Changing Monetary Policy Rules, Learning, and Real Exchange Rate Dynamics

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Abstract

When the exchange rate is priced by uncovered interest parity and central banks set nominal interest rates according to a reaction function such as the Taylor rule, the real exchange rate will be determined by expected inflation and the output gap or the unemployment gap of the home and foreign countries. This paper examines the implications of these Taylor-rule fundamentals for real exchange rate determination. Because the true parameters in central bank policy rules are unknown to the public and change over time, the model is presented in the context of a least-squares learning environment. This simple learning model captures the volatility and the major swings in the real deutschemark/euro–dollar exchange rate from 1976 to 2007.

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Introduction

Understanding exchange rate dynamics has been a research challenge ever since Meese and Rogoff (1983) found macroeconomic fundamentals and exchange rates to be unrelated. While some econometric progress has been made in modeling long-horizon exchange rate returns, the basic predictions of open economy macroeconomic theory—that the exchange rate is determined by the levels of variables such as domestic and foreign prices, money supplies, and income–does not enjoy strong empirical support.[1]

In this paper, I investigate the linkage between the exchange rate and an alternative set of fundamentals that are implied when monetary policy is guided by a nominal interest rate reaction function commonly referred to as the 'Taylor rule.' The Taylor (1993) approach predicts that the exchange rate is determined by expected inflation and the output or unemployment gaps of the home and foreign countries instead of levels of macroeconomic fundamentals as is often assumed in empirical exchange rate work. A successful implementation of the Taylor-rule approach suggests that at least some of the frustration encountered in earlier work may stem from a focus on the wrong set of fundamentals.

I study an environment where market participants do not know the exact values of the Taylor-rule coefficients and employ least-squares learning rules to acquire that information.[2] The learning environment is motivated by the fact that the central banks under study have not explicitly informed the public about the monetary policy rule that they adhere to and by evidence that the parameters of the rules change over time. I apply the model to understand the real exchange rate dynamics for the DM (deutschemark) price of the dollar (1976-1998) and then the euro price of the dollar (1999-2007). The implied least-squares learning path is generally able to account for the volatility and the major swings of the real exchange rate data.[3] I focus on the DM–dollar exchange rate primarily because the Bundesbank is one of the non-US central banks identified by Clarida et al. (1998) as having conducted monetary policy by following a variant of the Taylor rule. While emphasizing the role of interest rates in the study of real exchange rate determination is not new,

[1] See Mark (1995), Mark and Sul (2001), Groen (2000,2002), and Rapach and Wohar (2002) who report econometric evidence on the long-horizon predictability of exchange rate returns from standard macro pricing errors. The relation between exchange rates and levels of macro fundamentals is predicted by theory ranging from disequilibrium Keynesian models of Dornbusch (1976), Mussa (1982) and Obstfeld (1985) to the new open-economy macroeconomics of Obstfeld and Rogoff (1995).

[2] Lewis (1989a, b) conducts an analysis of Bayesian learning in the foreign exchange market to examine the 1979 changes in the Fed's operating procedures. She focused on shifts in the stochastic process governing monetary aggregates. In the monetary policy literature, Bullard and Mitra (2002) study conditions under which the rational expectations equilibrium is learnable while Orphanides (2003) examines whether the Fed's imperfect knowledge of and attempts to learn the natural rate of unemployment responsible for the inflationary buildup of the 1970s.

[3] These are the dollar cycle of the last 10 years and the the dollar cycle of the 1980s. The 1980s cycle has been referred to as the 'great appreciation' and 'great depreciation' by Papell (2002). Engel and Hamilton (1990) called these fluctuations 'long-swings,' and Frankel (1985) referred to the strong dollar of the 80s as the 'dazzling dollar.'

the innovation associated with the Taylor-rule approach is that it sets up a multivariate structure with a rich set of dynamics for interest rate forecasts that are potentially more informative than those obtained from univariate time-series specifications.[4]

My motivation for incorporating learning comes from two sources. The first is the rather poor empirical performance of macroeconomic-based rational expectations models of the exchange rate. This evidence suggests that relaxing some of the strong informational assumptions typically required—that market participants already know the very structure that econometricians themselves struggle to learn—may be a useful avenue to explore. Both direct evidence of structural instability and indirect evidence through the modest to poor out-of-sample fit of econometric exchange rate models are hints that parameter uncertainty is an important feature of the environment that should be explicitly accounted for. Adaptive learning schemes provide a plausible and tractable strategy for modeling market participants who operate in such an environment.

The other motivation draws upon changes in the way that central banks have responded to expected inflation–such as the change that occurred with the appointment of Paul Volker to the Federal Reserve chairmanship in 1979. Clarida et. al. (2000) reports evidence in this vein having found that in the pre-1979 data, an increase in expected inflation led to a reduction in the real interest rate because the Fed typically reacted by raising the nominal interest rate by less than the increase in expected inflation. Following the appointment of Paul Volker as Chairman of the Federal Reserve System, they found the real interest rate to be increasing in expected inflation because the Fed reacted more aggressively by raising the nominal interest rate by more than the increase in expected inflation. I also estimate reaction functions for the Bundesbank/ECB and the Fed and find similar instabilities as did Clarida et al. Shifts such as these represent significant transformations in the economic environment. Presumably it takes time for the public to perceive that the change has occurred and then to understand the nature of the change. If so, then allowing for structural change and modeling the transitional learning that goes on should be productive.

Apart from the learning aspect, this paper joins a growing literature that has studied the role of interest rate reaction functions in exchange rate determination. Engel and West (2006) construct the rational expectations time path of the real exchange rate implied by reaction function fundamentals and report a correlation of 0.32 between the implied rational expectations real DM–dollar rate and the historically observed real exchange rate from 1979 to 1998. Molodstova and Papell (2007) find that Taylor-rule fundamentals have significant out-of-sample predictive power for future exchange rates. Engel et al. (2007) examine predictive ability of Taylor-rule

[4]See Frankel (1979), Meese and Rogoff (1988), Edison and Pauls (1993), Campbell and Clarida (1987), and Baxter (1994). Mark and Moh (2004) consider nonlinear (threshold) models for real interest rate differentials and find that the implied rational expectations path for the real exchange rate has very little power to explain historical movements in the real exchange rate. For evidence on the importance of a multivariate approach, see Clarida and Taylor (1997) who show that information in the term structure of the forward premium provides significant out-of-sample predictive power for the exchange rate.

fundamentals in a panel regression framework and Clarida and Waldman (2007) show that the exchange rate response to higher than expected inflation is consistent with the operation of the Taylor rule. In related work, Groen and Matsumoto (2004) calibrate a dynamic general equilibrium model to the UK economy where monetary policy operates through interest rate reaction functions.

The remainder of the paper is as follows. The next section describes the data. Section 2 presents estimates of the Taylor rule for the Bundesbank/ECB and for the Fed. The purpose of reporting these results is twofold. First, the evidence provides the basic support for modeling German and US interest rates with this specification and second, it provides evidence that the Taylor-rule coefficients change over time. Section 3 presents a rational expectations model of the real exchange rate based on real interest parity and the learning algorithm that agents employ to understand the model. Section 4 presents the main empirical results and Section 5 concludes.

1 The data

The complete data set consists of quarterly observations spanning from 1960Q1 to 2007Q3. In the actual modeling of the exchange rate, however, I focus on the period from 1976Q1 to 2007Q3. Because the responsibility for German monetary policy shifted from the Bundesbank to the ECB (European Central Bank) in 1999, and because the ECB conducts policy not just for Germany but for the entire euro area, euro-area wide observations are used from 1999Q1 to 2007Q1. Germany (the euro-area) is viewed as the 'home' country and an increase in the real exchange rate signifies a real DM (euro) depreciation. Although the home country is Germany before 1999 and the euro-area afterwards, I will simply refer to the home country as Germany.

The US Federal funds rate and GDP data were obtained from FRED, the St. Louis Fed's economic data web site. All other data are from the OECD's Economic Outlook database. Inflation is measured by the rate of change in the German CPI from 1960Q1 to 1998Q4 and the rate of change in the Euro area harmonized CPI from 1999Q1 onwards.

I consider two definitions of the economic activity gap. The first is a measure of the output gap which is constructed from a recursively Hodrick-Prescott (1997) detrended real GDP series. The second is a measure of the unemployment gap which is formed from a recursively Hodrick-Prescott detrended unemployment rate. Quarterly inflation, the output and unemployment gaps, the nominal exchange-rate return and interest rates are stated as raw numbers.

