Understanding Spot and Forward Exchange Rate Regressions

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May 1993.

This version: December 4, 1995

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Abstract

Theories of disequilibrium exchange-rate dynamics and speculative behavior in asset markets suggest a permanent-transitory representation for spot and forward exchange rates. We show that such a model can explain a number of important features of the data. We model spot and forward rates as having a common trend while their transitory components evolve according to a vector ARMA(1,1) process. The model is estimated by maximum-likelihood using the Kalman filter. We find that the deviation of the forward rate from the rationally expected future spot rate implied by the estimated model is reasonable in magnitude, persistent, fluctuates from positive to negative in a sensible way, and covaries negatively with the implied expected depreciation. The implied forward premium covaries negatively with the future depreciation as well.

Understanding Spot and Forward Exchange Rate Regressions

This paper demonstrates that an unobserved components model of spot and forward exchange rates is able to characterize a number of important features of the data. The permanent-transitory components model that we employ requires both the spot and forward exchange rates to follow a common trend but allows each to have heterogeneous stationary components. We model the permanent component as a driftless random walk and the transitory dynamics as a vector ARMA(1,1) process. The model can be rationalized by the models of disequilibrium exchange rate dynamics [Mussa (1982)], or by models of speculative behavior that has appeared in the asset-pricing literature [Campbell and Shiller, (1988) and Summers (1986)].

The structural time-series model is estimated by maximum-likelihood using the Kalman filter with exchange rates between the U.S. dollar and the pound, the French franc, and the yen. We show that the estimated models are able to match a number of moments in the data that are not explicitly used in estimating the models. Values of the expected excess return implied by the estimated model (*i.e.,* the deviation of the forward rate from the rationally expected future spot rate), are reasonable in magnitude, persistent, fluctuate from positive to negative in a sensible way, and covary negatively with the expected depreciation.

We key in on three aspects of the data that useful models of spot and forward exchange rates should be able to explain. The first of these is that the forward premium is a persistent, but stationary series. Although spot and forward rates appear to be I(1) they also appear to be cointegrated because the forward premium appears to be I(0) for the currencies that we study.[1] The second aspect of the data that concerns us is that estimates of slope coefficients in cointegrating regressions of the future log spot rate on the log forward rate are insignificantly different from 1. These 'levels' regressions were originally fitted by researchers interested in testing foreign-exchange market efficiency [e.g., Frenkel (1981), Bilson (1981), Cornell (1980) and others].[2] The idea is that the absense of profitable arbitrage opportunities requires that the foward rate be the best predictor of the future spot rate if foreign exchange market

[1]We use the standard notation I(d) to denote that a time-series is d-*th* order integrated and requires differencing d times to induce stationarity.

[2]See Hodrick (1987) and Boothe and Longworth (1986), for surveys of this literature.

participants are risk neutral.[3] The early studies estimated the regression slope coefficients and drew inference with standard OLS procedures. It is now known that the OLS t-ratio is not asymptotically standard normal if the spot and forward exchange rates are I(1) but we shall demonstrate below that the hypothesis that the slope coefficient is 1 survives even when the appropriate cointegrating vector estimators and test statistics are employed.

These cointegrating regressions can be transformed into regressions of the future depreciation on the forward premium by subtracting the current log spot rate from both the regressor and the regressand. In contrast to the 'levels' regressions, however, regressions of the future depreciation on the forward premium yield estimated slope coefficients that are significantly *negative*. This result was first reported in the literature by Cumby and Obstfeld (1982) and Fama (1984) and has been viewed as an anomaly in international finance. The result implies that the forward premium, or equivalently, the nominal interest rate differential, helps to predict future changes in spot rates but enters with the 'wrong' sign. Fama demonstrates that these negative estimates are produced by expected excess returns in foreign exchange that are both negatively correlated with, and more volatile than the expected depreciation. Thus, the third aspect of the data that we want our model to conform to is that regressions of the future depreciation on the forward premium yield significantly negative estimates of the slope coefficients.

Previous attempts to understand these aspects of the data include Hodrick and Srivastava's (1986) demonstration that the negative correlation between the forward premium and the future depreciation is possible in Lucas's (1982) two-country model, Backus, Gregory, and Telmer's (1993) calibrations of that model, Froot and Frankel's (1989) argument for expectational errors, McCallum's (1992) policy reaction model, and Evans and Lewis's (1992) 'peso problem' model.

The paper is organized as follows. The next section discusses those empirical regularities of foreign-exchange data from the float that concern us. In section 3, we illustrate how a simple permanent-transitory component model whose transitory part follows an AR(1)

[3] Engel (1984) and Frenkel and Razin (1980) point out that it is the real forward rate that is the optimal predictor of the real futures spot rate under risk neutrality. Empirical studies have shown that it makes little difference whether real or nominal rates are used. Accordingly, our analysis employs only nominal exchange rates.

process can, in principle, generate a unit slope coefficient in a cointegrating regression of the future spot rate on the forward rate, and a negative slope coefficient in a regression of the future depreciation on the forward premium. The AR(1) model is instructive. It cannot, however, provide a complete characterization of the data because it carries with it the counterfactual implication that exchange rate changes are *negatively* correlated at all horizons when the autocorrelation coefficient is positive.[4] In the empirical work, we adopt a vector ARMA(1,1) specification for the transitory components. We find that this model is sufficiently rich to capture most of the salient features of the data. Section 4 describes the maximum-likelihood estimation and the design of a Monte Carlo experiment that we employ to evaluate the model. The empirical results are presented in section 5 and concluding remarks are contained in section 6.

2 Review of Spot and Forward Exchange Rate Behavior

This section reviews and documents those features of the data that we seek to understand. We examine monthly observations on spot, 1-month, and 3-month forward U.S. dollar prices of the pound, French franc, and the yen. We follow Hansen and Hodrick (1983) by starting our sample in 1976:1 following the Rambouillet Conference. The sample ends on 1992:8. These data are taken from the Harris Bank *Weekly Review*, and are drawn from those Fridays occuring nearest to the end of the calendar month. All observations are logarithms multiplied by 100 so that returns are expressed in percent.

Recent research has uncovered many empirical regularities during the float, but we are mainly concerned with the following three.

1. The forward premium is a persistent, but stationary process.

2. Future log-spot rates and log forward rates are cointegrated with a cointegrating regression slope coefficient of 1.

[4]Huizinga (1987) and Grilli and Kaminsky (1991), for example, show that while spot rate changes are negatively correlated at long horizons, they are positively correlated at short horizons.

3. The slope coefficient in regressions of the future depreciation on the forward premium are negative and significantly less than 1.

These are robust characteristics of the data, having been found in various subperiod analyses and surviving over time. We illustrate points (1) and (2) through the application of unit root and cointegration tests. We recognize the impossibility of discriminating between processes that are I(0) and nearly I(1) with finite amounts of data [Blough (1992), Faust (1993), Cochrane (1991)]. We view these tests as diagnostic tools and use the results simply to argue that points (1) and (2) are a reasonable way to characterize the data.

2.A The Forward Premium

Table 1 reports studentized coefficients for augmented Dickey-Fuller (1979) tests and Phillips-Perron (1988) tests that the forward premium contains a unit root. The sample consists of 200 monthly observations so the 5% and 10% critical values are -2.88 and -2.57 as reported in Fuller (1975). The augmented Dickey-Fuller test rejects the hypothesis that the forward premium is I(1) at the 5% level for the pound and at the 10% level for the franc and yen for both monthly and quarterly forward premia. The Phillips-Perron test rejects at the 5% level for monthly forward premia in all three currencies. With quarterly forward premia, the Phillips-Perron test rejects the I(1) null at the 5% level for the the pound and franc, and at the 10% level for the yen. We proceed under the supposition that these forward premia are I(0).[5]

We also note that the six series display a fair amount of persistent. For the pound, franc, and yen, the first-order autocorrelation coefficients for monthly forward premia are $0.796, 0.674$, and 0.920, and for the quarterly forward premia are, $0.761, 0.429$, and 0.778.

