes from the $k = 3$ DOLS regression for the yen where the point estimate is 1·01 and is statistically (if not economically) significantly different from 1.

While the residual based tests are able to reject the hypothesis of no cointegration, it is also of interest to examine whether the forward premium contains a unit root. Here, imposing rather than estimating the cointegration vector yields tests with higher power. For our 200 observations, the 5% and 10% critical values for the ADF and PP tests obtained from Fuller (1976, p. 373) are −2·88 and −2·57, respectively. We also employ the DF–GLS$^\mu$ test proposed by Elliott, Rothenberg, and Stock (1996), who show that this test has high power and low size distortion relative to the ADF and PP tests. The DF–GLS$^\mu$ test has the same distribution as the ADF test without mean and its 5% and 10% critical values, also from Fuller, are −1·95 and −1·62, respectively. The results, reported in Table III, provide strong evidence against the hypothesis that the forward premium contains a unit root. All three tests reject the hypothesis that the forward premium is I(1) at the 5% level for the pound and the franc for both monthly and quarterly horizons. The evidence for the yen is only slightly weaker where the null hypothesis can be rejected only at the 10% level.

Finally, we provide an alternative summary measure of persistence by computing confidence intervals for the largest autoregressive root of the forward premium. In Table IV we report the implied largest autoregressive root (ρ), as well as the lower (ρ_l) and upper (ρ_u) 95% confidence bands and the medium value (ρ_m), obtained using Table A.1 of Stock (1991). As can be seen, the forward premia in general are quite persistent. The estimates of the largest autoregressive root

Table III. Unit root tests on forward premia

Currency	$k = 1$			$k = 3$		
	τ(ADF)	τ(PP)	DF–GLS$^\mu$	τ(ADF)	τ(PP)	DF–GLS$^\mu$
Pound	−3·352	−4·694	−2·361	−2·924	−2·888	−2·358
Franc	−2·995	−6·238	−2·612	−3·770	−4·628	−2·588
Yen	−2·674	−2·929	−1·867	−3·150	−2·691	−1·797

Augmented Dickey–Fuller τ(ADF), Phillips–Perron τ(PP) and the DF–GLS$^\mu$ statistics to test the hypothesis that $f^p_{k,t}$ contains a unit root. The lag length for the ADF regressions is chosen optimally following Campbell and Perron (1991), while for the PP it is fixed at 6.

Table IV. Implied largest root of forward premia (95% confidence interval (ρ_l, ρ_u) and median estimate ρ_m)

	Largest root	ρ_l	ρ_m	ρ_u
$k = 1$				
Pound	0·812	0·829	0·900	0·976
Franc	0·798	0·861	0·926	1·004
Yen	0·923	0·883	0·943	1·009
$k = 3$				
Pound	0·926	0·868	0·931	1·005
Franc	0·825	0·813	0·872	0·953
Yen	0·923	0·845	0·913	0·992

ranges from 0·798 for the franc at $k = 1$ to 0·926 for the pound at $k = 3$ while the median estimates range from 0·872 for the franc at $k = 3$ to 0·943 for the yen at $k = 1$. The estimated 95% confidence bands are relatively large, and contain the value 1 for three of the series.

The weight of the evidence supports the view that spot and forward exchange rates are cointegrated with a cointegration vector (1, −1).[7] But our heavy reliance on unit root tests deserves a word of caution since authors such as Blough (1992), Cochrane (1991), and Faust (1993) have argued that the near observational equivalence between I(0) and I(1) processes in finite samples render generic unit root tests powerless to discriminate between the two. We face potential pitfalls in falsely assuming the presence of a unit root because the estimators employed may be biased. On the other hand, using the distribution theory for stationary time series when the observations are highly persistent typically leads to an understatement of the standard errors.

2.2. Regressing the Depreciation Rate on the Forward Premium

Prior to the advent of cointegrating regression estimation, concern that non-stationary spot and forward rates would lead to the wrong inferences in OLS regressions of equation (1) led some

[7] Evans and Lewis (1992) argue that the forward premium is I(1), but that the I(1) component is small and not detectable with standard unit root test procedures with data from the post-float era.

Table V. Forward premium regressions (OLS estimates of $\Delta_k s_{t+k} = \alpha_1 + \beta_1 f^p_{k,t} + v_{k,t}$)

Currency	$\hat{\alpha}_1$	(s.e.)	t-ratio $H_0 : \alpha_1 = 0$	$\hat{\beta}_1$	(s.e.)	t-ratio $H_0 : \beta_1 = 0$	t-ratio $H_0 : \beta_1 = 1$
$k = 1$							
Pound	−0·003	0·003	−1·038	−1·440	0·713	−2·020	−3·423
Franc	−0·002	0·003	−0·691	−0·766	0·729	−1·050	−2·422
Yen	0·011	0·003	3·416	−2·477	0·836	−2·965	−4·162
$k = 3$							
Pound	−0·016	0·008	−2·030	−2·367	0·895	−2·645	−3·763
Franc	−0·003	0·012	−0·259	−0·284	1·104	−0·257	−1·162
Yen	0·032	0·009	3·449	−2·398	0·613	−3·913	−5·544

investigators of foreign-exchange market efficiency to induce stationarity by transforming the data. For example, Cumby and Obstfeld (1984) and Fama (1984) regressed the future deprecia-tion on the forward premium,

$$\Delta_k s_{t+k} = \alpha_1 + \beta_1 f^p_{k,t} + v_{k,t} \qquad (2)$$

But instead of finding $\hat{\beta}_1 = 1$, these researchers obtained estimates that were negative. Table V reports our own estimates of this equation where we obtain estimated slope coefficients that are negative for each currency and, with the exception of the $k = 3$ regression for the franc, significantly less than 1.[8] Moreover, a left-tailed test rejects the hypothesis $\beta_1 = 0$ for the pound and the yen at both $k = 1$ and 3. These results are puzzling because if the forward exchange rate is an unbiased and optimal predictor of the future spot rate, both β_0 and β_1 should be 1. The forward premium thus appears to help predict future changes in the spot rate but enters with the 'wrong' sign.

The statistical explanation for this result is that the error term in equation (2) is correlated with the forward premium. Fama (1984) develops intuition for this result along the following lines. Let the nominal interest rate on k-period dollar- and foreign currency-denominated debt be $i_{k,t}$ and $i^*_{k,t}$, respectively. By covered interest arbitrage $f^p_{k,t} = i_{k,t} - i^*_{k,t}$, the expected excess return from forward speculation is equal to the excess return on dollar-denominated assets, or equiva-lently, the deviation from uncovered interest parity, $r_{k,t} = (i_{k,t} - i^*_{k,t}) - (E_t \Delta_k s_{t+k})$. Now letting $\delta_{t+k} \equiv s_{t+k} - E_t s_{t+k}$ denote the rational expectations forecast error, the k-period-ahead spot rate and the k-period-forward rate are seen to be related by

$$s_{t+k} = f_{k,t} + (\delta_{t+k} - r_{k,t}) \qquad (3)$$

Cointegrating regressions produce slope coefficient estimates near 1 because the bias induced from the correlation of the I(0) variable $r_{k,t}$ with the I(1) variable $f_{k,t}$ is of second order in importance and vanishes asymptotically. But subtracting s_t from both sides of equation (3) yields

$$\Delta_k s_{t+k} = f^p_{k,t} + e_{k,t} \qquad (4)$$

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[8] For $k = 3$, MA(2) serial correlation is induced into the regression error but does not affect the consistency of OLS. We use Newey and West (1987) with the truncation lag of the Bartlett window set to 15 to estimate consistent standard errors.

where $e_{k,t} \equiv \delta_{t+k} - r_{k,t}$. The omitted variables bias is seen to remain in equation (4) because both $r_{k,t}$ and $f^p_{k,t}$ are I(0).

