Online Appendix
Demographics and Monetary Policy Shocks

Kimberly Berg, Chadwick Curtis, Steven Lugauer, and Nelson Mark

Appendix A THE CONSUMPTION DATA

Our consumption data is taken from the quarterly Consumer Expenditure Survey Interview Sample from 1984Q1 to 2007Q4, compiled by the U.S. Bureau of Labor Statistics. The data are found in the Family Characteristics and Income (FAMILY) files.

Table A1: CEX consumption categories and CPI categories

	CEX		CPI	
	Category Name	CEX Code (CQ)	Category Name	CPI Code
1	Food	FOODPQ	Food	SAF1
2	Alcohol beverages	ALCBEVCQ	Alcoholic beverages	SAF116
3	Tobacco	TOBACCCQ	Tobacco and smoking products	SEGA
4	Utilities	UTILCQ	Fuels and utilities	SAH2
5	Personal care	PERSCACQ	Personal care	SAG1
6	Household operations	HOUSOPCQ	Household furnishings and operations	SAH3
7	Public transportation	PUBTRACQ	Public transportation	SETG
8	Gas and motor oil	GASMOCQ	Motor fuels	SETB
9	Apparel	APPARCQ	Apparel	SAA
10	Education	EDUCACQ	Tuition expenditures	SEEB
11	Reading	READCQ	Recreational reading material	SERG
12	Health Care	HEALTHCQ	Medical care	SAM
13	Miscellaneous expenditures	MISCCQ	Miscellaneous personal services	SEGD
14	Entertainment	ENTERTCQ	Entertainment	SA6/SAR†
15	House equipment	HOUSEQCQ	Household furnishings and operations	SAH3
16	Vehicles	TRANSCQ-GASMOCQ -PUBTRACQ	Private Transportation	SAT1‡
17	Other lodging	OTHLODCQ	Shelter	SAH1
18	Owned dwelling	OWNDWECQ	Shelter	SAH1
19	Rented dwelling	RENDWECQ	Rent of primary residence	SEHA

Notes: The CPI codes are matched with CEX consumption categories following Krueger and Perri (2006) with the exceptions of entertainment and vehicles.
† Prior to 1998 this is SA6 (Entertainment). From 1998 on it is SAR (recreation).
‡ We combine purchases and vehicle maintenance into Vehicles in the CEX category and use private transport in the CPI.

Following Krueger and Perri (2006), we construct real consumption expenditures by deflating the household's expenditures for each of the 19 categories listed in Table A1 by its category-specific deflator. The table also lists the BLS CPI code. Total household consumption is the sum of expenditures on components 1-19.

Let $c_{i,t}$ be total household consumption divided by the number of members in household

i at time t. Let H_t be the total number of household observations in the group. Following Anderson et al. (2016), within-group aggregate consumption is

$$c_t = \left(\frac{1}{H_t} \sum_{i=1}^{H_t} c_{i,t} \right).$$

Generating Quarterly Consumption: The CEX is a rotating survey where respondents are interviewed up to 5 times. Respondents are interviewed once a quarter, but the interview can occur in any of the 3 months within that quarter. The first interview collects information on the household's characteristics, but not it's consumption expenditures. Hence, each household has at most 4 usable observations. In the subsequent interviews, the household reports expenditures over the previous 3 months. There is a difference between the calendar quarter and the interview quarter, and we discuss here how we calculate calendar quarter observations, which is illustrated in Table A2.[1]

Table A2: Interview month and calendar-quarter

		Month of Interview			
Calendar - Quarter	Month of Expenditure Recorded	Jan. 2001 HH-1	Feb. 2001 HH-2	Mar. 2001 HH-3	Apr. 2001 HH-4
2000 Q4	Oct.	✓			
	Nov.	✓	✓		
	Dec.	✓	✓	✓	
2001 Q1	Jan.		✓	✓	✓
	Feb.			✓	✓
	Mar.				✓

The table shows 4 fictitious households, HH-1 through HH-4, each interviewed in a different month. HH-1 is interviewed in January about its October through December expenditures. HH-2 is interviewed in February about its November-January expenditures, and so forth.

Table A3: Calendar-Quarter Consumption

	HH-1	HH-2	HH-3	HH-4
2000 Q4	(Oct.+Nov.+Dec.)$\times\frac{3}{3}$	(Nov.+Dec.)$\times\frac{3}{2}$	Dec.$\times\frac{3}{1}$	
2001 Q1		Jan.$\times\frac{3}{1}$	(Jan.+Feb.)$\times\frac{3}{2}$	(Jan.+Feb.+Mar.)$\times\frac{3}{3}$

To align the interview-quarter expenditures to calendar-quarters, we treat expenditures in a given month as representative of expenditures for the calendar-quarter. We illustrate in Table A3. HH-1's interview provides information for the entire 4th quarter of 2000, so the interpretation is clear. HH-2 is reporting spending for 2 months in 2000Q4. We multiply that spending number by 3/2 and that becomes HH-2's 2000Q4 consumption. HH-2 reports spending for 1 month in 2001Q1. That spending number is multiplied by 3 and that becomes HH-2's 2001Q1 consumption. The adjustments for HH-3 and HH-4 follow analogously.[2]

Appendix B ADDITIONAL EMPIRICAL RESULTS

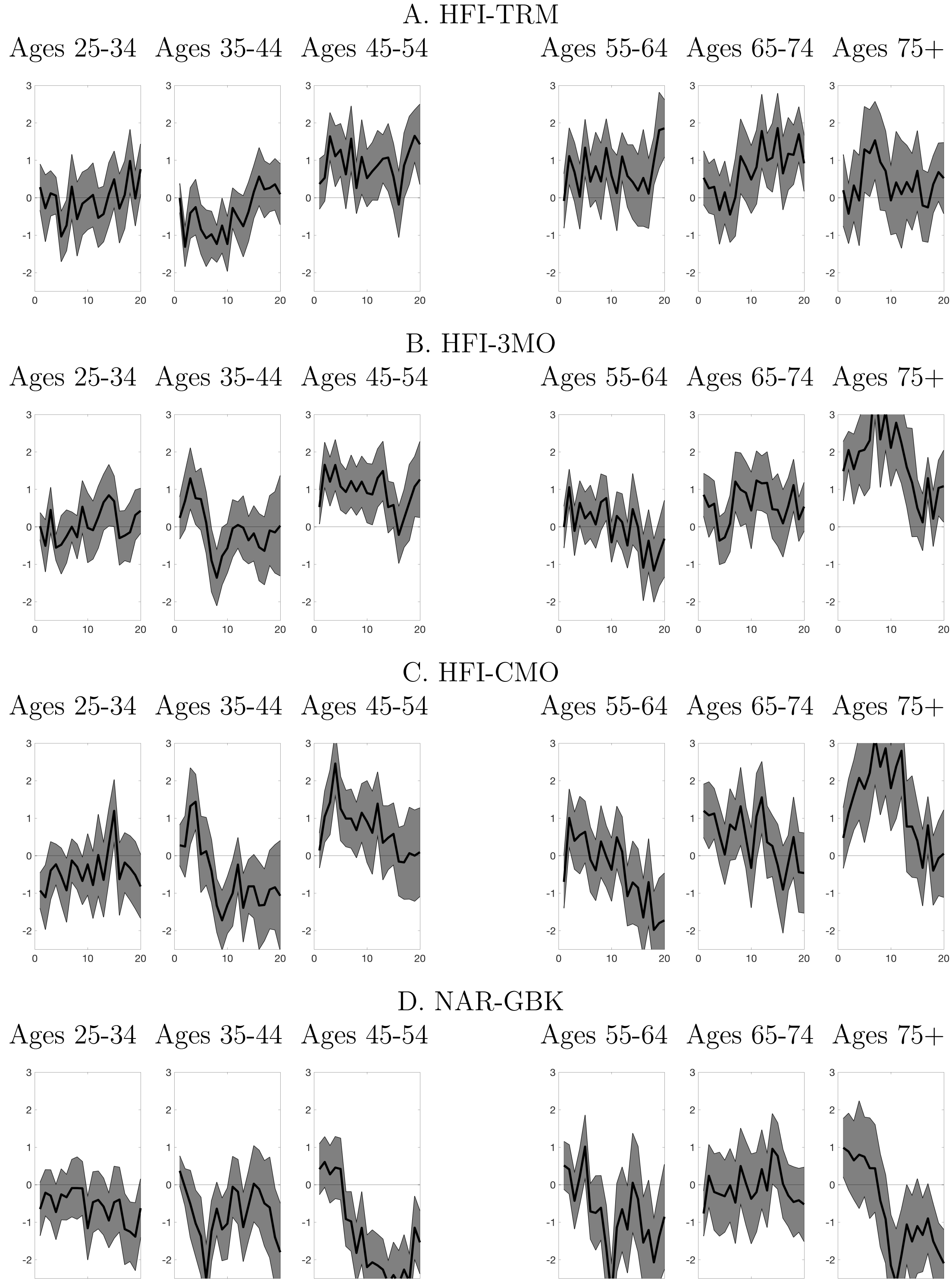
This section reports the following additional empirical results using local projections.