2.B Cointegration of Forward and Future Spot Rates

We next investigate whether the k-period forward rate, $f_{t,k}$ and the future spot rate s_{t+k} are cointegrated.[6] It is well known that both spot and forward rates exhibit I(1) behavior

[5] Evans and Lewis (1992) argue that the forward premium is I(1), but that the I(1) component is small and not detectable with standard unit root procedures with data from the post-float era.

[6] All exchange rates are measured in natural logs.

so we dispense with unit root tests for these data.[7] Table 2 reports the results of augmented Dickey-Fuller and Phillips-Perron tests on the residuals from the OLS regression,

$$s_{t+k} = \alpha_o + \beta_o f_{t,k} + u_{t+k,k}, \tag{1}$$

for $k = 1, 3$. For all three currencies, the null hypothesis that these spot and forward rates are *not* cointegrated is easily rejected at the 5% level for both monthly and quarterly horizons.

Notice that the estimated slope coefficients are near 1. Indeed, research on foreign-exchange market efficiency was originally pursued by estimating β_o by OLS and testing the hypothesis that β_o=1 with standard OLS t-ratios [*e.g.,* Frenkel (1981), Bilson (1981), Cornell (1980), and others]. However, when $\{s_{t+k}\}$ and $\{f_{t,k}\}$ are cointegrated, OLS suffers from second-order asymptotic biased, and its t-ratio is not asymptotically standard normal. The bias complicates the task of constructing useful test statistics involving the OLS estimator [Stock (1987)].

Numerous procedures are available that provide consistent estimates and tests of the cointegrating regression slope coefficient, β_o. But because evidence is sketchy at this time as to which estimator has the best sampling properties, we consider three that have recently been proposed. They are Stock and Watson's (1992) dynamic OLS (DOLS) and dynamic GLS (DGLS) estimators, and Park's (1992) canonical cointegrating regression (CCR) estimator. Stock and Watson's regressions purge the correlation between the cointegrating regression residual and the regressors that causes OLS to be biased by adding leads and lags of *changes* in the forward rate to the regression. Park's CCR achieves the asymptotically equivalent result by running regressions on suitable transformations of the data. A description of the estimators is given in the appendix.

Estimates of the cointegrating regression slope coefficients are reported in table 3 for $k = 1, 3$. As can be seen, most of the point estimates of the slope coefficients are (qualitatively) near 1. Using the asymptotic distributions, the hypothesis that $\beta_o = 1$ generally cannot be rejected at the 5% level. The exception is the DOLS regression for the yen. Although this estimate of the cointegrating regression slope coefficient is significantly different from 1, its

[7] See, for example, Baillie and Bollerslev (1989), Liu and Maddala (1992), Mark (1990), or Clarida and Taylor (1993).

deviation from 1 seems so small as to be economically irrelevant. The weight of the evidence, then, is favorable to the hypothesis that the cointegrating vector is 1.

2.C Regressions of the Future Depreciation on the Forward Premium

Prior to the advent of cointegrating regression estimation, concern that nonstationary spot and forward rates would lead to the wrong inferences in OLS regressions of (1) led some investigators of foreign-exchange market efficiency to induce stationarity by transforming the data. For example, Cumby and Obstfeld (1982) and Fama (1984) regressed the future depreciation on the forward premium.

$$s_{t+k} - s_t = \alpha_1 + \beta_1(f_{t,k} - s_t) + \nu_{t+k,k} \tag{2}$$

Instead of finding $\hat{\beta}_1 = 1$ however, these researchers obtained estimates that were significantly *negative.* Table 4 reports our own estimates of this equation for $k = 1, 3$ where we find that the estimated slope coefficients are negative in every case. Using the asymptotic distribution, the slopes are all significantly less than 1 except for the franc at the quarterly horizon and are significantly negative for the pound and the yen at both the monthly and quarterly horizons. [8]

This is a paradoxical result because if the forward exchange rate is the optimal predictor of the future spot rate, the slope coefficients in both the cointegrating regressions of the future spot rate on the forward rate and regressions of the future depreciation on the forward premium are 1. While this appears to be true for the levels regressions ($\beta_o = 1$), it is not in the forward premium regressions ($\beta_1 < 0$). The forward premium appears to help predict future changes in the spot rate but enters with the 'wrong' sign.

The statistical explanation for this result is that the error term in (2) is correlated with the forward premium. To develop an economic interpretation, Fama shows that the negative estimates of β_1 are caused by an expected excess return that is more volatile than, and is

[8] For $k = 3$, Newey and West (1987) standard errors with the truncation lag of the Bartlett window set to 15 were used to compute the asymptotic t-ratios.

negatively correlated with the expected depreciation. To see this, let $r_{t,k} \equiv f_{t,k} - E_t s_{t+k}$ be the expected excess return, and $\delta_{t+k} \equiv s_{t+k} - E_t s_{t+k}$ be the rational expectations forecast error.[9] Then the relations among spot and forward rates and the expected excess return can be represented as

$$s_{t+k} = f_{t,k} + (\delta_{t+k} - r_{t,k}), \tag{3}$$

$$s_{t+k} - s_t = (f_{t,k} - s_t) + (\delta_{t+k} - r_{t,k}). \tag{4}$$

If the forward premium and the expected excess return are both I(0) and correlated with each other, regressions of the future depreciation on the forward premium suffer from an omitted variables problem. The expected excess return is impounded into the error term which becomes correlated with the regressor and causes the slope coefficient to be biased away from 1. The error term in (1) is correlated with the forward rate as well but because the forward rate is I(1) the bias is of second order in importance.

3 A Model of Spot and Forward Rates

In this section, we put forth a permanent-transitory components model for spot and forward exchange rates. We suppress the horizon subscript k to simplify the notation and state the model as

$$s_t = z_t + x_{s,t}, \tag{5}$$

$$f_t = z_t + x_{f,t}, \tag{6}$$

$$z_t = z_{t-1} + \varepsilon_{z,t}, \tag{7}$$

where $\{\varepsilon_{z,t}\} \overset{i.i.d.}{\sim} N(0, \sigma_z^2)$, and $\{(x_{s,t}, x_{f,t})'\}$ is a stationary bi-variate stochastic process.

The representation of the spot-rate can be deduced from Mussa's (1982) model of ex-

[9] Fama (1984) defined $f_{t,k} - E_t s_{t+k}$ to be the 'risk premium.' It should be noted that the forward rate can deviate from the expected future spot rate for reasons unrelated to risk, as in Domowitz and Hakkio (1985) or Kaminsky and Peruga (1990). These authors study models in which expected excess returns can be nonzero and time-varying under risk neutrality when the underlying data generating process is log-normal. We avoid using this term because to legitmiately call r_t a risk premium, it should arise from an economic model that prices foreign exchange risk.

change rate dynamics. In that model, the spot rate is the sum of two components. The first component is the equilibrium exchange rate, which is the value that the spot rate would be in the absence of any nominal rigidities. It is equal to the expected present value of future nominal and real exogenous variables in the model. We represent this equilibrium value with the random walk, $\{z_t\}$. The second component of Mussa's exchange rate solution is a stationary deviation from the equilibrium value which we represent with the process, $\{x_{s,t}\}$.

Mussa's original formulation assumes uncovered interest parity holds which implies zero expected excess returns. However, an *ad hoc* expected excess return, or 'risk premium' can easily be incorporated to the model. This *ad hoc* risk premium can then be impounded into $\{x_{s,t}\}$. The persistence of this variable will then depend on the persistence of the expected excess return as well as on the speed of the economy's adjustment towards equilibrium.[10]

Next, assume that foreign-exchange traders set the forward rate to eliminate covered interest arbitrage profits by equating the forward premium to the interest differential. We represent the outcome of this pricing strategy as (6). The forward rate is constrained to be driven by the same random walk as the spot rate to conform to the evidence that future spot and forward rates are cointegrated with a slope coefficient of 1. The evolution of the forward premium is given by, $f_t - s_t = x_{f,t} - x_{s,t}$.

