REAL AND NOMINAL EXCHANGE RATES IN THE LONG RUN: AN EMPIRICAL INVESTIGATION

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This paper reports on econometric tests of the hypothesis that purchasing power parity holds as a long-run relationship using data on eight industrialized countries during the flexible exchange rate period. The empirical work proceeds by (i) testing whether nominal exchange rates and relative price levels are co-integrated and (ii) conducting impulse response analysis of long-run exchange rate and relative price level changes. To explain the data, Keynesian models suggest that shocks during the estimation period were due principally to exogenous shifts in aggregate demand, while equilibrium models suggest that monetary factors have been relatively more important.

1. Introduction

The relative price of the home country's consumption basket in terms of the foreign country's consumption basket is often referred to as the real exchange rate. The doctrine of purchasing power parity is generally viewed as the open economy extension of the quantity theory and it implies that pure monetary disturbances leave the real exchange rate unaffected.

Purchasing power parity, as an exact relationship, has been studied extensively and the research in this area has left little doubt that significant violations of the doctrine are persistent and common.¹ Although the validity of purchasing power parity as a short-run relation is doubtful, it may still be valid as a long-run relation. For example, in models of exchange rate determination that highlight differential speeds of adjustment in asset and commodity markets, monetary shocks produce short-run violations of purchasing power parity [e.g. Aizenman (1984, 1986), Dornbusch (1976), and Mussa (1982)]. These models imply short-run movements of nominal

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¹Some recent contributions include Frenkel (1978, 1981a), Krugman (1978), and Hakkio (1984). No attempt is made here to survey the extensive literature on purchasing power parity. For surveys, see Dornbusch (1987) and Officer (1976).

exchange rates that are short-run movements of real exchange rates. However, offsetting movements of commodity price levels occur over time to leave the real exchange rate unchanged in the long run. If permanent real disturbances are an important source of variability, however, long-run violations of purchasing power parity will occur in these models as well as in models with zero transactions costs and full commodity price flexibility. In equilibrium models, supply shocks can alter the relative price between traded and non-traded goods [Stulz (1987)], or the relative price between domestic and foreign output [Lucas (1982)]. Differential rates of technical progress may also necessitate long-run changes in real exchange rates [Hsieh (1982)].

Thus, if long-run movements in real exchange rate are found to be consistent with purchasing power parity, this would imply that real disturbances have not been an important source of variability. On the other hand, rejections of long-run purchasing power parity would imply that real sources of variability have been important. Furthermore, it may be possible to identify the underlying real disturbances likely to have caused the rejections and to determine their importance relative to nominal disturbances within the context of a particular model.

This paper undertakes empirical tests of purchasing power parity as a long-run relationship and documents some empirical regularities concerning movements of real exchange rates, nominal exchange rates, and commodity price levels in the long run. The methodology used here is specifically designed to address long-run issues. The paper employs data from the modern experience with floating exchange rates for eight O.E.C.D. countries. The examination of bilateral relations using the United States, the United Kingdom, and Germany as 'home countries' produces little support for longrun purchasing power parity. Given this apparent violation of long-run purchasing power parity, three popular models of exchange rate determination are called upon to explain these findings. They are Dornbusch's (1976) log-linear version of the Mundell-Fleming model, Mussa's (1982) stochastic generalization of the Dornbusch model, and Lucas's (1982) two-country equilibrium model. In the context of Dornbusch's model, it is argued that shocks to aggregate demand are consistent with the observed long-run dynamics of real and nominal exchange rates and relative price levels. Mussa's model suggests that the main source of uncertainty has been due to variability in real factors. The Lucas model suggests the opposite conclusion: that variability of monetary factors have been relatively more important than variability of real factors as a source of uncertainty.

The paper is organized as follows. Section 2 describes the data employed and reports on some useful summary statistics. Section 3 examines the longrun purchasing power parity hypothesis by testing whether nominal exchange rate and relative national price levels are co-integrated sequences. It is found that the null hypothesis of *no* co-integration cannot be rejected at standard significance level. These results suggest that the long-run behavior of real exchange rates is inconsistent with purchasing power parity and the paper proceeds under this supposition. Section 4 conducts an impulseresponse analysis that documents the long-run dynamics of real and nominal exchange rates and national price levels. From this analysis, it is seen that both a unit innovation in the logarithm of the nominal exchange rate and a unit innovation in the logarithm of relative commodity price levels lead to a unit long-run change in the logarithm of the real exchange rate, but in opposite directions. This suggests that nominal exchange rates and commodity price levels are, by and large, unrelated in the long run, since innovations to each lead to permanent changes in the real exchange rate. An interpretation of the results is suggested in section 5. This section attempts to explain the violations of purchasing power parity in the long run and the documented empirical regularities in terms of the Dornbusch, Mussa, and Lucas models. Finally, section 6 discusses some limitations of the analysis and provides some concluding remarks.

2. The data and summary statistics

All variables are expressed in logarithmic form. Let p and p^* be the logarithms of the home and foreign price levels, and define the logarithm of the *relative* price level to be $\pi \equiv p - p^*$. Let e be the logarithm of the domestic currency price of a unit of foreign exchange. Then the absolute version of purchasing power parity implies that the logarithm of the real exchange rate, q, is zero. That is,

$$q_t = e_t - \pi_t = 0. \tag{1}$$

The relative version of purchasing power parity is eq. (1) in first differences. That is,

$$\Delta q_t = \Delta e_t - \Delta \pi_t = 0. \tag{2}$$

Monthly observations from June 1973 through February 1988 were used in estimation. Consumer price index data from the IMF's International Financial Statistics were used for commodity price data. Exchange rates were obtained from the Harris Bank's Foreign Exchange Weekly Review, which reports Friday closing prices in London. Because most items in the U.S. C.P.I. are sampled mid-month, exchange rates were chosen from the Friday occurring nearest to the 15th of each month to coincide with mid-month sampling of the C.P.I. The countries involved are Belgium, Canada, France, Germany, Italy, Japan, and the United Kingdom. Three sets of bilateral

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Cross-correlations of changes in logarithms of nominal exchange rates and relative price levels: $corr(\Delta e_t, \Delta \pi_{t-k})$.

	k = lag of prices relative to exchange rate						
Country	-3	-2	-1	0	1	2	3
A. U.S. doll	ar as bas	se currenc	у				
Canada	0.082	0.035	0.073	0.033	0.002	0.049	0.102
U.K.	0.085	0.120	0.021	0.140	0.104	0.052	0.029
Belgium	0.037	0.107	0.031	0.021	0.079	0.030	0.034
France	0.108	0.172	0.076	0.006	0.042	0.172	0.064
Germany	0.090	0.108	0.068	0.048	0.011	0.049	0.043
Japan	0.035	0.036	0.011	0.035	0.051	0.080	0.055
Italy	0.023	0.135	0.133	0.110	0.149	0.098	0.023
B. British p	ound as	base curre	ency				
Canada	0.025	0.077	0.013	0.182	0.019	0.072	0.058
Belgium	0.047	0.007	0.000	0.030	0.094	0.007	0.028
France	0.034	0.029	0.062	0.011	0.102	0.032	0.046
Germany	0.056	0.045	0.026	0.039	0.153	0.034	0.040
Japan	0.076	0.034	0.052	0.176	0.025	0.037	0.036
Italy	0.032	0.072	0.039	0.078	0.106	0.008	0.013
C. German	mark as	base curr	ency				
Canada	0.094	0.057	0.015	0.003	0.013	0.004	0.055
Belgium	0.010	0.008	0.052	0.033	0.095	0.031	0.130
France	0.092	0.019	0.027	0.044	0.018	0.033	0.094
Japan	0.059	0.019	0.100	0.020	0.082	0.087	0.039
Italy	0.036	0.035	0.175	0.186	0.196	0.069	0.025

relations are examined with the United States (U.S.), the United Kingdom (U.K.), and Germany each serving as the home country.