1. Classification of households into six age groups.
2. Response of durable goods expenditure.
3. Response of consumption less housing expenditure.
4. Application of CEX weights in age group consumption aggregation.
5. Omitting lagged consumption and the real federal funds rate.

B.1 Six Age Groups

Here, we consider 6 separate groups: 25-34, 35-44, 45-54, 55-64, 65-74 and 75+. Figure B1 reports the consumption responses based on the local projections and one standard deviation expansionary HFI-TRM, HFI-3MO, HFI-CMO, and NAR-GBK monetary policy shocks. Once we split the sample into finer groups, we again see the old respond most to monetary policy shocks, although in a few instances the 65-74 age group responds most. The responses, in general, get progressively stronger as households age.

Figure B1: Local Projection Impulse Response of Cumulated Consumption Growth to Expansionary Monetary Policy Shock by Age Group

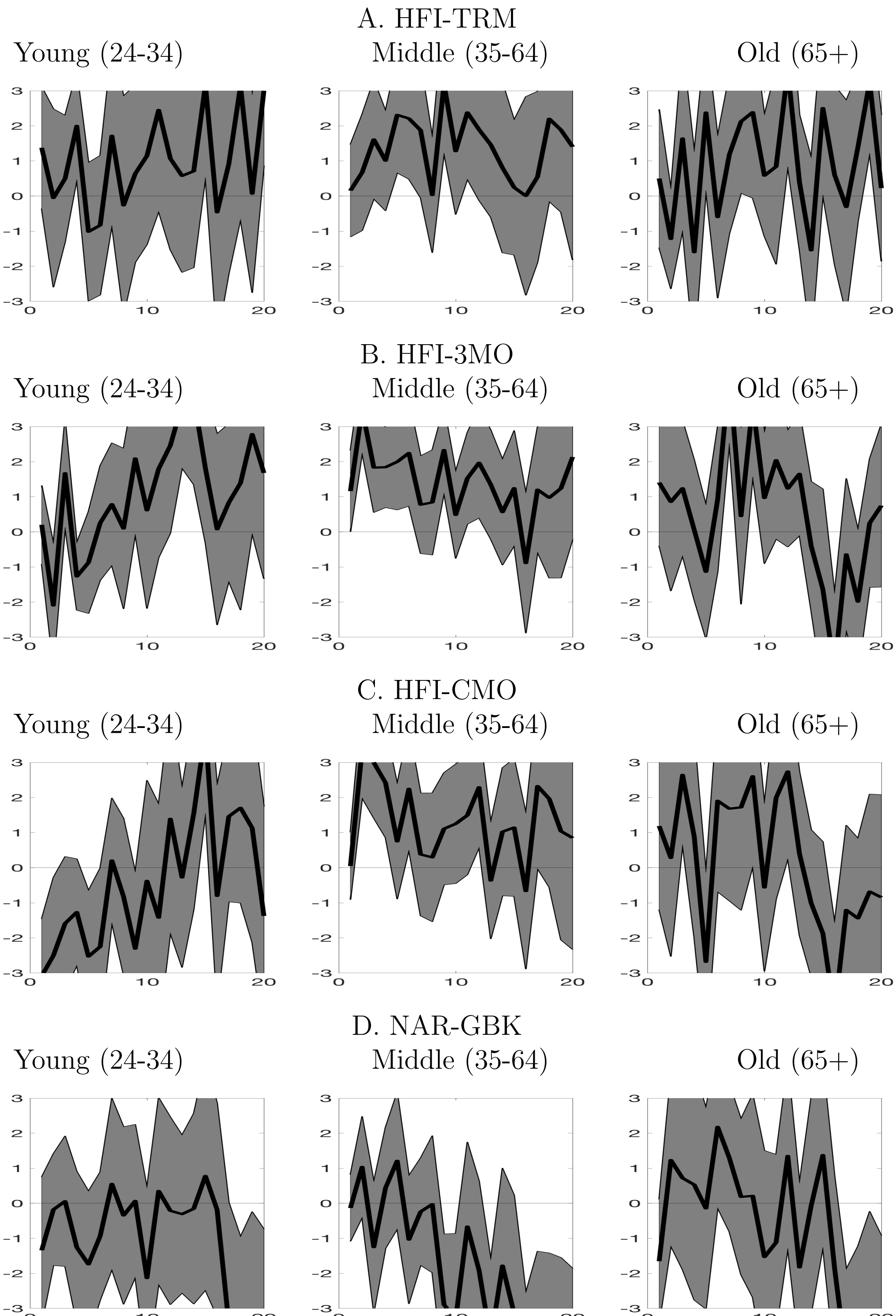


Notes: This figure plots $-\beta_h$ from Equation (1) in the main text. Shaded areas are $\pm$ one standard error Newey and West (1987) confidence bands. The horizontal axis indicates the number of quarters for up to five years after the shock. The vertical axis is measured in percent.

B.2 Durable Consumption

For Figure B2 we report responses of consumption on durable goods from our local projections. In the data, the oldest age groups spend a larger share on non-durable consumption than the consumption shown here: at the latter portions of the life-cycle, households already have durable goods acquired through life and are possibly downsizing their ownership of these goods. As a result, the data shows a lot of variation when we sub-divide consumption expenditures to include this smaller component of total consumption.

Figure B2: Local Projection Impulse Response of Cumulated Durable Consumption Growth to Expansionary Monetary Policy Shock by Age Group

