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Source: *The Review of Economics and Statistics*, Vol. 67, No. 4 (Nov., 1985), pp. 681-684
Published by: The MIT Press
Stable URL: https://www.jstor.org/stable/1924814
Accessed: 15-10-2020 19:56 UTC

A NOTE ON INTERNATIONAL REAL INTEREST RATE DIFFERENTIALS

Nelson C. Mark*

Abstract—This note empirically examines the issue of real interest rate equalization across countries. The equality of real interest rates is implied by two frequently employed concepts of equilibrium in international asset (capital) and commodity markets and imposes certain time series restrictions on ex post real interest rates. Statistical tests of these restrictions strongly reject the hypothesis that real interest rates have been equal across countries.

I. Introduction

This note undertakes statistical tests of the hypothesis that ex ante real interest rates are equal across countries (hereafter, the equality hypothesis). Apart from being of interest in its own right, the issue of real interest rate equalization has important implications for the use of monetary policy in the open economy. This is because the way in which money is viewed to affect real economic activity in almost every theory of the macroeconomy is by altering the real interest rate or the intertemporal terms of trade. In an open economy, a necessary condition for monetary policy to operate through this channel is that real interest rates diverge internationally; otherwise, the ability of the authorities to influence their own real interest rate would be limited to the extent to which they can influence the world real interest rate. This is not a sufficient condition, however, so that finding violations of the equality hypothesis would not constitute proof that monetary policy is effective in influencing the real interest rate.

The equality hypothesis is an implication of two parity conditions frequently employed in models of the open economy to represent a high degree of integration in international goods and asset (capital) markets.[1] These are uncovered interest parity and an ex ante version of relative purchasing power parity. Consequently, tests of the equality hypothesis can be thought of as tests of the joint validity of these parity conditions.[2]

Received for publication December 19, 1983. Revision accepted for publication March 6, 1985.

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This paper is adapted from the author's Ph.D. dissertation. The author wishes to acknowledge helpful comments and suggestions from Michael L. Mussa, Jacob A. Frenkel, John Huizinga, and an anonymous referee. They are not responsible for any remaining errors.

[1] See, for example, Frenkel (1976), Bilson (1978), Cox (1980), and Flood and Marion (1982).

[2] The international equalization of real interest rates has been empirically examined by a number of authors employing regression techniques with varying results. Hodrick (1980) and Cumby and Obstfeld (1983) test against a limited specification of time variation in the real interest rate differential. Mishkin (1984) requires special assumptions concerning the error process and Mark (1983) restricts economic agents to employing linear prediction rules. This note considers so-called "weak form" tests which focus on the time series restrictions on ex post real interest rates implied by the equality hypothesis. This methodology has not previously been employed in studies on real interest rate equality.

II. The Equality Hypothesis

At time $t + k$, the k-period ex post real interest rate in the domestic country, r_{t+k}, is defined as

$$r_{t+k} = i_{t+k} - \theta_{t+k}, \tag{1}$$

where i_{t+k} is the nominal interest rate on a k-period domestic bond issued at t which matures at $t + k$ and θ_{t+k} is the rate of change in the general price level from t to $t + k$. At t, people make predictions as to what the ex post real interest rate will be at $t + k$ based on the information set, I_t, available to them at t. This prediction is the ex ante real interest rate at t. Assuming that people's expectations are "rational," the ex ante real interest rate is obtained by taking expectations on both sides of equation (1) conditioned on I_t. That is,

$$E(r_{t+k}|I_t) = i_{t+k} - E(\theta_{t+k}|I_t), \tag{2}$$

where $E(y|x)$ is the mathematical expectation of y conditioned on x.[3] Similarly, the k-period ex ante real interest rate at time t in the foreign country, r^*_{t+k}, is

$$E(r^*_{t+k}|I_t) = i^*_{t+k} - E(\theta^*_{t+k}|I_t), \tag{3}$$

where stars denote variables of the foreign country.

