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TIME-VARYING BETAS AND RISK PREMIA IN THE PRICING OF FORWARD FOREIGN EXCHANGE CONTRACTS

Nelson C. MARK*

Ohio State University, Columbus, OH 43210-1172, USA

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This paper specifies the single-beta capital asset pricing model for the pricing of forward foreign exchange contracts from the point of view of a U.S. investor. Parametric specification of the betas as ARCH-like processes explicitly allows for time variation as well as sign variation of the risk premium in the forward foreign exchange market. I estimate the model jointly for four currencies, using a generalized method of moments procedure. The results show significant time variation for the betas and tests of the overidentifying restrictions are generally favorable to the model.

1. Introduction

Exchange-rate volatility has been a subject of interest and concern since the major industrialized nations moved to floating rates in 1973. Considerable research has focused on the conditional bias of the forward exchange rate as a predictor of the future spot rate. This research asks whether price determination in the foreign exchange market is efficient. Theoretical international finance models developed by Stockman (1978), Fama and Farber (1979), Hodrick (1981), Roll and Solnik (1977), Stulz (1981, 1984), and Hodrick and Srivastava (1986) consider the pricing of forward exchange contracts in much the same way as that of other financial assets. In these models, the forward exchange rate generally differs from the expected future spot rate by a risk premium. The available empirical evidence on models of foreign exchange risk, however, has been mixed. Thus whether the conditional bias of the forward exchange rate as a predictor of the future spot rate can be interpreted as a risk premium is debatable.[1] For example, some recent empirical studies suggest

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[1]See Boothe and Longworth (1986) for a survey on these issues and historical references. More recently, explicit structural models of the pricing of forward exchange contracts have been estimated and tested by Cumby (1986), Engle and Rodrigues (1987), and Kaminsky and Peruga (1987). Meese (1986) and Evans (1986) conclude that exchange rates have been influenced by speculative bubbles, while Frankel and Froot (1986) report violations of rational expectations from an analysis of survey data.

that exchange-rate movements have been dominated by speculative bubbles or that expectations of foreign exchange market participants may not be rational.

I argue in this paper that there are important dimensions along which empirical models of the risk premium have yet to be investigated and that it is premature to abandon the risk premium interpretation of the conditional bias of the forward rate as a predictor of the future spot rate. Specifically, I consider the pricing of forward foreign exchange contracts in the context of the single-beta capital asset pricing model (CAPM). The single-beta representation can be derived from a variety of economic environments and has been the subject of previous empirical investigations of the pricing of forward foreign exchange contracts. In contrast to much of this work, however, I consider generalizations in two important dimensions. First, the CAPM is specified in a *conditional* environment where the beta, which is the ratio of a conditional covariance and a conditional variance, is parameterized following Engle's (1982) ARCH modeling strategy.[2] Although a constant beta specification can be consistent with time variation in the risk premium as a result of time variation in the expected excess return on the reference asset, sign variation in such an environment can occur *only* as a result of sign variation in the expected excess return on the reference asset. The specification I consider conveniently admits both time and sign variation in the risk premium. Second, some of the previous research assumes that the risk premium is priced relative to a pure currency portfolio [e.g., Roll and Solnik (1977)]. This paper treats the pricing of forward contracts relative to a broader portfolio of assets that is likely to be held by a representative investor – namely, one involving equity returns.

The paper is organized as follows. The next section motivates the single-beta CAPM specification for the risk premium. Section 3 discusses the data used. The empirical specification and estimation procedures used are discussed in section 4, and the empirical results are reported in section 5. Some concluding remarks are reserved for section 6.

2. The single-beta representation

Let P_{t-1}^i be the net-of-dividend nominal price of asset i at time $t-1$ and V_t^i be its with-dividend nominal price at t. Also, let r_t^p and r_t^f denote the nominal rates of return from $t-1$ to t of a reference portfolio p and asset f, whose return is conditionally uncorrelated with r_t^p. r_t^f will be referred to as the conditionally risk-free rate. A generic specification of the price level form of

[2] The method used here has recently been applied to the pricing of equities by Bodurtha and Mark (1987).

the single-beta CAPM in a conditional environment can be written as

$$P_{t-1}^{i} = \frac{\mathrm{E}_{t-1}(V_t^i) - \beta_{t-1}^{i,p}\mathrm{E}_{t-1}(r_t^p - r_t^f)}{1 + r_t^f}, \tag{1}$$

where

$$\beta_{t-1}^{i,p} \equiv \frac{\mathrm{cov}_{t-1}(V_t^i, r_t^p)}{\mathrm{var}_{t-1}(r_t^p)},$$

and $\mathrm{E}_{t-1}(\cdot)$, $\mathrm{cov}_{t-1}(\cdot)$, and $\mathrm{var}_{t-1}(\cdot)$ are the mathematical expectation, covariance, and variance conditioned on the information available to investors at $t-1$. Investors are assumed to have rational expectations, so these mathematical conditional moments correspond to investors' subjective conditional moments.[3] This conditional specification can be motivated by recent empirical findings that time variation of conditional means, variances, and covariances is an important feature of financial market data.[4]

Eq. (1) corresponds to the equilibrium representation of asset prices in the Sharpe (1964) analysis of the one-period problem confronting a mean-variance optimizing investor. The beta, $\beta_{t-1}^{i,p}$, is the marginal contribution to portfolio (variance) risk of asset i. The reference portfolio, p, in this case is the market portfolio – the portfolio of all traded assets.[5]

Now consider taking an open position in a forward foreign exchange contract. Let S and F be the nominal spot and one-period forward prices of the foreign currency. Since there is a zero net investment at $t-1$, the net-of-dividend value of the contract at $t-1$ is zero and the speculative profit or the with-dividend value at t is $S_t - F_{t-1}$. Let $\rho_t \equiv (S_t - F_{t-1})/F_{t-1}$ be the ex post nominal speculative profit from the forward contract normalized by the forward rate. Now, pricing the forward contract according to (1) implies

$$\mathrm{E}_{t-1}(\rho_t) = \beta_{t-1}^{\rho,p}\mathrm{E}_{t-1}(r_t^p - r_t^f), \tag{2}$$

[3]Hansen, Richard, and Singleton (1982) demonstrate that the standard results from efficient-set mathematics in an unconditional environment carry over to a conditional environment.

[4]See Diebold and Nerlove (1986), Cumby (1986), Campbell (1987), Engle, Lilien, and Robins (1987), Bollerslev, Engle, and Wooldridge (1988), Engle and Rodrigues (1987), and French, Schwert, and Stambaugh (1987).

[5]In the empirical finance literature, the model is usually stated in an unconditional environment and tested by examining cross-sectional relationships of asset returns on their betas, which are first estimated from time-series regressions of individual asset returns on the return of the reference portfolio. Hansen and Richard (1987) demonstrate that the omission of conditioning information in the standard approach may be undesirable. The standard methodology has also been criticized by Roll (1979) and is subject to an explicit error-in-variables problem since the regressors were themselves estimated. Work by Gibbons (1982) and Shanken (1985) tries to salvage this methodology. This paper uses time-series methods, which avoid many of these problems.

where

$$\beta_{t-1}^{\rho,P} = \frac{\mathrm{cov}_{t-1}(\rho_t, R_t^P)}{\mathrm{var}_{t-1}(r_t^P)}.$$

The ex ante payoff, $\mathrm{E}_{t-1}(\rho_t)$, is the risk premium. In this sense, the pricing of a forward contract is similar to the pricing of any other asset and the risk premium on a currency is proportional to the expected excess return on the appropriate reference portfolio. The factor of proportionality or the beta is the contribution of the forward position to overall portfolio risk.