Using the above relations, Fama showed that the slope coefficient β_1 can be decomposed as

$$\begin{aligned} \beta_1 &= \frac{\text{Cov}[\Delta_k s_{t+k}, f^p_{k,t}]}{\text{Var}[f^p_{k,t}]} \\ &= \frac{\text{Var}[E_t(\Delta_k s_{t+k})] + \text{Cov}[r_{k,t}, E_t(\Delta_k s_{t+k})]}{\text{Var}[r_{k,t}] + \text{Var}[E_t(\Delta_k s_{t+k})] + 2\text{Cov}[r_{k,t}, E_t(\Delta_k s_{t+k})]} \end{aligned} \tag{5}$$

Negative values of β_1 thus imply that $\text{Cov}[r_{k,t}, E_t(\Delta_k s_{t+k})] < 0$ and is larger in absolute value than $\text{Var}[E_t(\Delta_k s_{t+k})]$. Furthermore,

$$1 - \beta_1 = \frac{\text{Var}[r_{k,t}] + \text{Cov}[r_{k,t}, E_t(\Delta_k s_{t+k})]}{\text{Var}[r_{k,t}] + \text{Var}[E_t(\Delta_k s_{t+k})] + 2\text{Cov}[r_{k,t}, E_t(\Delta_k s_{t+k})]} > 1$$

which implies that $\text{Var}[r_{k,t}] > \text{Var}[E_t(\Delta_k s_{t+k})]$.

Thus, Fama showed that negative estimates of β_1 imply that the expected excess currency return is both negatively correlated with and more volatile than the expected depreciation. Since we are interested in obtaining credible estimates of $r_{k,t}$ that exhibit these properties we need a model that accurately represents the time-series behaviour of spot and forward rates. We now turn to developing such a model.

3. A MODEL OF SPOT AND FORWARD RATES

We draw on Mussa's (1982) stochastic generalization of the well-known Dornbusch (1976) exchange-rate overshooting model to motivate our empirical work.[9] In the Mussa model, the operation of frictionless asset markets combined with sluggish commodity price adjustments leads to the two-component representation for the exchange rate

$$s_t = z_t + \theta \zeta_t \tag{6}$$

The first component, z_t, is the implied value of the exchange rate in the absence of nominal rigidities and can be thought of as the 'fundamental' or 'long-run equilibrium' value of the exchange rate. It can be shown that z_t is the expected present value of future realizations of the model's economic fundamentals—differentials in domestic and foreign money stocks, income, and aggregate demand shocks. The macroeconomic fundamentals are standard and are similar to those implied by other popular theories.[10] Since these variables are in turn likely to be I(1), z_t is modelled as the permanent component of the exchange rate.

In the second component, ζ_t measures the state of disequilibrium in the goods market and θ is the inverse of the economy's speed of adjustment coefficient which depends on other parameters

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[9] We emphasize Mussa's model over the more familiar Dornbusch (1976) model because the fundamental value in the exchange rate evolves stochastically, whereas in Dornbusch's model it is constant.

[10] For example, the monetary approach of Bilson (1978), Frankel (1976), and Mussa (1976), and the complete markets general equilibrium model of Lucas (1982).

of the model.[11] An attractive feature of this model over equilibrium theories is its explicit provision of a theory for the transitory but persistent deviations from the fundamental value.

The forward exchange rate is assumed to be determined by traders who eliminate covered interest arbitrage profits by equating the forward premium to the interest differential. To comply with the evidence on cointegration of Section 2, we require that spot and forward exchange rates be driven by a common random walk. Suppressing the horizon subscript k to simplify the notation, the forgoing considerations lead us to the empirical specification for the spot and forward exchange rate:

$$s_t = z_t + x_{s,t} \tag{7}$$

$$f_t = z_t + x_{f,t} \tag{8}$$

$$z_t = z_{t-1} + \varepsilon_{z,t} \tag{9}$$

where $\{\varepsilon_{z,t}\} \overset{\text{i.i.d.}}{\sim} N(0, \sigma_z^2)$, $\{(x_{s,t}, x_{f,t})'\}$ is a stationary bivariate stochastic process, $x_{s,t} = \theta \zeta_t$, and $f_t^p = x_{f,t} - x_{s,t}$.

The theory imposes no restrictions on the behaviour of $(x_{s,t}, x_{f,t})$ beyond being I(0). To strike a balance between flexibility and model parsimony, we represent these transitory deviations from the fundamental values by a vector ARMA process:

$$\begin{pmatrix} \phi_{ss}(L) & \phi_{sf}(L) \\ \phi_{fs}(L) & \phi_{ff}(L) \end{pmatrix} \begin{pmatrix} x_{s,t} \\ x_{f,t} \end{pmatrix} = \begin{pmatrix} c_s \\ c_f \end{pmatrix} + \begin{pmatrix} \theta_{ss}(L) & \theta_{sf}(L) \\ \theta_{fs}(L) & \theta_{ff}(L) \end{pmatrix} \begin{pmatrix} \varepsilon_{s,t} \\ \varepsilon_{f,t} \end{pmatrix} \tag{10}$$

with

$$\begin{pmatrix} \varepsilon_{s,t} \\ \varepsilon_{f,t} \end{pmatrix} \overset{\text{i.i.d.}}{\sim} N\left(\begin{bmatrix} 0 \\ 0 \end{bmatrix} \quad \begin{bmatrix} \sigma_s^2 & \rho_{sf}\sigma_s\sigma_f \\ \rho_{sf}\sigma_s\sigma_f & \sigma_f^2 \end{bmatrix} \right) \tag{11}$$

where c_s and c_f are constants and the $\phi(L)$'s and $\theta(L)$'s are polynomials in the lag operator, L.

3.1. An Example with AR(1) Transient Dynamics

We can gain some insight into the model's ability to account for the data by examining the special case where the transient components follow univariate AR(1) processes with contemporaneously correlated innovations — a simplification that allows interpretation of the analytic formulae. We present the considerably simpler formulae for $k = 1$ but note that the intuition carries over to $k = 3$ as well. Now proceed by setting $\phi_{ss}(L) = 1 - \phi_s L$, $\phi_{ff}(L) = 1 - \phi_f L$, $\theta_{ss}(L) = \phi_{ff}(L) = 1$, and $\phi_{fs}(L) = \phi_{sf}(L) = \theta_{fs}(L) = \theta_{sf}(L) = 0$ in equation (10). Also, to lighten the notational burden, let

$$\gamma = \frac{\sigma_{fs}}{\sigma_s^2} = \rho_{sf}\frac{\sigma_f}{\sigma_s}$$

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[11] The long-horizon regressions of Mark (1995) exploited the idea that deviations of the spot rate from its equilibrium value provide useful information for predicting future exchange rate movements. The two-component model has also been used to describe the evolution of stock prices where the random walk represents the rationally expected present value of future dividends (the fundamentals solution), and the deviation represents price 'fads'. See, for example, Summers (1986), Fama and French (1988), Campbell and Shiller (1988).