The equality of domestic and foreign ex ante real interest rates is obtained from two parity conditions often imposed in models of the open economy to represent international asset (capital) and commodity market equilibria. These are uncovered interest parity (UIP) and an ex ante version of relative purchasing power parity ($EAPPP$) given in equations (4) and (5).

$$i_{t+k} - i^*_{t+k} = E(s_{t+k} - s_t|I_t) \qquad (UIP) \tag{4}$$

$$E(\theta_{t+k} - \theta^*_{t+k}|I_t) = E(s_{t+k} - s_t|I_t) \qquad (EAPPP). \tag{5}$$

Here, s_t is the logarithm of the spot exchange rate, expressed as the domestic currency price of a unit of foreign exchange. UIP is a capital market equilibrium condition which requires that the expected net return on domestic and foreign currency denominated bonds of similar default risk be equal. $EAPPP$ represents international commodity market equilibrium and states that

[3] At time t, i_{t+k} is known.

the domestic currency price of goods at home and abroad change at the same rate.[4] Upon subtracting equation (5) from (4), the equality of ex ante real interest rates is obtained. That is,

$$0 = [i_{t+k} - E(\theta_{t+k}|I_t)] - [i^*_{t+k} - E(\theta^*_{t+k}|I_t)] = E[(r_{t+k} - r^*_{t+k})|I_t]. \quad (6)$$

III. Methodology

The sampling interval of the data is chosen to coincide with the time to maturity on the financial assets under consideration so that k can be taken to be unity. To economize on notation, let $R \equiv r - r^*$ denote the ex post real interest rate differential. Equations (2) and (3) imply that this differential can be represented as

$$R_{t+1} = E(R_{t+1}|I_t) + u_{t+1} \quad (7)$$

where $u_{t+1} = (\theta^*_{t+1} - \theta_{t+1}) - E[(\theta^*_{t+1} - \theta_{t+1})|I_t]$ is economic agents' forecast error of the domestic–foreign inflation differential. Since u_t is contained in I_t and is orthogonal to I_{t-j}, $(j \geq 1)$, it follows by an iterated expectations argument that

$$E(u_t) = 0, \quad (8)$$

$$E(u_{t+j}u_t) = 0, \qquad (j \neq 0). \quad (9)$$

That is, the forecast error is an unconditionally zero mean and uncorrelated process. It is assumed that the forecast error is wide sense stationary so that

$$E(u_t^2) = \sigma^2 = constant, \quad (10)$$

and also that the data are ergodic. Thus $\{u_t\colon\ t = 0, \pm 1, \pm 2, \dots\}$ is white noise. Under the equality hypothesis, equation (7) reduces to

$$R_{t+1} = u_{t+1}. \quad (11)$$

The following three tests of the equality hypothesis are put forth.

The first test examines the possibility that real interest rates across countries diverge by a constant differential and tests the hypothesis that the unconditional mean of R is zero.

The second test is a portmanteau test based on the Box-Ljung (1978) Q-statistic,

$$Q(p) = T(T+2)\sum_{k=1}^{p}(T-k)^{-1}c_k^2, \quad (12)$$

where c_k is the k^{th} order sample autocorrelation of the ex post real interest rate differential and T is the sample size. Under the equality hypothesis, $Q(p)$ is distributed as χ^2_p in large samples.

[4] Roll (1979) has derived *EAPPP* as an equilibrium condition arising from intertemporal and international commodity arbitrage and provides limited empirical support for it. Additional evidence supporting *EAPPP* can be found in Frenkel (1981).

The third test is based on the cumulated periodogram test. Let $f(\lambda)$ be the spectral density function and $F(\lambda)$ be the spectral distribution function of the ex post real interest rate differential. Under the equality hypothesis, the ex post differentials are white noise and $f(\lambda) = \sigma^2/\pi$, $0 \leq \lambda \leq \pi$, and $F(\lambda) = \int_0^\lambda f(s)\,ds = (\sigma^2\lambda)/\pi$, $0 \leq \lambda \leq \pi$. The periodogram is

$$I_{T,j} = |X(2\pi j/T)|^2, \qquad 1 \leq j \leq T,$$

where

$$X(2\pi j/T) = (1/\sqrt{T})\sum_{t=1}^{T} R_t \exp[-i(2\pi j/T)t]$$

is the finite Fourier transform of the ex post real interest rate differential and $i = \sqrt{-1}$. Since