In the standard specification of the CAPM in an unconditional environment, the beta is written as the ratio of the unconditional covariance and variance and, therefore, is constant. Moreover, the ex ante return on the reference portfolio is also treated as constant. This implies that the risk premium is constant – a hypothesis that has been soundly rejected. Here, a constant beta is still consistent with time variation in the risk premium, due possibly to time variation in the ex ante excess return on the reference portfolio. Although this excess return may be negative ex post, in the context of the CAPM it should not be negative ex ante. Thus allowing the beta to change sign seems to be a satisfactory way to allow the risk premium to change sign.

The theory provides little guidance on the appropriate reference portfolio for empirical work. In the CAPM with perfect markets, both domestic and foreign investors agree that the world market portfolio is mean–variance efficient. As Stulz (1984) argues, however, violations of the perfect market assumption, due to differential taxation and transactions costs or differential political risks, cause domestic and foreign agents to differ in their assessment of what constitutes the efficient portfolio. Consequently, neither the domestic nor the foreign agent holds the world market portfolio.

The CAPM representation in (1) can be viewed as a partial equilibrium asset pricing condition for the representative domestic agent where *his* mean–variance efficient portfolio or his reference portfolio is the one used. In the empirical work, three candidate reference portfolios are considered. Each is an all-equity portfolio; where they differ is in the weight assigned to foreign equities. The exclusion of assets in these candidate portfolios can be considered to be severe in that nonequity assets as well as assets from most of the countries of the world are omitted. Whether these are good choices is largely an empirical issue. The appropriateness of a candidate reference portfolio thus forms part of the composite null hypothesis when inferences from the model are drawn.

The single-beta representation with nominal returns can also be derived from an intertemporal setting under certain circumstances.[6] Suppose that the

[6]I am grateful to John Long for suggesting this line of reasoning.

domestic representative investor can invest in assets p and f in any amount with no restrictions on short sales. The first-order conditions from his intertemporal optimization problem imply that $E_{t-1}[(r_t^p - r_t^f)U_t] = 0$, where r_t^p and r_t^f are as defined above and U_t is the agent's marginal utility of nominal income in period t. This in turn implies

$$\mathrm{cov}_{t-1}\left[\left(r_t^p - r_t^f\right), U_t\right] = -E_{t-1}U_t E_{t-1}\left(r_t^p - r_t^f\right). \tag{3}$$

Now consider taking an open position in a forward foreign exchange contract at $t-1$. Since there is a zero net investment at $t-1$, the first-order condition is $E_{t-1}[(S_t - F_{t-1})U_t] = 0$. This first-order condition in turn implies

$$\mathrm{cov}_{t-1}[\rho_t, U_t] = -E_{t-1}U_t E_{t-1}\rho_t. \tag{4}$$

Combining (3) and (4) yields

$$E_{t-1}\rho_t = \frac{\mathrm{cov}_{t-1}[\rho_t, U_t]}{\mathrm{cov}_{t-1}\left[\left(r_t^p - r_t^f\right), U_t\right]} E_{t-1}\left(r_t^p - r_t^f\right). \tag{5}$$

Now consider the linear least-squares projection of U_t on $r_t^p - r_t^f$ and ρ_t. That is,

$$U_t = c_0 + c_1\left(r_t^p - r_t^f\right) + c_2\rho_t + e_t, \tag{6}$$

where e_t is the least-squares projection error. Exploiting the fact that e_t is orthogonal to $(r_t^p - r_t^f)$ and ρ_t by construction, (6) can be substituted into (5) to obtain

$$E_{t-1}\rho_t = \frac{c_1 \mathrm{cov}_{t-1}\left(r_t^p - r_t^f, \rho_t\right) + c_2 \mathrm{var}_{t-1}(\rho_t)}{c_1 \mathrm{var}_{t-1}\left(r_t^p - r_t^f\right) + c_2 \mathrm{cov}_{t-1}\left(r_t^p - r_t^f, \rho_t\right)} E_{t-1}\left(r_t^p - r_t^f\right). \tag{7}$$

Now suppose that ρ contains no information for predicting U beyond what is contained in $r^p - r^f$. That is, $c_2 = 0$ in (6). Since r_t^f is known at $t-1$, it follows that (7) reduces to the single-beta representation (2). [Notice that (2) also follows directly from (5) if ρ and $(r^p - r^f)$ are perfectly conditionally correlated.] Thus, when viewed in an intertemporal setting, an equity portfolio return can be used for r^p and the Treasury-bill return for r^f. The appropriateness of doing so rests on the assumption that the excess return on the empirical reference portfolio contains all the information that ρ has for predicting marginal utility, and this assumption will form part of the composite null hypothesis when inferences are drawn. These issues are addressed further in section 4.[7]

3. The data

Observations are sampled at monthly intervals. The e ange rates used are the German mark (DM), Swiss franc (SF), Japanese yen (JY), and British pound (BP), all in relation to the U.S. dollar. The exchange-rate data are Friday closing quotations from the Harris Bank *Foreign Exchange Weekly*. Speculative profits on open positions in forward foreign exchange contracts are constructed by sampling spot rates on those Fridays falling nearest to the end of the calendar month and thirty-day forward rates observed four weeks earlier.

The reference portfolios are constructed from stock return indices from the U.S., Germany, Switzerland, Japan, and Britain. The U.S. (domestic) returns are the valued-weighted returns of stocks listed on the New York Stock Exchange, obtained from the Center for Research in Security Prices (CRSP) at the University of Chicago. Foreign returns are dividend inclusive stock index returns from Capital International's *Perspectives*.[8] Three candidate reference portfolios are constructed from these indices. They are (i) the CRSP index of domestic securities, (ii) a value-weighted portfolio with weights summing to unity, and (iii) an equal-weighted international portfolio. The U.S. receives by far the largest weight in the value-weighted portfolio, although its relative size declines steadily throughout the sample, while the size of the Japanese market increases. For example, the weights for the U.S., Britain, Germany, Japan, and Switzerland at the beginning of the sample are 0.74, 0.10, 0.04, 0.10, and 0.01, respectively, and are 0.60, 0.10, 0.04, 0.23, and 0.02 at the end of the sample. The one-month Treasury-bill return serves as the conditionally risk-free rate. The sample extends from July 1973 through December 1985 to coincide roughly with the recent period of floating exchange rates. The end of the sample is dictated by the availability of equity return data.