The random-walk-AR(1) model implies the following moments:

$$\text{Cov}(\Delta s_{t+1}, f_t^p) = \sigma_s^2(\phi_s - 1)\left(\frac{\gamma}{1 - \phi_f\phi_s} - \frac{1}{1 - \phi_s^2}\right) \tag{12}$$

$$\text{Var}(f_t^p) = \left(\frac{\sigma_f^2}{1 - \phi_f^2} - \frac{\sigma_{fs}}{1 - \phi_s\phi_f}\right) + \left(\frac{\sigma_s^2}{1 - \phi_s^2} - \frac{\sigma_{fs}}{1 - \phi_s\phi_f}\right) \tag{13}$$

$$\text{Cov}(f_t^p, f_{t-1}^p) = \phi_f\left(\frac{\sigma_f^2}{1 - \phi_f^2} - \frac{\sigma_{fs}}{1 - \phi_s\phi_f}\right) + \phi_s\left(\frac{\sigma_s^2}{1 - \phi_s^2} - \frac{\sigma_{fs}}{1 - \phi_s\phi_f}\right) \tag{14}$$

$$\text{Var}(E_t\Delta s_{t+1}) = \sigma_s^2\left(\frac{(1 - \phi_s)^2}{1 - \phi_s^2}\right) \tag{15}$$

$$\text{Var}(r_t) = \frac{\sigma_f^2}{1 - \phi_f^2} + \sigma_s^2\left(\frac{\phi_s^2}{1 - \phi_s^2} - \frac{2\phi_s\gamma}{1 - \phi_f\phi_s}\right) \tag{16}$$

and

$$\text{Cov}(E_t\Delta s_{t+1}, r_t) = \sigma_s^2(\phi_s - 1)\left(\frac{\gamma}{1 - \phi_f\phi_s} - \frac{\phi_s}{1 - \phi_s^2}\right) \tag{17}$$

The ratio of expressions (12) and (13) gives the population value of β_1. If the transitory components are positively autocorrelated, the last grouped term in equation (12) must be positive to conform with the observation that $\beta_1 < 1$. This in turn requires the transitory component of the forward rate to be more persistent ($\phi_f > \phi_s$) or its innovation to be more volatile ($\sigma_f > \sigma_s$) than that of the spot rate. From equation (17) we see that satisfaction of these conditions implies that the expected excess return will covary negatively with the expected depreciation.

From equation (14) the first-order autocovariance of the forward premium is seen to depend on the forward premium variance weighted by the autoregressive parameters. Persistence in the transitory components clearly induces persistence in the forward premium. Equations (15) and (16) suggest why expected excess currency returns may be more volatile than the expected depreciation. The variance of the expected depreciation in equation (15) has a limiting value of 0 as the autoregressive parameter ϕ_s goes to 1 while the variance of the expected excess return in equation (16) has a limiting value of $2\sigma_f^2(1 + \gamma)$ as both ϕ_f and ϕ_s approach 1.

The formulae show that $\beta_1 = 1$ is a very special case. A particular set of restrictions that produce this result is for both spot and forward rates to be generated by a random walk plus noise where the noise terms have contemporaneous correlation equal to the ratio of their standard deviations ($\phi_s = \phi_f = 0$ and $\sigma_{fs} = \sigma_{2f}^2$). This implies that the expected excess return will evolve as an i.i.d. process with variance σ_f^2 and its covariance with the expected depreciation will be $-\sigma_f^2$.

4. MAXIMUM LIKELIHOOD ESTIMATES

Using the Kalman filter, we estimate the two-component model for spot and forward exchange rates by maximum likelihood.[12] While the AR(1) model discussed above is instructive, estimation results for that model proved to be unsatisfactory as the point estimates implied a positive covariance between the forward premium and the future depreciation. To enrich the transitory dynamics, we consider the vector ARMA(1, 1) process:

$$\begin{pmatrix} 1 - \phi_{ss}L & -\phi_{sf}L \\ -\phi_{fs}L & 1 - \phi_{ff}L \end{pmatrix} \begin{pmatrix} x_{s,t} \\ x_{f,t} \end{pmatrix} = \begin{pmatrix} c_s \\ c_f \end{pmatrix} + \begin{pmatrix} 1 + \theta_{ss}L & \theta_{sf}L \\ \theta_{fs}L & 1 + \theta_{ff}L \end{pmatrix} \begin{pmatrix} \varepsilon_{s,t} \\ \varepsilon_{f,t} \end{pmatrix} \tag{18}$$

where the innovation vector is normally and independently distributed as in equation (11). Table VI reports the maximum likelihood estimates and asymptotic standard errors for this model. The top panel reports estimates from the spot and 1-month-forward rate systems and the bottom panel shows estimates from the spot and 3-month-forward rate systems.

To check on the adequacy of the specification, we perform the Ljung and Box (1978) portmanteau test applied to the vector ARMA model, as proposed in Lutkepohl (1993, p. 300). The test statistic, denoted by $Q(p)$, is computed using the sample autocorrelation matrix of the model residuals, where p is the number of residual sample autocorrelations used. Under the null hypothesis that the model is correctly specified, $Q(p)$ has an asymptotic χ^2-distribution, with the degrees of freedom equal to n^2p minus the number of estimated coefficients in the vector ARMA, where n is the number of equations. We report $Q(12)$ and $Q(24)$ in Table VI along with their associated p-values. We see that the null hypothesis cannot be rejected at the 5% level for the pound and the yen, in both the 1-month and 3-month systems. Test results for the franc are mixed in that the $Q(12)$ statistic rejects the null hypothesis at the 5% level, while $Q(24)$ does not reject the null at conventional significance levels. Overall these results seem to suggest that our model is reasonably well specified.[13]

The asymptotic standard errors are generally small relative to the point estimates, suggesting that the parameters are precisely estimated. But due to the persistence of the transitory parts, these results should probably be viewed with some caution as the asymptotic standard errors may understate the true sampling variability. The estimates also indicate that exchange-rate variability is dominated by the random walk component.[14] The sample standard deviations of percentage changes in the pound, franc, and yen rates are 3·36, 3·28, and 3·42, respectively, while the estimated standard deviation of the random walk innovation for these currencies in the 1-month system are 3·13, 3·09, and 2·80. The estimates are consistent with the common failure in forecasting studies to outperform the random walk (e.g. Diebold and Nason, 1990; Engel, 1994) because exchange rate dynamics are dominated by unpredictable changes in the permanent component. These results may be viewed as an indictment of the failure of macroeconomic models to explain the exchange rate to the extent that the innovation variance of the permanent

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[12] A full description of the estimation strategy can be found in the working paper version of this paper, which is available from the authors upon request.