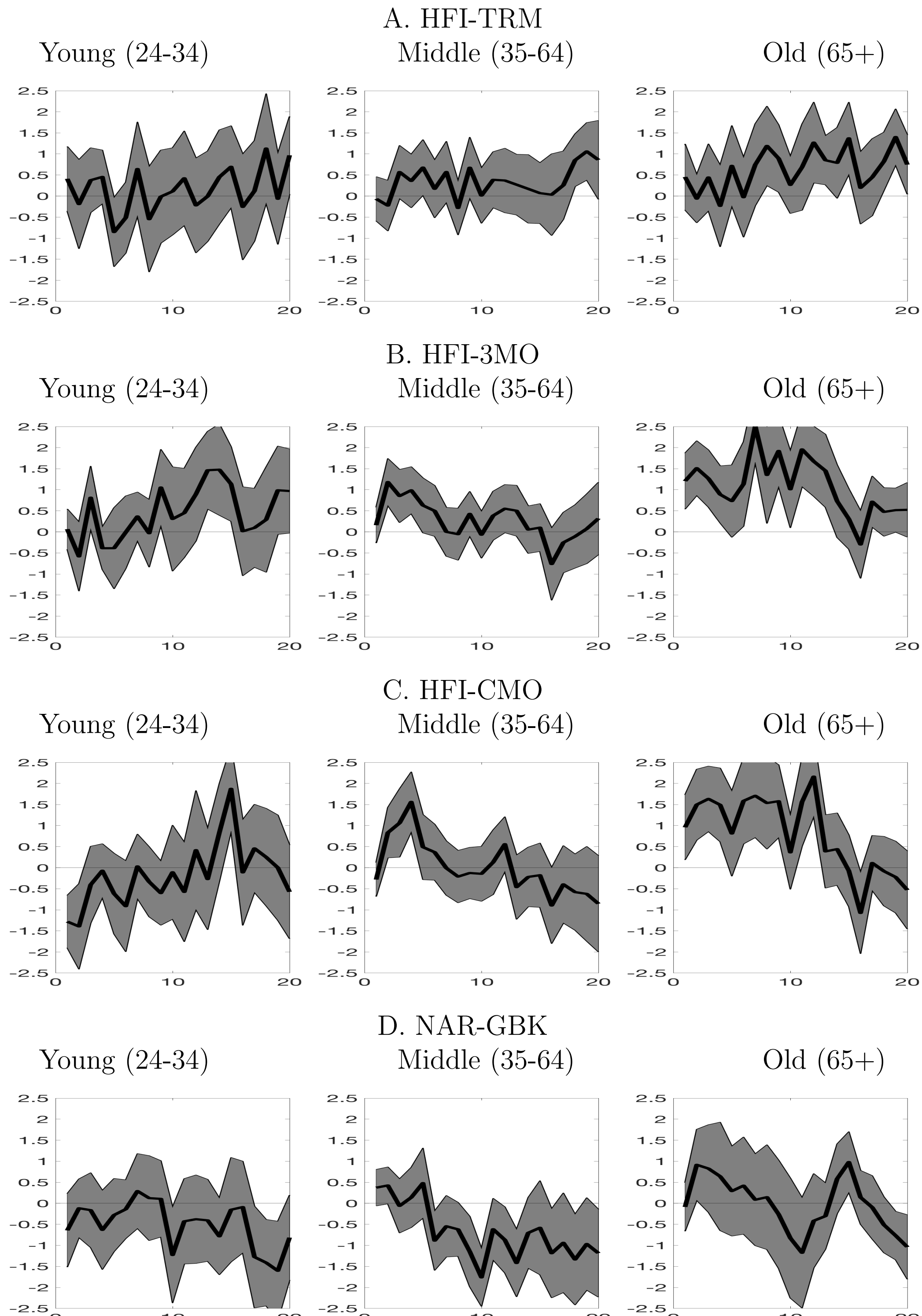


Notes: This figure plots $-\beta_h$ from Equation (1) in the main text. Shaded areas are $\pm$ one standard error Newey and West (1987) confidence bands. The horizontal axis indicates the number of quarters for up to five years after the shock. The vertical axis is measured in percent.

B.3 Consumption Less Housing

Another aspect to consider is the inclusion of housing rents and owner occupied dwelling expenditures. As these categories tend to be rather large, it may be informative to consider consumption expenditures without these categories. We use this consumption measure in our local projections in Figure B3. These results are closely in line with our baseline results, suggesting housing expenditures are not solely driving our results.

Figure B3: Local Projection Impulse Response of Cumulated Consumption Less Housing Expenditures Growth to Expansionary Monetary Policy Shock by Age Group

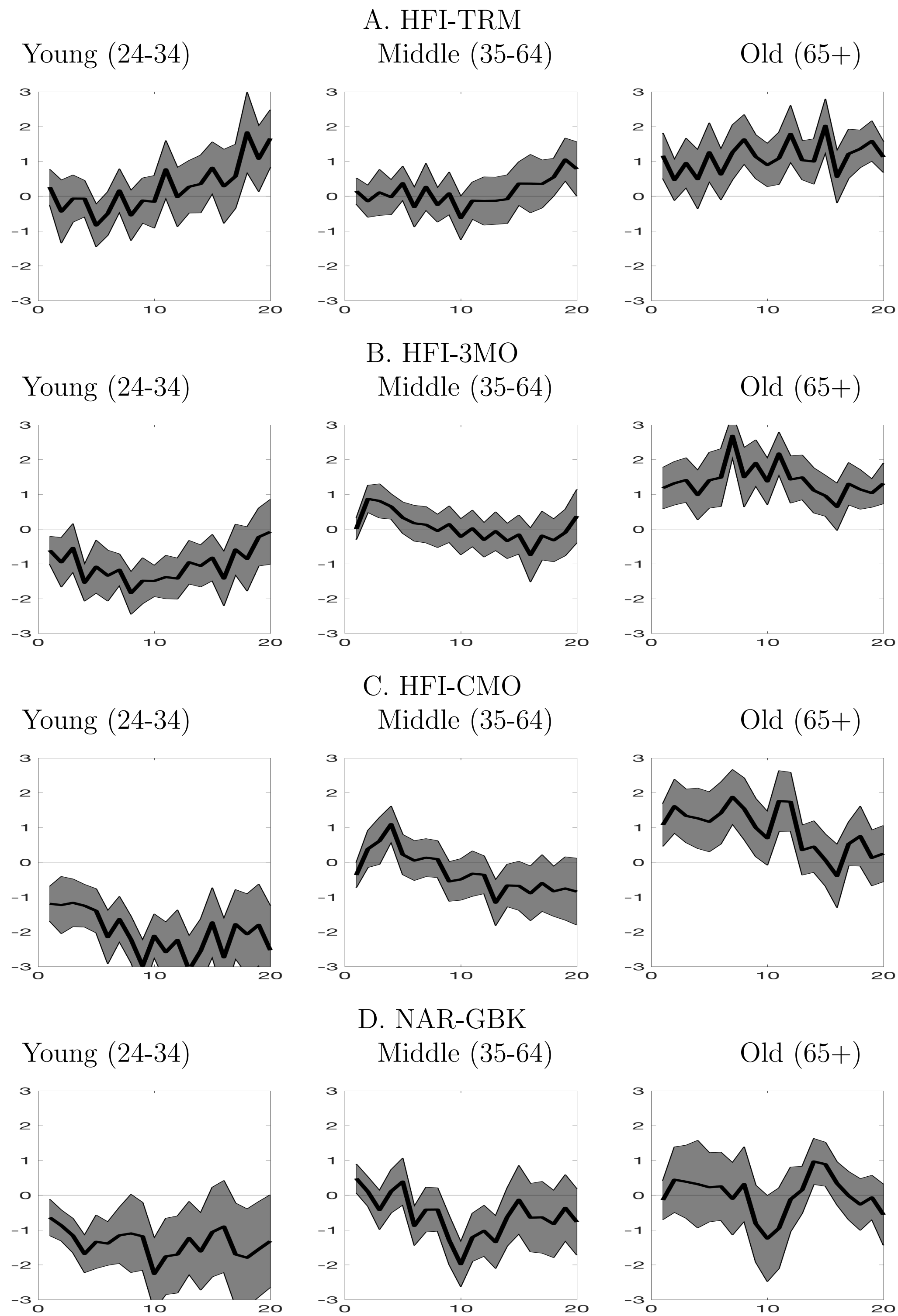


Notes: This figure plots $-\beta_h$ from Equation (1) in the main text. Shaded areas are ± one standard error Newey and West (1987) confidence bands. The horizontal axis indicates the number of quarters for up to five years after the shock. The vertical axis is measured in percent.

B.4 Alternative within Age Group Aggregation of Consumption Expenditures

Here, we aggregate consumption by age using the CEX provided sampling weights. Our consumption measure aggregates consumption by age group using the weights, and we transform this into a per-person measure by dividing total consumption for each age group by the weighted cohort sizes. The unattractive aspect of using this measure is that the weights are constructed based on numerous household characteristics to calculate aggregate consumption across all age groups, not just by household age. Nonetheless, Figure B4 shows the responses to consumption by this measure from our local projections. The results are close to our baseline specification.

Figure B4: Local Projection Impulse Response of Cumulated Consumption (Using CEX Weights) Growth to Expansionary Monetary Policy Shock by Age Group



Notes: This figure plots $-\beta_h$ from Equation (1) in the main text. Shaded areas are $\pm$ one standard error Newey and West (1987) confidence bands. The horizontal axis indicates the number of quarters for up to five years after the shock. The vertical axis is measured in percent.