Table 1 reports the time-series means, standard deviations, and first six autocorrelations of the data. The index of U.S. equities has by far the lowest average return during the sample. On average, its return was 17% of the return on the value-weighted international portfolio and only 1.8% of the average return on the equal-weighted portfolio. By contrast, the standard deviation of the value-weighted portfolio is the smallest (0.0399) and the equal-weighted

[7] Merton (1973) shows that the single-beta representation is an implication of an intertemporal continuous-time model when investors have logarithmic utility or when the investment opportunity set is fixed. Hansen, Richard, and Singleton (1982) obtain the single-beta representation as an implication of a discrete-time intertemporal model, but the appropriate reference portfolio and risk-free rate in their paper are unobservable unless explicit assumptions are made about investors' utility.

[8] These data were graciously provided by Jim Bodurtha.

Table 1

Time-series means, standard deviations, and autocorrelations of candidate reference portfolio[a] excess returns and ex post currency speculative profits[b] from July 1973 to December 1985.

		Reference portfolio			Speculative profits on forward currency contracts			
		NYSE	VWR	EWR	DM	SF	JY	BP
Mean		0.0034	0.0199	0.1884	−0.0008	−0.0004	0.0010	0.0002
Std. dev.		0.0476	0.0399	0.1833	0.0312	0.0352	0.0315	0.0299
Auto-	1:	0.02	0.06	0.14	0.12	0.14	0.19	0.08
correlation	2:	−0.04	−0.02	0.01	0.04	0.11	−0.05	0.13
at lags	3:	0.06	0.09	0.15	0.06	−0.06	0.15	0.00
	4:	0.05	0.04	−0.01	−0.02	−0.01	0.09	−0.05
	5:	0.15	0.09	−0.07	−0.08	−0.02	0.01	0.16
	6:	−0.10	−0.12	−0.05	−0.12	−0.11	−0.07	0.01

[a]The reference portfolios are defined as follows. NYSE is the CRSP value-weighted index of equity returns listed on the New York Stock Exchange. VWR is the value-weighted international equity portfolio with weights summing to unity. EWR is the equal-weighted international equity portfolio.

[b]DM, SF, JY, and BP are the differences between the current spot rate and the forward rate observed four weeks earlier normalized by the forward rate for the German mark, Swiss franc, Japanese yen, and British pound, respectively.

portfolio displays the largest standard deviation (0.1833) and the highest first-order serial correlation.

Average profits on the forward foreign exchange contracts are small and statistically insignificant from zero. This suggests in part why attempts to model a constant risk premium on forward foreign exchange contacts have generally been difficult and unsuccessful. By and large, the time-series properties of these profits are similar across the currencies.

4. Estimation

Let $r^e \equiv r^p - r^f$ denote the excess return on the reference portfolio and let i ($i = \mathrm{DM, SF, JY, BP}$) index the currencies under consideration. Decomposing $\{r_t^e\}$ and $\{\rho_t^i\}$ into their forecastable and unforecastable components, we have

$$\rho_t^i = \mathrm{E}_{t-1}(\rho_t^i) + u_t^i, \qquad i = \mathrm{DM, SF, JY, BP}, \tag{8}$$

$$r_t^e = \mathrm{E}_{t-1}(r_t^e) + \varepsilon_t, \tag{9}$$

where u_t^i and ε_t are the one-step-ahead forecast errors of ρ_t^i and r_t^e, respectively, which are orthogonal to date $t-1$ information. The CAPM implies

$$\mathrm{E}_{t-1}(\rho_t^i) = \frac{\mathrm{cov}_{t-1}(\rho_t^i, r_t^p)}{\mathrm{var}_{t-1}(r_t^p)} \mathrm{E}_{t-1}(r_t^e). \tag{10}$$

Now, using eqs. (8) and (9) in the definition of the conditional covariance yields

$$\begin{aligned}\operatorname{cov}_{t-1}(\rho_t^i, r_t^p) &= \operatorname{cov}_{t-1}(\rho_t^i, r_t^e)\\ &= \mathrm{E}_{t-1}(\rho_t - \mathrm{E}_{t-1}\rho_t)(r_t^e - \mathrm{E}_{t-1}r_t^e) \qquad (11)\\ &= \mathrm{E}_{t-1}(u_t^i\varepsilon_t),\end{aligned}$$

where the first equality in (11) exploits the assumption that the conditionally risk-free return is in the information set at $t-1$. Similarly, the conditional variance of the excess return on the reference portfolio can be expressed as

$$\begin{aligned}\operatorname{var}_{t-1}(r_t^p) &= \operatorname{var}_{t-1}(r_t^e)\\ &= \mathrm{E}_{t-1}(r_t^e - \mathrm{E}_{t-1}r_t^e)^2 \qquad (12)\\ &= \mathrm{E}_{t-1}\varepsilon_t^2.\end{aligned}$$

Decomposing the sequences $\{u_t^i\varepsilon_t\}$ and $\{\varepsilon_t^2\}$ into their forecastable and unforecastable components, eqs. (8)–(12) imply

$$\rho_t^i = \left[\frac{\mathrm{E}_{t-1}(\varepsilon_t u_t^i)}{\mathrm{E}_{t-1}(\varepsilon_t^2)}\right]\mathrm{E}_{t-1}(r_{t-1}^e) + u_t^i, \qquad i = \mathrm{DM, SF, JY, BP}, \qquad (13a)$$

$$r_t^e = \mathrm{E}_{t-1}(r_{t-1}^e) + \varepsilon_t, \qquad (13b)$$

$$u_t^i\varepsilon_t = \mathrm{E}_{t-1}(u_t^i\varepsilon_t) + \eta_t^i, \qquad i = \mathrm{DM, SF, JY, BP}, \qquad (13c)$$

$$\varepsilon_t^2 = \mathrm{E}_{t-1}(\varepsilon_t^2) + \nu_t, \qquad (13d)$$

where η_t^i and ν_t are the one-step-ahead forecast errors of $u_t^i\varepsilon_t$ and ε_t^2, respectively, and are orthogonal to date $t-1$ information. For any n currencies under consideration, (13a)–(13d) becomes an estimable system of $2(n+1)$ equations [an analog of (13a) and (13c) for each of the n currencies plus (13b) and (13d)] once the conditional expectations are parameterized. I defer until a later section the specific parameterizations to be investigated.

Simultaneous estimation of the $2n+2$ equations is performed using Hansen's (1982) generalized method of moments (GMM) procedure as follows. Denote the q-dimensional parameter vector of interest by β, its true value by β_0, and the system's p-dimensional innovation vector by

$$\omega_t(\beta_0) \equiv \{u_t^1(\beta_0), \eta_t^1(\beta_0), \ldots, u_t^n(\beta_0), \eta_t^n(\beta_0), \varepsilon_t(\beta_0), \nu_t(\beta_0)\}'.$$