Table 1 reports the cross correlations of nominal exchange rate changes and relative inflation rates estimated from 3 leads to 3 lags. These calculations reveal that exchange rate changes and relative price level changes are, by and large, uncorrelated at these leads and lags. Similarly, contemporaneous movements in nominal exchange rates and relative price levels appear to be uncorrelated. The contemporaneous sample correlations range from 0.14 for U.S.-U.K. to 0.006 for U.S.-France. The largest contemporaneous correlation is 0.18 for U.K.-Canada. Table 2 reports sample crosscorrelations between changes in real and nominal exchange rates from 3 leads to 3 lags. Here, the contemporaneous movements in real and nominal exchange rates are almost perfectly correlated for each of the seven currencies, while correlations at non-zero leads and lags are close to zero. Table 3 reports sample standard deviations of changes in real and nominal exchange rates and the inflation differential. Real exchange rates are slightly more variable than nominal rates except for the U.S.-Italy and Germany-Italy pairs. The nominal exchange rate is roughly four times more variable than the inflation differential. The ratio of the standard deviation of $\Delta \pi$ to

Table	2
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eal excha	nge rate t	o nomina	l exchange	e rate		
-3	-2	-1	0	1	2	3
lar as bas	se currenc	;y				
0.006	0.017	0.093	0.951	0.070	0.043	0.012
0.085	0.008	0.070	0.973	0.051	0.024	0.099
0.111	0.063	0.046	0.991	0.040	0.074	0.112
0.140	0.057	0.006	0.994	0.002	0.057	0.146
0.006	0.065	0.070	0.993	0.079	0.083	0.021
0.101	0.013	0.113	0.966	0.103	0.042	0.078
0.064	0.053	0.082	0.983	0.079	0.060	0.064
ound as	base curre	ency				
0.081	0.006	0.157	0.969	0.149	0.007	0.074
0.075	0.104	0.090	0.960	0.064	0.104	0.054
0.078	0.006	0.073	0.968	0.063	0.005	0.075
0.055	0.095	0.030	0.969	0.013	0.075	0.052
0.162	0.035	0.062	0.964	0.070	0.016	0.151
0.020	0.024	0.154	0.949	0.133	0.044	0.014
mark as	base curr	ency				
0.003	0.035	0.124	0.991	0.120	0.043	0.001
0.011	0.055	0.131	0.955	0.175	0.043	0.047
0.070	0.058	0.023	0.977	0.025	0.061	0.031
0.016	0.009	0.083	0.959	0.135	0.039	0.011
0.185	0.092	0.195	0.959	0.188	0.102	0.188
	eal excha: -3 lar as bas 0.006 0.085 0.111 0.140 0.006 0.101 0.064 bound as 0.081 0.075 0.078 0.055 0.162 0.020 mark as 0.003 0.011 0.070 0.016 0.185	eal exchange rate t -3 -2 lar as base currence 0.006 0.017 0.085 0.008 0.111 0.063 0.140 0.057 0.006 0.065 0.101 0.013 0.064 0.053 pound as base currence 0.081 0.081 0.006 0.075 0.104 0.075 0.104 0.075 0.095 0.162 0.035 0.020 0.024 mark as base currence 0.035 0.011 0.055 0.020 0.024	eal exchange rate to nomina -3 -2 -1 lar as base currency0.0060.0170.0930.0850.0080.0700.1110.0630.0460.1400.0570.0060.0060.0650.0700.1010.0130.1130.0640.0530.08200und as base currency0.0810.0060.0750.1040.0900.0750.0950.0300.1620.0350.0620.0200.0240.154mark as base currency0.0030.0350.1240.0110.0550.1310.0700.0580.0230.0160.0090.0830.1850.0920.195	eal exchange rate to nominal exchange -3 -2 -1 0lar as base currency 0.006 0.017 0.093 0.951 0.085 0.008 0.070 0.973 0.111 0.063 0.046 0.991 0.140 0.057 0.006 0.994 0.006 0.065 0.070 0.993 0.101 0.013 0.113 0.993 0.101 0.013 0.113 0.966 0.064 0.053 0.082 0.983 bound as base currency 0.081 0.006 0.157 0.081 0.006 0.073 0.968 0.075 0.104 0.090 0.960 0.078 0.006 0.073 0.968 0.055 0.095 0.030 0.969 0.162 0.035 0.124 0.991 0.003 0.035 0.124 0.991 0.011 0.055 0.131 0.955 0.070 0.058 0.023 0.977 0.016 0.009 0.083 0.959 0.185 0.092 0.195 0.959	all exchange rate to nominal exchange rate -3 -2 -1 0 lar as base currency 0.006 0.017 0.093 0.951 0.070 0.085 0.008 0.070 0.973 0.051 0.111 0.063 0.046 0.991 0.040 0.140 0.057 0.006 0.994 0.002 0.006 0.065 0.070 0.993 0.079 0.101 0.013 0.113 0.966 0.103 0.064 0.053 0.082 0.983 0.079 0.011 0.013 0.113 0.966 0.149 0.075 0.104 0.090 0.960 0.064 0.075 0.104 0.090 0.960 0.064 0.078 0.006 0.073 0.968 0.063 0.055 0.095 0.030 0.969 0.113 0.162 0.035 0.124 0.991 0.120 0.011 0.055 0.131 0.955 0.175 0.070 0.058 0.023 0.977 0.025 0.016 0.009 0.083 0.959 0.135 0.185 0.092 0.195 0.959 0.188	a lex change rate to nominal exchange rate -3 -2 -1 0 1 2 lar as base currency 0.006 0.017 0.093 0.951 0.070 0.043 0.085 0.008 0.070 0.973 0.051 0.024 0.111 0.063 0.046 0.991 0.040 0.074 0.140 0.057 0.006 0.994 0.002 0.057 0.006 0.065 0.070 0.993 0.079 0.083 0.101 0.013 0.113 0.966 0.103 0.042 0.064 0.053 0.082 0.983 0.079 0.060 pound as base currency 0.081 0.006 0.157 0.969 0.149 0.007 0.075 0.104 0.090 0.960 0.064 0.104 0.078 0.006 0.073 0.968 0.063 0.005 0.055 0.995 0.030 0.969 0.113 0.075 0.162 0.035 0.124 0.991 0.120 0.043 0.011 0.055 0.131 0.955 0.175 0.043 0.070 0.058 0.023 0.977 0.025 0.061 0.016 0.009 0.083 0.959 0.135 0.039 0.185 0.092 0.195 0.959 0.188 0.102

Cross-correlations of changes in logarithms of nominal exchange rates and real exchange rates: $corr(\Delta e_{t}, \Delta q_{t-k})$.

the standard deviation of Δe ranges from 32.8 to 11.0 percent with the U.S. as home country, from 32.7 to 24.8 percent with the U.K. as home country, and from 29.5 to 13 percent when Germany is the home country.