B.5 Local Projections Without Controls for Lagged Consumption of the Real Federal Funds Rate

Figure B5: Local Projection Old Minus Young and Old Minus Middle Impulse Responses of Cumulated Consumption Growth to Expansionary Monetary Policy Shock without Controls for Lagged Consumption and Federal Funds Rate

Notes: Shaded areas are ± one standard error Newey and West (1987) confidence bands. The horizontal axis indicates the

Appendix C STRUCTURAL VECTOR AUTOREGRESSIONS

Here we re-estimate the consumption response to a monetary policy shock using structural VARs, similar to those employed by Anderson et al. (2016) and Ramey (2011), who study consumption responses to fiscal policy shocks. The three variables in the VAR are, (i) $g_{c,j,t} = 100\Delta\ln(c_{j,t})$, the quarterly percentage growth rate of average real per capita consumption of age group $j = \{\text{young, middle, old}\}$ at time t, (ii) s_t, the monetary policy shock and (iii) r_t, the real federal funds rate. We estimate separate VARs for young (25-34), middle (35-64), and old (65+) households. To avoid clutter in this exposition, we suppress the age group j subscript.

Consumption growth is allowed to respond to contemporaneous monetary policy shocks and changes in the real federal funds rate. The real federal funds rate is affected by contemporaneous policy shocks but not contemporaneous consumption growth. Because the monetary policy shocks are exogenous, neither lags of the shock nor lags of other variables appear in the equation for s_t. Suppressing the constants, we impose these conditions in the structural VAR as,

$$\begin{pmatrix} 1 & a_{12} & a_{13} \\ 0 & 1 & 0 \\ 0 & a_{32} & 1 \end{pmatrix} \begin{pmatrix} g_{c,t} \\ s_t \\ r_t \end{pmatrix} = \sum_{p=1}^{k} \begin{pmatrix} b_{p,11} & b_{p,12} & b_{p,13} \\ 0 & 0 & 0 \\ b_{p,31} & b_{p,32} & b_{p,33} \end{pmatrix} \begin{pmatrix} g_{c,t-p} \\ s_{t-p} \\ r_{t-p} \end{pmatrix} + \begin{pmatrix} u_{c,t} \\ u_{s,t} \\ u_{r,t} \end{pmatrix}, \tag{C1}$$

where the structural error terms are serially uncorrelated and have diagonal covariance matrix, $E(u_t u_t') = D$. Multiplying both sides of Equation (C1) by A^{-1} gives the reduced form VAR,

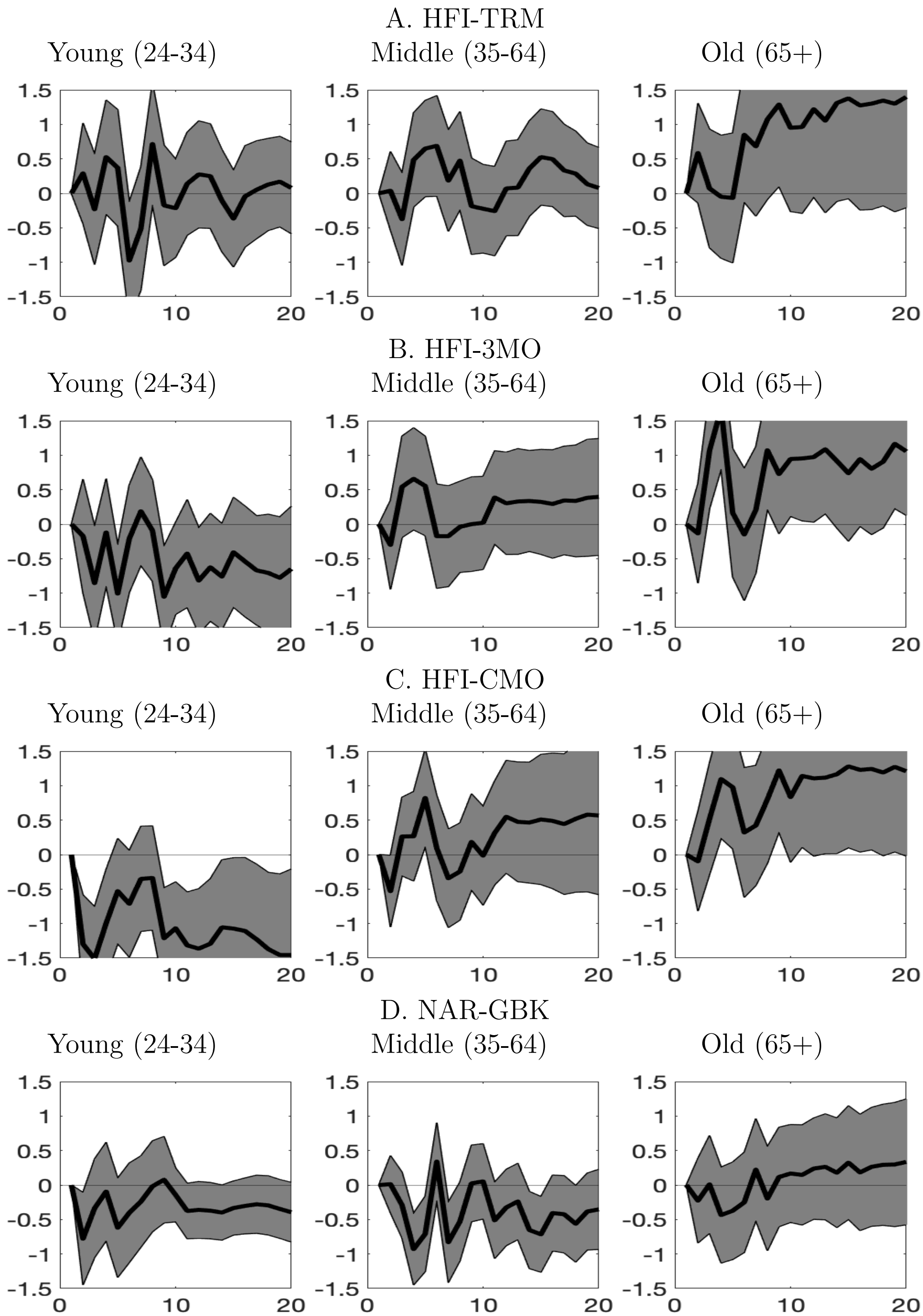
$$\begin{pmatrix} g_{c,t} \\ s_t \\ r_t \end{pmatrix} = \sum_{p=1}^{k} \begin{pmatrix} c_{p,11} & c_{p,12} & c_{p,13} \\ 0 & 0 & 0 \\ c_{p,31} & c_{p,32} & c_{p,33} \end{pmatrix} \begin{pmatrix} g_{c,t-p} \\ s_{t-p} \\ r_{t-p} \end{pmatrix} + \begin{pmatrix} \epsilon_{1,t} \\ \epsilon_{2,t} \\ \epsilon_{3,t} \end{pmatrix}. \tag{C2}$$

Due to the relatively short time-span of the data, imposing these theoretical restrictions lightens the parameterization of the VAR and preserves degrees of freedom.

We estimate the VARs with $k = 8$ lags. Figure C1 shows impulse response functions (IRFs) of cumulated consumption growth by age group to a negative (expansionary) one standard deviation monetary policy shock. The horizontal axis measures time in quarters, up to five years after the shock. The vertical axis measures the consumption response in percent. The shaded areas are plus and minus one asymptotic standard error confidence bands, commonly used in monetary policy VARs (e.g., Romer and Romer (2004)).

To summarize, for each of the four monetary policy shocks, we uncover heterogeneity in consumption responses across age groups with the old having the largest response. The impulse responses may not be statistically significant at every point (our sample sizes are relatively small); however, the difference between age groups is striking and is consistent with the local projections.

Figure C1: Structural VAR Impulse Response of Cumulated Consumption Growth to Expansionary Monetary Policy Shock by Age Group



Notes: The shock is a one standard deviation decrease in the monetary policy shock series. The monetary policy shock series are normalized such that one standard deviation changes in the series match a one standard deviation change in the real federal funds rate. Shaded areas are ± one standard error asymptotic confidence bands. The horizontal axis indicates quarters following the shock. The vertical axis is measured in percent.

Appendix D HOUSEHOLD (MICRO)-LEVEL REGRESSIONS

This section of the Appendix provides additional details for the estimated impulse responses by age group to monetary policy shocks based on household-level regressions, which follow the empirical strategy employed by Coibion et al. (2017) and Wong (2018). This alternative methodology generates responses qualitatively similar to the aggregate approaches (local projections) studied in the main paper. The micro-level analysis also finds the old to have the largest estimated response. The young generally have the second highest response, but not always.