To complete the characterization of spot and forward rates, the transitory deviations from the equilibrium values are driven by the vector ARMA process,

$$\begin{pmatrix} \phi_{ss}(L) & \phi_{sf}(L) \\ \phi_{fs}(L) & \phi_{ff}(L) \end{pmatrix} \begin{pmatrix} x_{s,t} \\ x_{f,t} \end{pmatrix} = \begin{pmatrix} c_s \\ c_f \end{pmatrix} + \begin{pmatrix} \theta_{ss}(L) & \theta_{sf}(L) \\ \theta_{fs}(L) & \theta_{ff}(L) \end{pmatrix} \begin{pmatrix} \varepsilon_{s,t} \\ \varepsilon_{f,t} \end{pmatrix}, \tag{8}$$

where

$$\begin{pmatrix} \varepsilon_{s,t} \\ \varepsilon_{f,t} \end{pmatrix} \overset{i.i.d.}{\sim} N\left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} \sigma_s^2 & \rho_{sf}\sigma_s\sigma_f \\ \rho_{sf}\sigma_s\sigma_f & \sigma_f^2 \end{bmatrix} \right) \tag{9}$$

[10] The long-horizon regressions of Mark (1993) exploited the idea that deviations of the spot rate from its equilibrium value provide useful information for predicting future exchange rate movements. The two-component model has also been used to describe the evolution of stock prices where the random walk represents the rationally expected present value of future dividends (the fundamentals solution), and the deviation represents price 'fads.' See, for example, Summers (1986), Fama and French (1988), Campbell and Shiller (1988)

where c_s and c_f are constants and the $\phi(L)'s$ and $\theta(L)'s$ are polynomials in the lag operator, L.

3.A An Example with AR(1) Transient Dynamics

Suppose that the transient components follow univariate AR(1) processes with contemporaneously correlated innovations. AR(1) transient dynamics are simple enough that analytic formulae for the relevant moments provide insight as to how the two-component model can in principle account for the moments in the data. We present the formulae for spot and $k = 1$ period forward rates because they are considerably simpler. The intuition carries over to the $k = 3$ period horizon.

The AR(1) model is obtained from (8) by setting $\phi_{ss}(L) = 1 - \phi_s L$, $\phi_{ff} = 1 - \phi_f L$, $\theta_{ss}(L) = \theta_{ff}(L) = 1$, and $\phi_{fs}(L) = \phi_{sf}(L) = \theta_{fs}(L) = \theta_{sf}(L) = 0$. To shorten the notation, let $\gamma = \frac{\sigma_{fs}}{\sigma_s^2} = \rho_{sf}\frac{\sigma_f}{\sigma_s}$. The random-walk-AR(1) model implies the following moments.

$$Cov[\Delta s_{t+1}, (f_t - s_t)] = \sigma_s^2(\phi_s - 1)\left(\frac{\gamma}{1-\phi_f\phi_s} - \frac{1}{1-\phi_s^2}\right) \tag{10}$$

$$Var(f_t - s_t) = \left(\frac{\sigma_f^2}{1-\phi_f^2} - \frac{\sigma_{fs}}{1-\phi_s\phi_f}\right) + \left(\frac{\sigma_s^2}{1-\phi_s^2} - \frac{\sigma_{fs}}{1-\phi_s\phi_f}\right) \tag{11}$$

$$Cov[(f_t - s_t), (f_{t-1} - s_{t-1})] = \phi_f\left(\frac{\sigma_f^2}{1-\phi_f^2} - \frac{\sigma_{fs}}{1-\phi_s\phi_f}\right) + \phi_s\left(\frac{\sigma_s^2}{1-\phi_s^2} - \frac{\sigma_{fs}}{1-\phi_s\phi_f}\right) \tag{12}$$

$$Var(E_t\Delta s_{t+1}) = \sigma_s^2\left(\frac{(1-\phi_s)^2}{1-\phi_s^2}\right) \tag{13}$$

$$Var(r_t) = \frac{\sigma_f^2}{1-\phi_f^2} + \sigma_s^2\left(\frac{\phi_s^2}{1-\phi_s^2} - \frac{2\phi_s\gamma}{1-\phi_f\phi_s}\right) \tag{14}$$

$$Cov(E_t\Delta s_{t+1}, r_t) = \sigma_s^2(\phi_s - 1)\left(\frac{\gamma}{1-\phi_f\phi_s} - \frac{\phi_s}{1-\phi_s^2}\right). \tag{15}$$

(10) is the population value of the numerator in the OLS slope coefficient from the regression of the future depreciation on the forward premium. If the transitory components are positively autocorrelated, it can be seen that the slope coefficient will be negative provided that the last term in (10) is positive, since $\phi_s < 1$. This could happen, for example, if the transitory component of the forward rate is sufficiently more persistent ($\phi_f > \phi_s$) or if its innovation is sufficiently more volatile ($\sigma_f > \sigma_s$) than that of the spot rate. From (15) we see that these conditions also imply that the expected excess return will covary negatively with the expected depreciation.

Notice that persistence in the transitory components induce persistence in the forward premium. This can be seen from the first-order autocovariance of the forward premium (12) which is composed of the components of the forward premium variance, weighted by the autoregressive parameters.

Similarly, (13)-(14) show that depending on the particular values of the parameters, the simple AR(1) model generates an expected excess return that is more volatile than the expected depreciation. The variance of the expected depreciation in (13) has a limiting value of 0 as the autoregressive parameter ϕ_s goes to 1 while the variance of the expected excess return in (14) has a limiting value of $2\sigma_f^2(1+\gamma)$ as both ϕ_f and ϕ_s approach 1.

The slope coefficient in the regression of the future depreciation on the forward premium will, in general, deviate from 1. However, one instance in which this regression has a slope coefficient of 1 is when both the spot and forward rates are generated by a random walk plus noise where the noise terms have contemporaneous correlation equal to the ratio of their standard deviations ($\phi_s = \phi_f = 0$ and $\sigma_{fs} = \sigma_f^2$). This implies that the expected excess return will evolve as an i.i.d. process with variance σ_f^2 and its covariance with the expected depreciation will be $-\sigma_f^2$.

3.B Vector ARMA(1,1) Transient Dynamics

The AR(1) model is instructive because it illustrates how the permanent-transitory components model could potentially account for the data. That model cannot be taken seriously, however, because it carries with it the counterfactual implication that for positive values ϕ_s and ϕ_f, changes in spot and forward rates are negatively serially correlated at all horizons.

Instead, we estimate a model in which the transitory components are governed by the vector ARMA(1,1) process,

$$\begin{pmatrix} 1-\phi_{ss}L & -\phi_{sf} \\ -\phi_{fs} & 1-\phi_{ff}L \end{pmatrix} \begin{pmatrix} x_{s,t} \\ x_{f,t} \end{pmatrix} = \begin{pmatrix} c_s \\ c_f \end{pmatrix} + \begin{pmatrix} 1+\theta_{ss}L & \theta_{sf} \\ \theta_{fs} & 1+\theta_{ff}L \end{pmatrix} \begin{pmatrix} \varepsilon_{s,t} \\ \varepsilon_{f,t} \end{pmatrix}. \quad (16)$$

The innovation vector is normally and independently distributed as in (9).

4 Empirical Methodology

Section 4.A describes the estimation strategy that we pursue. After obtaining parameter estimates of the two-component trend-vector ARMA(1,1) model, the question we ask is: Does the fitted model match important moments of the data that were not explicitly imposed in estimation? We are particularly interested in examining the ability of the model to match the moments documented in section 2. Section 4.B describes the Monte Carlo methodology that we employ to evaluate the model's ability to match these moments using classical inference procedures.