These statistics suggest why purchasing power parity breaks down in the short run, and why it may be reasonable to represent real exchange rates as a martingale.² The low variability in inflation rate differentials relative to nominal exchange rate movements implies that real exchange rate movements are dominated by nominal exchange rate movements. Thus, in the short run nominal exchange rate changes are, by and large, real exchange rate changes in relative price levels. Little in the way of a systematic relationship between commodity price level movements and nominal exchange rate movements appears to be present in the short run. Since nominal exchange rates approximately follow a martingale, it is not surprising that real exchange rates can also be approximated by the martingale model. It has

²Adler and Lehmann (1983), Frenkel (1981b), and Roll (1979) report evidence that the real exchange rate follows a martingale. Darby (1983) estimates ARIMA models for the real exchange rate and finds that first differencing is required. Cumby and Obstfeld (1984) report evidence inconsistent with the martingale specification.

	Inflation	Change in logarithm of nominal exchange	Change in logarithm of real exchange
	differential	rate	rate
Country	(Δπ)	(<i>∆e</i>)	(<i>∆q</i>)
A. Dollar as	base currency		
Canada	0.00394	0.01200	0.01277
U.K.	0.00777	0.03120	0.03326
Belgium	0.00454	0.03367	0.03418
France	0.00367	0.03335	0.03371
Germany	0.00368	0.03197	0.03246
Japan	0.00894	0.03263	0.03412
Italy	0.00570	0.03086	0.03070
B. Pound as	base currency		
Canada	0.00828	0.03047	0.03306
Belgium	0.00773	0.02683	0.02819
France	0.00699	0.02773	0.02872
Germany	0.00705	0.02840	0.02958
Japan	0.00970	0.03238	0.03548
Italy	0.00864	0.02644	0.02841
C. DM as ba	ise currency		
Canada	0.00421	0.03218	0.03250
Belgium	0.00382	0.01246	0.01315
France	0.00357	0.01681	0.01701
Japan	0.00878	0.02968	0.03103
Italy	0.00573	0.02068	0.02043

Sample stand	dard deviation	s of inflation	differentials	and	changes	in
lo	garithms of rea	al and nomin	al exchange	rates.	-	

Table 3

been argued that the inability to reject the martingale model for real exchange rates is evidence that purchasing power parity is violated in the long run, since real exchange rate innovations represent permanent changes. Attention is now directed explicitly toward the long-run issues.

3. Co-integration tests

The methodology employed in this section is based on Engle and Granger (1987), and the interested reader is referred there for details. In this paper, two sequences of random variables $\{x_t\}$ and $\{y_t\}$ will be said to be co-integrated if:

(1) they are non-stationary in levels,

(2) they are stationary in first differences, and

(3) there exists a linear combination of the levels, $u_t = x_t + \beta y_t$, which is stationary.

 β will be called the co-integrating constant. Engle and Granger's treatment of co-integrated sequences is more general in that they allow for higherordered non-stationarities in the levels of the observations, but the above definition will suffice for the purposes at hand.

In applications, economic theory might imply an exact relationship between two variables, say $x_t + \beta y_t = 0$. At any point in time, however, it is likely that the system will display deviations from the long-run equilibrium, and that the exact relationship is violated. A test of the long-run consequences of the theory can be undertaken by examining whether $\{x_t\}$ and $\{y_t\}$ are co-integrated. This approach views short-run deviations from equilibrium as the rule rather than the exception, but requires that these deviations be transient. Although observations on $\{x_t\}$ and $\{y_t\}$ will drift over time, if they are co-integrated, they cannot drift far apart from one another. Thus, a longrun interpretation arises naturally: if $\{x_t\}$ and $\{y_t\}$ are co-integrated, they share a common long-run component or stochastic trend.

To test for co-integration, $\{x_t\}$ and $\{y_t\}$ must first be determined to be non-stationary in levels, but stationary in first differences. Next, x_t is regressed on y_t or vice versa. Asymptotically, it does not matter which regression is used. Call this the co-integrating regression and let $\{u_t\}$ be the residual from this regression. Least squares provide a consistent estimator of the co-integrating constant, β , and convergence occurs rapidly [Engle and Granger (1987), Stock (1987)]. Finlly, an augmented Dickey-Fuller test for a unit root is performed on the equilibrium error sequence, $\{u_t\}$, implied by the co-integrating regression. The procedure is as follows. Let

$$(1-\theta_1 L - \theta_2 L^2)(1-\rho L)u_t = v_t, \tag{3}$$

where $\{v_t\}$ is an i.i.d. sequence and L is the lag operator. The objective is to test the hypothesis that $\rho = 1$, or that the equilibrium error sequence has a unit root. Eq. (3) can be rewritten as:

$$u_{t} = -\phi_{1}u_{t-1} + \phi_{2}\Delta u_{t-1} + \phi_{3}\Delta u_{t-2} + v_{t}, \qquad (4)$$

where $\phi_1 = (1 - \rho)(1 - \theta_1 - \theta_2)$. Under the null hypothesis of *no* co-integration, $\phi_1 = 0$. The usual *t*-statistic or studentized coefficient for ϕ_1 , denoted by τ , is used to conduct inference. The distribution of τ in this case is not standard. When $\{u_t\}$ are observations on data, critical values in Fuller (1976) are appropriate. When $\{u_t\}$ is estimated from a regression, these critical values reject the null too often, but Engle and Granger have tabulated the appropriate critical values and these are used instead.