A preliminary look at the data is provided in Figure 1 which plots the log real DM-dollar rate and a 3-quarter moving average of the German-US inflation differential. From 1960Q2 to 1979Q3, (except for a brief period from 1969Q3 to 1972Q2), rising relative US inflation coincides with a weakening of the dollar (correlation = 0.29). This association is weakened somewhat from 1979Q3 to 1998Q4 (correlation = 0.11) and strengthens again (correlation= 0.26) 1999Q1 to 2007Q3.

Two preliminary conclusions can be drawn from this informal examination of the data. First, the figure suggests that shifting the emphasis on exchange rate determinants away from relative levels of macroeconomic fundamentals towards variables such as the differences in differences of log national price levels may be a sensible thing to do. Second, one gets the impression from the changing correlation that the relationship between the real exchange rate and the inflation differential may have changed sometime in the late 1970s or early 1980s. An obvious candidate for such a regime shift, is the change in the conduct of monetary policy. We now turn to an examination of the regime shift in the context of real exchange rate determination.

2 Interest rate reaction functions

Let Germany be country '1' and the US country '2.' Then the inflation rate, interest rate and activity gaps for Germany and the US are denoted $\pi_{1,t}$, $\pi_{2,t}$,$i_{1,t}, i_{2,t}$, and $x_{1,t}, x_{2,t}$ respectively. The log real DM–dollar exchange rate is denoted by q_t.

My specification of the interest rate reaction function for the Bundesbank/ECB draws on Clarida et al. (1998) who estimate monetary policy reaction functions for the Bundesbank and several other countries using data spanning from 1979 to 1993. The Bundesbank/ECB is assumed to sets the deviation of the targeted interest rate from the desired rate $\left(i_{1,t}^T - \overline{i_1}\right)$, in response to the deviation of the public's expected inflation rate from the inflation target $(E_t\pi_{1,t+1} - \overline{\pi}_1)$ and to the activity gap $x_{1,t}$. When x is the output gap (unemployment gap), we expect the coefficient μ to be positive (negative) so that an economy operating below potential will trigger a loosening of credit. Because the Bundesbank/ECB occasionally intervenes in the foreign exchange market, I allow it to react to nominal exchange rate deviations from its 'natural level,' which is given by purchasing-power parity. Clarida et al. (1998) found that the feedback from the exchange rate to the German interest rate was statistically significant but quantitatively very small. Thus, the German interest rate target is set by the rule,

$$i_{1,t}^T = \bar{\imath}_1 + \theta_1 \left(E_t\pi_{1,t+1} - \overline{\pi}_1\right) + \mu_1 x_{1,t} + \sigma q_t,$$

The actual interest rate $i_{1,t}$ is subject to an exogenous and i.i.d. policy shock $\eta_{1,t}$. It is set according to a partial adjustment mechanism to reflect the central bank's desire to limit interest rate volatility,

$$i_{1,t} = (1 - \rho_1)\, i_{1,t-1} + \rho i_{1,t}^T + \eta_{1,t}.$$

The Fed sets the target Federal funds rate in an analogous fashion, but without a reaction to the exchange rate,

$$i_{2,t}^T = \bar{i}_2 + \theta_2 \left(E_t\pi_{2,t+1} - \overline{\pi}_2\right) + \mu_2 x_{2,t},$$

where the actual interest rate is subject to an exogenous and i.i.d. policy shock $\eta_{2,t}$ and a partial adjustment,

$$i_{2,t} = (1-\rho_2) i_{2,t}^T + \rho_2 i_{2,t-1} + \eta_{2,t}.$$

We thus have the empirical specification of the reaction functions,

$$\begin{aligned} i_{1t} &= \delta_1 + (1-\rho_1)\, i_{1t-1} + \rho_1 \left(\theta_1 E_t \pi_{1,t+1} + \mu_1 x_{1,t} + \sigma q_t\right) + \eta_{1,t}, && (1) \\ i_{2t} &= \delta_2 + (1-\rho_2)\, i_{2t-1} + \rho_2 \left(\theta_2 E_t \pi_{1,t+1} + \mu_2 x_{1,t}\right) + \eta_{2,t}, && (2) \end{aligned}$$

where $\delta_1 \equiv \rho_1 \left(\bar{i}_1 - \theta_1 \overline{\pi}_1\right)$, and $\delta_2 \equiv \rho_2 \left(\bar{i}_2 - \theta_1 \overline{\pi}_2\right)$.

To estimate the reaction function say for the Fed, add and subtract $\rho_2 \theta_2 \pi_{2,t+1}$ to the right side of (2) and rearrange to obtain

$$i_{2,t} = \delta_2 + \rho_2 \left(\theta_2 \pi_{2,t+1} + \mu_2 x_{2,t}\right) + (1-\rho_2)\, i_{2,t-1} + \eta'_{2,t}, \tag{3}$$

where $\eta'_{2,t} = \eta_{2,t} - \rho_2 \theta_2 \left(\pi_{2,t+1} - E_t \pi_{2,t+1}\right)$. Under rational expectations, the composite error term η'_{2t} is uncorrelated with date t information so (3) can be estimated by GMM (generalized method of moments). The instrumental variables that I employ are a constant, the current value and three lags of inflation, the current value and three lags of the activity gap, and four lags of the nominal interest rate. For the Bundesbank/ECB, I also use these instruments plus four lags of the real exchange rate.

We begin with the Bundesbank/ECB. Estimation results over three subperiods using the output gap are presented in panel A of Table 1. In the first subsample (1960Q2 to 1979Q2), German monetary policy appears to have been accommodating to inflation as the response coefficient to inflation is negative but insignificant. Notice also that the response coefficient on the exchange rate has the wrong sign. The second subsample is timed to coincide with the appointment of Paul Volker as the chairman of the Federal Reserve. Clarida et al. (2000) report evidence of a significant structural shift in the Fed's reaction function at that time. To the extent that there is coordination in monetary policy, we might expect to observe a similar shift for the Bundesbank and the estimates are consistent with such a story. According to the 2.60 point estimate of the inflation response coefficient, the Bundesbank adhered to the Taylor principle and reacted aggressively to inflation over this period. The point estimate on the response to the output gap is also much higher but not statistically significant over this period. The third subsample covers ECB monetary policy. Judging from the estimates over this period, ECB policy looks remarkably similar to pre-1979 Bundesbank policy. In each subperiod, the specification appears to be adequate as Hansen's J–test of the overidentifying restrictions is never rejected.

To formally examine the evidence for a structural shift, I run Hodrick and Srivastava's (1984) GMM test for structural change.[5] These results are shown in the last two columns of the table.

[5] The $k-$dimensional coefficient vector estimated from subsample $j = 1,2$ be $\widehat{\beta}_j$ is has asymptotic distribution $\sqrt{T_j}\left(\widehat{\beta}_j - \beta_j\right) \sim N\left(0, \Omega_j\right)$. If the observations from the two subsamples are independent, then under null hypothesis of no structural change $H_0 : \beta_1 = \beta_2$ the test statistic $HS = \left(\widehat{\beta}_1 - \widehat{\beta}_2\right)' \left(\frac{\Omega_1}{T_t} + \frac{\Omega_2}{T_2}\right)^{-1} \left(\widehat{\beta}_1 - \widehat{\beta}_2\right)$ is asymptotically distributed as a chi-square variate with k degrees of freedom.

The hypothesis of no structural change in any of the coefficients between subsamples 1 and 2, and between subsamples 2 and 3, are strongly rejected.

For the Fed, I conform to Clarida et al. (2000) and allow a single break point to coincide with the Volker appointment.[6] Before 1979Q3, the point estimate of the inflation response coefficient is significantly greater than zero and lies below 1. It is significantly greater than zero and lies above 1 after 1979Q3. The coefficient on the output gap is estimated with the correct sign and is significant in the second subsample. The specification appears to be adequate, and the test of structural stability strongly rejects the hypothesis of no change.

Panel C reports the Wald test of the hypothesis that the German coefficients are equal to the US coefficients (excepting σ). In the first subsample, these homogeneity restrictions appear to be satisfied, but they are strongly rejected in the second subsample.

The table also shows estimation results using the unemployment gap for x_t. For both the Bundesbank/ECB and the Fed, the estimated coefficients on the unemployment gap have the predicted sign. These estimates exhibit a similar (to those using the output gap) pattern across the subperiods. There is substantial evidence that a structural shift had taken place across the subsamples, and the cross-country homogeneity restrictions are strongly rejected in the 1979Q3-1998Q4 subsample.

Figure 2 provides a visual account of the fit for the Bundesbank/ECB and Figure 3 for the Fed. These are plots of the actual interest rate and fitted values using the output gap. It can be seen that this simple specification works reasonably well but perhaps more so for the Fed in describing the dynamics of short-term interest rates.