In the CEX, households are interviewed quarterly for up to 5 consecutive quarters. There are at most 4 quarters of consumption observations per household because the first interview is only to obtain household-level characteristics but not information on consumption. Due to staggering of interview months, not all observations are based on consumption from all 3 months in the quarter. We keep only those consumption observations that have data for all 3 months in the quarter. Most households have 3 or 4 quarters of consumption observations.

The variable of interest is the percent change in quarterly consumption per household, i, at time t, $\Delta ln(c_{i,t}) = ln(c_{i,t}) - ln(c_{i,t-1})$. Real consumption per household is obtained by deflating the household's consumption in each specific category with the category-specific CPI and aggregating these together, as described in the main text and Appendix A. By working with growth rates, we lose one observation to differencing, leaving either 2 or 3 usable observations per household.

We further restrict the sample to households reporting non-zero expenditures on food. We also trim the sample by those consumption growth observations below -2.5 (change in log consumption) and above 2.5. If these criteria are not satisfied for even one household observation, the household is dropped from the sample. We consider the same four monetary policy shock series (HFI-TRM, HFI-3MO, HFI-CMO, and NAR-GBK).

Appendix E LIFE-CYCLE MODEL DETAILS

This section provides additional technical details for the life-cycle model, including the income process and consumption choice at the individual level.

E.1 The Income Process

The income process used in the model simulations is as follows. Let there be N individuals per cohort. In each quarterly time period t, living cohorts are indexed by $z \in [1, 248]$. Cohort $z = 1$ begins economic life as a 25 year old household, cohort $z^* = 161$ are newly retired, and cohort $z = 248$ are in the last possible quarter of life.

Working age household i of cohort $z < z^*$ draws labor income $(W_{i,z,t})$ and each retiree $(z \geq z^*)$ draws pension income $(S_{i,z,t})$. Both labor and retirement income have a permanent component $(Y_{i,z,t})$ and a transitory component $(e^{u_{i,z,t}})$.

The transitory income shock $u_{i,z,t}$, is drawn from a mixture of a normal distribution and a low-probability event of zero income for that quarter

$$u_{i,z,t} = \begin{cases} N\left(\mu_u, \sigma_u^2\right) & \text{with probability } (1-p) \\ -\infty & \text{with probability } p \end{cases} \tag{E1}$$

where p is the probability of drawing zero income, and $\mu_u = \frac{-\sigma_u^2}{2} - \ln(1-p)$.

Recalling that cohort z^* has just retired, the labor income for people in their working years is

$$W_{i,z,t} = Y_{i,z,t} e^{u_{i,z,t}} \text{ for } z < z^* \tag{E2}$$

and pension income for retired cohorts is

$$S_{i,z,t} = \begin{cases} Y_{i,z,t} & z = z^* \\ Y_{i,z,t} e^{u_{i,z,t}} & z > z^* \end{cases}. \tag{E3}$$

During the working years, wage growth is driven in part by a common secular component, whose gross growth rate is M_g, and also by the individual's movement along the age-earnings profile. The gross growth rate along this profile at cohort z is G_z. In retirement, both M_g and G_z become 1.

Let $n_{i,z,t} \overset{iid}{\sim} N\left(\mu_n, \sigma_n^2\right)$ be the shock to permanent income $Y_{i,z,t}$ and p_{rr} be the replacement rate on pension income. Then the life-cycle of permanent income evolves according to

$$Y_{i,z,t} = \begin{cases} Y_{i,z-1,t-1} M_g G_z e^{n_{i,z,t}} & z < z^* \\ p_{rr} Y_{i,z^*,t-1} & z = z^* \\ Y_{i,z-1,t-1} e^{n_{i,z,t}} & z > z^* \end{cases} . \tag{E4}$$

Note that in the retirement period, the household receives p_{rr} with certainty, after which income resumes its risky evolution.

We estimate the income process from biennial waves of the Panel Study of Income Dynamics (PSID). We select data between 1986 and 2007 to align with the span of our CEX sample. We use the same definition of household income in the PSID as Blundell et al. (2008) and Storesletten et al. (2007). Our estimation method follows Choi et al. (2017), who build on Zeldes (1989), Carroll (1992), and Carroll (1997).

From the raw PSID income data, we first remove the aggregate time trend, predictable life-cycle or occupation dependent fluctuations, and household fixed effects. The remaining variation is used to estimate the parameters (σ_n, σ_u, p) separately for young, middle, and old households. The gross secular growth rate of household income M_g is given by average real income growth across households over the entire sample period. We estimate the age-income profile G_z using variation in income by age. The age-income profile is assumed to be constant over time.

The resulting annual gross growth rate of the common secular component is 1.006. The parameters governing the age-specific shock processes (σ_n, σ_u, p) are shown in Table E1. There are some differences across age groups. The old are most likely to experience a near

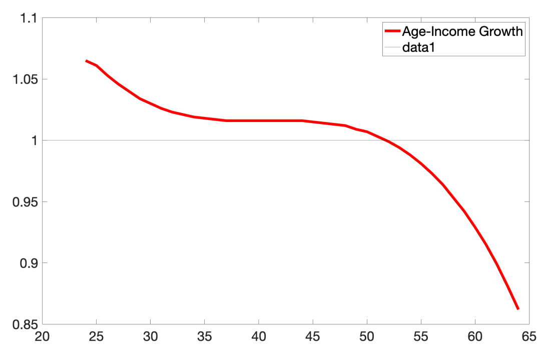
Table E1: Permanent and Transitory Income Parameters

	Young 25-35	Middle 36-64	Old 65+
p	0.185	0.231	0.308
σ_u	0.471	0.467	0.482
σ_n	0.144	0.120	0.126

Notes: p is the probability of zero income, σ_u is the standard deviation of transitory income, and σ_n is the standard deviation of permanent income.

zero income event with $p = 0.31$, whereas volatility of permanent income is highest for the young, with $\sigma_n = 0.14$.

Figure E1: Estimated Age-Income Profile: Gross Income Growth Rate



For the age-income profile, the data allow direct estimation for household heads aged 25-60. The estimated age-income profile, which we assume to be constant over time, is obtained from observed variation in income by age. Given these estimates, we 'forecast' values for ages 61-64 with a cubic trend regression. Figure E1 displays the results. As seen from the figure, income growth peaks early in the career. Income continues to grow but at decreasing rates until age 51, after which income declines. The numerical estimates are reported in Table E2.

Table E2: Annual Income Growth Estimates

Age-Income Gross-Growth Rates					
Age	Growth	Age	Growth	Age	Growth
24	1.065	38	1.016	52	0.999
25	1.061	39	1.016	53	0.994
26	1.053	40	1.016	54	0.988
27	1.046	41	1.016	55	0.981
28	1.040	42	1.016	56	0.973
29	1.034	43	1.016	57	0.964
30	1.030	44	1.016	58	0.953
31	1.026	45	1.015	59	0.942
32	1.023	46	1.014	60	0.929
33	1.021	47	1.013	61	0.915*
34	1.019	48	1.012	62	0.899*
35	1.018	49	1.009	63	0.881*
36	1.017	50	1.007	64	0.862*
37	1.016	51	1.003		
Gross Secular Growth: 1.006					

Notes: * are values forecasted by cubic trend. p is the probability of zero income, σ_u is the standard deviation of transitory income, and σ_n is the standard deviation of permanent income.

E.2 Preferences and Budget Constraints

We next lay out the equations for the life-cycle model incorporating notations denoting cohorts and individuals. This notation was suppressed in the main paper.

Let $C_{i,z,t}$ denote consumption of household i, with cohort z, at time t. Labor supply is $L_{i,z,t}$ and, normalizing the time endowment to 1, leisure is $(1 - L_{i,z,t})$.