To test purchasing power parity as a long-run relationship, I test for cointegration of $\{e_t\}$ and $\{\pi_t\}$. The first two columns of table 4 report

	Augmented Ditests for unit rand π	ckey–Fuller oots in <i>e</i>	Augmented Dickey–Fuller tests for unit roots in residuals from co-integrating regression			
Country	u is the nominal exchange rate $(u=e)^a$	u is the relative price level $(u=\pi)^{b}$	u is the real exchange rate $(u=q)^a$	<i>u</i> is residual from regression of <i>e</i> on π^{b}	<i>u</i> is residual from regression of π on e^{b}	
A. U.S. as he	ome country					
Canada	1.1834	0.3597	1.3397	1.3401	0.9923	
U.K.	1.6661	4.2316*	-	-	_	
Belgium	1.1115	0.9640	1.0720	1.1234	1.0026	
France	1.1307	1.4760	1.1920	1.2049	0.9628	
Germany	1.0389	0.8966	1.3359	1.1641	1.1641	
Japan	0.4376	0.7481	0.7347	0.9772	1.2648	
Italy	1.6817	1.9945	1.2621	1.0185	0.6149	
B. U.K. as h	iome country					
Canada	1.8183	4.2195*	-	-	_	
Belgium	2.0300	2.5600	1.3683	1.8768	1.3562	
France	1.7621	3.0188*	-	-	_	
Germany	1.3085	4.5491*	-	-		
Japan	0.2193	0.6197	1.1959	1.4733	1.5970	
Italy	1.5177	0.5044	2.0227	2.0301	1.1769	
C. Germany	as home country					
Canada	0.8386	0.8653	1.4454	1.3889	1.2847	
Belgium	0.0175	2.5796	2.0831	2.3614	3.1807*	
France	0.5245	2.1839	2.8161	2.8832	2.9463	
Japan	0.7437	6.7620*	-	-	-	
Italy	1.7406	2.5360	i.1441	2.4942	2.3433	

Table	4
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Co-integration tests: Studentized coefficients for ϕ_1 in the regression $\Delta u_i = -\phi_1 u_{i-1} + \phi_2 \Delta u_{i-1} + \phi_3 \Delta u_{i-2} + v_i$.

Note: An asterisk indicates significance at the 5% level.

^aA constant is included in these regressions. The critical values [Fuller (1976)] are 2.89 at the 5% level and 3.14 at the 1% level.

^bNo constant in these regressions. Critical values tabulated by Engle and Granger (1987) are 3.17 at the 5% level and 3.73 at the 1% level. – indicates the second stage of the co-integration test was not required since the unit root hypothesis could be rejected for relative price levels but not for nominal exchange rates for these bi-lateral relations.

studentized coefficients on ϕ_1 in augmented Dickey-Fuller tests for unit roots on $\{e_t\}$ and $\{\pi_t\}$. It is seen that the logarithms of nominal exchange rates appear uniformly to be non-stationary in levels, but stationary in first differences. For logarithms of relative price levels, the unit root hypothesis can be rejected at the 5 percent level for the U.S.-U.K., U.K.-Canada, U.K.-France, U.K.-Germany and Germany-Japan pairs. This implies from the outset that there is no co-integration in these cases, and these country pairs are excluded from further analysis.

The tests of co-integration are next performed by constraining the cointegrating constant to unity, which is just the test for whether the real exchange rate has a unit root or not. Column 3 of table 4 reports the studentized coefficients on ϕ_1 when the co-integrating constant is constrained to unity. The 5 percent critical value is 2.89, and there is little evidence of co-integration. The larger statistics are 1.339 for ther U.S.-Canada, 2.02 for the U.K.-Italy, and 2.81 for Germany-France.

Alternatively, it might be of interest to estimate the co-integrating constant and then test for co-integration. Although a non-unity valued co-integrating constant is not implied by the theory, if it is established that $\{e_i\}$ and $\{\pi_i\}$ are co-integrated, one can in principle go back and test the hypothesis that the co-integration constant is equal to one, since Stock (1987) shows how the distribution of the OLS estimator can be computed in this case. Column 4 of table 4 shows the studentized coefficients, τ , on the coefficient ϕ_1 , where the sequence $\{u_t\}$ is estimated from a regression of e_t on π_t , and column 5 reports the results when $\{u_t\}$ is obtained from a regression of π_t on e_t . The null hypothesis of no co-integration can still not be rejected at the 5 percent level when the U.S. or the U.K. are viewed as the home countries. There is some evidence against the null hypothesis in the Germany-Belgium pair, as can be seen from column 5. However, when $\{u_t\}$ is obtained as the residual from the regression of e on π , this null hypothesis cannot be rejected at the 5 percent level, so the evidence against the null is apparently weak. The tentative conclusion to be drawn is that nominal exchange rates and relative price levels are not co-integrated.

It might be mentioned at this point that the tests performed above have low power against local alternatives. For example, the logarithm of nominal exchange rates and relative price levels would technically be co-integrated if the coefficient on $-u_{t-1}$ is 0.001 (instead of zero), but the ability to detect such small departures from the null hypothesis is limited with the data currently available.³ The inability to reject a null hypothesis does not imply its acceptance and these results are not conclusive proof that the real exchange rate has a unit root. However, these results do suggest that shocks to the real exchange rate are persistent enough so as to raise doubts as to whether a return to purchasing power parity occurs in the long run.⁴

4. Impulse-response analysis

This section documents the long-run dynamics of real and nominal

³Engle and Granger report some power calculations for this test for a limited number of alternatives based on Monte-Carlo methods. In a related context, Hakkio (1986) has investigated the random-walk hypothesis for nominal exchange rates by studying the ability of commonly used tests for unit roots to discriminate against various alternative hypotheses.

⁴Given the current availability of data, the lack of power, while problematic, comes with the territory. As a result, other researchers, employing other methods, have come to different conclusions. For example, Huizinga (1987) reports evidence that the real exchange rate is not a random walk but displays mean-reverting tendencies and Rush and Husted (1985) and Edison (1987) find evidence that purchasing power parity does hold as a long-run relation.

exchange rates and relative commodity price levels through an analysis of their impulse-response functions. The results of the previous section suggest that purchasing power parity is violated in the long run. It is possible, with the passage of time and the accumulation of more observations, that the hypothesis of no co-integration will be rejected at small significance levels. In any event, this section proceeds under the supposition that logarithms of nominal exchange rates and relative commodity price levels are not cointegrated and inquires as to the nature of the dependence between these two sequences and their relationship to the real exchange rate.