[13] The Monte Carlo simulations of Kwan and Wu (1996) showed that many portmanteau tests for univariate time series, including the Ljung and Box test, have large size distortion when p is chosen to be small. We note that the multiple-time-series version of our residual diagnostic tests are somewhat sensitive to the choice p, but because the finite-sample properties of the test are unknown we choose not to rely exclusively on these results but to combine them with the simulation results below in assessing the adequacy of the specification.

[14] Campbell and Clarida (1987) also use the Kalman filter and find that exchange rate movements are dominated by the random walk component.

Table VI. Maximum likelihood estimates of the trend–VARMA(1, 1) model

c	Φ		Θ		σ_s	σ_f	ρ_{sf}	σ_z
Spot and one-month-forward exchange rates								
Pound log likelihood = −501·85								
0·0827	0·9020	−0·0525	0·4598	0·1204	0·9700	0·8828	0·9818	3·1303
(0·0041)	(0·0030)	(0·1197)	(0·0400)	(0·1732)	(0·0173)	(0·0133)	(0·0021)	(0·0038)
0·0338	0·1386	0·7141	0·3131	0·3127	Q(12) = 52·609 p-value = 0·087			
(0·0042)	(0·0281)	(0·1197)	(0·0382)	(0·0113)	Q(24) = 78·120 p-value = 0·765			
Franc log likelihood = −515·44								
0·0405	0·9380	−0·0119	0·0765	0·3380	0·9519	0·7590	0·9999	3·0852
(0·0016)	(0·0027)	(0·2217)	(0·0118)	(0·0037)	(0·0036)	(0·0051)	(0·0000)	(0·0017)
−0·0353	0·3032	0·6513	−0·1447	0·6998	Q(12) = 56·890 p-value = 0·040			
(0·0016)	(0·0160)	(0·0138)	(0·0020)	(0·0018)	Q(24) = 81·593 p-value = 0·672			
Yen log likelihood = −388·22								
−0·0297	0·8436	0·0816	0·1346	0·1695	1·9188	1·9233	0·9986	2·8017
(0·0042)	(0·0216)	(0·1236)	(0·0090)	(0·0155)	(0·0069)	(0·0053)	(0·0001)	(0·0056)
0·0340	0·0703	0·8635	0·1260	0·1677	Q(12) = 46·880 p-value = 0·211			
(0·0043)	(0·1679)	(0·0153)	(0·0226)	(0·0537)	Q(24) = 102·384 p-value = 0·140			
Spot and three-month-forward exchange rates								
Pound log likelihood = −592·26								
0·1816	0·9998	−0·0808	0·4254	0·1884	1·0716	1·0434	0·9700	3·1144
(0·0340)	(0·0009)	(0·0418)	(0·0574)	(0·1392)	(0·0761)	(0·0510)	(0·0070)	(0·0345)
−0·0633	0·2428	0·7136	0·4218	0·2973	Q(12) = 43·485 p-value = 0·325			
(0·0350)	(0·0621)	(0·0240)	(0·0949)	(0·0522)	Q(24) = 76·869 p-value = 0·796			
Franc log likelihood = −670·27								
0·0642	0·9994	−0·0571	0·3254	0·1117	1·0322	0·6130	0·9999	3·0455
(0·0423)	(0·0015)	(0·0321)	(0·0601)	(0·2056)	(0·0402)	(0·0432)	(0·0001)	(0·0406)
−0·2354	0·4045	0·6033	0·3480	0·2143	Q(12) = 56·197 p-value = 0·046			
(0·0388)	(0·0265)	(0·0312)	(0·0462)	(0·0374)	Q(24) = 78·894 p-value = 0·746			
Yen log likelihood = −558·60								
−0·0848	0·9914	−0·0683	0·3864	0·0472	1·6318	1·6293	0·9898	2·9394
(0·0043)	(0·0006)	(0·0153)	(0·0042)	(0·1938)	(0·0044)	(0·0044)	(0·0006)	(0·0045)
0·1336	0·2220	0·7304	0·3655	0·0333	Q(12) = 41·005 p-value = 0·426			
(0·0043)	(0·0106)	(0·0029)	(0·0092)	(0·1092)	Q(24) = 82·384 p-value = 0·649			

$\mathbf{y}_t = \iota z_t + \mathbf{x}_t$, where $\mathbf{y}_t = (s_t, f_{k,t})'$, $k = 1, 3$, $\iota = (1, 1)'$, $z_t = z_{t-1} + \varepsilon_{z,t}$, $\varepsilon_{z,t} \overset{\text{i.i.d.}}{\sim} N(0, \sigma_z^2)$,

$\mathbf{x}_t = \mathbf{c} + \Phi \mathbf{x}_{t-1} + \varepsilon_t + \Theta \varepsilon_{t-1}$, $\varepsilon_t = (\varepsilon_{s,t}, \varepsilon_{f,t})' \overset{\text{i.i.d.}}{\sim} N(0, \Sigma)$, with

$$\sum = \begin{pmatrix} \sigma_s^2 & \rho_{sf}\sigma_s\sigma_f \\ \rho_{sf}\sigma_s\sigma_f & \sigma_f^2 \end{pmatrix}$$

$\mathbf{c}$ a (2×1) constant vector, and Φ and Θ being (2×2) parameter matrices. Asymptotic standard errors are in parentheses. Q(p) are pth-order Ljung–Box statistics for serial correlation of the vector $\{\varepsilon_t\}$. Q(12) $\sim \chi_{40}^2$, Q(24) $\sim \chi_{88}^2$. Asymptotic standard errors are in parentheses.

Table VII. Sample and implied moments from maximum likelihood estimates

	Pound		Franc		Yen	
	Sample	Implied	Sample	Implied	Sample	Implied
$\mathrm{Cov}(\Delta s_{t+1}, f^p_{1,t})$	−0·153	−0·023	−0·077	−0·094	−0·200	−0·078
$\mathrm{Var}(f^p_{1,t})$	0·011	0·106	0·101	0·094	0·081	0·074
$\rho(f^p_{1,t}, f^p_{1,t-1})$	0·796	0·786	0·674	0·652	0·920	0·916
β_1	−1·440	−0·213	−0·766	−1·002	−2·477	−1·049
$\mathrm{Var}[E_t(\Delta s_{t+1})]$	n.a.	0·322	n.a.	0·125	n.a.	0·404
$\rho[E_t(\Delta s_{t+1}), E_{t-1}(\Delta s_t)]$	n.a.	0·497	n.a.	0·471	n.a.	0·541
$\mathrm{Var}(r_{1,t})$	n.a.	0·474	n.a.	0·407	n.a.	0·634
$\mathrm{Cov}(E_t \Delta s_{t+1}, r_{1,t})$	n.a.	−0·345	n.a.	−0·219	n.a.	−0·482
$\mathrm{Cov}(\Delta_3 s_{t+3}, f^p_{3,t})$	−0·635	−0·675	−0·192	−0·492	−1·494	−0·710
$\mathrm{Var}(f^p_{3,t})$	0·691	0·674	0·675	0·632	0·623	0·586
$\rho(f^p_{3,t}, f^p_{3,t-3})$	0·761	0·773	0·429	0·547	0·778	0·815
β_1	−2·367	−1·001	−0·284	−0·778	−2·398	−1·212
$\mathrm{Var}(E_t \Delta_3 s_{t+3})$	n.a.	1·105	n.a.	0·596	n.a.	1·736
$\rho[E_t(\Delta_3 s_{t+3}), E_{t-3}(\Delta_3 s_t)]$	n.a.	0·783	n.a.	0·827	n.a.	0·850
$\mathrm{Var}(r_{3,t})$	n.a.	3·130	n.a.	2·211	n.a.	3·743
$\mathrm{Cov}(E_t \Delta_3 s_{t+3}, r_{3,t})$	n.a.	−1·753	n.a.	−1·077	n.a.	−2·447

component exceeds the innovation variance of observable economic fundamentals. We note also that the contemporaneous correlation between the transitory component innovations $(\varepsilon_{s,t}, \varepsilon_{f,t})$ are all estimated to be near 1.