A notable difference in these results with those of Clarida et al. (1998) is that my estimates of the Bundesbank/ECB response to the exchange rate typically have the wrong sign. In the one instance where the estimate is positive and significant, its magnitude is negligible. Since the empirical analysis produces such mixed results for σ, it will be set to zero in the remainder of the analysis.

3 Modeling real exchange rate dynamics with learning

This section describes the MSV (minimum state variable) rational expectations equilibrium and the methodology that the public employs to learn about the equilibrium. In what follows, the public understands the structure of the economic environment but does not know the numerical values of relevant coefficients and/or parameters. The public's beliefs about those values are then formed with least-squares learning rules.

I adopt a relatively unstructured and partial equilibrium approach in the sense that inflation and the activity gap are viewed as exogenously generated by a bivariate vector autoregression

[6] See also Gerlach and Schnabel (1999) who estimate monetary policy reaction functions for an average of the EMU countries over a sample spanning from 1990 to 1998.

(VAR). Market participants view the interest rate rules in conjunction with the VARs as the data generating process which they use to forecast of future inflation, activity gaps, and interest rates. Details about the derivations are relegated to the appendix.

3.1 Rational expectations equilibrium

The economic model of the exchange rate is uncovered interest parity. For the log nominal DM price of the dollar s_t, we have

$$s_t = E_t s_{t+1} - (i_{1t} - i_{2t}). \tag{4}$$

To price the real exchange rate, add and subtract the expected inflation differential $E_t(\pi_{1,t+1} - \pi_{2,t+1})$ on the right side of (4) and rearrange to get the model of the real exchange rate,

$$q_t = E_t q_{t+1} - (i_{1,t} - E_t \pi_{1,t+1}) + (i_{2,t} - E_t \pi_{2,t+1}). \tag{5}$$

Inflation and the activity gap are generated by a fourth-order VAR.[7] For $j = 1, 2$, let $Y'_{j,t} = (\pi_{j,t}, \ldots, \pi_{j,t-3}, x_{j,t}, \ldots, x_{j,t-3})$, and let the VAR have the companion representation,

$$Y_{j,t} = \alpha_j + A_j Y_{j,t-1} + v_{j,t}. \tag{6}$$

Define e_1 and e_2 to be selection vectors such that $\pi_{j,t} = e_1 Y_{j,t}$ recovers the inflation rate and $x_{j,t} = e_2 Y_{j,t}$ recovers the activity gap. Then the one-step ahead forecast of the inflation rate is

$$E_t \pi_{j,t+1} = e_1 (\alpha_j + A_j Y_{j,t}). \tag{7}$$

Substituting (7), (1) and (2) into (5) gives a first-order stochastic difference equation in q_t. The MSV rational expectations solution to this equation is,

$$q_t = a_0 + a_1 i_{1,t} + a_2 i_{2,t} + a_3 Y_{1,t} + a_4 Y_{2,t} \tag{8}$$

where

$$a_1 = -\frac{1}{\rho_1}, \tag{9}$$

$$a_2 = \frac{1}{\rho_2}, \tag{10}$$

$$a_3 = (e_1 (I - \theta_1 A_1) - \mu_1 v_1) A_1 (I - A_1)^{-1}, \tag{11}$$

$$a_4 = -(e_2 (I - \theta_2 A_2) - \mu_2 v_2) A_2 (I - A_2)^{-1}. \tag{12}$$

Notice the dependence of the coefficient vectors a_3 and a_4 on the inflation response coefficient θ. Shifts in the response coefficient might explain changes in the correlation between the real

[7] In the empirical work that follows, the BIC rule identifies a 4-th order VAR as appropriate.

exchange rate and the German–US inflation differential over time. Because $\theta_1, \theta_2 < 1$ in the pre-Volker sample, a decline in the expected German-US inflation might have led the public to expect an increase in the German-US real interest differential and a real depreciation of the dollar whereas with $\theta_1, \theta_2 > 1$ after 1979Q3, a decline in the expected inflation differential may have led the public to expect a decline in the German-US interest differential and a real appreciation of the dollar.

3.2 Learning the rational expectations equilibrium

In the learning environment, agents know the relevant functional forms so of the model but do not know the values of the policy rule parameters or the true coefficient values of the VAR that governs the dynamics of inflation and output (unemployment) gap. In 'real time,' the public proceeds as a would-be econometrician who acquires knowledge of the relevant coefficients using least-squares learning rules [Evans and Honkapohja (2001)].

The learning path is obtained by solving (5) using expectations formed from people's perceived law of motion. At time t, given the coefficients $\alpha_{j,t-1}, A_{j,t-1}$, agents use the VAR

$$Y_{j,t} = \alpha_{j,t-1} + A_{j,t-1} Y_{j,t-1} + v_{j,t}, \tag{13}$$

to forecast future values $E_t(Y_{j,t+1}) = \alpha_{j,t-1} + A_{j,t-1} Y_{j,t}, E_t(Y_{j,t+2}) = \alpha_{j,t-1} + A_{j,t-1} E_t(Y_{j,t+1})$. Inflation and output gap forecasts follow directly $E_t \pi_{j,t+k} = e_1 E_t(Y_{j,t+k})$, for $k = 1, 2$, and $E_t(x_{j,t+1}) = e_2 E_t(Y_{j,t+1})$. Believing that the rational expectation solution is (8), agents form their PLM (perceived law of motion) for the exchange rate as

$$q_t = a_{1,t-1} i_{1,t} + a_{2,t-1} i_{2,t} + a'_{3,t-1} Y_{1,t} + a'_{4,t-1} Y_{2,t}, \tag{14}$$

The PLM for interest rates, based on the Taylor rules (1) and (2) are,

$$\begin{aligned} i_{1,t} &= b'_{t-1}(1, i_{1,t-1}, E_t(\pi_{1,t+1}), x_{1,t}) + \eta_{1,t}, \\ i_{2,t} &= c'_{t-1}(1, i_{2,t-1}, E_t(\pi_{2,t+1}), x_{2,t}) + \eta_{2,t} \end{aligned}$$

where $b_{t-1} = \left(b_{1,t-1}, \ldots, b'_{4,t-1}\right)$ and $c_{t-1} = \left(c_{1,t-1}, ..., c'_{4,t-1}\right)$ Agents then form interest rate forecasts based on the PLM,

$$E_t i_{1,t+1} = b'_{t-1}(1, i_{1,t}, E_t(\pi_{1,t+2}), E_t(x_{1,t+1})) \tag{15}$$

$$E_t i_{2,t+1} = c'_{t-1}(1, i_{2,t}, E_t(\pi_{2,t+2}), E_t(x_{2,t+1})) \tag{16}$$

Use (13), (15) and (16) to obtain the expected exchange rate

$$E_t q_{t+1} = a'_{t-1}(E_t(i_{1,t+1}), E_t(i_{2,t+1}), E_t(Y_{1,t+1}), E_t(Y_{2,t+1})). \tag{17}$$

Now plug (17) and the expected inflation rates into (5) to get the ALM (actual law of motion),

$$
\begin{aligned}
q_t &= (a_{1,t-1}b_{1,t-1} - 1)\, i_{1,t} + (1 + a_{2,t-1}c_{1,t-1})\, i_{2,t} \\
&\quad + ((e_1 + a_{1,t-1}b_{3,t-1}e_2 + a_{3,t-1} + a_{1,t-1}b_{2,t-1}e_1 A_{1,t-1}))\, A_{1,t-1}Y_{1,t} \\
&\quad + (a_{2,t-1}c_{2,t-1}e_1 A_{2,t-1} + a_{2,t-1}c_{3,t-1}e_2 + a_{4,t-1} - e_1)\, A_{2,t-1}Y_{2,t}
\end{aligned} \tag{18}
$$

Then the coefficients $a_{t-1}, b_{t-1}, c_{t-1}, \alpha_{1,t-1}, \alpha_{2,t-1}, A_{1,t-1}, A_{2,t-1}$ are updated for analysis next period. I employ a constant gain g least-squares updating algorithm, the details of which are described in the appendix. Notice that the learning path and coefficient updating is generated using observations of π_t, x_t, and i_t from the data, but not with exchange rate data.