The examination of the dependence among real and nominal exchange rates and relative price levels requires that the properties of only two of these time series be studied. That is because the logarithm of the real exchange rate is, by construction, the sum of the logarithms of the nominal exchange rate and relative price levels, so that there are only two independent variables in this system of three. I have chosen to examine the nominal exchange rate and relative price levels, although this choice is purely arbitrary. Also, I employ only those country pairs for which the analysis of section 3 could not reject the unit root hypothesis for both $\{e_t\}$ and $\{\pi_t\}$.⁵

Assume that $y_t \equiv (\Delta e_t, \Delta \pi_t)'$ is a stationary, linearly indeterministic stochastic process that has the vector autoregressive representation:

$$A(L)y_t = u_t, \tag{5}$$

where $A(L) = \sum_{0}^{k} A_{j}L^{j}$ is a 2×2 matrix polynomial in the lag operator, L, $A_{0} = I$, $E(u_{i}u'_{i}) = \Sigma$, and $E(u_{it}u_{js}) = 0$ for $t \neq s$. Now invert A(L) to obtain the moving average representation:

 $y_t = A(L)^{-1}u_t.$ (6)

As is well known, to decompose the dynamic response of the elements of y unambiguously into innovations of $\{\Delta e_t\}$ and $\{\Delta \pi_t\}$ an orthogonal transformation of the innovations $\{u_t\}$ is required. Thus, the following transformation is made: let $\varepsilon_t = Bu_t$, where B is a lower triangular matrix chosen such that $B\Sigma B' = \Gamma$, and Γ is a diagonal matrix. It follows that $E\varepsilon_{it}\varepsilon_{js} = 0$, for all t, s where $i \neq j$. To preserve the scaling of the innovations, B is chosen such

⁵The country pairs omitted from further data analysis are the U.K.-U.S., U.K.-Canada, U.K.-Germany, and Germany-Japan. The previous section found that the nominal exchange rate is non-stationary in levels and stationary in first differences but could not reject the hypothesis that relative price levels are stationary in levels for these pairs.

that the diagonals of Σ and Γ are the same. Now (6) can now be expressed as:

$$y_t = C(L)\varepsilon_t,\tag{7}$$

where

$$C(L) = A(L)^{-1}B^{-1} = \begin{bmatrix} c_{11}(L) & c_{12}(L) \\ c_{21}(L) & c_{22}(L) \end{bmatrix} = \begin{bmatrix} \sum_{j=0}^{\infty} c_{11,j}L^j & \sum_{j=0}^{\infty} c_{12,j}L^j \\ \sum_{j=0}^{\infty} c_{21,j}L^j & \sum_{j=0}^{\infty} c_{22,j}L^j \end{bmatrix}$$

Consider a unit shock to Δe_i . The ultimate change in the nominal exchange rate following this shock is given by $c_{11}(1)$, and the ultimate change in the relative price levels is given by $c_{21}(1)$. These changes are interpreted as the *long-run* changes following a Δe_i shock. Similarly, long-run changes in the nominal exchange rate and relative price levels following a unit shock to $\Delta \pi_i$ are given by $c_{12}(1)$ and $c_{22}(1)$, respectively. The long-run change in the real exchange rate due to a unit innovation in Δe and in $\Delta \pi$ are given in (8) and (9), respectively:

$$c_{11}(1) - c_{21}(1), (8)$$

$$c_{12}(1) - c_{22}(1). \tag{9}$$

The system is estimated in its autoregressive form, and inverted to obtain the moving-average representation. Akaike's (1974) information criterion was employed to select lag lengths for estimation of the vector autoregressive systems. Notice that if purchasing power parity held in the long run, the differences (8) and (9) would be identically zero, and hence det [C(z)] would have a root on the unit circle so that the autoregressive representation for $\{y_t\}$ would not exist. However, the previous results on co-integration suggest that this is not the case.

The cumulative responses to Δe and $\Delta \pi$ shocks are calculated as $C(1) = A(1)^{-1}B^{-1}$. Standard errors for these cumulative impulse responses were calculated from 1,000 Monte-Carlo simulations of the estimated vector autoregressive process.⁶

Table 5 reports the long-run responses of nominal exchange rates and

⁶To compute the standard errors, initial startup values for y are taken as given. First, a sequence of 177 u's are drawn from a bivariate normal distribution calibrated to the estimated model. Given the u's, the initial y values, and the autoregressive parameters estimated from the data, a sequence of 177 y's are generated. The parameters are then re-estimated from this artificial data, and C(1) re-computed and recorded. This is repeated 1,000 times. The resulting sample of C(1) forms the Monte-Carlo distribution of C(1) from which the standard errors are computed.

Ta	ble	5

Cumulative impulse response changes – vector autoregressions include 11 seasonal durnmies. $\Delta e_t = c_{11}(L)\varepsilon_t^e + c_{12}(L)\varepsilon_t^{\pi}; \ \Delta \pi_t = c_{21}(L)\varepsilon_t^e + c_{22}(L)\varepsilon_t^{\pi}.$

		• •••	· · · · · · ·	•••••		•	
Country	Lag	$c_{11}(1)$ Δe response to ε^{e} shock (S.E.)	$c_{12}(1)$ Δe response to ε^{π} shock (S.E.)	$c_{21}(1)$ $\Delta \pi$ response to ε^{e} shock (S.E.)	$c_{22}(1)$ $\Delta \pi$ response to ε^{π} shock (S.E.)	$c_{11}(1)-c_{21}(1)$ Δq response to ε^{e} shock (S.E.) ^a	$c_{12}(1)-c_{22}(1)$ Δq response to ε^{π} shock (S.E.) ^a
A. U.S. as I	nome cou	ntry	•				
Belgium	1	1.056	0.869	0.017	2.104 (0.292)	1.039*	-1.235
Canada	1	0.945	0.226	-0.014	1.211	0.959*	-0.958*
France	2	1.136	3.302	0.032	1.841	1.104*	1.461
Germany	2	1.167	2.971	-0.005	1.890	1.172*	1.081
Italy	2	1.256	2.272	0.071	1.904	1.185*	0.368
Japan	1	1.152 (0.107)	-0.134 (0.555)	0.033 (0.034)	1.290 (0.129)	1.119 * (0.110)	-1.424^{*} (0.557)
B. U.K. as	home co	untrv		. ,	. ,	. ,	. ,
Belgium	1	1.122 (0.102)	0.521 (0.615)	0.032 (0.048)	1.693 (0.207)	1.090* (0.102)	-1.172^{*}
Italy	1	1.307 (0.136)	0.298	0.083	1.616	1.224*	-1.318^{*}
Japan	1	1.146 (0.106)	0.046 (0.542)	-0.018 (0.045)	1.500 (0.169)	1.164* (0.116)	- 1.454* (0.576)
C. German	y as hom	e country					
Belgium	1	0.877 (0.063)	-0.332 (0.367)	0.066 (0.043)	1.494 (0.162)	0.811* (0.078)	1.826 * (0.389)
Canada	1	1.125	0.274	0.009	1.098	1.116*	0.824
France	1	1.044	0.526	0.016	1.601	1.028*	-1.075
Italy	1	1.308 (0.140)	0.612 (0.590)	0.128 (0.055)	1.543 (0.181)	(0.089) 1.180* (0.127)	-0.928 (0.536)

Note: Standard errors are computed from 1,000 Monte Carlo simulations of the vector autoregressive system. Seasonal dummies were included.