4.A Maximum Likelihood Estimation

Let $\mathbf{y_t} = (s_t, f_t)'$.[11] We first rewrite the model using standard state-space notation in the state-space form

$$\mathbf{y_t} = \mathbf{Z}\alpha_\mathbf{t}, \quad (17)$$

$$\alpha_\mathbf{t} = \mathbf{c} + \mathbf{T}\alpha_\mathbf{t-1} + \mathbf{R}\eta_\mathbf{t}, \quad (18)$$

$$\mathbf{Q} = E(\eta_\mathbf{t}\eta_\mathbf{t}'). \quad (19)$$

(17) is the measurement equation, (18) is the transition equation, $\alpha_\mathbf{t}$ is the state vector, $\mathbf{c}$ is a vector of constants, and $\mathbf{Z}$, $\mathbf{T}$, $\mathbf{R}$, and $\mathbf{Q}$, are matrices of constants. For our vector

[11] We estimate systems composed of the spot rate and 1-month forward rate and the spot rate and 3-month forward rate separately. Ideally, one would want to estimate one joint system to capture the term structure dynamics, but we found the three variable system to be computationally intractable.

ARMA(1,1) model we have,

$$\alpha_{\mathbf{t}} = \begin{pmatrix} z_t \\ x_{s,t} \\ x_{f,t} \\ \varepsilon_{s,t} \\ \varepsilon_{f,t} \end{pmatrix}, \ \mathbf{Z} = \begin{pmatrix} 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 \end{pmatrix}, \ \mathbf{T} = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & \phi_{ss} & \phi_{sf} & \theta_{ss} & \theta_{sf} \\ 0 & \phi_{fs} & \phi_{ff} & \theta_{fs} & \theta_{ff} \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix},$$

$$\eta_{\mathbf{t}} = \begin{pmatrix} \varepsilon_{z,t} \\ \varepsilon_{s,t} \\ \varepsilon_{f,t} \end{pmatrix}, \ \mathbf{R} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}.$$

Let $\mathbf{a_{t-1}} = E(\alpha_{t-1}|\mathbf{y}_{t-1}, \mathbf{y}_{t-2}, ..., \mathbf{y}_1)$ be the optimal estimate of the state at date t-1, α_{t-1}, given the data up to that point, and $\mathbf{P}_{t-1} = E(\alpha_{t-1} - \mathbf{a}_{t-1})(\alpha_{t-1} - \mathbf{a}_{t-1})'$ be its covariance matrix. Also let $\mathbf{a}_{t|t-1}$ be the optimal estimate of the state at date t, given the data available at date t-1, and $\mathbf{P}(t|t-1)$ be its covariance matrix. The likelihood function is constructed with the Kalman filter which consists of the two prediction equations,

$$\mathbf{a}_{t|t-1} = \mathbf{c} + \mathbf{T}\mathbf{a}_{t-1} \tag{20}$$

$$\mathbf{P}_{t|t-1} = \mathbf{T}\mathbf{P}_{t-1}\mathbf{T}' + \mathbf{R}\mathbf{Q}\mathbf{R}', \tag{21}$$

and the updating equations,

$$\begin{aligned} \mathbf{a}_t &= \mathbf{a}_{t|t-1} + \mathbf{P}_{t|t-1}\mathbf{Z}'\mathbf{F}_t^{-1}(\mathbf{y}_t - \mathbf{Z}\mathbf{a}_{t|t-1}), \\ \mathbf{P}_t &= \mathbf{P}_{t|t-1} - \mathbf{P}_{t|t-1}\mathbf{Z}'\mathbf{F}_t^{-1}\mathbf{Z}\mathbf{P}_{t|t-1}, \\ \mathbf{F}_t &= \mathbf{Z}\mathbf{P}_{t|t-1}\mathbf{Z}'. \end{aligned}$$

Let $\mathbf{Y}_{t-1} = \{\mathbf{y}_{t-1}, \mathbf{y}_{t-2}, ..., \mathbf{y}_1\}$. Then it follows that,

$$
\begin{aligned}
E(\mathbf{y}_t|\mathbf{Y}_{t-1}) &= \mathbf{Z}\mathbf{a}_{t|t-1} \\
\mathbf{F}_t &= E[\mathbf{y}_t - E(\mathbf{y}_t|\mathbf{Y}_{t-1})][\mathbf{y}_t - E(\mathbf{y}_t|\mathbf{Y}_{t-1})]'.
\end{aligned}
$$

Denoting $p(\mathbf{y}_t|\mathbf{Y}_{t-1})$ as the log p.d.f. of $\mathbf{y}_t$ conditioned on $\mathbf{Y}_{t-1}$, the log-likelihood function is,

$$
\begin{aligned}
\mathcal{L}(\mathbf{y};\psi) &= \sum_{t=1}^{T} p(\mathbf{y}_t|\mathbf{Y}_{t-1}) \qquad (22) \\
&= -Tln(2\pi) - \frac{1}{2}\sum_{t=1}^{T} ln|\mathbf{F}_t| - \frac{1}{2}\sum_{t=1}^{T}(\mathbf{y}_t - \mathbf{Z}\mathbf{a}_{t|t-1})'\mathbf{F}_t^{-1}(\mathbf{y}_t - \mathbf{Z}\mathbf{a}_{t|t-1})
\end{aligned}
$$

where $\psi = (c_s, c_f, \phi_{ss}, \phi_{sf}, \phi_{fs}, \phi_{ff}, \theta_{ss}, \theta_{sf}, \theta_{fs}, \theta_{ff}, \sigma_s, \rho_{sf}, \sigma_f, \sigma_\eta)$ is the vector of parameters to be estimated.

In addition to parameter estimation, we are interested in estimating the expected excess return and expected depreciation. This is accomplished by estimating the individual components of the state vector using all of the available data with the full-sample smoother. The equations that form the full-sample smoother are,

$$
\begin{aligned}
\mathbf{a}_{t|T} &= \mathbf{a}_t + \mathbf{P}_t^*(\mathbf{a}_{t+1|T} - \mathbf{a}_{t+1|t}) && (23) \\
\mathbf{P}_t^* &= \mathbf{P}_t\mathbf{T}'\mathbf{P}_{t+1|t}^{-1}, \quad t = T-1, T-2, \ldots, 1, && (24) \\
\mathbf{a}_{T|T} &= \mathbf{a}_T. && (25)
\end{aligned}
$$

4.B The Monte Carlo Experiment

Given the maximum likelihood estimates of the parameter vector, $\hat{\psi}$, we ask whether the estimated two-component model can account for the spot and forward exchange rate regressions. We are asking whether the model can match a set of moments that were not explicitly imposed in estimation.

We address this question by generating parametric bootstrap distributions of the slope-coefficient estimators and their asymptotic t-ratios under the null of the estimated two-component model. These bootstrap distributions are built up by the following Monte Carlo experiment of 5000 trials. For each trial i (i=1,...,5000), we

1. Draw a scaler sequence of observations $\{\varepsilon^i_{z,t}\}_{t=1}^T$ from a normal distribution with mean 0 and variance $\hat{\sigma}_z^2$.

2. Draw a vector sequence of observations $\{(\varepsilon^i_{s,t}, \varepsilon^i_{f,t})'\}_{t=1}^T$ from a bi-variate normal distribution with mean 0 and covariance matrix, $\begin{pmatrix} \hat{\sigma}_s^2 & \hat{\rho}_{sf}\hat{\sigma}_s\hat{\sigma}_f \\ \hat{\rho}_{sf}\hat{\sigma}_s\hat{\sigma}_f & \hat{\sigma}_f^2 \end{pmatrix}$

3. Generate sequences of observations $\{z^i_t\}_{t=1}^T$, and $\{(x^i_{s,t}, x^i_{f,t})'\}_{t=1}^T$ according to (7) and (16). These sequences are then combined to construct sequences of log-levels of spot and forward rates, $\{(s^i_t, f^i_t)'\}_{t=1}^T$.

4. Use the computer-generated observations to estimate the cointegrating vector β_o with DOLS, DGLS, and CCR, and the slope coefficient in the regression of the future depreciation on the forward premium, β_1. Call these estimates $\hat{\beta}^i_{o,DOLS}$, $\hat{\beta}^i_{o,DGLS}$, $\hat{\beta}^i_{o,CCR}$, and $\hat{\beta}^i_1$.