Let $z_{t-1}(\beta_0)$ be an m-dimensional vector of date $t-1$ information, uncorrelated with $\omega_t(\beta_0)$, to serve as instrumental variables. Since $\omega_t(\beta_0)$ has the interpretation of being a vector of prediction errors, this specification implies a family of orthogonality conditions, $E(\omega_t(\beta_0) \otimes z_{t-1}(\beta_0)) = 0$, which are used in estimation. The GMM estimator of β_0, b_T, minimizes the quadratic criterion function

$$\phi(\beta) = \left[\frac{1}{T}\sum_{t=1}^{T}(\omega_t(\beta) \otimes z_{t-1}(\beta))\right]'(S_T)^{-1} \times \left[\frac{1}{T}\sum_{t=1}^{T}(\omega_t(\beta) \otimes z_{t-1}(\beta))\right], \tag{14}$$

where

$$S_T = \frac{1}{T}\sum_{1}^{T}(\omega_t(b)\omega_t(b)' \otimes z_{t-1}(b)z_{t-1}(b)'),$$

and b is a consistent estimator of β_0. Asymptotically, $\sqrt{T}(b_T - \beta_0)$ is normally distributed with mean zero and covariance matrix

$$\Omega = (D'S^{-1}D)^{-1}, \tag{15}$$

where

$$D = E[\partial(\omega_t(\beta_0) \otimes z_{t-1}(\beta_0))/\partial\beta'],$$

and

$$S = E[\omega_t(\beta_0)\omega_t(\beta_0)' \otimes z_{t-1}(\beta_0)z_{t-1}(\beta_0)']$$

is the spectral density matrix of $(\omega_t(\beta_0) \otimes z_{t-1}(\beta_0))$ at frequency 0. The matrices D and S are consistently estimated by their sample logs,

$$D_T = \frac{1}{T}\sum_{1}^{T}[\partial(\omega_t(b_T) \otimes z_{t-1}(b_T))/\partial\beta'],$$

and

S_T as defined above.

This choice of the weighting matrix S_T yields a heteroskedasticity-consistent estimate of the covariance matrix of b_T.

In what follows, $\omega_t(\beta)$ depends on past $\omega(\beta)$'s, so derivatives of $\omega_t(\beta)$ with respect to the parameter vector, β, will involve derivatives of $\omega_{t-1}(\beta), \omega_{t-2}(\beta), \ldots$ Because of this recursive structure, analytical derivatives are cumbersome and numerical derivatives are used in minimizing $\phi(\beta)$ as well as in calculating the standard errors according to (15). In estimation, initial values of $\omega_t(\beta)$ (i.e., observations prior to the sample period) are set to their theoretical values of zero, under the null hypothesis. The standard two-step procedure is used in estimation. An initial guess value of β is used to construct the weighting matrix, S_T, and (14) is minimized. This first-step estimator is consistent, but does not have the desired asymptotic distribution. Final estimates are obtained by repeating the minimization of (14) using the first-step estimates in the construction of the weighting matrix S_T.

The econometric specification represents a joint hypothesis that includes the CAPM as an appropriate model in pricing forward foreign exchange contracts, a particular information structure, specifications for the conditional expectations, rational expectations, and the appropriateness of the data used. To test this joint hypothesis, Hansen's specification test is used. Because the first-order conditions of the estimation procedure set q linear combinations of the mp orthogonality conditions to zero, the model is exactly identified when $q = mp$. When $mp > q$, there are $mp - q$ linearly independent orthogonality conditions that should be close to zero if the model is correctly specified. Hansen has shown that $T\phi(b_T)$ is asymptotically distributed as a chi-square variate with $mp - q$ degrees of freedom under the null hypothesis. This chi-square test is used to test the model's overidentifying restrictions.

5. Empirical results

Two special cases of the system (13a)–(13d) are considered. Owing to the size of the system and the considerable nonlinearities involved, a strong attempt is made to maintain model parsimony.

5.1. Autoregressions

The first model to be considered is

$$\rho_t^i = \left[\frac{\alpha_0^i + \alpha_1^i \varepsilon_{t-1} u_{t-1}^i}{\gamma_0 + \gamma_1 \varepsilon_{t-1}^2}\right]\left[a_0 + a_1 r_{t-1}^e\right] + u_t^i, \qquad i = \mathrm{DM, SF, JY, BP}, \tag{16a}$$

$$r_t^e = a_0 + a_1 r_{t-1}^e + \varepsilon_t, \tag{16b}$$

$$u_t^i \varepsilon_t = \alpha_0^i + \alpha_1^i u_{t-1}^i \varepsilon_{t-1} + \eta_t^i, \qquad i = \mathrm{DM, SF, JY, BP}, \tag{16c}$$

$$\varepsilon_t^2 = \gamma_0 + \gamma_1 \varepsilon_{t-1}^2 + \nu_t. \tag{16d}$$

That is, the conditional covariances, the excess return on the reference portfolio, and the reference portfolio's square forecast error are assumed to follow AR(1) processes. The autoregressive specification of the sequences $\{\varepsilon_t^2\}$ and

$\{\varepsilon_t u_t^i\}$ follows from the ARCH modeling strategy of Engle (1982). This specification is adopted to keep the problem tractable and reflects the idea that past observations on a variable provide the most useful information in predicting its future. To ensure stationarity in the product sequences, it is required that $|\alpha_1^i| < 1$, $i = 1,\ldots,4$, and that $0 < \gamma_1 < 1$.

Obviously, the set of legitimate instruments is large, but is kept small here for three reasons. First, the system is quite large, and expanding the instrument set imposes a greater computational burden. Second, it is likely that, with a sufficient search over instruments and a large enough instrument set, the econometric specification can be rejected, but proceeding in this fashion provides little useful information about the limitations and usefulness of the theory. Third, Tauchen (1986) reports evidence, based on simulation results, that expansion of the instrument set introduces serious bias to the GMM estimator in small samples. He finds for the single-equation model he studies that the test of the overidentifying restrictions is not affected. However, it is not clear how this specification test would be affected in a simultaneous-equation setup with cross-equation restrictions like the one considered here. To err on the side of caution, I keep the instrument set small.

The model is estimated for the four currencies simultaneously. Examination of (16) indicates that this will be a system of ten equations. Owing to the size of this system, estimation is performed using a constant, the squared innovation on the reference portfolio, and the average product of the forecast error in the reference portfolio and each of the currencies. That is, $z_{t-1}(\beta) = \{\text{constant}, \varepsilon_{t-1}^2(\beta), \varepsilon_{t-1}(\beta)\sum_1^4 u_{t-1}^j(\beta)/4\}$.[9] These three instruments generate a family of thirty orthogonality conditions; the results are reported in table 2.

For each choice of the reference portfolio, the test of the overidentifying restrictions, given by the chi-square statistic with eighteen degrees of freedom, does not reject the model at standard significance levels. For example, the largest chi-square statistic is 21.35 with a confidence level of 0.7479 when the equal-weighted international portfolio is used as the reference portfolio. All the parameters appear to be reasonable and most are estimated with precision. The significance of the parameters of the conditional variance and covariances (γ_0, γ_1, α_0, and α_1) implies that there is significant time variation in the betas. This effect is strongest for the DM and BP and weakest for the JY. Further, the results do not depend on the choice of reference portfolio.

[9] The model was initially estimated for each currency on an individual basis with a constant, the squared innovation on the reference portfolio, and the product of the innovation on the reference portfolio and the individual currency as instruments (not reported to economize on space). This seemed a natural choice for the instrumental variables, since these variables appear explicitly in the model. Simultaneous estimation across currencies was first attempted using a constant, the squared innovation in the reference portfolio, and each of the products of the currency and reference portfolio innovations as instrumental variables, which is a set of six instrumental variables and hence sixty orthogonality conditions. These attempts ran into convergence problems and proved unmanageable. This led to the use of the instrumental variables described in the text.