4 Implied exchange rate paths

The real exchange rate behaves differently under a flexible exchange rate regime than it does under a fixed regime [e.g., Mussa (1986), Baxter and Stockman (1989)]. Also, because exchange controls were in place during the 1960's and early 1970s, uncovered interest parity would not be expected to work well prior to the float. Although the data extend back to 1960, I generate the implied rational expectations real exchange rate beginning in 1976Q1 to coincide with the Rambouillet conference at which the move to floating was ratified by the major industrialized countries.[8]

4.1 Learning paths

If the public believes that the environment is subject to continual and unannounced change, then a constant gain specification makes sense. I follow Orphanides and Williams (2003) by using a constant gain of $g = 0.02$. This is the value they obtained by calibrating expectational adjustments of professional forecasters. Observations from 1960Q2 to 1975Q4 are used to estimate initial values for the covariance matrices and least-squares coefficients.

Table 2 reports report correlations between the implied paths and the data and the volatility of the implied exchange rates relative to the data. The calculations are carried out for observations in log levels and for 1,4,8, and 16 quarter returns. The volatility of the implied one-period return using the output gap measure is somewhat high, but otherwise, the volatility of the learning paths match up reasonably well to the volatility in the data.

The model does a better job at matching movements of the exchange rate at long horizons. Using the output gap, the levels correlation with the data is 0.30 and the 16-period return correlation is 0.45 whereas the 1-period return correlation is only -0.01 and insignificant.

[8] See Hansen and Hodrick (1983).

The implied learning paths and the data are plotted in Figure 4 (output gap) and Figure 5 (unemployment gap). Looking first at Figure 4, the learning path captures the major swings of the exchange rate. It shows a weakening dollar in the 70s, the appreciation and subsequent decline in the 80s, and a dollar weakening from 2001 through 2006. However, the turning points of the learning path do not match exactly with the turning points in the data, which probably accounts for the relatively low correlations in Table 2. While the dollar continued climbing from 1983 to 1985, the learning path shows the dollar falling. Another missed turning point occurs in 1993Q3 when the learning path shows the dollar gaining while the actual bottom of this cycle occurs in 1995.

Using the unemployment gap, as seen in Figure 5, also generates a learning path that broadly captures the major swings–the dollar depreciation of the 70s, dollar gains in the early 1980s and mid 1990s. This learning path shows a more sustained strengthening of the dollar in the early 1980s than the path using the output gap, although it too misses the turning point by beginning the appreciation 8 quarters late and by beginning the dollar appreciation in the mid 1990s 6 quarters early.

To sum up, both of the learning paths capture the broad swings in the real DM–dollar. The model is less able to correctly time the turning points in these swings, however. Because the US–German interest differential begins a sustained rise in 1992Q3, both learning paths show a real dollar appreciation beginning at that time whereas the actual real dollar began its ascension in 1995. The other event that the model does not explain well is the timing of the dollar appreciation that began in 1980. As argued earlier, this was a period of substantial structural instability. Although the learning model is implemented to deal with this instability, the results suggest that there remains room for improvement.

4.2 Rational paths

The implied rational expectations paths are generated using the sub-sample estimated values of the coefficients in the Taylor rules and the VARs. I assume that market participants know that parameter shifts occurred in 1979Q3 in both the Fed and Bundesbank policy rules and also in 1999Q1 for the Bundesbank/ECB. The estimated coefficients are plugged into (9)–(12). I then feed the data values of interest rates, inflation, and the output gap (unemployment gap) into (8) to generate the implied rational path.

The last two columns of Table 2 show the correlations between the implied paths and the data and the volatility of the implied exchange rates relative to the data. The rational path generated with the output gap does not produce enough volatility but the rational path generated with the unemployment gap is about as volatile as the corresponding learning path.

Neither of the rational paths are highly correlated with the data either in levels or in returns form. Looking at the plots in Figures 6 and 7 reveals why. From 1976 through 1992 the rational paths show a general downward trend in the dollar and largely misses the cycle of the early 1980s. The rational paths match up better with the smaller exchange rate fluctuations in the mid 1990s and capture the rise in the dollar from 1995 to 2001.

To sum up, the learning model provides a better explanation of the data than the rational model by more closely matching the volatility in the data and more closely approximating the four major turning points (1979Q3, 1984Q4, 1995Q1, and 2002Q1) in the real DM-dollar rate.

4.3 Comparison to Engel and West (2006)

Engel and West (2006) calibrate a rational expectations version of the Taylor–rule exchange rate model. There are a number differences between our analyses which may be useful to point out. First, Engel and West use monthly data from 1979:10 to 1998:12 and they assume a single regime for the Taylor rule. Second, they do not estimate the parameter values of the Taylor rule but use estimated values reported in other studies. Third, they do not allow for interest rate smoothing by including the lagged interest rate in the policy rule. Fourth, they include the contemporaneous real exchange rate in the Taylor rule. Fifth, their Taylor rule depends on the expected one-year ahead inflation. Sixth, they use a trivariate VAR for π, x, and i. Seventh, their activity gap is quadratically detrended industrial production, and finally, they impose equality of the Taylor rule coefficients across countries and work in terms of German–US differentials.

To expand on the comparison with Engel–West, I adapt their monthly model over 1979.10–1998.12 to my quarterly data. They stated interest rates as annual rates and assumed that the authorities react to expected annual inflation. The quarterly version of their Taylor rule is

$$4i_t = \delta + \theta E_t \left(\pi_{t+4} + \pi_{t+3} + \pi_{t+2} + \pi_{t+1} \right) + \mu x_t + \sigma q_t + \eta_t, \tag{19}$$

where the variables are German–US differentials ($\pi_t = \pi_{1,t} - \pi_{2,t}$, and so on). Let $Y_t = (\pi_t, ..., \pi_{t-3}, x_t, ..., x_{t-3}, 4i_t, ..$ then the rational expectations solution is

$$q_t = a_0 + a_1 \eta_t + a_2 Y_t,$$

where

$$a_2 = \left(e_1 \left(A - \theta \left(A^4 + A^3 + A^2 + A \right) \right) - \mu e_2 \right) \left((1+\sigma) I - A \right)^{-1}, \tag{20}$$

$$a_1 = \frac{-1}{1+\sigma}, \tag{21}$$

$$a_0 = -\frac{1}{\sigma} \left(\delta + \left(e_1 \left(\theta \left(4 + 3A + 2A^2 + \theta A^3 \right) - 1 \right) - a_2 \right) \alpha \right). \tag{22}$$

Engel and West set $\theta = 1.75$, $\mu = 0.25$, and $\sigma = 0.10$ and obtain a correlation between the rational and actual exchange rate of 0.32. To compare our two models, I generated the rational path according to their model using the output gap and the learning paths from my model over the sample 1979Q3 to 1998Q4, and report the results in Table 3.

Although my data are quarterly, I am able to come reasonably close to replicating Engel and West's results (my correlation is 0.26 while theirs is 0.32). The Engel and West model generates about the same volatility in the real exchange rate as the two learning models. The dominance of the correlation between the long horizon changes in the data and the implied paths from the learning model with the output gap over Engel and West indicates that the learning model does a better job of explaining the long swings in this subsample.

5 Conclusion

Standard open economy models predict that the exchange rate is determined by differences in the levels of macroeconomic variables. The traditional focus on standard macro fundamentals in exchange rate determination has perhaps led to a rush of judgment about the irrelevance of macro-modeling of exchange rates. In contrast, the fundamental determinants of the exchange rate are relative expected inflation gaps and relative output gaps when central banks conduct monetary policy by setting interest rates according to Taylor rules.

A relatively new and growing literature shows the relevance and importance of Taylor-rule fundamentals in exchange rate determination. This paper contributes to this nascent literature by presenting evidence that the real DM–dollar exchange rate is linked to Taylor-rule fundamentals. To deal with occasional structural change in the Taylor rules, market participants are placed in a learning environment.

As a general statement, the performance of the learning paths dominated the rational paths in explaining the volatility and the actual movements of the real DM–dollar rate. While not a 'slam dunk,' this simple learning framework provides a reasonably good macro-fundamentals driven explanation of major swings in the real DM–dollar exchange rate spanning from 1976 to 2007. While alternative approaches based on multiple equilibria (e.g., Flood and Rose (1999)) or micro market structure (Lyons and Evans (2003)) are worthwhile research directions to pursue, the analysis in this paper also suggests that additional work in the macroeconomic context will be constructive.