The 5000 observations of $\hat{\beta}^i_{o,DOLS}$, $\hat{\beta}^i_{o,DGLS}$, $\hat{\beta}^i_{o,CCR}$, and $\hat{\beta}^i_1$ and their asymptotic t–ratios form the parametric bootstrap distribution for these estimators under the null hypothesis that our estimated permanent-transitory components model is the true data generating mechanism. Since inference is typically drawn from the asymptotic t–ratio, we compute the bootstrap distribution of the asymptotic t's for this purpose. We examine the bootstrap distribution of the slope coefficients to determine the extent of small-sample bias.

We also provide a test, based on a quadratic measure of distance, that the four asymptotic t–ratios (or the four slope coefficients) estimated from the data were jointly drawn from our data generating process. Let $\hat{\theta}$ be the (4×1) vector of interest estimated from the data. To perform the joint test, we compute the bootstrap distribution for the statistic,

$$J = (\hat{\theta} - \bar{\theta})'\Sigma_\theta^{-1}(\hat{\theta} - \bar{\theta}), \tag{26}$$

where $\bar{\theta}$ and Σ_θ are the mean vector and covariance matrix from the bootstrap distribution.

5 Results

In section 5.A, we report the maximum likelihood estimates of the unobserved components model and examine its ability to match the levels and forward premium regression coefficients as described in section 2. In section 5.B, we go on to examine the behavior of the expected excess return implied by the estimated model.

5.A ML Estimates and Moment Matching

The maximum likelihood estimates and asymptotic standard errors for our model are displayed in table 5. The top panel reports estimates from the spot and 1-month forward rate systems and the bottom panel shows estimates from the spot and 3-month forward rate systems. We make three observations about this table.

First, the variability of the exchange rate is apparently dominated by the random walk component.[12] The sample standard deviations of percent changes in the pound, franc, and yen rates are 3.36, 3.28, and 3.42 respectively, while the estimated standard deviation of the random walk innovation for these currencies in the 1-month system are 3.13, 3.09, and 2.80. Second, most of the parameters apparently are estimated with a high degree of precision, as their asymptotic standard errors are small relative to the estimates. Third, even though the random walk components for the spot and forward rates are constrained to be identical, estimates of the correlation between the innovations to the transitory components are large. The correlation for the franc is 1 (due to rounding). The near perfect correlation of the innovations does not, however, mean that the processes $\{x_{s,t}\}$ and $\{x_{f,t}\}$ are perfectly correlated since the estimated vector ARMA coefficients differ in magnitude.

Table 6 displays various population moments implied by the ML estimates. Using the 'eyeball' metric, the model does a credible job of matching these moments. In particular, the implied slope coefficients from regressing the future depreciation on the forward premium, β_1, are much less than 1 and are negative for each currency. The implied expected excess returns are substantially more volatile than the expected depreciation, and they are negatively

[12]Campbell and Clarida (1987) also use the Kalman filter and find that exchange rate movements are dominated by the random walk component.

correlated with each other. The implied forward premia are persistent, as can be seen from the large values of their first-order autocorrelations. The implied forward premium variance is seen to match up with the sample variances as well. These sample variances are 0.111, 0.101, 0.081, for the pound, franc, and yen 1-month forward premia and 0.691, 0.675, and 0.623 for the 3-month forward premia.

We now turn to the Monte Carlo results. For the spot and 1-month forward rate systems, table 7 displays the lower 2.5, 50, and 97.5 percentiles of the bootstrap distribution for the four slope-coefficient estimators that we consider and their asymptotic t-ratios. The asymptotic t's are constructed under the hypothesis that the slope coefficient is 1 as they were in tables 3 and 4. p-values are the proportion of the empirical distribution that lies to the right of the values estimated from the data. Table 8 reports the same information for the spot and 3-month forward rate systems.

The small-sample bias in the estimators, while generally small, is greater in the spot and 3-month forward rate system. The medians for the cointegrating regression slope coefficients are close to the true value of 1 while the medians for the forward premium regressions are negative. The median values of $\hat{\beta}_1$ are $-0.24, -1.06$, and -1.22 in the spot and 1-month forward rate system, and $-1.17, -0.84$, and -1.39 in the spot and 3-month forward rate system and are close to the implied population values shown in table 6. None of the p-values lie outside the interval $(0.025, 0.975)$ so that our model cannot be rejected at the 5% level. In particular, the joint test for the four slope coefficients does not reject the model for any of the currencies nor does the joint test on the four t-ratios.

Although not the main focus of our investigation, tables 7 and 8 do provide interesting information about the sampling properties of the three cointegrating vector estimators. All of the cointegrating vector estimators are biased downward, as the medians from each of the distributions are less than 1. This bias is most severe for CCR, which translates into a substantial downward bias in its asymptotic t-ratios. For our particular application, CCR apparently is the least efficient and most biased of the three cointegrating vector estimators. The spread between the lower and upper 2.5% tails among the three cointegrating vector estimators is greatest for CCR. On the other hand, the lower and upper 2.5% tails of the the CCR asymptotic t-ratio distribution has the smallest spread.

The bootstrap distributions of the asymptotic t-ratios appear to be poorly approximated by the standard normal for our model with a sample size of 200. There is considerable size distortion, as the lower and upper 2.5% tails of the asymptotic t-ratios differ from the standard normal's values of ± 1.96. For example, the lower and upper t-ratio tails for DOLS in for the yen regressions in table 8 is -8.21 and 3.09. More detailed examinations of the distributions than that reported in the table indicate that they do not appear to be particularly skewed in either direction however.

5.B Implied Expected Excess Returns

As we discussed in section 4.A, we estimate the expected future spot rate with the smoothed Kalman filter. The smoothed estimates provide an estimate of the state, α_t given the entire sample. This estimate is then subtracted from the forward rate to obtain the implied expected excess return. Figures 1-3 plot estimates of the quarterly expected depreciation and expected excess return for the three currencies.[13]

We note that there are a number of similarities across the figures. First, it is visually apparent that the expected excess return is both negatively correlated with and is more volatile than the expected depreciation. Second, the expected excess return fluctuates from positive to negative values but is quite persistent. Looking across the 3 currencies, these expected excess returns are high and positive for much of the decade spanning the mid 1970's to the mid 1980's, and negative for the final 3 years or so of the sample.[14]

A plausible story to tell about the behavior of the expected excess return begins by recognizing that $r_{t,k}$ is the expected return from buying dollars forward.[15]If $r_{t,k}$ is large and positive, the dollar must be risky because the market is compensating agents who buy the dollar forward. Now suppose that total expenditures of U.S. residents are negatively correlated with the exchange rate. Then in a 'bad' year, expenditures fall while the exchange rate increases. Even with the domestic purchasing power of the dollar unchanged, the dollar

[13]Plots at the $k = 1$ horizon reveal that monthly expected excess return and expected depreciation are qualitatively similar, but as one would expecte, somewhat noisier. We suppress these plots to economize on space.

[14]In a related context, LeBaron (1992) finds that to match moving average trading rule results requires a persistent, but stationary risk premium.

[15]Hodrick (1987) and Engel (1992) provide the foundation for telling this story.

buys fewer foreign goods in this state of nature. If both foreign and domestic goods are valued in consumption, then states in which total expenditures and the exchange rate are negatively correlated are states in which the dollar is 'risky,' because it provides a poor hedge against the bad expenditure shock. It thus makes sense that these episodes are associated with an expected weakening of the dollar.[16]

6 Conclusions

The behavior of expected excess foreign exchange returns has been the subject of extensive empirical research. Complete markets general equilibrium settings provide an interpretation of the expected excess return as a risk premium, but these models have done a poor job in quantifying its behavior. Similarly, finance based models that characterize risk as a covariance that fluctuates according to ARCH or its generalizations, have not generated expected excess foreign exchange returns of reasonable volatility and magnitude.[17]

This paper has taken a structural time series model that draws its motivation from the disequilibrium dynamics of the exchange rate analyzed by Mussa (1982). The simple two-component model that we estimate for spot and forward exchange rates is able to match many key features of the data that were not explicitly used in estimating the model.