Working age household $z < z^*$ utility is,

$$V_{i,z,t} = \left\{ (1-\beta) \left(C^{\nu}_{i,z,t} \left(1 - L_{i,z,t}\right)^{1-\nu} \right)^{(1-\rho)} + \beta \left[\left(E_t \left[V^{1-\gamma}_{i,z+1,t+1} \right] \right)^{\frac{(1-\rho)}{(1-\gamma)}} \right] \right\}^{\frac{1}{(1-\rho)}} \tag{E5}$$

$\rho > 0$, $\gamma > 0$, $0 \leq \nu \leq 1$, and $0 < \beta < 1$. β is the subjective discount factor. ρ^{-1} is the intertemporal elasticity of substitution.

The net number of bonds held by the household is $A_{i,z,t}$. Upon retirement, households face a possibility of death and must have non-negative assets in retirement to ensure that they do not die in debt. The price of the bond is the inverse of the interest rate, $P^a_t = 1/r_t$. Current wealth for working-aged households consists of the net bond coupon ($A_{i,z,t}$) plus the market value of the bonds plus labor income less consumption. Their budget constraints are

$$P^a_t A_{i,z+1,t+1} = A_{i,z,t} + P^a_t A_{i,z,t} + L_{i,z,t} W_{i,z,t} - C_{i,z,t}, \tag{E6}$$

which can be written in a more familiar form,

$$A_{i,z+1,t+1} = A_{i,z,t} + r_t \left(A_{i,z,t} + L_{i,z,t} W_{i,z,t} - C_{i,z,t} \right). \tag{E7}$$

Retired households have a bequest motive, supply no labor and face an uncertain death where the cohort z specific probability of surviving to age $z+1$ is $\delta_{z,t}$. Following Gomes and Michaelides (2005), we model the bequest motive of retirees as,

$$\frac{1}{1-\gamma} E_t \left(\left(\frac{1}{b} \frac{A_{i,z+1,t+1}}{r_t} \right)^{\nu} \right)^{1-\gamma}.$$

Hence, utility for retired households, aged $z^* \leq z < Z = 248$, is

$$V_{i,z,t} = \left\{ (1 - \beta\delta_{z,t}) \left(C^{\nu}_{i,z,t} \right)^{(1-\rho)} + \beta \left[\delta_{z,t} E_t \left(V^{1-\gamma}_{i,z+1,t+1} \right) + \frac{(1-\delta_{z,t})}{1-\gamma} E_t \left(\left(\frac{1}{b} \frac{A_{i,z+1,t+1}}{r_t} \right)^{\nu} \right)^{1-\gamma} \right]^{\frac{1-\rho}{1-\gamma}} \right\}^{\frac{1}{1-\rho}}, \tag{E8}$$

where $\delta_{z,t}$ is the cohort z specific probability of surviving to age $z+1$. In the last quarter of life, $z = Z = 248$, and $\delta_{248,t} = 0$, so utility is

$$V_{i,z,t} = \left\{ \left(C^{\nu}_{i,z,t} \right)^{(1-\rho)} + \beta \left[\frac{1}{1-\gamma} E_t \left(\left(\frac{1}{b} \frac{A_{i,z+1,t+1}}{r_t} \right)^{\nu} \right)^{1-\gamma} \right]^{\frac{1-\rho}{1-\gamma}} \right\}^{\frac{1}{1-\rho}}. \tag{E9}$$

Retired households face budget constraints

$$A_{i,z+1,t+1} = A_{i,z,t} + r_t \left(A_{i,z,t} + S_{i,z,t} - C_{i,z,t}\right)$$

with $A_{i,z,t} \geq 0$.

E.3 Stationary Representation of the Life-Cycle Model

This section outlines the stationary representation of the life-cycle model. Because the income process has a unit root, the state space becomes unbounded. To solve the model, we induce stationarity by normalizing income and utility by last period's permanent income. We suppress the individual subscript to avoid clutter. Normalization of the income process follows,

$z < z^*$	$\frac{W_{z,t}}{Y_{z-1,t-1}} = \frac{Y_{z,t}e^{u_t}}{Y_{z-1,t-1}} = M_g G_{z,t} e^{n_{z,t}} e^{u_{z,t}}$	working
$z = z^*$	$\frac{W_{z^*,t}}{Y_{z^*-1,t-1}} = \frac{Y_{z^*,t}}{Y_{z^*-1,t-1}} = p_{rr}$	retirement quarter 1
$z > z^*$	$\frac{W_{z,t}}{Y_{z-1,t-1}} = e^{n_{z,t}} e^{u_{z,t}}$	retirement

Let $\tilde{v}_{z,t} = \frac{V_{z,t}}{Y^{\nu}_{z-1,t-1}}$ and $\tilde{c}_{z,t} = \frac{C_{z,t}}{Y_{z-1,t-1}}$. Normalized utility during the working years is,

$$\tilde{v}_{z,t} = \left\{ (1-\beta) \left(\tilde{c}^{\nu}_{z,t} \left(1 - L_{z,t}\right)^{1-\nu} \right)^{1-\rho} + \beta \left(M_g G_{z,t} e^{n_{z,t}} \right)^{\frac{\nu(1-\rho)}{(1-\gamma)}} \left[E_t \tilde{v}^{1-\gamma}_{t+1} \right]^{\frac{(1-\rho)}{(1-\gamma)}} \right\}^{\frac{1}{1-\rho}}. \tag{E10}$$

Let $\tilde{w}_{z,t} = \frac{W_{z,t}}{Y_{z-1,t-1}}$ as defined above and $\tilde{a}_{z,t} = \frac{A_{z,t}}{Y_{z-1,t-1}}$. The normalized budget constraint is,

$$\tilde{a}_{z+1,t+1} M_g G_{z,t} e^{n_{z,t}} = \tilde{a}_{z,t} + r_t \left(\tilde{a}_{z,t} + \tilde{w}_{z,t} L_{z,t} - \tilde{c}_{z,t} \right). \tag{E11}$$

In the retirement years, normalized utility and normalized budget constraints are,

$$\tilde{v}_{z,t} = \left\{ (1-\beta) \left(\tilde{c}^{\nu}_t \right)^{1-\rho} + \beta \left(M_g G_{z,t} e^{n_{z,t}} \right)^{\frac{\nu(1-\rho)}{(1-\gamma)}} \left[\delta_t E_t \left(\tilde{v}^{1-\gamma}_{t+1} \right) + \frac{(1-\delta_t) e^{n_{z,t}}}{1-\gamma} E_t \left(\frac{\tilde{a}_{t+1}}{r_t b} \right)^{\nu(1-\gamma)} \right]^{\frac{(1-\rho)}{(1-\gamma)}} \right\}^{\frac{1}{1-\rho}}, \tag{E12}$$

and

$$\tilde{a}_{z,t+1}e^{n_{z,t}} = \tilde{a}_{z,t} + r_t\left(\tilde{a}_{z,t} + \tilde{w}_{z,t} - \tilde{c}_{z,t}\right). \tag{E13}$$