Table 2

Generalized method-of-moments estimates of single-beta representation from July 1973 to December 1985 on forward contracts for the German mark, Swiss franc, Japanese yen, and British pound in relation to the U.S. dollar.[a] Reference portfolio excess returns modeled as AR(1). Time-series averages of estimated conditional betas, test of the model's overidentifying restrictions, and Wald test of constant betas.

$$\rho_t^i = \left[\left(\alpha_0^i + \alpha_1^i u_{t-1}^i \varepsilon_{t-1}\right) / \left(\gamma_0 + \gamma_1 \varepsilon_{t-1}^2\right)\right]\left[a_0 + a_1 r_{t-1}^e\right] + u_t^i \quad (17a)$$

$$u_t^i \varepsilon_t = \alpha_0^i + \alpha_1^i u_{t-1}^i \varepsilon_{t-1} + n_t^i \quad (17b)$$

$$r_t^e = a_0 + a_1 r_{t-1}^e + \varepsilon_t \quad (17c)$$

$$\varepsilon_t^2 = \gamma_0 + \gamma_1 \varepsilon_{t-1}^2 + \nu_t \qquad i = \text{DM, SF, JY, BP} \quad (17d)$$

Reference portfolio					Currency				Test of overidentifying restrictions[c]	Wald test of constant betas[d]
a_0	a_1	γ_0	γ_1		DM	SF	JY	BP	(p-value)	(p-value)
					(A) *Reference Portfolio: Domestic Equities (NYSE*					
0.0022	0.0778	0.0020*	0.1020	α_0	0.20 × 10^−3*	0.12 × 10^−3*	0.13 × 10^−3	0.10 × 10^−3	14.04	26.83*
(0.454)	(0.906)	(7.079)	(1.649)		(3.627)	(2.193)	(1.252)	(1.790)	(0.273)	(0.999)
				α_1	0.1268*	0.0618	−0.0735	0.1877*		
					(2.313)	(0.910)	(0.727)	(3.526)		
	Average beta[b]				0.1046	0.0597	.0567	0.0665		
					(B) *Reference Portfolio: Value-Weighted International Equities*					
0.0090*	0.1704*	0.0013*	0.1365*	α_0	0.15 × 10^−3*	0.86 × 10^−4*	0.12 × 10^−3	0.11 × 10^−3	14.88	30.94*
(2.881)	(1.949)	(5.513)	(3.156)		(2.228)	(1.045)	(1.665)	(1.630)	(0.329)	(1.000)
				α_1	0.1368*	0.0946	0.0653	0.2191*		
					(2.889)	(1.529)	(0.597)	(4.438)		
	Average beta[b]				0.1163	0.0656	0.0887	0.1003		
					(C) *Reference Portfolio: Equally-Weighted International Equities*					
0.[illegible]18*	0.1374*	0.0154*	0.2146*	α_0	−0.14 × 10^−5	0.33 × 10^−4	0.31 × 10^−3	0.89 × 10^−4	22.73	83.29*
(10.15)	(2.237)	(3.439)	(4.213)		(0.017)	(0.160)	(0.809)	(0.261)	(0.798)	(1.000)
				α_1	0.1299*	0.1296*	−0.1157	0.2125*		
					(3.392)	(2.993)	(1.050)	(5.093)		
	Average beta[b]				0.0025	0.0034	0.0149	0.0103		

[a] Absolute value of asymptotic t-ratios in parentheses. An asterisk (*) indicates significance at the 5% level. The instrument set is $z_{t-1} = \{\text{constant}, \varepsilon_{t-1}^2, \varepsilon_{t-1}(\sum_{i=1}^4 u_{t-1}^i)/4\}$.

[b] This is the time-series average of the estimated conditional beta.

[c] Test of the overidentifying restrictions implied by the model reported here. Test statistic is distributed as $\chi^2(18)$.

[d] Test of the restrictions $\gamma_1 = \alpha_1^i = 0$, i = DM, SF, JY, BP, implied by a constant beta specification. Test statistic is distributed as $\chi^2(5)$.

To further investigate the importance of the time-varying beta specification, I perform a Wald test of a constant beta version of the model that sets $\gamma_1 = \alpha_1^i = 0$, $i = \mathrm{DM, SF, JY, BP}$. This constant beta specification still allows time variation in the risk premium, since the expected excess return on the reference portfolio varies over time. The Wald statistic for this test is distributed as a chi-square variate with five degrees of freedom and is also reported in table 2. The constant beta specification can be rejected at better