Appendix

Relation between regressions and the VAR companion form

For $j = 1, 2$, let the regression form of the VAR be

$$\begin{aligned} \pi_{j,t} &= B_j' \left(1, Y_{j,t-1}'\right)' + \epsilon_{j,t} \\ x_{j,t} &= C_j' \left(1, Y_{j,t-1}'\right)' + \omega_{j,t} \end{aligned}$$

where $Y_{j,t}' = (\pi_{j,t}, \ldots, \pi_{j,t-3}, x_{j,t}, \ldots, x_{j,t-3})$ and B_j' and C_j' are 1×9 vectors of least-squares coefficients. The constant vector and coefficient matrix for the companion form of the bivariate VAR(4) in (6) is

$$\begin{aligned} \alpha_j &= \begin{pmatrix} B_{j,1} & 0 & 0 & 0 & C_{j,1} & 0 & 0 & 0 \end{pmatrix}' \\ A_j &= \begin{pmatrix} B_{j,1} & B_{j,2} & B_{j,3} & B_{j,4} & B_{j,5} & B_{j,6} & B_{j,7} & B_{j,8} \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ C_{j,1} & C_{j,2} & C_{j,3} & C_{j-J,4} & C_{j,5} & C_{\mathrm{j},6} & C_{j,7} & C_{j,8} \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{pmatrix} \end{aligned}$$

Rational expectations solution

Begin with the basic difference equation, which is reproduced here for convenience,

$$q_t = E_t q_{t+1} - (i_{1,t} - E_t \pi_{1,t+1}) + (i_{2,t} - E_t \pi_{2,t+1}) \,. \tag{23}$$

In the model considered in the text, the Taylor rules do not depend on the real exchange rate. When we iterate forward on the basic difference equation the real exchange rate is represented as the expected present value of future interest differentials with a discount factor of 1.

$$q_t = \sum_{j=0}^{\infty} E_t \left((i_{2,t+j} - \pi_{2,t+j+1}) - (i_{1,t+j} - \pi_{1,t+j+1}) \right)$$

Since there is no discounting, we assume equality of the unconditional mean real interest rates so that the unconditional mean real exchange rate $E(q_t) = \sum E(i_{2,t+j} - \pi_{2,t+j}) - E(i_{1,t+j} - \pi_{1,t+j}) =$

0 is finite. Upon substituting the Taylor rules (1) and (2) for the interest rates and the expected inflation (7) into (23) gives a first-order stochastic difference equation in q_t,

$$\begin{aligned} q_t &= E_t q_{t+1} + (\delta_2 - \delta_1) + e_1 \left((1 - \rho_1 \theta_1) \alpha_1 - (1 + \rho_2 \theta_2) \alpha_2 \right) \\ &\quad + (1 - \rho_2) i_{2t-1} - (1 - \rho_1) i_{1t-1} + \left(\eta_{2,t} - \eta_{1,t} \right) \\ &\quad + \left((\rho_2 \theta_2 - 1) e_1 A_2 + \rho_2 \mu_2 e_2 \right) Y_{2,t} + \left((1 - \rho_1 \theta_1) e_1 A_1 + \rho_1 \mu_1 e_2 \right) Y_{1,t}. \end{aligned} \tag{24}$$

Using the method of undetermined coefficients, the rational expectations solution to (24) is (8) given in the text. The constant term is omitted from the solution since it is not identified. This identification problem stems from the fact that the real exchange rate does not enter into the German Taylor rule for if it did, the present value representation would have a discount factor that is less than one. This would allow the unconditional mean of the real exchange rate to differ from zero. The nonidentifyability of the constant means that the rational expectations equilibrium is not unique. By focusing on the zero constant solution, we focus on one particular solution.

Learning

The ALM (18) presented in the text is obtained as follows. The least-squares updating algorithm proceeds as follows. At time t, the coefficient vectors $B_{j,t-1}, C_{j,t-1}$ $(j = 1, 2)$ are obtained from the regression form of the VAR,

$$\pi_{j,t} = B'_{j,t-1} \left(1, Y'_{j,t-1} \right)' + \epsilon_{j,t}, \tag{25}$$

$$x_{j,t} = C'_{j,t-1} \left(1, Y'_{j,t-1} \right)' + \omega_{j,t}. \tag{26}$$

We then construct the companion form

$$Y_{j,t} = \alpha_{j,t-1} + A_{j,t-1} Y_{j,t-1} + v_{j,t}, \tag{27}$$

for $j = 1, 2$. It follows that

$$\begin{aligned} E_t \left(Y_{j,t+1} \right) &= \alpha_{j,t-1} + A_{j,t-1} Y_{j,t}, \\ E_t \left(Y_{j,t+2} \right) &= \alpha_{j,t-1} + A_{j,t-1} \left(\alpha_{j,t-1} + A_{j,t-1} Y_{j,t} \right). \end{aligned}$$

Expected inflation and output gaps are then for $k = 1, 2$

$$\begin{aligned} E_t \left(\pi_{j,t+k} \right) &= e_1 E_t \left(Y_{j,t+k} \right), \\ E_t \left(x_{j,t+k} \right) &= e_2 E_t \left(Y_{j,t+k} \right). \end{aligned}$$

Agents believe that the rational expectation solution is (8) and form the PLM for the exchange rate as

$$q_t = a'_{t-1}\left(i_{1,t}, i_{2,t}, Y'_{1,t}, Y'_{2,t}\right)' \tag{28}$$

where $a_{t-1} = \left(a_{1,t-1}, \ldots, a'_{4,t-1}\right)'$. The PLMs for interest rates, based upon the Taylor rules are

$$\begin{aligned} i_{1,t} &= b'_{t-1}\left(1, i_{1,t-1}, E_t\left(\pi_{1,t+1}\right), e_2 Y_{1,t}\right) + \eta_{1,t} \\ i_{2,t} &= c'_{t-1}\left(1, i_{2,t-1}, E_t\left(\pi_{2,t+1}\right), e_2 Y_{2,t}\right) + \eta_{2,t} \end{aligned}$$

where $b_{t-1} = \left(b_{1,t-1}, \ldots b'_{4,t-1}\right)'$, $c_{t-1} = \left(c_{1,t-1}, ..., c'_{4,t-1}\right)'$. The PLMs imply the one-step ahead expected interest rates,

$$\begin{aligned} E_t i_{1,t+1} &= b'_{t-1}\left(1, i_{1,t}, E_t\left(\pi_{1,t+2}\right), E_t\left(x_{1,t+1}\right)\right) \\ E_t i_{2,t+1} &= c'_{t-1}\left(1, i_{2,t}, E_t\left(\pi_{2,t+2}\right), E_t\left(x_{2,t+1}\right)\right) \end{aligned}$$

Advance the time subscript in (28) and take expectations conditional on date t information. This gives the expected exchange rate

$$E_t q_{t+1} = a'_{t-1}\left(E_t\left(i_{1,t+1}\right), E_t\left(i_{2,t+1}\right), E_t\left(Y_{1,t+1}\right), E_t\left(Y_{2,t+1}\right)\right)'. \tag{29}$$

Plugging the inflation forecast and (29) into (5) gives (18) in the text.

The least-squares updating of the coefficients proceeds as follows. Let us define

$$\begin{aligned} Z_{1,t} &= \left(1, Y'_{1,t}\right) \\ Z_{2,t} &= \left(1, Y'_{2,t}\right) \\ Z_{3,t} &= \left(1, i_{1,t-1}, E_t\left(\pi_{1,t+1}\right), x_{1,t}\right) \\ Z_{4,t} &= \left(1, i_{2,t-1}, E_t\left(\pi_{2,t+1}\right), x_{2,t}\right) \\ Z_{5,t} &= \left(1, i_{1,t}, i_{2,t}, Y'_{1,t}, Y'_{2,t}\right) \end{aligned}$$

$$\begin{aligned} \beta_{1,t-1} &= B_{1,t-1} \\ \beta_{2,t-1} &= C_{1,t-1} \\ \beta_{3,t-1} &= b_{t-1} \\ \beta_{4,t-1} &= c_{t-1} \\ \beta_{5,t-1} &= a_{t-1} \end{aligned}$$

and

$$\begin{aligned} y_{1,t} &= \pi_{1,t} \\ y_{2,t} &= \pi_{2,t} \\ y_{3,t} &= i_{1,t} \\ y_{4,t} &= i_{2,t} \end{aligned}$$

For $j = 1, ..., 4,$

$$\begin{aligned} R_{j,t} &= R_{j,t-1} + g\left(Z_{j,t-1}Z'_{j,t-1} - R_{j,t-1}\right) \\ \beta_{j,t} &= \beta_{j,t-1} + gR_{j,t}^{-1}Z_{j,t-1}\left(y_{j,t} - Z'_{j,t-1}\beta_{j,t-1}\right) \end{aligned}$$

We can now construct the ALM for the exchange rate according to (18) in the text. The coefficients for the exchange rate PLM are then updated as

$$\begin{aligned} R_{5,t} &= R_{5,t-1} + g\left(Z_{5,t-1}Z'_{5,t-1} - R_{5,t-1}\right) \\ a_t &= a_{t-1} + gR_{5,t}^{-1}Z_{5,t-1}\left(q_t^{alm} - q_{t-1}^{plm}\right) \end{aligned}$$

where

$$q_{t-1}^{plm} = a'_{t-1}Z_{5,t-1}$$

Notice that the learning path of the real exchange rate is generated only with data on the output gaps, inflation, and interest rates.