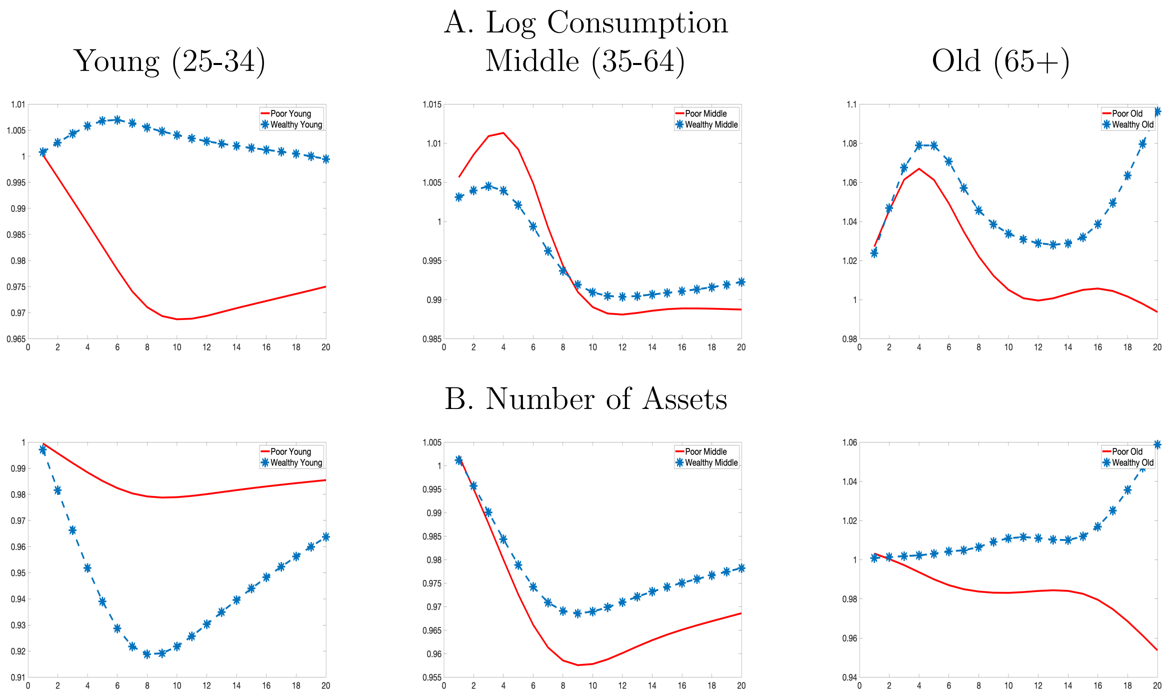
Adjustments for one-period lived assets. The price of the asset is $P_t^a = (1 + r_t)^{-1}$. The (unnormalized) budget constraints become

$$A_{z,t+1} = (1 + r_t)\left(A_{z,t} + L_{z,t}W_{z,t} - C_{z,t}\right). \tag{E14}$$

E.4 Additional Features of the Life-Cycle Model

The baseline model includes neither endogenous interest rates nor a short term asset, both of which could affect the model's ability to quantitatively match the data. As mentioned in the main text, the model simulation is not meant as an all-encompassing quantitative exercise. Instead, we use the model to show that our proposed wealth channel can work within a standard life-cycle set-up. However, to engender confidence in the model mechanism, we here report some additional properties of the model to demonstrate its qualitative plausibility.

Figure E2: Relative Mean Responses by Wealthy and Poor

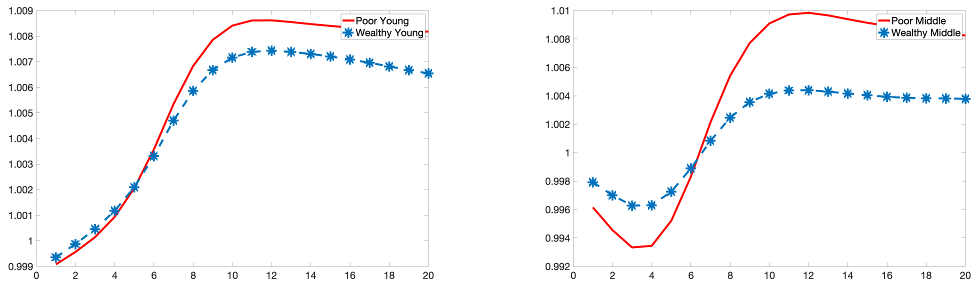


Notes: The figure shows simulated un-normalized relative responses of mean log consumption and mean asset holdings across age groups for the wealthy and poor to a decline in the long-term bond rate. If the shock occurs at time $t*$, the relative response for consumption is $\ln(C_t/C_{t*})$ and for asset holdings is A_t/A_{t*}. Wealthy households are those with above median asset holdings. The horizontal axis indicates the number of quarters for up to five years after the shock.

Figure E2 divides the model agents into 'wealthy' and 'poor' for each age group according to whether their asset holdings are above or below the median. Wealthy old increase their consumption more than poor old, especially at longer horizons. Wealthy young also increase consumption more than poor young, whose consumption declines. There is little difference between middle poor and wealthy (note the scale). From Panel B, wealthy old consumption response is driven mainly by the wealth effect. Young wealthy consumption patterns are driven by asset sales to realize capital gains. Poor young hang on to their assets and reduce consumption modestly. At ten quarters, the difference in consumption of wealthy and poor young is roughly three percent. The difference in wealthy and poor young consumption is mainly from effects on assets and not labor income. Figure E3 shows the difference in labor response between poor and wealthy young are trivial in size.

Figure E3: Relative Mean Log Labor Responses by Wealthy and Poor

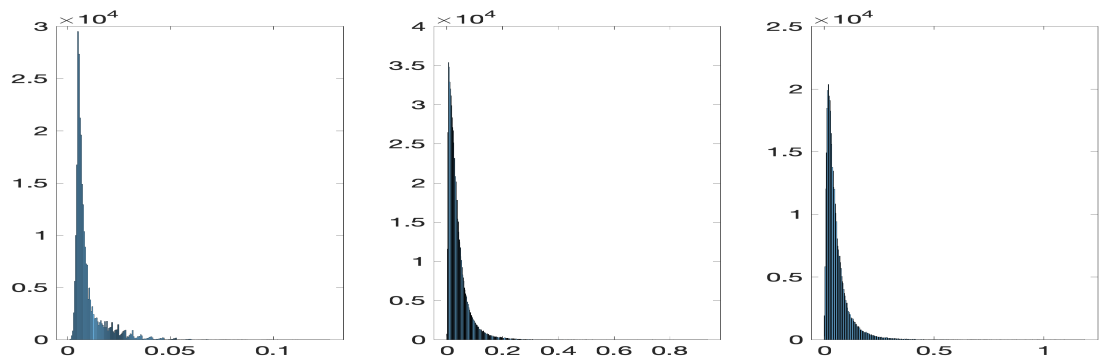
A. Young (25-34) B. Middle (35-64)



Notes: The figure shows simulated un-normalized relative responses of mean log labor across age groups for the wealthy and poor to a decline in the long-term bond rate. If the shock occurs at time $t*$, the relative response for labor is $\ln(L_t/L_{t*})$. Wealthy households are those with above median asset holdings. The horizontal axis indicates the number of quarters for up to five years after the shock.

Figure E4: Net Asset Position Histograms

A. Young (25-34) B. Middle (35-64) C. Old (65+)



Notes: This figure shows the histograms of the asset holding positions for the three age groups. The horizontal axis denotes the asset holding position per individual and the vertical axis denotes the number of individuals.

Figure E4 shows the histograms of the asset holdings for the three age groups. The distributions are heavy in the right tail (note the difference in scale). Some people are very lucky and hence, wealthy. Because the intertemporal elasticity of substitution is greater than 1, the young and middle-aged always maintain positive net worth.

Figure E5: Life-Cycle Patterns: Medians

A. Consumption B. Asset Value to Consumption C. Labor

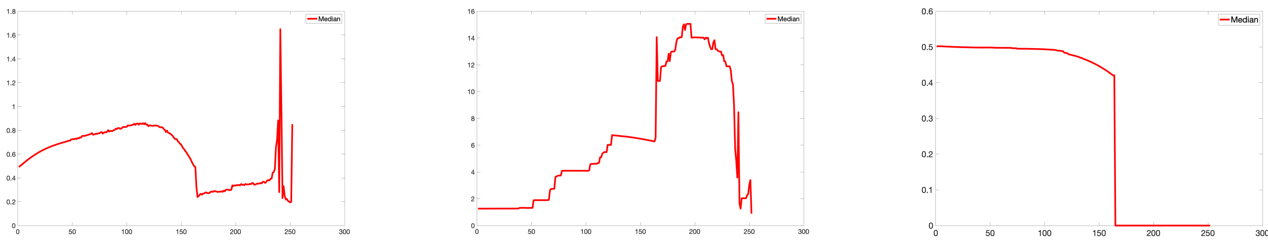


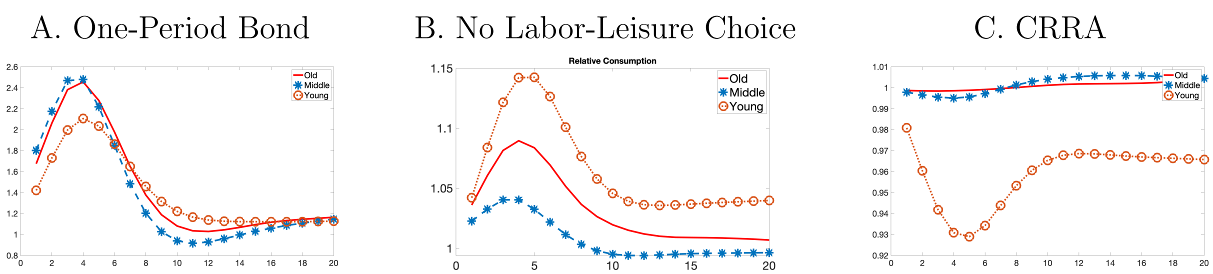
Figure E5 shows median consumption, median wealth to consumption ratio, and median labor over the life-cycle. Consumption, in Panel A, displays the familiar hump shape over the working years. Consumption declines as people approach retirement due to lower wages in latter years of the age-earnings profile. As in the data, consumption in retirement is lower than during the working years. The massive spikes in near end-of-life consumption is the result of only a small number of surviving people (only about 6% make it the entire 86 years) combined with a few of those surviving people being extremely lucky having accumulated massive wealth. Confronting a hard terminal date in the near future, they undertake an immense asset drawdown which is used for an enormous consumption binge.