^aAn asterisk indicates that estimate is more than two standard errors from zero.

relative price levels implied by the impulse response functions and the longrun responses of the real exchange rate as implied by (8) and (9).⁷ In column 1 it is seen that a unit innovation in the nominal exchange rate is followed approximately by a unit increase in the exchange rate in the long

⁷The alternative orthogonalizion (not reported) yielded almost no difference in the results, as the residual correlations are very small.

run. These responses range from 0.877 for Germany-Belgium to 1.308 for Germany-Italy. Relatively small and insignificant long-run changes in relative price levels follow the exchange rate shocks, as seen from column 3. The exchange rate responses to unit relative price level shocks are somewhat erratic and are reported in column 2. The dollar/French franc exchange rate rises by 3.302, while the DM/Belgian franc rate falls by 0.332. These estimates are not very precise, however, and none of them are more than two standard errors from zero. Column 4 displays the long-run change in relative price levels following a unit shock to relative price levels. In all cases a unit shock to relative price levels is followed by greater than a unit increase in these relative price levels in the long run. Column 5 reports the long-run change in the real exchange rate following a unit innovation in Δe_r . This cumulative change is near one in each case, and is more than two standard errors from zero. The point estimates indicate that a unit increase in the nominal exchange rate is followed by a unit increase in the long-run real exchange rate. The cumulative, long-run change in the real exchange rate following a unit shock to $\Delta \pi$, is shown in column 6. In most cases this change is negative, and point estimates are insignificantly different from minus one.

These results suggest that the long-run dynamic relationship between nominal exchange rates and relative price levels is weak. Innovations in nominal exchange rates appear important for explaining nominal exchange rate movements but not for relative price level movements, and vice versa. Innovations in nominal exchange rate changes are followed by permanent changes in real exchange rates. It also appears that innovations in relative commodity price levels lead to permanent changes in the real exchange rate.⁸

5. Interpretation

At this point, it is useful to summarize the findings from sections 2-4.

(1) Nominal exchange rate movements and relative price-level movements are for the most part unrelated in the long run as well as in the short run. The evidence in favor of purchasing power parity in the long run is weak.

(2) Changes in nominal exchange rates and changes in relative price levels are permanent. In the long run, a unit increase in the nominal exchange rate leads to a unit increase in the real exchange rate, while a unit increase in the relative price level leads to a unit decrease in the real exchange rate.

(3) A large proportion of long-run real exchange rate movements occur in

⁸In an analysis of variance decompositions, which are nor reported to conserve space, over 95 percent of the 48-month-ahead forecast error variance of Δe and $\Delta \pi$ was attributable to innovations in the variable's own past and only about 5 percent to the innovations in the past of the other variable.

response to long-run nominal exchange rate movements. This is because the variability of nominal exchange rate changes exceeds the variability of relative price-level changes in the long run as well as in the short run. Consequently, the variability of real exchange rate changes is approximately equal to the variability of nominal exchange rate changes, and these changes are highly correlated in the long run as well as in the short run.

In this section, three popular models of exchange rate determination are called upon to explain these observations.⁹ Specifically, this section asks under what circumstances do these models imply real and nominal exchange rate behavior and relative commodity price-level behavior that is broadly consistent with the above findings. First, the long-run specification of Dornbusch's (1976) log-linear version of the Mundell-Fleming model is considered. Second, Mussa's (1982) stochastic generalization of the Dornbusch model is considered. Third, Lucas's (1982) two-country model is considered. This last model is a representative of the class of equilibrium models with perfect markets.

In Dornbusch's model, i is the domestic nominal interest rate and i^* is the exogenous world nominal interest rate. The nominal money supply, price level, output, and nominal exchange rate, in logarithms, are denoted by m, p, y, and e, respectively. The long-run specification of this model is:

$$i=i^*,\tag{10}$$

$$m - p = \phi y - \eta i, \tag{11}$$

$$(1-\gamma)y = u + \delta(e-p) - \sigma i. \tag{12}$$

Eq. (10) is the interest parity condition, (11) is the LM curve, and (12) is the IS curve. u is a shift parameter in aggregate demand to which a fiscal policy interpretation may be ascribed. The logarithm of the foreign price level is normalized to unity so that the logarithm of the real exchange rate is q=e-p. The comparative statics for changes in the exogenous variables, y, i^* , and u, are summarized in the following matrix:¹⁰

	de	dp	dq
dy		_	+
di*	+	+	+
du		0	—

⁹In their survey article, Obstfeld and Stockman (1985) identify conditions in which exchange rate models generate nominal exchange rate variability that is higher than nominal price-level variability.

¹⁰Monetary shocks are not considered here since purchasing power parity would hold in the long run if this was the sole source of variability.

For this model, it can be seen that only shocks to aggregate demand are broadly consistent with the actual experience. With output and the world interest rate given, the long-run commodity price level is determined by the quantity of money so that dp/du=0. This is consistent with the low variability of long-run relative price-level changes. However, an increase in domestic absorption raises the relative price of domestic goods, or the real exchange rate, and this can only be achieved through a decline in the nominal exchange rate by the same amount. That is, $de/du = dq/du = -1/\delta$, so that domestic shocks to aggregate demand yield nominal exchange rate changes that are entirely real in the long run.

Supply shocks and world interest rate shocks, on the other hand, yield predictions that are inconsistent with the data. This is because a supply shock causes the real exchange rate and the nominal exchange rate to change in opposite directions while world interest rate shocks cause the price level and the real exchange rate to move in the same direction.¹¹

The Dornbusch model suggests that demand shocks have been empirically important, but because it is deterministic, it is difficult to assess the relative importance of nominal and real variability required to explain the data. Mussa's (1982) stochastic generalization of the Dornbusch model permits such an assessment within a Keynesian framework. Let logarithms of the nominal money supply, domestic goods price, foreign goods price, and nominal exchange rate be denoted by m, p, p^* , and e and the levels of the domestic and foreign nominal interest rates be denoted by i and i^* , respectively. The basic equations of the Mussa model are:

$$P_t = \lambda p_t + (1 - \lambda)(e_t + p_t^*) \quad \text{(domestic price level)}, \tag{13}$$

$$q_t = e_t + p_t^* - p_t$$
 (real exchange rate),¹² (14)

$$m_t = k_t + P_t - \eta i_t$$
 (money market equilibrium), (15)

¹¹This model views the world interest rate as exogenous but the source of the change in i^* is important. A two-country representation might characterize the rest of the world by a similar IS-LM structure, where the LM curve is $m^* - p^* = \phi y^* - \eta i^*$, and the IS curve is $(1-\gamma)y^* = u^* - \sigma i^*$. It is assumed that a change in q has a negligible effect on the aggregate demand for the goods of the rest of the world. This is often assumed to reflect the 'smallness' of the domestic economy. Otherwise, the other parameters are assumed to be the same in the rest of the world as at home. The change in i^* is now assumed to be caused by changes in u^* and changes in y^* . It can be seen now that $de/du^* = dq/du^*$ and $d\pi/du^* = 0$, so that world aggregate demand shocks are also broadly consistent with actual behavior. It is likely, however, that world supply shocks are not consistent with the findings above. For world supply shocks, $de/dy^* = -[(1-\gamma)(\lambda\delta + \sigma)]/\delta\sigma < 0$, $d\pi/dy^* = \phi$, and $dq/dy^* = -[(1-\gamma)(\lambda\delta + \sigma)]/\delta\sigma - \phi < 0$. Although this yields the right directional changes, it implies that the magnitude of real exchange rate changes exceeds that of nominal exchange rate changes, and that relative price variability may exceed nominal exchange rate variability for reasonable parameter values.