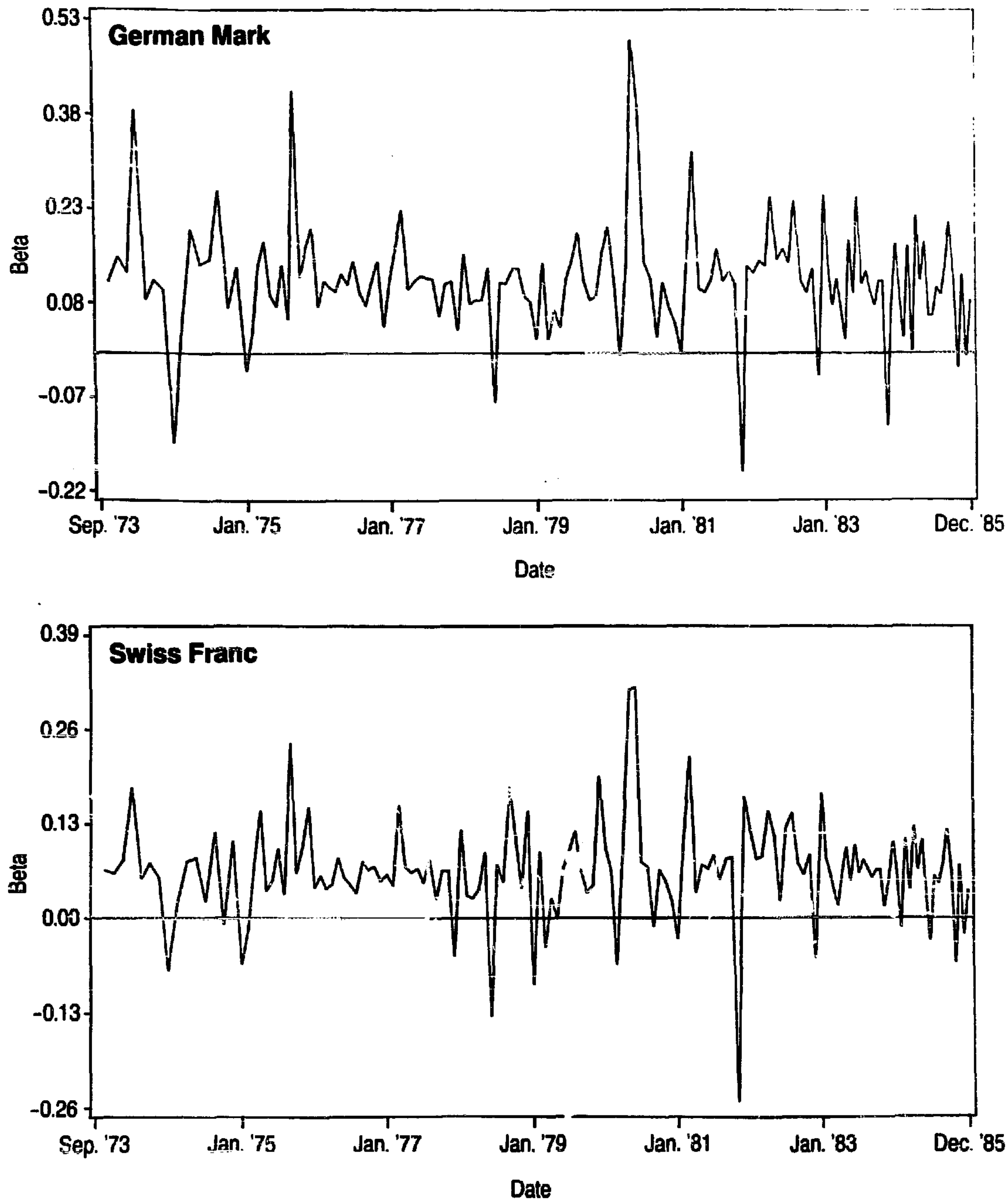
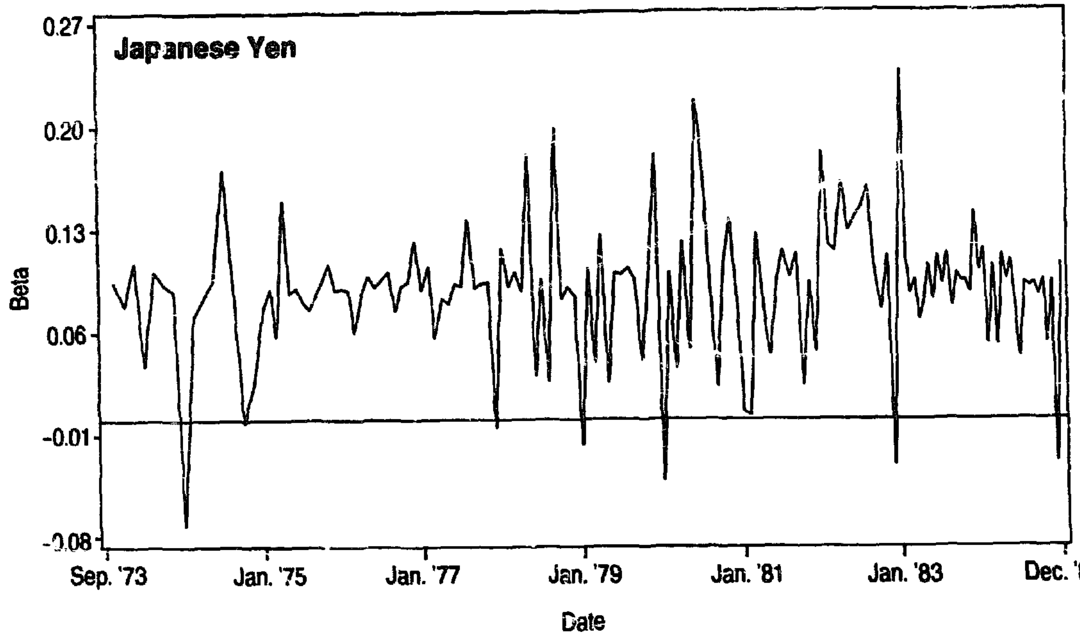


Fig. 1. Monthly conditional betas for open positions on forward foreign exchange contracts implied by estimates in table 2, panel B, 1973–1975. The beta is given by $(\alpha_0 + \alpha_1 u_{t-1}\epsilon_{t-1})/(\gamma_0 + \gamma_1 \epsilon_{t-1}^2)$.



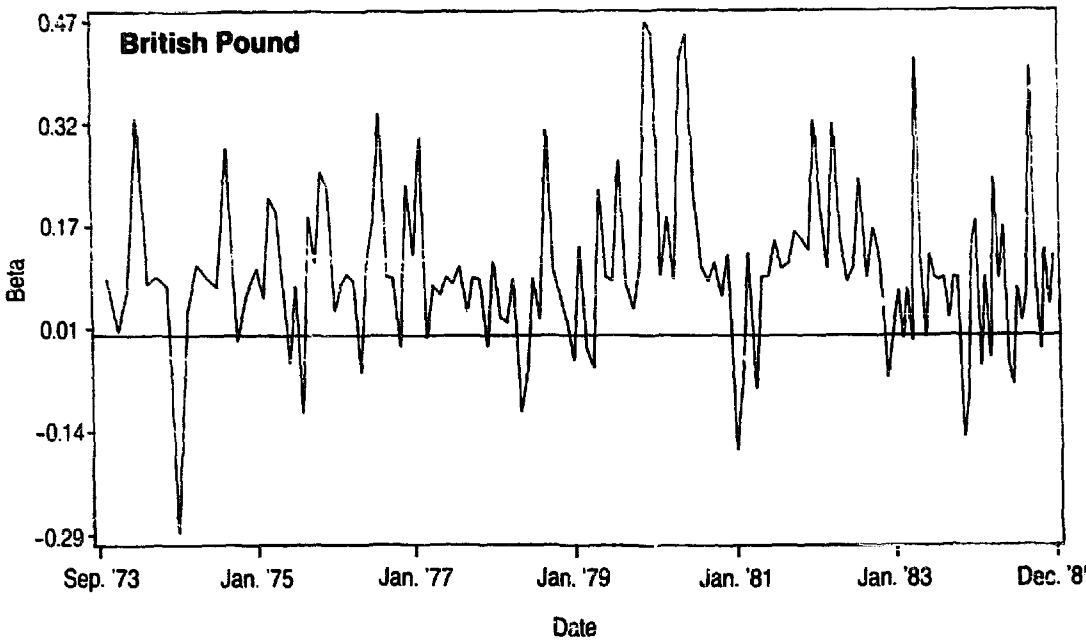


Fig. 1 (continued)

than the 1% level regardless of the reference portfolio used. These results reaffirm the importance of modeling the betas in a conditional setting.