We imposed the restriction that spot and forward rates follow a common trend or random walk process. This ensures both that the forward and the future spot rates are cointegrated with a cointegrating vector of 1 and that the forward premium is an I(0) process. The transitory components were modeled as a vector ARMA(1,1) processes with contemporaneously correlated innovations. This specification was shown to generate the negative correlation between the forward premium and future depreciation that has been found in the data. Using parametric bootstrap distributions generated by Monte Carlo methods, the model is not generally rejected at standard significance levels.

[16] Alternatively, periods in which expenditures and the exchange rate are positively correlated imply that the dollar is a safe currency. Cumby (1988) reports estimates of the conditional covariance between consumption growth and speculative foreign exchange returns that change sign over his sample.

[17] e.g.,Engel and Rodrigues (1987), and Mark (1988). See also Frankel (1988) who surveys estimates of the risk premium.

APPENDIX

This appendix provides a brief description of the DOLS, DGLS, and CCR cointegration vector estimators. Consider the triangular representation,

$$y_t = \theta x_t + e_t \tag{A.1}$$

$$\Delta x_t = u_t \tag{A.2}$$

For our purposes, $y_t = s_{t+k}$, $x_t = f_{t,k}$, and the cointegrating vector, θ is a scaler. Stock and Watson assume that $E(e_t|\{\Delta x_j\}_{j=1}^{\infty}) = d(L)\Delta x_t$, where $d(L)$ is a two-sided infinite polynomial in the lag operator L. Now add and subtract this term in eq. (A.1) to get,

$$y_t = \theta x_t + d(L)\Delta x_t + \tilde{e}_t \tag{A.3}$$

where $\tilde{e}_t = e_t - E(e_t|\{\Delta x_j\}_{j=1}^{\infty})$, which is orthogonal to $\{\Delta x_j\}_{j=1}^{\infty}$. Now, exploit the Wold representations, $u_t = c_{11}(L)\varepsilon_{1,t}$ and $\tilde{e}_t = c_{22}(L)\varepsilon_{2,t}$ to rewrite (A.1) and (A.2) as,

$$\Delta x_t = c_{11}(L)\varepsilon_{1,t} \tag{A.4}$$

$$y_t = \theta x_t + d(L)\Delta x_t + c_{22}(L)\varepsilon_{2,t} \tag{A.5}$$

Stock and Watson show that $\varepsilon_{1,t}$ is independent of $\varepsilon_{2,t}$ and x_t, so as long as there are no restrictions between the parameters of of $c_{11}(L)\varepsilon_{1,t}$ and $c_{22}(L)\varepsilon_{2,t}$, the cointegrating vector, θ can be estimated by applying either OLS or GLS to (A.5).

The OLS and GLS formulas can be conveniently expressed if we let $z_t = (z'_{1,t}, z'_{2,t}, z'_{3,t})'$, where $z_{1,t}$ contains all the I(0) terms, $z_{2,t}$ is a constant, and $z_{3,t}$ contains the I(1) terms. That is,

$$z_{1,t} = (\Delta x_{t+p}, \ldots, \Delta x_t, \ldots, \Delta x_{t-p})',$$

$$z_{2,t} = 1$$

$$z_{3,t} = y_{1,t}$$

If y_t is an $(n \times 1)$ vector, the OLS estimator is,

$$\delta_{OLS} = \left(\left[\sum z_t z_t'\right] \otimes I_n\right)^{-1} \left(\sum \left[z_t \otimes I_n\right] y_t\right) \tag{A.6}$$

The estimate of θ is obtained by picking out the appropriate element from δ_{OLS}. To test the null hypothesis $H_o : R\delta = r$ where r is $(h \times 1)$, form the Wald statistic,

$$W_{OLS} = (R\delta_{OLS} - r)' \left(R\left(\left[\sum z_t z_t'\right]^{-1} \otimes \hat{\Omega}_{22}\right) R'\right)^{-1} (R\delta_{OLS} - r) \sim \chi_h^2 \tag{A.7}$$

and $\hat{\Omega}_{22}$ is a consistent estimator of $\Omega_{22} = c_{22}(1)\Sigma_{22}c_{22}(1)'$, and $\Sigma = E(\varepsilon_{2,t}\varepsilon'_{2,t})$

Similarly, the GLS estimator is given by,

$$\delta_{GLS} = \left(\sum \tilde{z}_t \tilde{z}_t'\right)^{-1} \left(\sum \tilde{z}_t \tilde{y}_t\right) \tag{A.8}$$

where

$$\phi(L) = \Sigma_{22}^{-\frac{1}{2}} c_{22}(L)^{-1}$$

$$\tilde{y}_t = \phi(L) y_t,$$

$$\tilde{z}_t = z_t \nwarrow \phi(L)',$$

and Wald statistic for the GLS estimator is given by,

$$W_{GLS} = (R\delta_{GLS} - r)' \left(R \left[\sum \tilde{z}_t \tilde{z}_t'\right]^{-1} R'\right)^{-1} (R\delta_{GLS} - r) \sim \chi_n^2$$

In Park's CCR, the observations are first transformed in a way that asymptotically eliminates the long-run correlation between the innovations of the regressors and the regression errors. OLS performed on these transformed observations yield asymptotically efficient estimators and chi-square tests.

The model generating the data is assumed to be,

$$y_t = \pi_1' c_t + y_t^o, \tag{A.9}$$

$$x_t = \pi_2' c_t + x_t^o, \tag{A.10}$$

where $\{c_t\}$ is a $(k \times 1)$-dimensional deterministic sequence. The cointegration relation between the purely stochastic I(1) processes takes the form,

$$y_t^o = \alpha' x_t^o + e_t. \tag{A.11}$$

Let $w_t = (e_t, \Delta x_t^o)'$, - $= \Sigma + \Lambda + \Lambda'$ be its long-run covariance matrix, where $\Sigma = E(w_t w_t')$ and $\Lambda = \sum_{j=1}^{\infty} E(w_t w_{t+j}')$. $\Gamma = \Sigma + \Lambda$, and $\Gamma_2 = (\gamma_{12}', \Gamma_{22}')'$. The long-run variance of e_t is thus, $\omega_{11 \cdot 2} = \omega_{11} - \omega_{12}$- $_{22}^{-1}\omega_{21}$.

Next, we obtain the transformed observations,

$$x_t^* = x_t - (\Sigma^{-1}\Gamma_2)' w_t \tag{A.12}$$

$$y_t^* = y_t - (\Sigma^{-1}\Gamma_2 \alpha + (0, \omega_{12}\text{-}\ _{22}^{-1})')' w_t \tag{A.13}$$

and estimate the cointegrating vector by runing OLS with y_t^* on x_t^* and any deterministic variables. Let $\hat{\theta}^*$ be this least-squares estimator, and Z_* be the design matrix, containing the transformed observations and any deterministic variables. To test a null hypothesis

$H_o : \phi(\theta_o) = 0$, of h restrictions on the true value of the coefficient vector, θ_o, the statistic,

$$G(\hat{\theta}_n^*) = \frac{\phi(\hat{\theta}^*)(\phi(\hat{\theta}^*)(Z_*' Z_*)^{-1}\phi(\hat{\theta}^*)')^{-1}}{\omega_{11\cdot 2}} \tag{A.14}$$

is asymptoticaly chi-square with h degrees of freedom. Of course, to apply CCR, we use consistent estimates of the long-run covariances in place of their population values.

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Table 1: Unit Root Tests on Forward Premia

Tests that the forward premium, $y_t = f_{t,k} - s_t$, is I(1) for $k = 1, 3$. The table reports studentized coefficients on y_{t-1} in augmented Dickey-Fuller tests τ(ADF) for the regression, $\Delta y_t = \phi_0 + \phi_1 y_{t-1} + \sum_1^p \theta_j \Delta y_{t-j} + u_t$, and in Phillips-Perron tests τ(PP) in the regression $y_t = \phi_0 + \phi_1 y_{t-1} + u_t$ The sample extends from 1976:1 to 1992:8. We use a fixed lag-length, p=6. '*' and '**' indicate significance at the 5% and 10% levels respectively.