¹²In Mussa (1982) the real exchange rate is defined as $p-(e+p^*)$, so some of the signs here are reversed.

$$i_t = i_t^* + E_t(e_{t+1} - e_t)$$
 (uncovered interest parity), (16)

$$d_t = \lambda \alpha (\rho_t - r_t) + \beta (q_t - \tau_t) \quad \text{(domestic excess goods demand)}, \tag{17}$$

$$m_i^* = k_i^* + p^* - \eta i_i^*$$
 (foreign money market equilibrium), (18)

$$d_t^* = \alpha(\rho_t^* - r_t^*)$$
 (foreign excess goods demand), (19)

where $r_t = i_t - E_t(P_{t+1} - P_t)$ and $r_t^* = i_t^* - E_t(p_{t+1}^* - p_t^*)$ are the domestic and foreign money demand, ρ , ρ^* , and τ are the 'natural' rate of interest and real exchange rate which together equilibrate the goods market. These natural rates can vary over time. The foreign country serves as a proxy for the rest of the world and is assumed to be large in the sense that the exchange rate exerts a negligible effect on its price level and on the aggregate demand for the foreign output. As in the Dornbusch model, the asset market is continuously in equilibrium and short-run disequilibria can occur in the goods market due to the slow adjustment of commodity prices. To stay within the long-run spirit of the discussion, I will concentrate on the equilibrium real and nominal exchange rates implied by the model, i.e. those prices implied by the model assuming that $d = d^* = 0$.

Define the exogenous 'forcing variables' $\{z_t\}$ and $\{w_t\}$ to be, $z_t = \tau_t + (\alpha \lambda/\beta)$ $[r_t^* - \rho_t]$, and $w_t = m_t - k_t + \eta r_t^*$. z and w are composite variables comprised of the independently varying components, τ , ρ , m, k, and r^* . To consider a specific case, let $\{z_t\}$ and $\{w_t\}$ follow random walks, where $z_t = z_{t-1} + \varepsilon_t^z$, and $w_t = w_{t-1} + \varepsilon_t^w$, where $\{\varepsilon_t^z\}$ and $\{\varepsilon_t^w\}$ are zero mean, serially uncorrelated sequences. Then the equilibrium log-level of the nominal and real exchange rate can be shown to be:

$$e_t = w_t + \lambda z_t - p_t^*, \tag{20}$$

$$q_t = z_t. \tag{21}$$

Next, let $z_t^* = \rho_t^* + (1/\eta)(m_t^* - k_t^*)$, where, $z_t^* = z_{t-1}^* + \varepsilon_t^*$, and $\{\varepsilon_t^*\}$ is a serially uncorrelated sequence. Then the equilibrium foreign price level and real interest rates can be shown to be:

$$p_i^* = \eta z_i^*, \tag{22}$$

$$r_t^* = \rho_t^*. \tag{23}$$

Now substitute (22) and (23) and the definitions of z, z^* , and w into (20) and (21) to obtain:

$$e_t = n_t + \lambda z_t, \tag{24}$$

$$q_t = z_t, \tag{25}$$

where $n_t = (m_t - k_t) - (m_t^* - k_t^*)$ represents the 'nominal' factors of the model. By the previous assumptions, the nominal factors can be represented as a random walk, $n_t = n_{t-1} + \varepsilon_t^n$. Now take differences of (24) and (25) to obtain the equilibrium percentage changes in the nominal and real exchange rates:

$$\Delta e_t = \varepsilon_t^n + \lambda \varepsilon_t^z, \tag{26}$$

$$\Delta q_t = \varepsilon_t^z, \tag{27}$$

where $\{\varepsilon_t^z\}$ and $\{\varepsilon_t^n\}$ are zero mean, serially uncorrelated sequences with $E(\varepsilon_t^z)^2 = \sigma_z^2$, $E(\varepsilon_t^n)^2 = \sigma_n^2$, $E(\varepsilon_t^z \varepsilon_t^n) = \sigma_{nz}$, and $E(\varepsilon_t^z \varepsilon_s^n) = 0$, $t \neq s$. Denote by σ_e^2 and σ_q^2 , the variances of Δe and Δq , and σ_{eq} , the covariance between Δe and Δq . Then it follows that

$$\sigma_e^2 = \sigma_n^2 + \lambda^2 \sigma_z^2 + 2\lambda \sigma_{nz},\tag{28}$$

$$\sigma_q^2 = \sigma_z^2,\tag{29}$$

and

$$\sigma_{eq} = \sigma_{nz} + \lambda \sigma_z^2. \tag{30}$$

To be broadly consistent with actual experience, the model must produce real and nominal exchange rate changes that are roughly of equal variance and that are highly correlated. Let the data be characterized by:

$$\delta^2 = \sigma_e^2 / \sigma_q^2 \le 1 \tag{31}$$

and

$$\rho_{eq} \equiv \sigma_{eq} / (\sigma_e \sigma_a) \le 1. \tag{32}$$

The data suggest that δ and ρ_{eq} are close to one. Now, making the appropriate substitutions from (28)–(30) into (31) and (32) yields:

$$\sigma_n^2 = \theta_M^2 \sigma_z^2 \tag{33}$$

and

$$\rho_{nz} = (\delta \rho_{eq} - \lambda) / \theta_{\rm M},\tag{34}$$

where $\theta_{M}^{2} = \delta^{2} + \lambda^{2} - 2\lambda\delta\rho_{eq}$ is the variance ratio of nominal and real exogenous factors. Since λ is the share of the domestic goods price in the

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Table 6

Implied correlations between exogenous nominal and real factors and their variance ratios for the Mussa and Lucas models assuming $\delta^2 = \sigma_e^2/\sigma_a^2 = 0.95$, and $\rho_{eq} = 0.95$.

A. The Mussa model	$\theta_{14}^2 = \sigma_z^2 / \sigma_z^2$	0
	- M - N/- 2	P n2
0.50	0.300	0.822
0.60	0.199	0.731
0.70	0.144	0.596
0.80	0.108	0.382
0.90	0.093	0.085
B. The Lucas model		
R	$\theta_L^2 = \sigma_m^2 / \sigma_y^2$	ρ_{my}
0	1.000	1.00
1	0.950	0.950
2	1.096	0.814
3	1.439 ·	0.649
4	1.977	0.500
5	9.330	0.085

Note: λ is the share of domestic goods in the general price index. R is the coefficient of relative risk aversion.

construction of the price index, $(1-\lambda)$ can be interpreted as a measure of openness of the economy. If $\delta \rho_{nz} > \lambda$, which the data suggest is true, then $\partial \theta_{M}^2/\partial \lambda < 0$ and $\partial \rho_{nz}/\partial \lambda < 0$ so that the variance ratio and correlation between nominal and real factors required of the Mussa model to be consistent with actual experience increases with the openness of the economy. Panel A of table 6 shows the implied values of the correlation between nominal and real factors and their variance ratios assuming $\delta = 0.9715$ and $\rho_{eq} = 0.95$. For example, with $\lambda = 0.70$, the variance ratio of nominal to real factors, θ_{M}^2 , is 0.14, indicating that real variability is something like seven times nominal variability and the required correlation between nominal and real factors, ρ_{nz} , is 0.596.