The Engel-West model

Here, all variables are stated in terms of German–US differentials and the Taylor rule states interest rates expressed at annual rate so I multiply my interest rates by 4.

$$4i_t = \delta + \theta E_t\left(p_{t+4} - p_t\right) + \mu x_t + \sigma q_t + \eta_t, \tag{30}$$

where p is the relative price level. The real interest parity condition is

$$q_t = -4i_t + E_t\pi_{t+1} + E_t q_{t+1}. \tag{31}$$

In Engel–West, inflationary expectations are constructed from a fourth-order VAR in $(\pi, x, 4i)$. Let $Y_t = (\pi_t, \ldots, \pi_{t-3}, x_t, ..., x_{t-3}, 4i_t, ..., 4i_{t-3})'$. Conjecture the solution

$$q_t = a_0 + a_1\eta_t + bY_t. \tag{32}$$

Due to the dependence of the Taylor rule on the real exchange rate, the constant in the solution is identified. Advancing the time subscript in (32) and taking expectations gives $E_t q_{t+1} = a_0 + b\left(\alpha + AY_t\right)$. Substitute the guess solution, the Taylor rule (30) and inflationary expectations into (31). Upon equating coefficients, one obtains (20)–(22).

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Table 1: GMM Estimates of Central Bank Reaction Function

							Structural Change	
	θ	μ	σ	δ	$1-\rho$	J-Stat	All	θ
Sample	(t-ratio)	(t-ratio)	(t-ratio)	(t-ratio)	(t-ratio)	(p-value)	(p-value)	(p-value)
A. Bundesbank/ECB, output gap								
61.2-79.2	-0.512	**0.583**	-0.027	**0.003**	**0.076**	15.324	–	–
	(-0.904)	(**2.270**)	(-1.597)	(**2.151**)	(**2.764**)	(0.501)	–	–
79.3-98.4	2.601	1.699	-0.036	0.001	0.030	14.363	**14.685**	3.3097
	(1.611)	(1.255)	(-1.210)	(1.644)	(1.424)	(0.572)	(0.012)	(0.069)
99.1-07.3	-0.108	**0.661**	-0.003	**0.001**	**0.120**	8.087	**42.484**	2.7986
	(-0.875)	(**6.650**)	(-1.160)	(**5.634**)	(**6.922**)	(0.946)	(0.000)	(0.094)
B. FED, output gap								
61.2-79.2	**0.723**	0.104	–	**0.002**	**0.282**	11.073	–	–
	(**12.726**)	(1.377)	–	(**3.283**)	(**3.383**)	(0.271)	–	–
79.3-07.3	**2.082**	**0.403**	–	-0.001	**0.116**	5.973	**16.599**	**11.935**
	(**5.346**)	(**3.072**)	–	(-1.533)	(**5.507**)	(0.201)	(0.002)	(0.000)

C. Cross-equation restrictions

Sample	Test statistic	p-value
61.2-79.2	7.209	0.125
79.3-98.4	**910.055**	0.000

Table 1 continued

Sample	θ (t-ratio)	μ (t-ratio)	σ (t-ratio)	δ (t-ratio)	$1-\rho$ (t-ratio)	J-Stat (p-value)	Structural Change All (p-value)	Structural Change θ (p-value)
D. Bundesbank/ECB, unemployment gap								
61.2-79.2	-0.297	-2.496	-0.013	0.002	**0.058**	12.512		
	(-0.402)	(-1.771)	(-0.605)	(1.064)	(**2.353**)	(0.708)		
79.3-98.4	3.814	-3.958	-0.028	0.000	0.027	15.643	**23.475**	1.8795
	(1.312)	(-1.083)	(-0.932)	(0.647)	(1.173)	(0.478)	(0.000)	(0.170)
99.1-07.3	-0.121	**-0.710**	**0.004**	**0.001**	**0.224**	7.684	**66.304**	1.8318
	(-1.345)	(**-9.533**)	(**2.442**)	(**2.187**)	(**8.015**)	(0.958)	(0.000)	(0.176)
E. FED, unemployment gap								
61.2-79.2	**0.706**	**-0.379**	–	**0.001**	**0.250**	10.307	–	–
	(**12.801**)	(**-2.574**)	–	(**3.844**)	(**3.514**)	(0.326)	–	–
79.3-07.3	**2.252**	**-0.934**	–	-0.001	**0.107**	5.850	**20.960**	**13.942**
	(**5.489**)	(**-3.401**)	–	(-1.603)	(**5.125**)	(0.755)	(0.000)	(0.000)

F. Cross-equation restrictions

Sample	Test statistic	p-value
61.2-79.2	5.125	0.275
79.3-98.4	**21.458**	0.000

Notes: Estimation of the reaction functions for the Bundesbank/ECB, $i_{1t} = \delta_1 + (1-\rho_1)\, i_{1t-1} + \rho_1\left(\theta_1 E_t \pi_{1,t+1} + \mu_1 x_{1,t} + \sigma q_t\right) + \eta_{1,t}$, and for the Fed, $i_{2t} = \delta_2 + (1-\rho_2)\, i_{2t-1} + \rho_2\left(\theta_2 E_t \pi_{2,t+1} + \mu_1 x_{2,t}\right) + \eta_{2,t}$. J-Stat is Hansen's GMM test of the overidentifying restrictions. Structural change tests conducted for the Bundesbank/ECB between 61.2–79.2 and 79.3–98.4 and between 79.3–98.4 and 99.1–07.3. For the Fed, structural change test is conducted between 61.2–79.2 and 79.3–07.3. Cross equation restrictions tested: $\theta_1 = \theta_2, \mu_1 = \mu_2, \rho_1 = \rho_2, \delta_1 = \delta_2$.

Table 2: Correlations with Data and Relative Volatility

		Learning		Rational	
		output gap	unemployment gap	output gap	unemployment gap
Level	Rel Vol	0.621	0.803	0.541	0.745
	Corr	**0.250**	0.075	0.002	-0.084
	t-ratio	**2.889**	0.838	0.026	-0.956
1-qtr return	Rel Vol	0.806	0.963	0.595	0.872
	Corr	-0.009	-0.047	0.043	0.019
	t-ratio	-0.100	-0.520	0.482	0.215
4-qtr return	Rel Vol	0.623	0.862	0.464	0.761
	Corr	-0.107	-0.158	0.149	0.038
	t-ratio	-1.189	-1.757	1.680	0.424
8-qtr return	Rel Vol	0.553	0.817	0.409	0.611
	Corr	0.002	-0.149	0.048	-0.050
	t-ratio	0.027	-1.633	0.510	-0.549
16-qtr return	Rel Vol	0.477	0.802	0.374	0.528
	Corr	**0.340**	0.008	0.102	0.005
	t-ratio	**3.778**	0.084	1.080	0.052

Notes: Rel Vol is the volatility of the model implied exchange rate relative to the volality found in the data. Bold indicates significance at the 5 percent level.

Table 3: 1979Q3–1998Q4 Comparison to Engel–West

		Rational EW with output gap	Learning with output gap	Learning with unemployment gap
Levels	Rel Vol	0.624	0.614	0.803
	Corr	**0.257**	-0.016	0.075
	t-ratio	**2.058**	-0.136	0.838
1-qtr return	Rel Vol	0.440	0.749	0.963
	Corr	-0.068	0.075	-0.047
	t-ratio	-0.528	0.657	-0.520
4-qtr return	Rel Vol	2.291	0.614	0.862
	Corr	0.047	-0.140	-0.158
	t-ratio	0.367	-1.237	-1.757
8-qtr return	Rel Vol	1.409	0.576	0.817
	Corr	0.157	-0.074	-0.149
	t-ratio	1.235	-0.649	-1.633
16-qtr return	Rel Vol	1.118	0.534	0.802
	Corr	**-0.301**	**0.350**	0.008
	t-ratio	**-2.441**	**3.255**	0.084

Notes: Rel Vol is the volatility of the model implied exchange rate relative to the volality found in the data. Bold indicates significance at the 5 percent level.