	Studentized Coefficients			
	1-Month		3-Month	
Currency	τ(ADF)	τ(PP)	τ(ADF)	τ(PP)
pound	–3.306*	–4.694*	–2.924*	–2.888*
Franc	–2.693**	–6.238*	–2.634**	–4.628*
Yen	–2.674**	–2.929*	–2.572**	–2.691**

Table 2: Cointegration Tests

Residual based tests of the null that forward and future spot rates are not cointegrated. For $k = 1, 3$, the table reports the static OLS estimate, $\hat{\beta}$, of the cointegrating coefficient from the regression, $s_{t+k} = \alpha + \beta f_{t,k} + u_{t,k}$, the studentized coefficients, τ, for $u_{t-1,k}$ in the augmented Dickey-Fuller regression, $\Delta u_{t,k} = \phi_0 + \phi_1 u_{t-1,k} + \sum_1^p \theta_j \Delta u_{t-j,k} + w_t$, and in the Phillips-Perron regression, $u_{t,k} = d + \rho u_{t-1,k} + \omega_t$ where u is the cointegrating regression error. The sample extends from 1976:1 to 1992:8. The 5% critical value is -2.88, and the lag-length, p, is 6.

Forward Rate	1-Month			3-Month		
Currency	$\hat{\beta}_o$	τ(ADF)	τ(PP)	$\hat{\beta}_o$	τ(ADF)	τ(PP)
Pound	0.975	–4.288	–12.880	0.911	–3.670	–5.275
Franc	0.987	–3.986	–13.693	0.953	–3.342	–4.949
Yen	0.994	–4.587	–12.547	0.975	–3.224	–5.163

Table 3: Cointegrating Regressions

Estimates of cointegrating regression coefficient, $s_{t+k} = \alpha_o + \beta_o f_{t,k} + \varepsilon_{t+k,k}$ for $k = 1, 3$, using Stock and Watson's method with 6 leads and lags and Park's method of canonical cointegrating regression. t(β_o) is the asymptotic t-statistics to test the hypothesis $\beta_o = 1$. Marginal significance levels ($m.s.l.$) are for a two-tailed test and are computed from the t-ratio's asymptotic standard normal distribution. The sample extends from 1976:1 to 1992:8.

	DOLS			DGLS			CCR		
Currency	$\hat{\beta}_o$	t($\hat{\beta}_o$)	$m.s.l.$	$\hat{\beta}_o$	t($\hat{\beta}_o$)	$m.s.l.$	$\hat{\beta}_o$	t($\hat{\beta}_o$)	$m.s.l.$
					k=1				
Pound	0.997	–1.533	0.125	0.997	–0.908	0.364	0.992	–1.025	0.306
Franc	0.999	–0.828	0.408	0.998	–0.719	0.472	0.996	–0.780	0.435
Yen	1.003	1.947	0.052	1.001	0.676	0.500	1.001	0.195	0.845
					k=3				
Pound	0.992	–1.455	0.146	0.988	–1.315	0.189	0.969	–1.008	0.313
Franc	0.993	–1.439	0.150	0.992	–1.200	0.230	0.981	–1.059	0.290
Yen	1.010	2.186	0.029	1.003	0.519	0.604	1.000	–0.009	0.993

Table 4: Forward Premium Regressions

OLS estimates, asymptotic standard errors and t-ratios from the regression of future changes in the spot rate on the forward premium,

$$s_{t+k} - s_t = \alpha_1 + \beta_1(f_{t,k} - s_t) + \nu_{t+k,k}$$

for $k = 1, 3$ from 1976:1 to 1992:8.

Currency	$\hat{\alpha}_1$	(s.e.)	t–ratio $H_0 : \alpha_1 = 0$	$\hat{\beta}_1$	(s.e.)	t–ratio $H_0 : \beta_1 = 0$	t–ratio $H_0 : \beta_1 = 1$
				k=1			
Pound	–0.003	0.003	–1.038	–1.440	0.713	–2.020	–3.423
Franc	–0.002	0.003	–0.691	–0.766	0.729	–1.050	–2.422
Yen	0.011	0.003	3.416	–2.477	0.836	–2.965	–4.162
				k=3			
Pound	–0.016	0.008	–2.030	–2.367	0.895	–2.645	–3.763
Franc	–0.003	0.012	–0.259	–0.284	1.104	–0.257	–1.162
Yen	0.032	0.009	3.449	–2.398	0.613	–3.913	–5.544

Table 5: Maximum Likelihood Estimates of the Trend–VARMA(1,1) Model
The sample extends from 1976:1 to 1992:8. $\mathbf{y}_t = \iota z_t + \mathbf{x}_t$, where $\mathbf{y}_t = (s_t, f_{k,t})'$, $k = 1, 3$, $\iota = (1,1)'$, $z_t = z_{t-1} + \varepsilon_{z,t}$, $\varepsilon_{z,t} \overset{i.i.d.}{\sim} N(0, \sigma_z^2)$, $\mathbf{x}_t = \mathbf{c} + \Phi \mathbf{x}_{t-1} + \epsilon_t + \Theta \epsilon_{t-1}$, $\epsilon_t = (\varepsilon_{s,t}, \varepsilon_{f,t})' \overset{i.i.d.}{\sim} N(\mathbf{0}, \Sigma)$, with

$$\Sigma = \begin{pmatrix} \sigma_s^2 & \rho_{sf}\sigma_s\sigma_f \\ \rho_{sf}\sigma_s\sigma_f & \sigma_f^2 \end{pmatrix},$$

$\mathbf{c}$ a (2×1) constant vector, and Φ and Θ being (2×2) parameter matrices. Asymptotic standard errors in parentheses.

Spot and One-Month Forward Exchange Rates

c	Φ		Θ		σ_s	σ_f	ρ_{sf}	σ_z
0.0827	0.9020	-0.0525	0.4598	0.1204	0.9700	0.8828	0.9818	3.1303
(0.0041)	(0.0030)	(0.1197)	(0.0400)	(0.1732)	(0.0173)	(0.0133)	(0.0021)	(0.0038)
0.0338	0.1386	0.7141	0.3131	0.3127	Pound			
(0.0042)	((0.0281)	(0.1197)	(0.0382)	(0.0113)	Log likelihood: -501.85			
0.0405	0.9380	-0.0119	0.0765	0.3380	0.9519	0.7590	0.9999	3.0852
(0.0016)	(0.0027)	(0.2217)	(0.0118)	(0.0037)	(0.0036)	(0.0051)	(0.0000)	(0.0017)
-0.0353	0.3032	0.6513	-0.1447	0.6998	Franc			
(0.0016)	(0.0160)	(0.0138)	(0.0020)	(0.0018)	Log likelihood: -515.44			
-0.0297	0.8436	0.0816	0.1346	0.1695	1.9188	1.9233	0.9986	2.8017
(0.0042)	(0.0216)	(0.1236)	(0.0090)	(0.0155)	(0.0069)	(0.0053)	(0.0001)	(0.0056)
0.0340	0.0703	0.8635	0.1260	0.1677	Yen			
(0.0043)	(0.1679)	(0.0153)	(0.0226)	(0.0537)	Log likelihood: -388.22			

Spot and Three-Month Forward Exchange Rates

c	Φ		Θ		σ_s	σ_f	ρ_{sf}	σ_z
0.1816	0.9998	-0.0808	0.4254	0.1884	1.0716	1.0434	0.9700	3.1144
(0.0340)	(0.0009)	(0.0418)	(0.0574)	(0.1392)	(0.0761)	(0.0510)	(0.0070)	(0.0345)
-0.0633	0.2428	0.7136	0.4218	0.2973	Pound			
(0.0350)	(0.0621)	(0.0240)	(0.0949)	(0.0522)	Log likelihood: -592.2617			
0.0642	0.9994	-0.0571	0.3254	0.1117	1.0322	0.6130	0.9999	3.0455
(0.0423)	(0.0015)	(0.0321)	(0.0601)	(0.2056)	(0.0402)	(0.0432)	(0.0001)	(0.0406)
-0.2354	0.4045	0.6033	0.3480	0.2143	Franc			
(0.0388)	(0.0265)	(0.0312)	(0.0462)	(0.0374)	Log likelihood: -670.2740			
-0.0848	0.9914	-0.0683	0.3864	0.0472	1.6318	1.6293	0.9898	2.9394
(0.0043)	(0.0006)	(0.0153)	(0.0042)	(0.1938)	(0.0044)	(0.0044)	(0.0006)	(0.0045)
0.1336	0.2220	0.7304	0.3655	0.0333	Yen			
(0.0043)	(0.0106)	(0.0029)	(0.0092)	(0.1092)	Log likelihood: -558.5972			

Table 6: Implied Moments
Moments implied by VARMA(1,1) trend-cycle model estimates from the bivariate system.