Table VII displays various population moments implied by the point estimates. Under the 'eyeball' metric, the model does a credible if not an exact job of matching these moments. The implied slope coefficients from regressions of the future depreciation on the forward premium, β_1, are much less than 1 and are negative for each currency. The implied expected excess returns are negatively correlated with and are more volatile than the implied expected depreciation. The implied forward premia are persistent, as can be seen from the large values of their first-order autocorrelations. The implied forward premium variance is seen to match up with the sample variances. Although the implied values of β_1 do not match the data — they are not large enough in magnitude for the pound and yen, and too large for the franc — the next section shows that the differences are not statistically significant.

5. SIMULATIONS

In this section we supplement the diagnostic tests performed in the previous section by asking whether our model could plausibly have generated the data. Specifically, we ask whether the fitted model can match important functions of the data that were not explicitly imposed in estimation. We focus our attention on the ability of the model to match those features of the data reviewed in Section 2.

We address this question by generating simulation distributions of the slope-coefficient estimators and their asymptotic t-ratios where the data-generating process is the two-component

model with parameter values equal to the point estimates. These distributions are built up from simulations of 5000 trials where for each trial i ($i = 1, \ldots, 5000$), we

1. draw a scalar sequence of observations $\{\varepsilon_{z,t}^i\}_{t=1}^T$ from a normal distribution with mean 0 and variance $\hat{\sigma}_z^2$;
2. draw a vector sequence of observations $\{(\varepsilon_{s,t}^i, \varepsilon_{f,t}^i)'\}_{t=1}^T$ from a bivariate normal distribution with mean 0 and covariance matrix,

$$\begin{pmatrix} \hat{\sigma}_s^2 & \hat{\rho}_{sf}\hat{\sigma}_s\hat{\sigma}_f \\ \hat{\rho}_{sf}\hat{\sigma}_s\hat{\sigma}_f & \hat{\sigma}_f^2 \end{pmatrix}$$

3. generate sequences of observations $\{z_t^i\}_{t=1}^T$ and $\{(x_{s,t}^i, x_{f,t}^i)'\}_{t=1}^T$ according to equations (9) and (18). These sequences are then combined to construct sequences of log-levels of spot and forward rates, $\{(s_t^i, f_t^i)'\}_{t=1}^T$;
4. use the computer-generated observations to estimate the cointegrating vector β_0 with DOLS and DGLS, and the slope coefficient in the regression of the future depreciation on the forward premium, β_1. Call these estimates $\hat{\beta}_{0,\text{DOLS}}^i$, $\hat{\beta}_{0,\text{DGLS}}^i$, and $\hat{\beta}_1^i$.

The 5000 observations on $\hat{\beta}_{0,\text{DOLS}}^i$, $\hat{\beta}_{0,\text{DGLS}}^i$, and $\hat{\beta}_1^i$ and their asymptotic t-ratios form the empirical distribution for these estimators under the null hypothesis that the estimated permanent–transitory components model is the true data-generating mechanism. We generate the distribution of the asymptotic t-ratios since inference is typically drawn using this statistic. We also provide a test based on a quadratic measure of distance, that the three asymptotic t-ratios (or the three slope coefficients) estimated from the data were jointly drawn from our data-generating process. Let $\hat{\theta}$ be the (3×1) vector of interest estimated from the data. To perform the joint test, we compute the distribution for the statistic,

$$J = (\hat{\theta} - \bar{\theta})' \sum_{\theta}^{-1} (\hat{\theta} - \bar{\theta}) \tag{19}$$

where $\bar{\theta}$ and Σ_θ are the mean vector and covariance matrix from the empirical distribution.

5.1. Results

Table VIII displays the lower 2·5, 50, and 97·5 percentiles of the empirical distribution for $(\hat{\beta}_{0,\text{DOLS}}, \hat{\beta}_{0,\text{DGLS}}, \hat{\beta}_1)$ and their asymptotic t-ratios for $k = 1$. As in Tables II and V, the asymptotic t's are constructed under the hypothesis that the slope coefficient is 1. p-values are the proportion of the empirical distribution that lies to the right of the values estimated from the data. Table IX reports the same information for $k = 3$.

Although it is not the main focus of our investigation, the tables provide some interesting information about the sampling properties of the cointegrating vector estimators. Both DOLS and DGLS are biased downward, as the medians from each of the distributions are less than 1. The bias is slightly more severe for $k = 3$. The distributions of the asymptotic DOLS and DGLS t-ratios appear to be poorly approximated by the standard normal for our model with a sample size of 200. There is considerable size distortion, as the lower and upper 2·5% tails of the asymptotic t-ratios differ from the standard normal's values of ± 1.96. For example, the lower and upper t-ratio tails for DOLS in the yen regressions in Table VIII is -8.21 and 3.09. More

Table VIII. Features of the empirical distribution — monthly horizon

	Slope coefficient				Asymptotic t-ratios			
	2·5%	median	97·5%	p-value	2·5%	median	97·5%	p-value
Pound								
$\hat{\beta}_{0,\text{DOLS}}$	0·9907	0·9994	1·0082	0·7274	−3·4924	−0·2659	2·7934	0·7964
$\hat{\beta}_{0,\text{DGLS}}$	0·9911	0·9994	1·0077	0·7920	−2·4451	−0·1813	1·9640	0·7466
β_1	−2·3763	−0·8020	0·7646	0·7888	−4·3832	−2·2613	−0·2920	0·8642
J	0·1296	1·8722	12·9954	0·6010	0·1768	2·1825	10·8689	0·5106
Franc								
$\hat{\beta}_{0,\text{DOLS}}$	0·9853	0·9981	1·0088	0·4650	−5·3252	−0·8014	3·2728	0·5028
$\hat{\beta}_{0,\text{DGLS}}$	0·9857	0·9980	1·0083	0·4626	−4·5199	−0·7298	2·6568	0·4968
β_1	−2·5536	−1·0596	0·4875	0·3530	−4·6337	−2·6728	−0·6382	0·4024
J	0·1159	1·7743	13·1562	0·9435	0·1307	1·8944	11·7070	0·9908
Yen								
$\hat{\beta}_{0,\text{DOLS}}$	0·9806	0·9956	1·0078	0·1050	−8·1698	−1·9317	3·1185	0·0634
$\hat{\beta}_{0,\text{DGLS}}$	0·9816	0·9947	1·0048	0·0978	−5·1387	−1·4784	1·4534	0·0680
β_1	−3·2701	−1·2155	0·6144	0·8992	−4·1772	−2·2988	−0·3696	0·9738
J	0·1716	1·9558	11·3565	0·3346	0·1841	2·1020	10·6800	0·0958

Notes: Selected percentiles of the empirical distribution computed for the cointegrating vector estimators ($\hat{\beta}_{0,\text{DOLS}}$, $\hat{\beta}_{0,\text{DGLS}}$), the slope coefficient in regressions of the future depreciation on the forward premium ($\hat{\beta}_1$), and their asymptotic t-ratios. J is the joint test statistic described in equation (19). p-values are the proportion of the empirical distribution that lies above the values estimated from the data. The data-generating mechanism is the random-walk-vector ARMA(1, 1) components model fitted to spot and 1-month-forward exchange rates from 1976:1 to 1992:8.

