

# Where's the Risk? The Forward Premium Bias, the Carry-Trade Premium, and Risk-Reversals in General Equilibrium

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# Intro

- We study whether **three empirical regularities** of international currency returns can be understood within a **common framework**
- The empirical regularities
  - ① Downward forward premium bias
  - ② Carry trade return
  - ③ Long-run risk reversal
- The framework
  - ▶ Fairly standard two-country DSGE NK model
  - ▶ Role of asymmetric unit-root productivity
  - ▶ Role of monetary policy

## Downward forward premium bias

- Regression evidence against uncovered interest parity
- A long History.<sup>1</sup>  $\beta < 1$  (or sometimes  $(\beta < 0)$ ) in

$$\Delta \ln S_{1,2,t+1} = \alpha + \beta (i_{1,t} - i_{2,t}) + \epsilon_{t+1}$$

- Mainstream hypothesis: Caused by a time-varying risk premium

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<sup>1</sup>Bilson (1981), Fama (1984)

# Carry Trade

- Academic interest for about a decade<sup>2</sup>
- Go short the low interest currency, go long the high interest currency, ignore the exchange rate. Generates a **systematically profitable** trading strategy.
- High interest currency **pays** the excess return. Must be **risky**
  - ▶ Empirical work looks for **priced** risk factors.
  - ▶ Focused on the cross-section of carry returns
  - ▶ Less extensive: Modeling the carry
- Carry and forward premium bias are connected but not the same

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<sup>2</sup>Lustig-Verdelhan, Burnside et al.

## FP Bias v. Risk Premium

If  $n_{t+1}$  is nominal stochastic discount factor,

- Forward premium bias

$$n_{2,t+1} - n_{1,t+1} = \alpha_0 + \beta_0 (E_t (n_{2,t+1} - n_{1,t+1})) + \epsilon_{t+1}.$$

- Risk premium/carry trade excess return

$$(i_{1,t} - i_{2,t} - E_t \Delta \ln (S_{1,2,t+1})) = \frac{1}{2} (Var_t (n_{2,t+1}) - Var_t (n_{1,t+1})).$$

# Long-run risk reversal

- Relatively new idea, identified and explained by Charles Engel (2016)
  - ▶ Let there be forward premium bias or profitable carry
  - ▶ High interest country is risky because it pays an excess return
  - ▶ But high interest means strong currency.<sup>3</sup> Strength should indicate safety. Safe currency should pay negative premium.
- How to reconcile contradictory implications? Risk reversal

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<sup>3</sup>Riksbank 1992, Korea 1997

# Recent Literature

- Forward premium bias
  - ▶ Endowment models: Verdelhan (2010) (habit persistence), Bansal and Shaliastovich (2013) (LRR) Backus, Gavazzoni, Telmer, and Zin (2013). Inflation has no welfare effects in these models.
  - ▶ Small-open economy: Chinn and Zhang (2015).
- Carry trade
  - ▶ Empirical: Lustig-Verdelhan (2007), Lustig, Roussanov, and Verdelhan (2014), Burnside, Eichenbaum, Kleshchelski, and Rebelo, (2011), Della Corte, Riddiough, and Sarno (2016), Menkhoff, Sarno, Schmeling, and Schrimpf (2013), Berg and Mark (2017a, 2017b),
  - ▶ Structural Models: Hassan (2013), Ready, Roussanov, and Ward (2017)
- Risk reversal
  - ▶ Engel (2016), Valchev (2015)

# Our paper

- ① How far can 2-country DSGE NK macro model go in jointly explaining the three return facts?
- ② Can we model a common macro source of risk underlying these return patterns?

# Our paper asks

- Why General Equilibrium?
  - ▶ Risk is 'baked in' the endowment based LRR models
  - ▶ Evidence for LRR in consumption is tenuous, at best (Ma (2013))
  - ▶ People 'care' about inflation in GE
- Country-level heterogeneity
  - ▶ Productivity
  - ▶ Monetary policy—hence NK model
- Summary of results
  - ▶ Complete or incomplete markets model with productivity and monetary policy heterogeneity
  - ▶ High risk aversion

## General Features of the Model

- Two-country DSGE New Keynesian macro model
- Labor, no capital. Sticky goods prices (Calvo)
- Exporters engage in local-currency pricing (LCP)
- Recursive utility (Epstein-Zin)
- Productivity driven by unit-root process