Qualitatively, the estimated betas from the different systems behave similarly. Fig. 1 displays the time series of the betas generated by the estimates in panel B of table 2. The betas range roughly from -0.3 to 0.5, display a fair amount of time variation, and fluctuate from positive to negative for each currency. On *average*, however, each of the betas is positive over the estimation period. As can be seen from table 2, the time-series averages of the

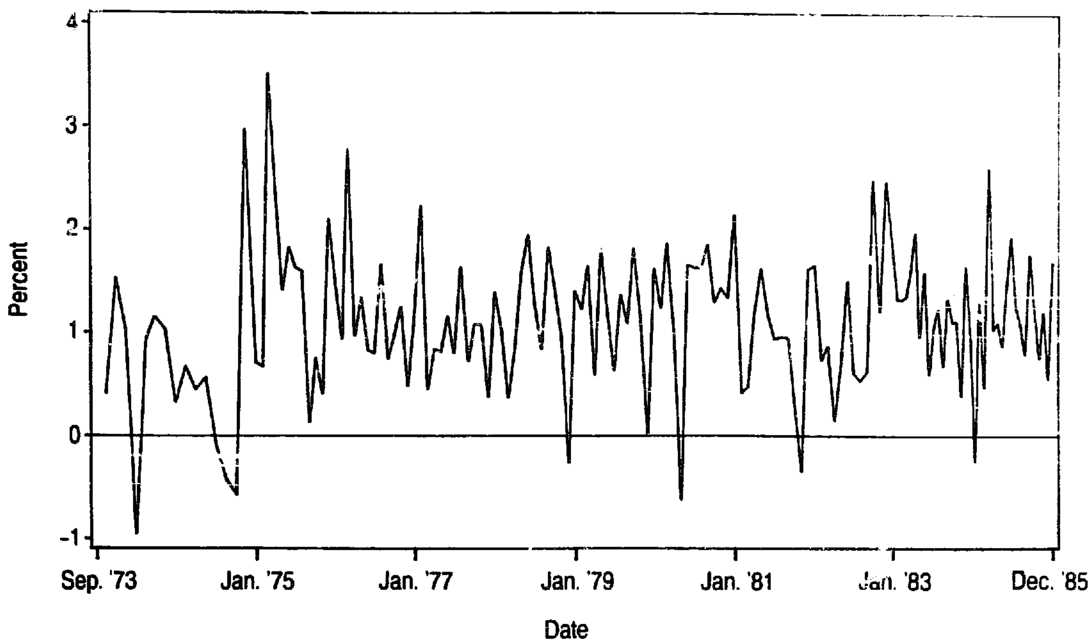


Fig. 2. Monthly risk premium for value-weighted international equity portfolio implied by estimates in table 2, panel B, 1973–1985. The reference portfolio's excess return, r_t^e, follows an AR(1) process.

$$r_t^e = a_0 + a_1 r_{t-1}^e + \varepsilon_t \qquad \text{(16b)}$$

estimated conditional betas are not much different from the implied unconditional betas, $(\alpha_0/\gamma_0)(1-\gamma_1)/(1-\alpha_1)$.

The results of this section suggest that the data are generally consistent with the model. The model's overidentifying restrictions cannot be rejected at better than the 25% level regardless of the reference portfolio used. One problem arises with this specification, however, because there are ex post negative values of the reference portfolio's excess return. The AR specification on the reference portfolio thus admits the possibility that the risk premium on the reference portfolio can be negative, which in the context of the CAPM is difficult to explain. In fact, some negative values are obtained. Fig. 2 displays the estimated risk premium on the value-weighted international reference portfolio obtained from panel B of table 2. Although this does not seem to be a serious problem, the next subsection considers an alternative specification of the risk premium on the reference portfolio that other researchers have found useful.

5.2. Reference portfolio excess returns modeled as ARCH in the mean

An alternative specification is now considered in which the excess return on the reference portfolio follows an ARCH in the mean process. Specifically, I assume that the conditional mean of the reference portfolio's excess return depends linearly on the logarithm of its conditional variance. That is,

$$\mathrm{E}_{t-1} r_t^e = b_0 + b_1 \log\left(\mathrm{E}_{t-1}\varepsilon_t^2\right). \tag{17}$$

This specification, intended to capture the intertemporal risk and return tradeoff in the reference portfolio, is suggested by the findings of Campbell (1987), Engle, Lilien, and Robins (1987), and French, Schwert, and Stambaugh (1987). If $b_1 > 0$, investors are rewarded with high expected returns during periods of high variance or risk. The AR(1) specifications on the conditional variance and covariance are maintained so that (16a) and (16b) are replaced by

$$\rho_t^i = \left[\frac{\alpha_0^i + \alpha_1^i \varepsilon_{t-1} u_{t-1}^i}{\gamma_0 + \gamma_1 \varepsilon_{t-1}^2}\right]\left[b_0 + b_1 \log\left(\gamma_0 + \gamma_1 \varepsilon_{t-1}^2\right)\right] + u_t^i, \tag{16a$'$}$$

$$i = \mathrm{DM}, \mathrm{SF}, \mathrm{JY}, \mathrm{BP},$$

$$r_t^e = b_0 + b_1 \log\left(\gamma_0 + \gamma_1 \varepsilon_{t-1}^2\right) + \varepsilon_t. \tag{16b$'$}$$

Table 3 reports results from estimating this model jointly across the currencies, using the same instrumental variables as before. Again, there are twelve parameters to estimate and thirty orthogonality conditions. Here, the model's overidentifying restrictions are not rejected when the reference portfolio is either the domestic equity or the value-weighted international equity portfolio. There is some evidence against the model when the equal-weighted international equity portfolio is used, however. The chi-square statistic in this case is 30.52, which rejects the model at better than the 4% level. The parameters γ_0 and γ_1 continue to be estimated with a fair amount of precision. The estimates of α_1 are also generally precise and are larger than those found in table 2. As in the previous subsection, the parameters associated with the JY are estimated with the least precision.

This specification for the reference portfolio's risk premium also admits the possibility of negative values for example, during periods when the conditional variance is very small. It turns out, however, that the risk premium estimates are all positive during the estimation period. Fig. 3 plots the estimated risk premium implied by the estimates in panel B of table 3, using the value-weighted international equity portfolio as the reference asset. Qualitatively, the betas obtained here are similar to those in fig. 1 and are suppressed to economize on space.

Once again, I perform a Wald test of a constant beta version of the model. In this case, when γ_1 and α_1 are set to zero, the model reduces to the unconditional version of the CAPM in which both the betas and the risk

Table 3

Generalized method-of-moments estimates of single-beta representation from July 1973 to December 1985 on forward contracts for the German mark, Swiss franc, Japanese yen, and British pound in relation to the U.S. dollar.[a] Reference portfolio excess returns modeled as arch in the mean. Time-series averages of estimated conditional betas, test of the model's overidentifying restrictions, and Wald test of constant betas.

$$\rho^i_t = \left[\left(\alpha^i_0 + \alpha^i_1 u^i_{t-1}\varepsilon_{t-1}\right)/\left(\gamma_0 + \gamma_1\varepsilon^2_{t-1}\right)\right]\left[b_0 + b_1\log\left(\gamma_0 + \gamma_1\varepsilon^2_{t-1}\right)\right] + u^i_t \quad (17a')$$

$$r^e_t = b_0 + b_1\log\left(\gamma_0 + \gamma_1\varepsilon^2_{t-1}\right) + \varepsilon_t, \quad (17b')$$

$$u^i_t\varepsilon_t = \alpha^i_0 + \alpha^i_1 u^i_{t-1}\varepsilon_{t-1} + n^i_t, \quad (17c)$$

$$\varepsilon^2_t = \gamma_0 + \gamma_1\varepsilon^2_{t-1} + v_t \qquad i = \mathrm{DM, SF, JY, BP} \quad (17d)$$