	Pound	Franc	Yen
$Cov[(s_{t+1} - s_t), (f_t - s_t)]$	–0.023	–0.094	–0.078
$Var(f_t - s_t)$	0.106	0.094	0.074
$\rho[(f_t - s_t), (f_{t-1} - s_{t-1})]$	0.786	0.652	0.916
β_1	–0.213	–1.002	–1.049
$Var[E_t(s_{t+1} - s_t)]$	0.322	0.125	0.404
$\rho[E_t(s_{t+1} - s_t), E_{t-1}(s_t - s_{t-1})]$	0.497	0.471	0.541
$Var(r_t)$	0.474	0.407	0.634
$Cov[E_t(s_{t+1} - s_t), r_{t,1}]$	–0.345	–0.219	–0.482
$Cov[(s_{t+3} - s_t), (f_t - s_t)]$	–0.675	–0.492	–0.710
$Var(f_t - s_t)$	0.674	0.632	0.586
$\rho[(f_t - s_t), (f_{t-3} - s_{t-3})]$	0.773	0.547	0.815
β_1	–1.001	–0.778	–1.212
$Var[E_t(s_{t+3} - s_t)]$	1.105	0.596	1.736
$\rho[E_t(s_{t+3} - s_t), E_{t-3}(s_t - s_{t-3})]$	0.783	0.827	0.850
$Var(r_t)$	3.130	2.211	3.743
$Cov[E_t(s_{t+3} - s_t), r_{t,3}]$	–1.753	–1.077	–2.447

Table 7: Features of the Monte Carlo Distribution

Selected percentiles of the Monte Carlo distribution computed for the cointegrating vector estimators ($\hat{\beta}_{o,DOLS}$, $\hat{\beta}_{o,DGLS}$, $\hat{\beta}_{o,CCR}$) and the slope coefficient in regressions of the future depreciation on the forward premium ($\hat{\beta}_1$) and their asymptotic t-ratios. J is the joint test statistic described in (26). p-values are the proportion of the empirical distribution that lies above the values estimated from the data. The data generating mechanism is the trend-vector ARMA(1,1) components model fitted to spot and 1-month forward exchange rates from 1976:1 to 1992:8.

	Slope coefficient				Asymptotic t-ratios			
	2.5%	median	97.5%	p-value	2.5%	median	97.5%	p-value
				Pound				
$\hat{\beta}_{o,DOLS}$	0.9896	1.0002	1.0114	0.7530	-3.1905	0.0806	3.2443	0.8574
$\hat{\beta}_{o,DGLS}$	0.9902	1.0003	1.0107	0.8102	-2.1575	0.0707	2.2579	0.8334
$\hat{\beta}_{o,CCR}$	0.9538	0.9939	1.0058	0.5866	-2.5783	-0.7637	0.8994	0.6314
$\hat{\beta}_1$	-1.6875	-0.2402	1.2411	0.9450	-3.6985	-1.6740	0.3146	0.9548
J	0.3307	2.3202	17.9333	0.3622	0.3808	3.0232	13.4668	0.3538
				Franc				
$\hat{\beta}_{o,DOLS}$	0.9853	0.9981	1.0088	0.4650	-3.4924	-0.2658	2.7934	0.7964
$\hat{\beta}_{o,DGLS}$	0.9857	0.9980	1.0083	0.4626	-2.4451	-0.1813	1.9640	0.7466
$\hat{\beta}_{o,CCR}$	0.9502	0.9920	1.0046	0.3132	-2.5044	-0.8302	0.7810	0.6072
$\hat{\beta}_1$	-2.5536	-1.0596	0.4875	0.3530	-4.3832	-2.2613	-0.2920	0.8642
J	0.3187	2.2252	18.4785	0.9472	0.3073	2.7521	13.7762	0.9932
				Yen				
$\hat{\beta}_{o,DOLS}$	0.9806	0.9956	1.0078	0.1050	-8.1698	-1.9317	3.1185	0.0634
$\hat{\beta}_{o,DGLS}$	0.9816	0.9947	1.0048	0.0978	-5.1387	-1.4784	1.4534	0.0680
$\hat{\beta}_{o,CCR}$	0.9417	0.9878	1.0034	0.0526	-3.4283	-1.3546	0.5153	0.0504
$\hat{\beta}_1$	-3.2701	-1.2155	0.6144	0.8992	-4.1772	-2.2988	-0.3696	0.9738
J	0.3769	2.4390	16.9470	0.3818	0.3766	2.9487	13.0354	0.1322

Table 8: Features of the Monte Carlo Distribution

Selected percentiles of the Monte Carlo distribution computed for the cointegrating vector estimators ($\hat{\beta}_{o,DOLS}$, $\hat{\beta}_{o,DGLS}$, $\hat{\beta}_{o,CCR}$) and the slope coefficient in regressions of the future depreciation on the forward premium ($\hat{\beta}_1$) and their asymptotic t-ratios. J is the joint test statistic described in (26). p-values are the proportion of the empirical distribution that lies above the values estimated from the data. The data generating mechanism is the trend-vector ARMA(1,1) components model fitted to spot and 3-month forward exchange rates from 1976:1 to 1992:8.

	Slope coefficient				Asymptotic t-ratios			
	2.5%	median	97.5%	p-value	2.5%	median	97.5%	p-value
Pound								
$\hat{\beta}_{o,DOLS}$	0.9465	0.9916	1.0277	0.4878	-6.7314	-1.2359	3.5059	0.5424
$\hat{\beta}_{o,DGLS}$	0.9461	0.9871	1.0161	0.4776	-4.2188	-1.1880	1.6043	0.5348
$\hat{\beta}_{o,CCR}$	0.8275	0.9665	1.0149	0.4690	-3.4013	-1.2263	0.6863	0.4114
$\hat{\beta}_1$	-2.9974	-1.1313	0.5959	0.9150	-5.9931	-2.7578	-0.5025	0.7620
J	0.4042	2.4399	16.7857	0.5950	0.3819	2.9163	13.6032	0.9592
Franc								
$\hat{\beta}_{o,DOLS}$	0.9553	0.9930	1.0230	0.4962	-6.2832	-1.1647	3.3078	0.5522
$\hat{\beta}_{o,DGLS}$	0.9565	0.9922	1.0200	0.4934	-4.9732	-1.0165	2.4198	0.5444
$\hat{\beta}_{o,CCR}$	0.8404	0.9692	1.0125	0.3440	-3.3172	-1.1993	0.6016	0.4432
$\hat{\beta}_1$	-2.4051	-0.8374	0.7524	0.2458	-5.6204	-2.5842	-0.3244	0.1126
J	0.3748	2.3302	18.4734	0.8820	0.3549	2.8152	14.2667	0.7602
Yen								
$\hat{\beta}_{o,DOLS}$	0.9445	0.9882	1.0222	0.0928	-8.2059	-1.8725	3.0926	0.0514
$\hat{\beta}_{o,DGLS}$	0.9446	0.9826	1.0101	0.0766	-5.0632	-1.7163	1.1217	0.0610
$\hat{\beta}_{o,CCR}$	0.8141	0.9599	1.0108	0.0730	-3.6366	-1.4019	0.5058	0.0730
$\hat{\beta}_1$	-3.5078	-1.3920	0.5098	0.8508	-6.0877	-2.8154	-0.5255	0.9598
J	0.4130	2.4925	17.1538	0.4324	0.4088	2.8878	13.4213	0.1062