In Panel B, the wealth to consumption ratio displays a bit of a hump during the working years. The ratio spikes in the retirement period because consumption decreases upon retirement. In the periods before retirement, the median household has accumulated six quarters of consumption in bond wealth.

E.5 Alternative Life-Cycle Model Specification Results

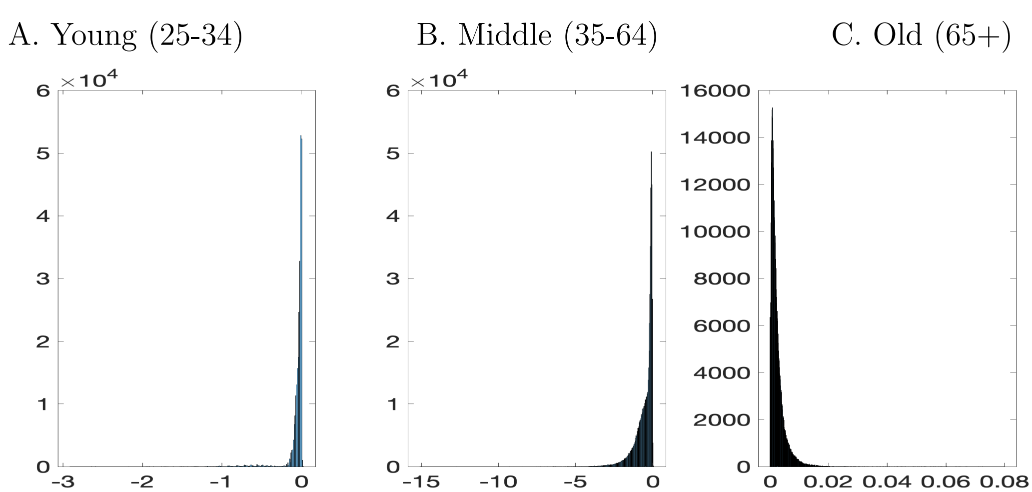
This section reports results from alternative specifications of the life-cycle model. Panel A of Figure E6 shows relative mean log consumption responses to negative interest rate shock by age group when the asset is a short-term (one-period) bond and other life-cycle model features are the same as those in the main paper. Panel B of Figure E6 shows relative mean log consumption responses to negative interest rate shock by age group when there is no labor-leisure choice and other life-cycle model features are the same as those in the main paper. Panel C of Figure E6 shows relative mean log consumption responses to negative interest rate shock by age group when there is constant relative risk aversion utility and other life-cycle model features are the same as those in the main paper.

Figure E6: Relative Mean Log Consumption Responses to Negative Interest Rate Shock



Notes: This figure shows the simulated un-normalized relative mean log consumption responses $\left(\ln\left(\frac{C_t}{C_{t*}}\right)\right)$ by age group to a decline in the long-term bond rate. The horizontal axis indicates the number of quarters up to five years after the shock.

Figure E7: Net Asset Position Histograms with CRRA

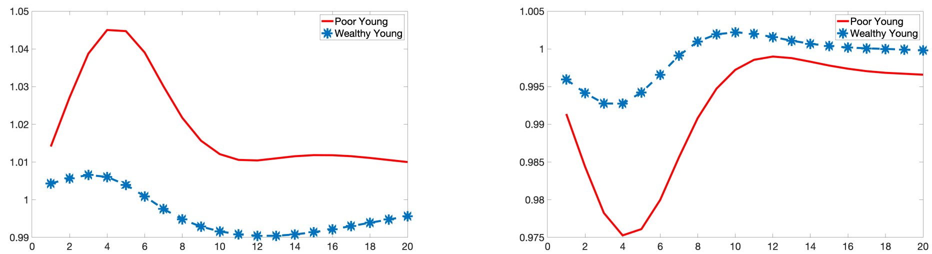


Notes: This figure shows the histograms of the asset holdings for the three age groups. The horizontal axis denotes the asset holding position per individual and the vertical axis denotes the number of individuals.

Figure E7 shows the histograms of the asset holdings by age group when there is constant relative risk aversion utility and other life-cycle model features are the same as those in the main paper.

Figure E8: Relative Mean Responses to Negative Interest Rate Shock by Poor Young and Wealthy Young when $\gamma = 12$

A. Log Consumption B. Log Labor

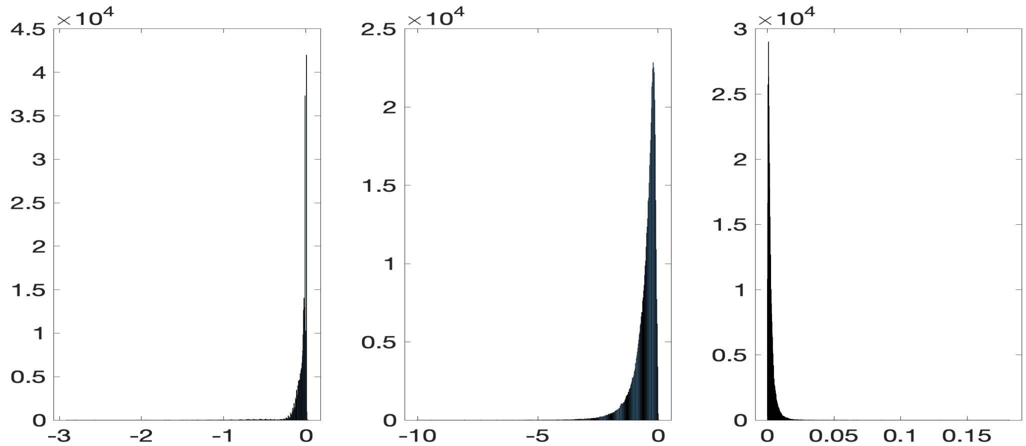


Notes: This figure shows the simulated un-normalized relative mean log consumption responses $\left(\ln\left(\frac{C_t}{C_{t*}}\right)\right)$ and un-normalized relative mean log labor responses $\left(\ln\left(\frac{L_t}{L_{t*}}\right)\right)$ for the poor young and the wealthy young to a decline in the long-term bond rate. Wealthy households are those with above median asset holdings. The horizontal axis indicates the number of quarters up to five years after the shock.

Panel A of Figure E8 shows relative mean log consumption responses to negative interest rate shock by poor young and wealthy young when $\gamma = 12$ and other life-cycle model features are the same as those in the main paper. Panel B of Figure E8 shows relative mean log labor responses to negative interest rate shock by poor young and wealthy young when $\gamma = 12$ and other life-cycle model features are the same as those in the main paper.

Figure E9: Net Asset Position Histograms when $\rho^{-1} = 0.5$

A. Young (25-34) B. Middle (35-64) C. Old (65+)



Notes: This figure shows the histograms of the asset holdings for the three age groups. The horizontal axis denotes the asset holding position per individual and the vertical axis denotes the number of individuals.

Figure E9 shows the histograms of the asset holdings by age group when $\rho^{-1} = 0.5$ and other life-cycle model features are the same as those in the main paper.

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