$$E(I_{T,j}) = \pi f(j/T) = \sigma^2,$$

the normalized cumulated periodogram,

$$C'(k) = (Ts^2)^{-1}\sum_{j=1}^{k} I_{T,j}, \qquad 1 \leq k \leq T,$$

is used as an estimate of the normalized spectral distribution function,

$$C(k) = \pi F(k/T)/\sigma^2 = k/T, \qquad 1 \leq k \leq T,$$

where s^2 is a consistent estimator of σ^2. The probability relationship between the cumulated periodogram and the spectral distribution function is the same as that between an empirical cumulative frequency function and a cumulative distribution function. Hence, the Kolmogorov-Smirnov test can be used to assess the probability that an observed deviation of $C'(k)$ from the theoretical value $C(k)$ for white noise has occurred under the equality hypothesis (i.e., the probability that $\max|C'(k) - C(k)| > \delta$ for some number $\delta > 0$).

IV. The Data

All data are monthly and seasonally unadjusted. The sample runs from May 1973 to August 1984, and the countries in the sample are the United States, Canada, Germany, Italy, the Netherlands, and the United Kingdom. The rate of change in the CPI is used to measure commodity price inflation. The CPI data are obtained from the IFS tape (line 64). One-month eurocurrency rates, obtained from the Harris Bank's *Foreign Exchange Weekly Review*, are used as nominal interest rates. These interest rates are sampled from the Friday which occurs nearest to the end of the calendar month. Ex post real interest rates are calculated as percent per

annum and the United States is viewed as the home country.[5]

V. Empirical Results

Table 1 reports estimates of the unconditional mean of the ex post real interest rate differentials. Except for the United States–United Kingdom differential (2.134 with standard error 0.99), there is little to suggest that real interest rates across countries are separated by a constant or that they diverge in the long run.

Q-statistics, calculated for $p = 10$, 20, and 30, and the max $|C'(k) - C(k)|$ are reported in table 2. With the exception of the United States–Canadian real interest rate differential, all of the Q-statistics have marginal significance levels less than 0.05. Similarly, the probability that the calculated cumulated periodograms are observed if the data are white noise is less than 0.05 except for the United States–Canadian case. Thus the equality hypothesis is strongly rejected for the United States vis-à-vis the European countries, but the equality of U.S. and Canadian real rates cannot be rejected at standard significance levels.

It is possible that seasonal fluctuations in the CPI inflation rates, although not indicative of unexploited profit opportunities, are responsible for rejecting the equality hypothesis. It has been argued (see Fama and Schwert (1979)) that in order to offset storage costs and to equalize returns across industries, seasonal fluctuations in the prices of some goods will occur and that these seasonal factors may find themselves in the aggregate inflation rate.

To investigate this possibility, suppose the domestic inflation rate is composed of a seasonal component, θ^s, and a nonseasonal component, θ^n, where $\theta = \theta^s + \theta^n$ (and similarly for the foreign country). The ex post real interest rate differential can be expressed as

$$R_{t+1} = E(R^n_{t+1}|I_t) + (\theta^{*s}_{t+1} - \theta^s_{t+1}) + u'_{t+1}, \qquad (13)$$

where

$$R^n \equiv (i - \theta^n) - (i^* - \theta^{*n})$$

is the nonseasonal component of the real interest rate differential and

$$u'_{t+1} = (\theta^{*n}_{t+1} - \theta^n_{t+1}) - E[(\theta^{*n}_{t+1} - \theta^n_{t+1})|I_t]$$

is a white noise process.[6] The idea here is that it is the nonseasonal part of the real interest rate that is important so that the equality hypothesis is stated that $E(R^n_{t+1}|I_t) = 0$.

[5] The Harris data set reports Friday market closing quotations. Two alternative sample selection procedures for nominal interest rates were tried. These were sampling on the Friday nearest to the 15th of the month to coincide with midmonth sampling of some items in the U.S. CPI, and sampling from the last Friday of the month. Neither of these produced substantive differences in the results and are not reported.

[6] The seasonal factors in commodity prices are postulated to occur to equalize returns across industries so that arbitrage profits cannot be made on these differential seasonals. Thus the *EAPPP* condition is modified so that the nonseasonal part of the domestic currency price of home and foreign goods change at the same rate. Nominal interest rates, on the other hand, display little or no seasonality so that *UIP* remains unchanged. This modification of *EAPPP* together with *UIP* motivates the specification in (13).