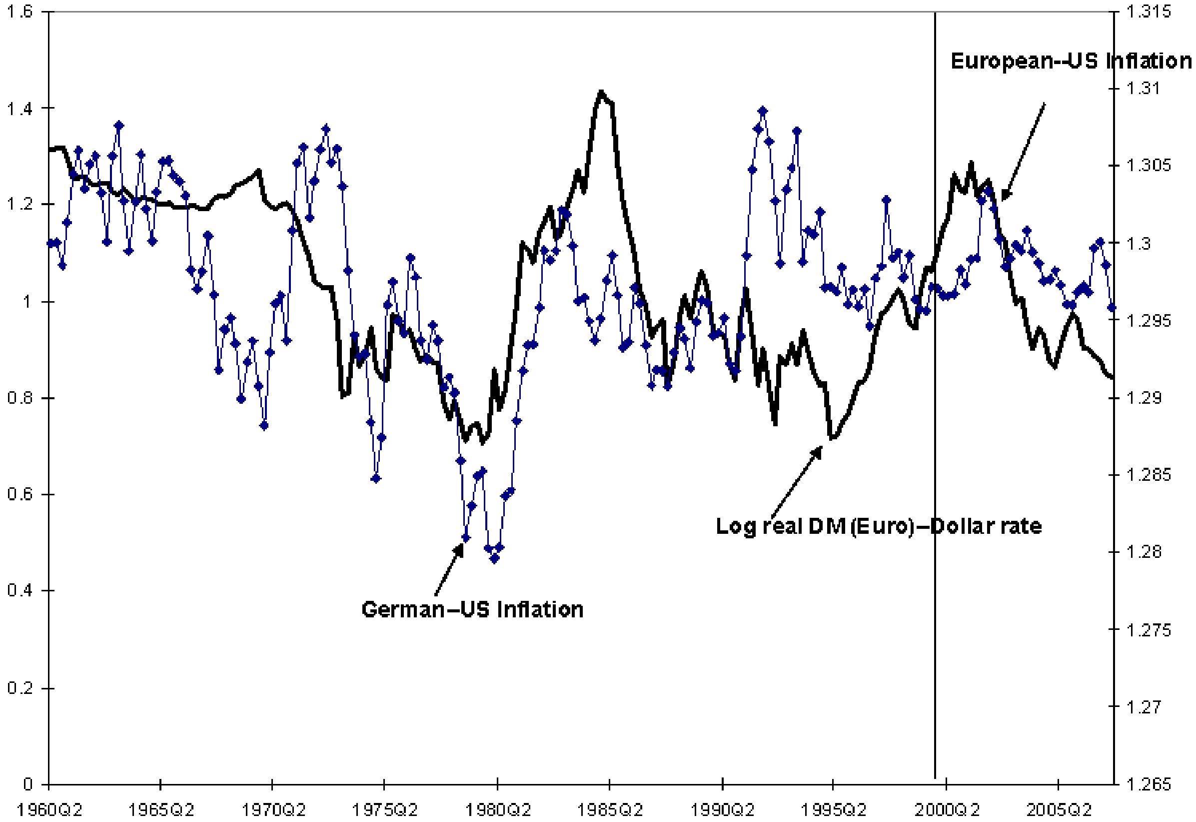

Figure 1: Log real DM–dollar rate and German–USA inflation before 1999.1 and log real euro-dollar rate with Euro-USA inflation afterwards.