To take a representative from the class of equilibrium models that feature perfect markets, consider Lucas's (1982) two-country model. The equilibrium exchange rate in this model is given by:

$$E = \frac{M}{M^*} \frac{Y^*}{Y} \frac{U_{c^*}(Y, Y^*)}{U_c(Y, Y^*)}.$$
(35)

Here, M and M^* are the domestic and foreign money supplies, Y and Y^* are exogenous domestic and foreign outputs, and $U_{c^*}(\cdot)$ and $U_c(\cdot)$ are the marginal utilities of consumption of the foreign and domestic commodity for

a representative consumer. In this model, prices are flexible, markets are complete, agents are fully informed and have rational expectations. When the population is the same in both countries and domestic and foreign consumers are endowed with equal wealth, they pool their specific risks and consume half of the world's output of each good in equilibrium. Hence, the equilibrium consumption bundle is $(Y/2, Y^*/2)$. Commodity price levels are given by a quantity equation with unit velocity, so that the real exchange rate, Q, is simply the marginal rate of substitution between domestic and foreign goods.¹³ That is,

$$Q = \frac{U_{c^{*}}(Y, Y^{*})}{U_{c}(Y, Y^{*})}.$$
(36)

To take a specific example, suppose that preferences are given by constant relative risk aversion utility, $U(C, C^*) = [\theta C^{(1-R)} + (1-\theta)C^{*(1-R)}]/(1-R)$, where R is the coefficient of relative risk aversion. Then in logarithmic form, the changes in nominal and real exchange rates can be expressed as:

$$\Delta e = \Delta m + (R - 1)\Delta y, \tag{37}$$

$$\Delta q = R \Delta y, \tag{38}$$

where $m \equiv \ln(M/M^*)$ and $y \equiv \ln(Y/Y^*)$. Let $\sigma_e^2, \sigma_q^2, \sigma_m^2$, and σ_y^2 denote the variance of the first differences in e, q, m, and y. Also, let σ_{eq} denote the covariance between Δe and Δq and σ_{my} be the covariance between Δm and Δy . Then it follows that

$$\sigma_e^2 = \sigma_m^2 + (1-R)^2 \sigma_y^2 + 2(R-1)\sigma_{my},$$
(39)

$$\sigma_a^2 = R^2 \sigma_v^2,\tag{40}$$

$$\sigma_{eg} = R\sigma_{mv} + R(R-1)h_v^2. \tag{41}$$

To be broadly consistent with the actual experience, this model must produce real and nominal exchange rate changes that are of roughly equal variance, and that are highly correlated. Let the actual experience again be

¹³Strictly speaking, the real exchange rate should be constructed as the relative price of the domestic consumption basket in terms of the foreign consumption basket. A utility function could be specified and the true price indices constructed. The main point of this section is not affected by doing this, however. For example, the arguments presented here are made in Stulz (1987), who defines the real exchange rate in terms of the true price indices in an equilibrium model with traded and non-traded goods.

characterized by setting $b \simeq 1$ and $\rho_{eq} \simeq 1$ in (31) and (32). Now, for eqs. (39)-(41) to jointly satisfy (31) and (32) requires that

$$\sigma_m^2 = \theta_{\rm L}^2 \sigma_{\rm v}^2 \tag{42}$$

and

$$\rho_{my} = [R\rho_{eq}\delta + 1 - R]/\theta_{\rm L},\tag{43}$$

where the variance ratio of nominal and real factors in the Lucas model is $\theta_L^2 = (1-R)[2\delta\rho_{eq}R+1-R] + \delta^2R^2$. Panel B of table 6 shows the implied values of this variance ratio and the correlation between relative money supplies and relative output levels, assuming again that $\delta = 0.9715$ and $\rho_{eq} = 0.95$. The Lucas model generally requires monetary variability to be more important than real variability. In the risk-neutral case (R=0), the variance ratio and the correlation between relative money supply and relative output are both equal to unity, which seems implausible. As individuals display risk aversion beyond the log-utility case (R>1), the implied variance ratio increases with the coefficient of relative risk aversion while the required correlation between nominal and real factors declines.

The models examined in this section are somewhat stylized and I have imposed auxiliary assumptions not assumed by the original authors. Thus, the implications drawn here are meant only to be suggestive. Furthermore, the Mussa and Lucas models are not directly comparable because different things serve as the exogenous monetary and real factors in the two models and some of the exogenous variables of the Mussa model are not directly observable. With these caveats in mind, however, it seems that both Keynesian and equilibrium theories are capable of explaining the data in the dimension of real and nominal exchange rate movements. For reasonable parameter values, the required correlation between the exogenous nominal and real factors in the Lucas and Mussa models are roughly the same. Where the two models differ is in the implied volatilities of the nominal and real factors. The Lucas model implies that monetary shocks have been a principal source of variability over the flexible exchange rate period, while the Dornbusch and Mussa models suggest that real shocks have been more important. Further assessments of the overall validity of the competing theories is an interesting topic for research but is beyond the scope of this paper.

6. Conclusions

This paper has employed methods specifically designed to address long-run issues and has found evidence unfavorable to the hypothesis that purchasing power parity holds in the long run. The results are robust across the three countries used as the 'home country' in bilateral relations using data on eight industrialized countries suggesting that the violation of long-run purchasing power parity is not solely a U.S. dollar problem. To be consistent with the evidence, Keynesian models suggest that real shocks have been a relatively more important source of variability than nominal shocks. Equilibrium models with perfect markets, on the other hand, suggest that monetary sources of variability have been relatively more important.

Some caution should be exercised in the interpretation of the empirical results. The paper examines the behavior of real and nominal exchange rates only during the modern experience with flexible exchange rates, and as a result could exploit only 16 years of data. One possibility which arises is that the 'true' long run may be longer than 16 years. If this is indeed the case, the sample used here would effectively represent less than one observation on long-run behavior, and the issue of committing type I errors becomes especially relevant. Extension of the data set backwards would be inappropriate since the methods employed here cannot deal with regime changes. As documented by Mussa (1986) and Stockman (1983), the behavior of real and nominal exchange rates has differed significantly across periods of fixed and flexible nominal exchange rate regimes. In particular, the variance of the changes in both real and nominal exchange rates are significantly larger during periods of flexible exchange rates.

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