TABLE 1.—TESTS OF UNCONDITIONAL EQUALITY OF REAL INTEREST RATES

	Canada	Germany	Italy	Netherlands	U.K.
$\bar{R}$	.489	.483	−.335	.742	2.134
(s.e.)	(.768)	(.748)	(.391)	(.815)	(0.990)

TABLE 2.—TESTS OF CONDITIONAL EQUALITY OF REAL INTEREST RATES

	Canada	Germany	Italy	Netherlands	U.K.
$Q(10)$[a]	14.44	35.46	31.97	44.99	21.28
(confidence)[b]	(0.846)	(1.000)	(1.000)	(1.000)	(0.980)
$Q(20)$[a]	27.11	68.87	36.81	80.17	51.77
(confidence)[b]	(0.868)	(1.000)	(0.988)	(1.000)	(1.000)
$Q(30)$[a]	38.76	90.58	57.46	113.72	65.98
(confidence)[b]	(0.869)	(1.000)	(0.998)	(1.000)	(1.000)
$\max\|C'(k) - C(k)\|$[c]	0.1386	0.2298	0.2757	0.1752	0.1845

[a] Box-Ljung Q-statistic. $Q(p)$ has a chi-square distribution with p degrees of freedom in large samples under the equality hypothesis.

[b] Confidence level of the test is the integral of the chi-squared variate with p degrees of freedom from 0 to $Q(p)$.

[c] The maximum absolute deviation of the cumulated periodogram from theoretical values for white noise. For sample size of 136, the Prob$(|C'(k) - C(k)|) > \delta$ are

Probability:	.01	05	.10	.25
δ	.1991	.1662	.1490	1246

TABLE 3.—TESTS OF CONDITIONAL EQUALITY OF REAL INTEREST RATES: INFLATION DIFFERENTIALS SEASONALLY ADJUSTED

	Canada	Germany	Italy	Netherlands	U.K.
$Q(10)$[a]	16.78	63.79	26.25	63.60	17.28
(confidence)[b]	(0.921)	(1.000)	(0.997)	(1.000)	(0.932)
$Q(20)$[a]	24.20	86.92	41.99	81.33	38.20
(confidence)[b]	(0.767)	(1.000)	(0.997)	(1.000)	(0.992)
$Q(30)$[a]	29.49	97.22	49.40	84.22	41.18
(confidence)[b]	(0.509)	(1.000)	(0.986)	(1.000)	(0.916)
$\max\lvert C'(k) - C(k)\rvert$[c]	0.2039	0.1810	0.2430	0.2196	0.1828

[a,b,c] See notes to table 2.

To allow for differential seasonal effects in the tests of the equality hypothesis, table 3 reports Q-statistics and cumulated periodogram calculations using ex post real interest rate differentials constructed by subtracting the residuals from regressing the inflation differential on twelve seasonal dummies from the nominal interest rate differential.[7] As before, the Box-Ljung test rejects the equality hypothesis (at the 5% level) for the German, Italian, and Netherlands differentials, while it is not possible to reject for the U.S.–Canadian differential. However, the evidence against equality of U.S. and U.K. real interest rates is now much weaker. Thus, in only one instance does it appear that differential seasonal factors might have led to a rejection of the equality hypothesis.

The periodogram test is generally consistent with the Box-Ljung test except for the U.S.–Canadian case ($\max|C'(k) - C(k)| = .2039$). This is because a large part of the variance of this series is concentrated at very low frequencies and is not captured in the Q-statistics for the lengths chosen for p. (The cumulated periodogram takes its largest jumps at frequencies 0.1386 and 0.1848 for the U.S.–Canadian differential.)[8]

VI. Conclusion

The statistical tests in this note reject the hypothesis that ex ante real interest rates have been equal across countries and thus have rejected the joint validity of *UIP* and *EAPPP*. Violations of *EAPPP* might come about through stickiness in commodity prices, which is a feature of models such as Mussa's (1982). Violations of *UIP* may be due to the presence of a risk premium in the forward foreign exchange market.

[7] These tests were also performed on residuals from regressing R on twelve seasonal dummies with almost no difference in the results.

[8] Since the equality hypothesis is not easily rejected for the United States and Canada, but is strongly rejected for the United States and European real interest rates, it might seem that only intercontinental divergences of real interest rates have occurred. This is not the case, however, as tests with Germany as the home country (not reported) strongly reject equality with real interest rates of the other European countries.

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