Table IX. Features of the empirical distribution — quarterly horizon

	Slope coefficient				Asymptotic t-ratios			
	2·5%	median	97·5%	p-value	2·5%	median	97·5%	p-value
Pound								
$\hat{\beta}_{0,\text{DOLS}}$	0·9445	0·9911	1·0275	0·4762	−6·7519	−1·2898	3·4376	0·5338
$\hat{\beta}_{0,\text{DGLS}}$	0·9430	0·9859	1·0156	0·4470	−4·2429	−1·2562	1·5241	0·5164
β_1	−3·0038	−1·1721	0·4895	0·9116	−6·1503	−2·8739	−0·6310	0·7352
J	0·1863	1·9749	12·1640	0·5034	0·1952	2·0933	11·0836	0·9508
Franc								
$\hat{\beta}_{0,\text{DOLS}}$	0·9553	0·9930	1·0230	0·4962	−6·2832	−1·1647	3·3078	0·5522
$\hat{\beta}_{0,\text{DGLS}}$	0·9565	0·9922	1·0200	0·4934	−4·9732	−1·0165	2·4198	0·5444
β_1	−2·4051	−0·8374	0·7524	0·2458	−5·6204	−2·5842	−0·3244	0·1126
J	0·1560	1·8400	12·4834	0·8806	0·1530	1·9305	12·2403	0·6332
Yen								
$\hat{\beta}_{0,\text{DOLS}}$	0·9445	0·9882	1·0222	0·0928	−8·2059	−1·8725	3·0926	0·0514
$\hat{\beta}_{0,\text{DGLS}}$	0·9446	0·9826	1·0101	0·0766	−5·0632	−1·7163	1·1217	0·0610
β_1	−3·5078	−1·3920	0·5098	0·8508	−6·0877	−2·8154	−0·5255	0·9598
J	0·1843	2·0299	12·2966	0·3644	0·1911	2·0826	11·0014	0·0662

Notes: see Table VIII.

detailed examinations of the distributions than those reported in the table indicate that they do not appear to be particularly skewed in either direction.

Turning now to $\hat{\beta}_1$, the median values are seen to be negative. For the pound, franc, and yen, their respective values are -0.80, -1.06, and -1.22 for $k = 1$ and -1.17, -0.84, and -1.39 for $k = 3$, and are close to the implied population values shown in Table VI. The sample estimates (Table V) for the pound and the yen lie to the left of the median values and to the right for the franc, but none of the individual p-values lie outside the interval $(0.025, 0.975)$. Thus, the hypothesis that the regression estimates of β_1 were drawn from the empirical null distribution cannot be rejected at standard significance levels.

The test of the joint hypothesis that $(\hat{\beta}_{0,\text{DOLS}}, \hat{\beta}_{0,\text{DGLS}}, \hat{\beta}_1)$ was drawn from the empirical null distribution similarly cannot be rejected at standard significance levels. The test that the vector of asymptotic t-ratios was drawn from our empirical null distribution can be rejected at the 10% level only for the yen, and yields little evidence against the model. The weight of the evidence, then, suggests that the two-component model provides a reasonably accurate representation of the process-generating spot and forward exchange rate observations.

6. IMPLIED EXPECTED EXCESS RETURNS

We employ the smoothed Kalman filter, which provides estimates of the state using the entire sample, to obtain an estimate of the expected future spot rate $E_t(s_{t+k})$ series. The estimated values are then subtracted from the forward rate to obtain the implied expected excess return series. Figures 1–3 plot our estimates of the quarterly expected depreciation and the expected excess returns for the three currencies.[15]

From these figures, it is visually apparent that $r_{3,t}$ is both negatively correlated with and more volatile than $E_t(\Delta_3 s_{t+3})$. $r_{3,t}$ is also seen to be persistent and to fluctuate between positive and negative values.[16] Looking across the three currencies, similarities in their behaviour are evident. The expected excess returns are seen to be negative in 1976 and 1977 for all three currencies, and positive for much of the latter 1970s for the franc and the yen. Similarly, the expected excess returns are positive for all three currencies during the 1981 recession and negative during the final three years of the sample. Furthermore, the general pattern exhibited by the implied expected depreciations appears plausible. The estimates imply that the dollar was expected to strengthen relative to the yen and the franc during the late 1970s and relative to the pound and the yen during the 1980s. We also find an expected weakening of the dollar relative to all three currencies during the mid-1980s and towards the end of the sample.

The time-variation of $r_{k,t}$ can emerge for a variety of reasons. One hypothesis that is frequently suggested is that it represents a rational risk premium. According to this hypothesis, the dollar is the risky currency when $r_{k,t} > 0$ since a premium is being paid to holders of dollar-denominated assets. Engel (1992) provides an explanation in which the risk premium is compensation for covariance risk between consumption and the exchange rate. His reasoning begins by noting that consumption includes expenditures on both domestic and foreign goods. Thus, if the value of the dollar is positively correlated with consumption, the dollar provides a poor hedge against bad

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[15] Plots at the $k = 1$ horizon reveal that monthly expected excess return and expected depreciation are qualitatively similar, but as one would expect, somewhat noisier. We suppress these plots to economize on space.