Reference portfolio					Currency				Test of overidentifying restrictions[c]	Wald test of constant betas[d]
b_0	b_1	γ_0	γ_1		DM	SF	JY	BP	(p-value)	(p-value)
					(A) Reference Portfolio: Domestic Equities (NYSE)					
0.0115	0.0016	0.0017*	0.3265*	α_0	0.12×10^{-3}	0.83×10^{-4}	0.12×10^{-3}	0.10×10^{-3}	11.08	31.44*
(0.896)	(0.793)	(5.100)	(2.297)		(0.805)	(0.566)	(1.240)	(0.878)	(0.109)	(1.000)
				α_1	0.4378*	0.4004*	0.0610	0.4276*		
					(3.416)	(3.038)	(0.557)	(4.502)		
Average beta[b]					0.0891	0.0575	0.0581	0.0721		
					(B) Reference Portfolio: Value-Weighted International Equities					
0.0246	0.0020	0.0011*	0.2925*	α_0	-0.45×10^{-4}	0.11×10^{-3}	0.59×10^{-3}	0.71×10^{-4}	12.20	30.33*
(1.670)	(0.956)	(4.847)	(4.514)		(0.453)	(1.005)	(0.599)	(0.890)	(0.163)	(1.000)
				α_1	0.5736*	0.5172*	0.1525	0.3572*		
					(4.578)	(4.197)	(0.760)	(4.177)		
Average beta[b]					0.0195	−0.0472	0.0545	0.0805		
					(C) Reference Portfolio: Equally-Weighted International Equities					
1.2518*	0.3018*	0.0285*	0.0505*	α_0	-0.11×10^{-3}	0.32×10^{-4}	0.29×10^{-3}	0.48×10^{-4}	30.52*	72.83*
(8.716)	(8.399)	(7.906)	(6.801)		(0.080)	(0.149)	(1.735)	(0.213)	(0.967)	(1.000)
				α_1	0.1325*	0.1370*	−0.1081	0.2144*		
					(7.260)	(6.176)	(1.534)	(7.002)		
Average beta[b]					−0.0010	0.0049	0.0085	0.0076		

[a]Absolute value of asymptotic t-ratios in parentheses. An asterisk (*) indicates significance at the 5% level. The instrument set is $z_{t-1} = \{\text{constant}, \varepsilon^2_{t-1}, \varepsilon_{t-1}(\Sigma^4_1 u^i_{t-1})/4\}$.

[b]This is the time-series average of the estimated conditional beta.

[c]Test of the overidentifying restrictions implied by the model reported here. Test statistic is distributed as $\chi^2(18)$.

[d]Test of the restrictions $\gamma_1 = \alpha^i_1 = 0$, $i =$ DM, SF, JY, BP, implied by a constant beta specification. Test statistic is distributed as $\chi^2(5)$.

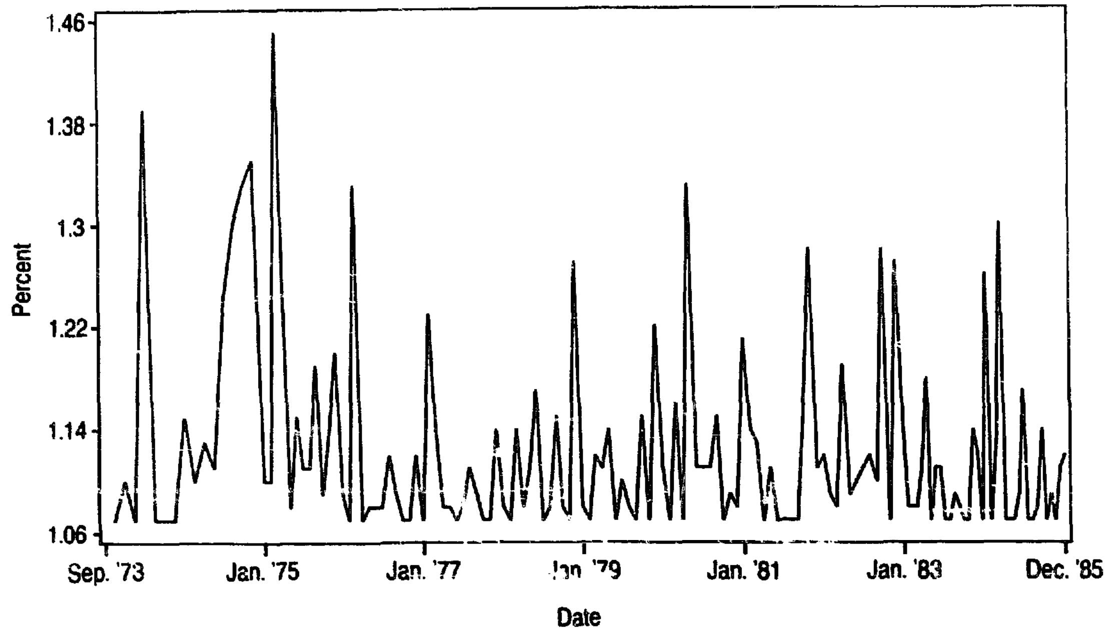


Fig. 3. Montly risk premium for value-weighted international equity portfolio implied by estimates in table 3, panel B, 1973–1985. The reference portfolio's excess return, r_t^e, follows an ARCH in the mean process.

$$r_t^e = b_0 + b_1 \log(\gamma_0 + \gamma_1 \varepsilon_{t-1}^2) + \varepsilon_t \qquad (16b')$$

premium on the reference portfolio are constant. The Wald statistic again is distributed as a chi-square variate with five degrees of freedom and is reported in table 3. As can be seen, the constant beta hypothesis can be rejected at better than the 1% level regardless of the empirical reference portfolio used.

To summarize, the data are generally consistent with the alternative specification as well. The reason for the similarity is that excess returns on the reference portfolio are difficult to predict. Since the forecast errors are large in relation to the explainable part (i.e., the R^2 is low), the resulting $\{\varepsilon_t^2\}$ process is similar despite the different specifications for the conditional mean of the reference portfolio's excess return. In one sense, the ARCH-M specification for the reference portfolio's excess return receives only modest support, as significant estimates for b_1 are obtained only when the empirical reference portfolio is the equal-weighted international portfolio. Although a formal analysis is not conducted, this specification may be preferred, since only positive estimates of the reference portfolio's risk premium are obtained.

6. Conclusions

This paper specifies and estimates a model of forward foreign exchange rate determination based on the single-beta capital asset pricing model. The model is specified from the perspective of a representative U.S. investor in a conditional setting that explicitly models time variation in the betas. Significant estimates of the parameters are obtained and tests of the overidentifying restrictions are not unfavorable to the model. The hypothesis that the betas are constant is strongly rejected. Moreover, the results generally hold up with variations in the empirical reference portfolio.

The evidence emerging here is consistent with the idea that the pricing of forward foreign exchange contracts is fundamentally no different from the pricing of any other financial asset. The evidence also supports the idea that deviations of the forward exchange rate from the expected future spot rate are due to a risk premium and not to irrationality among market participants.

There are number of issues not addressed here. For example, the information set that economic agents are assumed to condition on is somewhat restrictive. It might be useful to augment the conditioning set by including directly observable economic variables and to investigate the usefulness of non-ARCH processes for the conditional covariances and variances. Other models of the risk premium on the reference portfolio might also be investigated. Further, it may be useful to investigate other estimation strategies. The advantage of GMM is that it produces a robust estimator. One disadvantage is that it is not, in general, asymptotically efficient. These and other extensions are left for future work.

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