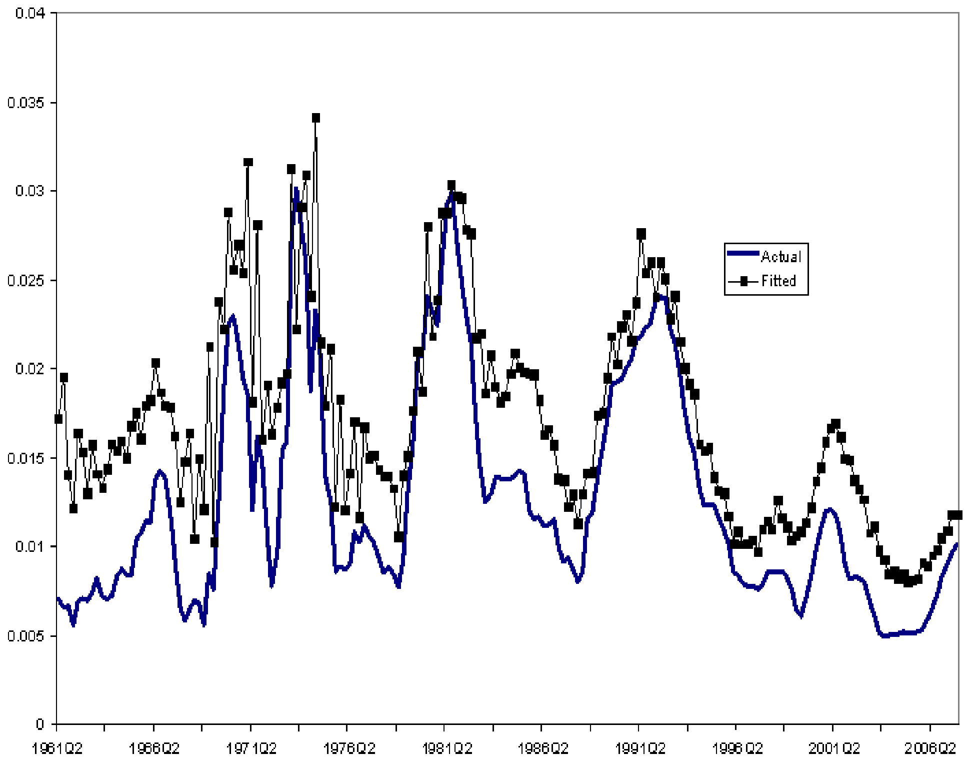

Figure 2: German interest rate and fitted value from estimated reaction function

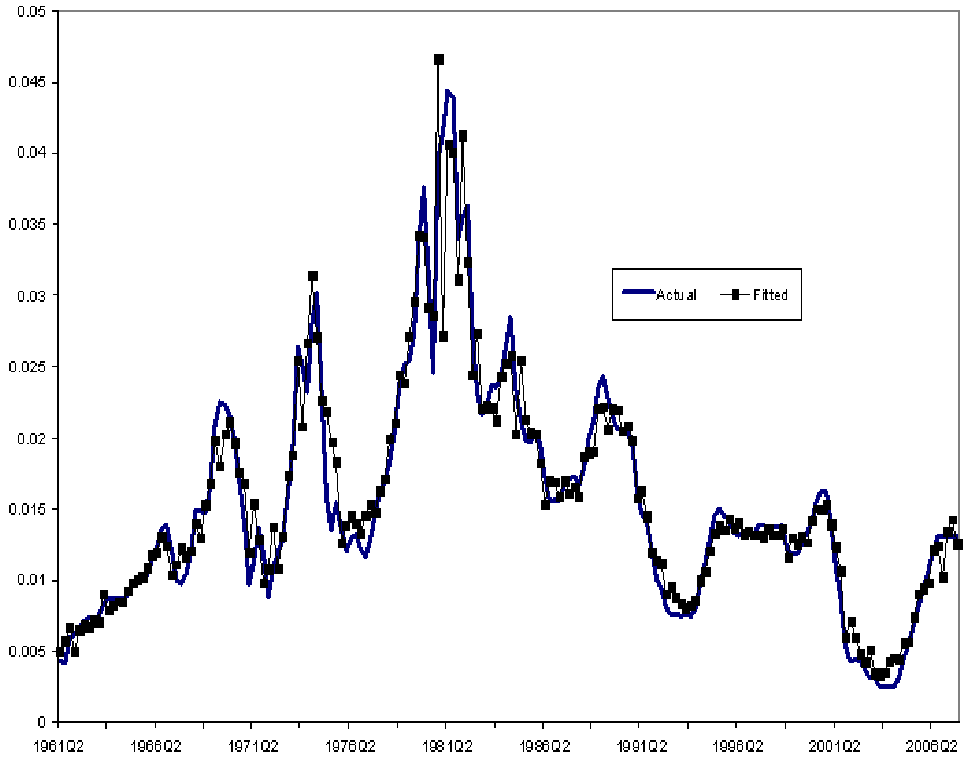

Figure 3: US interest rate and fitted values from estimated reaction function

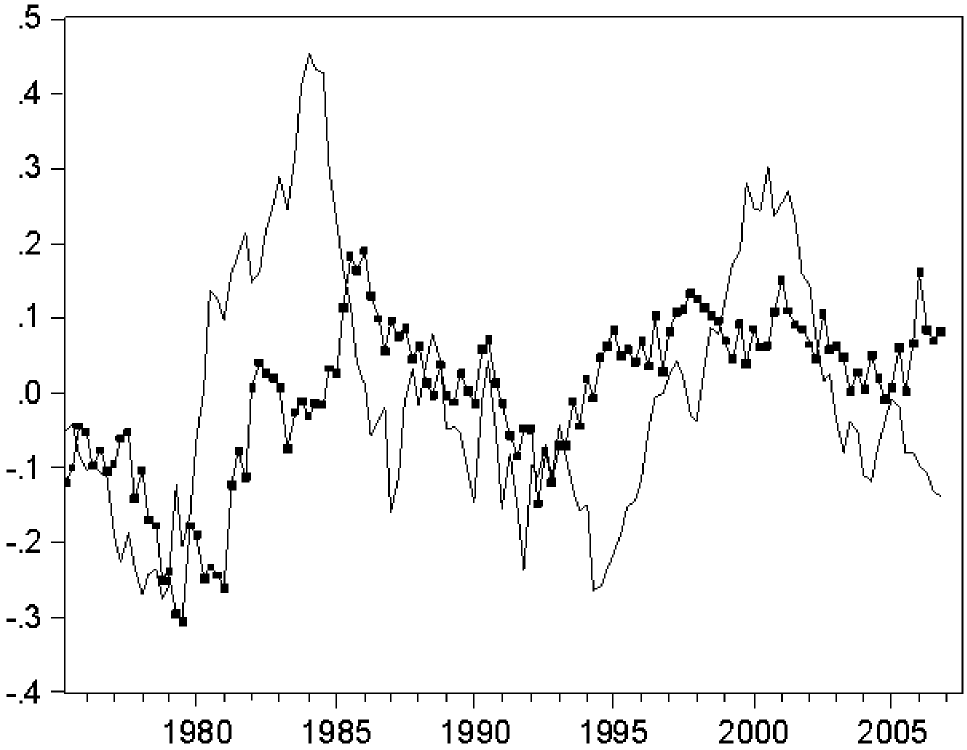

Figure 4 Learning path using output gap (boxes) and the data (solid).

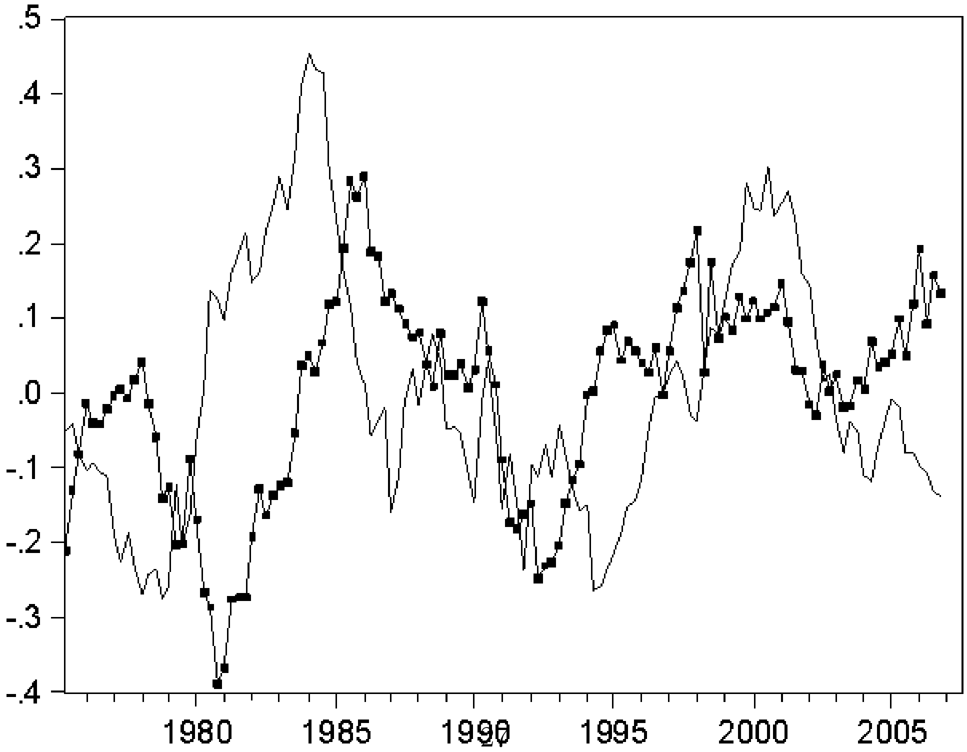

Figure 5: Learning path using unemployment gap (boxes) and the data (solid).

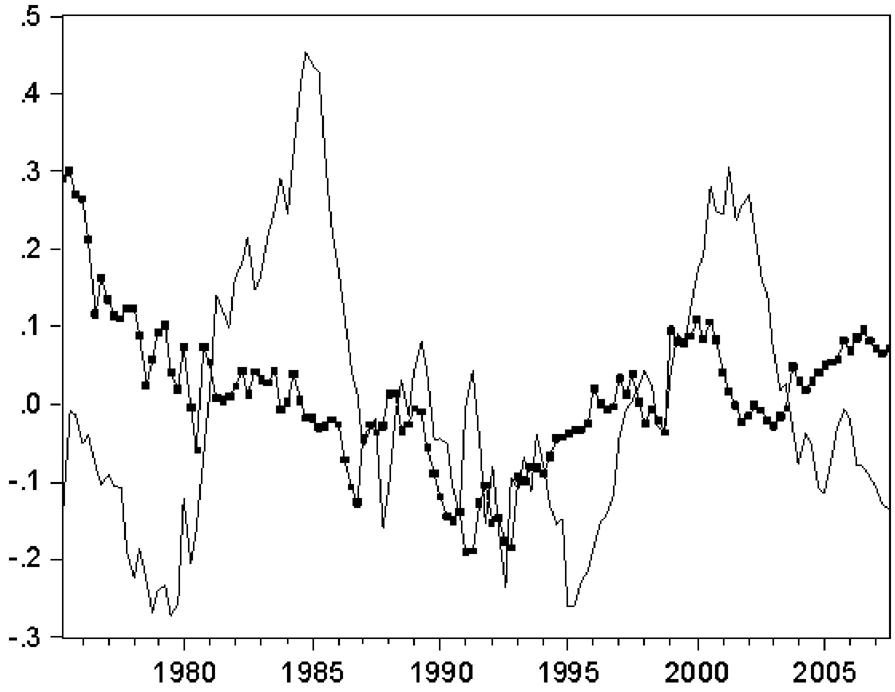

Figure 6: Rational path using output gap (boxes) and the data (solid).

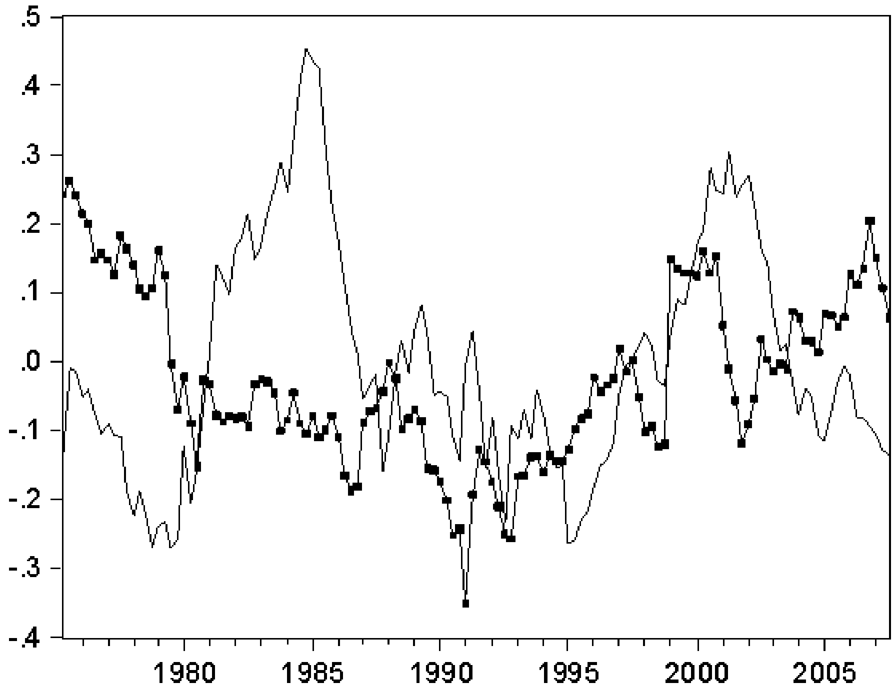

Figure 7: Rational path using unemployment gap (boxes) and the data (solid).