[16] In a related context, LeBaron (1992) finds that to match moving average trading rule results requires a persistent but stationary risk premium.
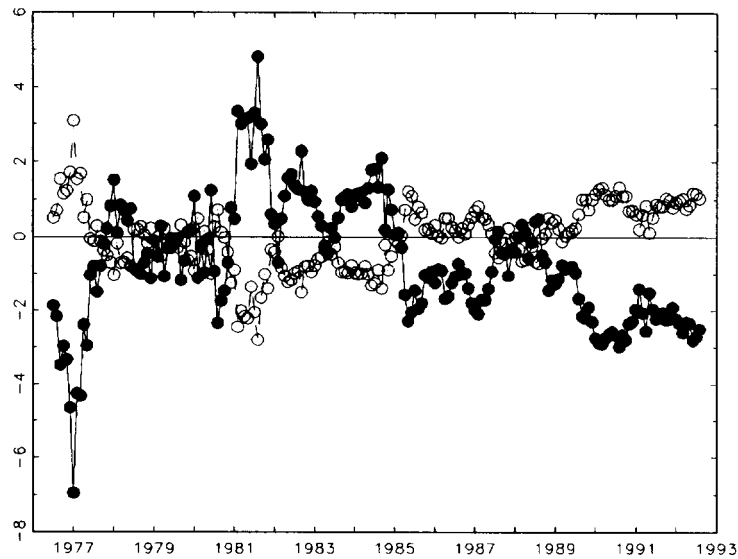
Figure 1. Quarterly expected depreciation (open circles) and excess return (solid circles) for the dollar–
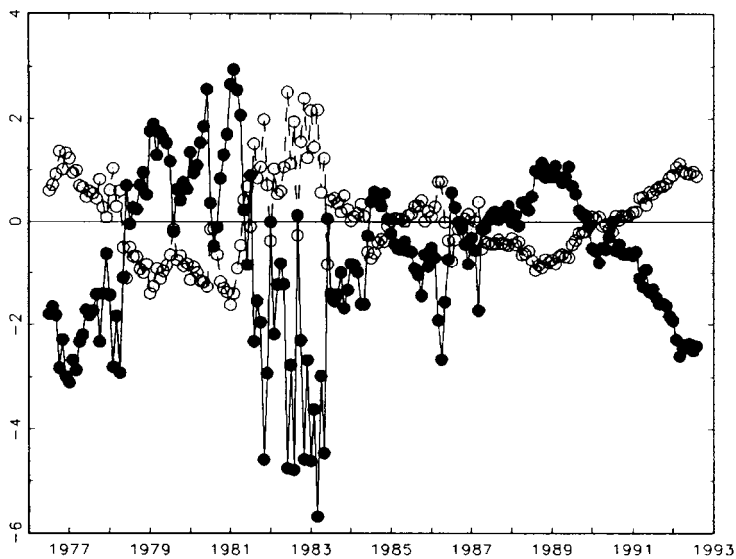pound rate

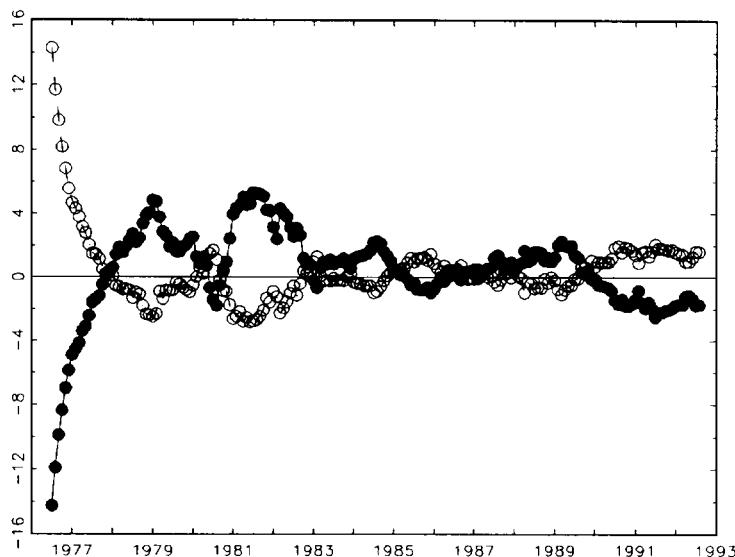

Figure 2. Quarterly expected depreciation (open circles) and excess return (solid circles) for the dollar–
franc rate

states of nature and is therefore risky. This particular story of the risk premium evidently implies that the sign of $r_{k,t}$ depends on whether the covariance between consumption and the exchange rate is positive or negative.

An alternative to the risk premium interpretation is provided by Gourinchas and Tornell (1996) who present a model in which agents who engage in learning about permanent and

Figure 3. Quarterly expected depreciation (open circles) and excess return (solid circles) for the dollar–yen rate

transitory dynamics of interest rates rationalizes the predictability of the expected excess return as well as the delayed overshooting of exchange rates following monetary shocks as documented by Eichenbaum and Evans (1995). Finally, yet another hypothesis is that $r_{k,t}$ is the consequence of foreign exchange market inefficiency or irrationality on the part of market participants as suggested by Frankel and Froot's (1989) study of survey expectations.[17]

Whether the behaviour of $r_{3,t}$ is consistent with it being a risk premium remains an open question. We observe negative $r_{3,t}$'s during the 1991–92 recessionary period but positive values during the 1981 recession. The expected excess return is also negative for the franc and pound during the late 1980s, which was a period of economic expansion. Thus, if $r_{3,t}$ is indeed a risk premium, the sign of the covariance between the exchange rate and consumption must be changing over time. An examination of the pattern of changing covariance between consumption and the exchange rate is a task beyond the scope of the present paper.

7. CONCLUSIONS

The behaviour of expected excess foreign exchange returns has been the subject of extensive empirical research. We have adopted a structural time-series approach in an effort to further understand the dynamics of these expected excess returns. The two-component model we estimate draws its motivation from the disequilibrium exchange rate dynamics of sticky-price models. Standard diagnostic tests and a simulation experiment were performed to gauge the adequacy of the representation.

The model helps to shed light on why the forward rate is an unbiased predictor of the future spot rate while at the same time increases in the forward premium predict a currency appreciation.

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[17] See also Domowitz and Hakkio (1985) and Kaminsky and Peruga (1990), who study models in which expected excess returns are non-zero and time-varying under risk neutrality when the underlying data-generating process is log-normal.

We use the model to obtain estimates of the expected currency excess return. It is unclear, however, whether the expected excess return emerges as compensation for risk bearing. A thorough investigation of this important question remains a topic for future research.

REFERENCES

Backus, D. K., A. W. Gregory and C. I. Telmer (1993), 'Accounting for forward rates in markets for foreign currency', *Journal of Finance*, **48**, 1887–908.

Baillie, R. T. and T. Bollerslev (1989), 'Common stochastic trends in a system of exchange rates', *Journal of Finance*, **44**, 167–81.

Bilson, J. F. O. (1978), 'Rational expectations and the exchange rate', in J. A. Frankel and H. G. Johnson (eds), *The Economics of Exchange Rates: Selected Studies*, Addison-Wesley, Reading, MA, 75–96.

Bilson, J. F. O. (1981), 'The "speculative efficiency" hypothesis', *Journal of Business*, **54**, 435–52.

Blough, S. R. (1992), 'The relationship between power and level for generic unit root tests in finite samples', *Journal of Applied Econometrics*, **7**, 295–308.

Boothe, P. and D. Longworth (1986), 'Foreign exchange market efficiency tests: implications of recent findings', *Journal of International Money and Finance*, **5**, 135–52.

Campbell, J. Y. and R. H. Clarida (1987), 'The dollar and real interest rates', *Carnegie-Rochester Conference Series on Public Policy*, **27**, 103–40.

Campbell, J. Y. and P. Perron (1991), 'Pitfalls and opportunities: what macroeconomists should know about unit roots', in O. J. Blanchard and S. Fisher (eds), *NBER Macroeconomics Annual*, Vol. 6, MIT Press, Cambridge, MA.

Campbell, J. Y. and R. J. Shiller (1988), 'Stock prices, earnings, and expected dividends', *Journal of Finance*, **43**, 661–76.

Clarida, R. H. and M. P. Taylor (1993), 'The term structure of forward exchange premia and the forecastibility of spot exchange rates: correcting the errors', NBER working paper No. 4442.

Cochrane, J. H. (1991), 'A critique of the application of unit root tests', *Journal of Economic Dynamics and Control*, **5**, 275–84.

Cornell, B. (1977), 'Spot rates, forward rates, and exchange market efficiency', *Journal of Financial Economics*, **5**, 55–65.

Cumby, R. E. and M. Obstfeld (1984), 'International interest rate and price-level linkages under flexible exchange rates: a review of recent evidence', in J. F. O. Bilson and R. C. Marston (eds), *Exchange Rate Theory and Practice*, University of Chicago Press for the National Bureau of Economic Research, Chicago, 121–51.

Dickey, D. A. and W. A. Fuller (1979), 'Distribution of the estimators for autoregressive time series with a unit root', *Journal of the American Statistical Associations*, **74**, 427–81.

Diebold, F. X. and J. A. Nason (1990), 'Nonparametric exchange rate prediction?' *Journal of International Economics*, **28**, 315–32.

Domowitz, I. and C. S. Hakkio (1995), 'Conditional variance and the risk premium in the foreign exchange market', *Journal of International Economics*, **19**, 47–66.

Dornbusch, R. (1976), 'Expectations and exchange rate dynamics', *Journal of Political Economy*, **84**, 1161–76.

Duffie, D. and K. Singleton (1993), 'Simulated moments estimation of Markov models of asset prices', *Econometrica*, **61**, 929–52.

Eichenbaum, M. and C. Evans (1995), 'Some empirical evidence on the effects of monetary policy shocks on exchange rates', *Quarterly Journal of Economics*, **110**, 975–1009.

Elliott, G., T. J. Rothenberg and J. Stock (1996), 'Efficient tests for an autoregressive unit root', *Econometrica*, **64**, 813–36.

Engel, C. (1984), 'Testing for the absence of expected real profits from forward market speculation', *Journal of International Economics*, **17**, 299–308.

Engel, C. (1992), 'On the foreign exchange risk premium in a general equilibrium model', *Journal of International Economics*, **32**, 305–19.

Engel, C. (1994), 'Can the Markov-switching model forecast exchange rates?' *Journal of International Economics*, **36**, 151–65.

Engle, R. F. and B. S. Yoo (1987), 'Forecasting and testing in co-integrated systems', *Journal of Econometrics*, **35**, 143–59.

Evans, M. D. D. and K. K. Lewis (1992), 'Are foreign exchange returns subject to permanent shocks?' Mimeo, University of Pennsylvania.

Fama, E. F. (1984), 'Forward and spot exchange rates', *Journal of Monetary Economics*, **14**, 319–38.

Fama, E. F. and K. R. French (1988), 'Dividend yields and expected stock returns', *Journal of Financial Economics*, **22**, 3–25.

Faust, J. (1993), 'Near observational equivalence and unit-root processes: formal concepts and implications', Mimeo, International Finance Division, Board of Governors of the Federal Reserve System.

Frankel, J. A. (1988), 'Recent estimates of time-variation in the conditional variance and in the exchange risk premium', *Journal of International Money and Finance*, **7**, 115–25.

Frankel, J. A. (1976), 'A monetary approach to the exchange rate: doctrinal aspects and empirical evidence', *Scandinavian Journal of Economics*, **78**, 200–24.

Frankel, J. A. (1981), 'Flexible exchange rates, prices, and the role of "news": lessons from the 1970s', *Journal of Political Economy*, **89**, 665–705.

Frankel, J. A. and A. Razin (1980), 'Stochastic prices and tests of efficiency of foreign exchange markets', *Economics Letters*, **6**, 165–70.

Froot, K. A. and J. A. Frankel (1989), 'Forward discount bias: is it an exchange risk premium?' *Quarterly Journal of Economics*, **104**, 139–61.

Fuller, W. A. (1976), *Introduction to Statistical Time Series*, John Wiley, New York.

Gourinchas, P. O. and A. Tornell (1996), 'Exchange rate dynamics and learning', NBER Working Paper No. 5530.

Hansen, L. P. and R. J. Hodrick (1983), 'Risk averse speculation in the forward foreign exchange market: an econometric analysis of linear models', in J. A. Frankel (ed.), *Exchange Rates and International Macroeconomics*, University of Chicago Press for the National Bureau of Economic Research, Chicago.

Hodrick, R. J. (1987), *The Empirical Evidence on the Efficiency of Forward and Futures Foreign Exchange Markets*, Harwood Academic Publishers, Chur, Switzerland.

Hodrick, R. J. and S. Srivastava (1986), 'The covariation of risk premiums and expected futures spot rates', *Journal of International Money and Finance*, **5**, S5–21.

Kaminsky, G. and R. Peruga (1990), 'Can a time-varying risk premium explain excess returns in the market for foreign exchange?' *Journal of International Economics*, **28**, 47–70.

Kwan, A. and Y. Wu (1996), 'A comparative study of the finite-sample distribution of some portmanteau tests for univariate time series models', *Communications in Statistics—Simulation and Computation*, forthcoming.

LeBaron, B. (1992), 'Do moving average trading rule results imply nonlinearities in foreign exchange markets?' University of Wisconsin SSRI working paper No. 9222.

Liu, P. C. and G. S. Maddala (1992), 'Rationality of survey data and tests for market efficiency in the foreign exchange markets', *Journal of International Money and Finance*, **11**, 366–81.

Ljung, G. M. and G. E. P. Box (1978), 'On a measure of lack of fit in time-series models', *Biometrika*, **65**, 297–303.

Lucas, R. E., Jr (1982), 'Interest rates and currency prices in a two-country world', *Journal of Monetary Economics*, **10**, 335–60.

Lutkepohl, H. (1993), *Introduction to Multiple Time Series*, Springer-Verlag, Berlin.

Mark, N. C. (1990), 'Real and nominal exchange rates in the long run: an empirical investigation', *Journal of International Economics*, **28**, 115–36.

Mark, N. C. (1995), 'Exchange rates and fundamentals: evidence on long-horizon prediction', *American Economic Review*, **85**, 201–18.

McCallum, B. T. (1994), 'A reconsideration of the uncovered interest parity relationship', *Journal of Monetary Economics*, **33**, 105–32.

Mussa, M. L. (1976), 'The exchange rate, the balance of payments, and monetary and fiscal policy under a regime of controlled floating', *Scandinavian Journal of Economics*, **78**, 229–48.

Mussa, M. L. (1982), 'A model of exchange rate dynamics', *Journal of Political Economy*, **90**, 74–104.

Newey, W. K. and K. D. West (1987), 'A simple, positive semi-definite, heteroskedasticity and auto-correlation consistent covariance matrix', *Econometrica*, **55**, 703–8.

Phillips, P. C. B. and P. Perron (1988), 'Testing for a unit root in time series regression', *Biometrika*, **75**, 335–46.

Stock, J. H. (1991), 'Confidence intervals for the largest autoregressive root in US macroeconomic time series', *Journal of Monetary Economics*, **28**, 435–59.

Stock, J. H. and M. W. Watson (1993), 'A simple estimator of cointegrating vectors in higher order integrated systems', *Econometrica*, **4**, 783–820.

Summers, L. H. (1986), 'Does the stock market rationally reflect fundamental values?' *Journal of Finance*, **41**, 591–601.