## Households

Utility:  $V_t = (1 - \beta) \left[ \ln(c_t) - \eta \frac{\ell_t^{1+\chi}}{1+\chi} \right] - \frac{\beta}{\alpha} \ln \left[ E_t \left( e^{-\alpha V_{t+1}} \right) \right]$

$\beta \in (0, 1)$ ,  $\eta > 0$ ,  $\chi > 0$ , Frisch elasticity  $1/\chi$ ,  $\alpha \in \mathbb{R}$ , IES=1.

$$RRA = \alpha + \left( \frac{1}{1 + \frac{\eta}{\chi}} \right).$$

Real SDF:  $M_{t+1} = \beta \left( \frac{c_t}{c_{t+1}} \right) \left( \frac{e^{-\alpha V_{t+1}}}{E_t(e^{-\alpha V_{t+1}})} \right)$

Nominal SDF:  $N_{t+1} = M_{t+1} e^{-\pi_{t+1}}$

# Complete Markets

Nominal state-contingent bond, in country 1 currency

$$\text{Nominal bond Euler: } \frac{1}{1+i_t} = E_t(M_{t+1} e^{-\pi_{t+1}})$$

$$\text{Nominal exra: } \frac{S_{1,2}(\omega^{t+1})}{S_{1,2}(\omega^t)} = \frac{M_2(\omega_{t+1}|\omega^t) e^{-\pi_2(\omega^{t+1})}}{M_1(\omega_{t+1}|\omega^t) e^{-\pi_1(\omega^{t+1})}}$$

$$\text{Real exra (def): } Q_{1,2}(\omega^t) = \frac{S_{1,2}(\omega^t) P_2(\omega^t)}{P_1(\omega^t)}$$

$$\text{Real exra: } \frac{Q_{1,2}(\omega^{t+1})}{Q_{1,2}(\omega^t)} = \frac{M_2(\omega_{t+1}|\omega^t)}{M_1(\omega_{t+1}|\omega^t)}$$

$$\text{Labor: } \eta \tilde{c}(\omega^t) \ell(\omega^t)^\chi = \frac{\tilde{W}(\omega^t)}{P(\omega^t)}$$

# Complete Markets

$$\frac{Q_{1,2}(\omega^{t+1})}{Q_{1,2}(\omega^t)} = \left( \frac{M_2(\omega_{t+1}|\omega^t)}{M_1(\omega_{t+1}|\omega^t)} \right) \left( \frac{G_1(\omega^t)}{G_2(\omega^t)} \right)$$

$$\frac{S_{1,2}(\omega^{t+1})}{S_{1,2}(\omega^t)} = \left( \frac{M_2(\omega_{t+1}|\omega^t) e^{-\pi_2(\omega^{t+1})}}{M_1(\omega_{t+1}|\omega^t) e^{-\pi_1(\omega^{t+1})}} \right) \left( \frac{G_1(\omega^t)}{G_2(\omega^t)} \right)$$

$$\frac{1}{1 + i(\omega^t)} = \left( \frac{\beta}{G(\omega^t)} \right) E_t \left( M(\omega_{t+1}|\omega^t) e^{-\pi(\omega^{t+1})} \right)$$

# Incomplete Markets

Countries each issue tradable, zero-net supply, nominal non-state contingent bond.

$$\text{Home Bond Euler: } \frac{1}{(1 + i_{i,t})} = E_t M_{i,t+1} e^{-\pi_{i,t+1}}$$

$$\text{Foreign Bond Euler: } \left( \frac{1 + \tau \left( \frac{Q_{i,j,t}}{P_{j,t}} \right) B_{i,j,t}}{(1 + i_{j,t})} \right) = E_t M_{i,t+1} e^{-\pi_{j,t+1}} \frac{Q_{i,j,t+1}}{Q_{i,j,t}}$$

$$\text{Nom Exra (implicit): } \frac{S_{1,2,t}}{S_{1,2,t-1}} = \frac{Q_{1,2,t}}{Q_{1,2,t-1}} \frac{e^{\pi_{1,t}}}{e^{\pi_{2,t}}} \quad (1)$$

## Goods Demand

- Continuum of intermediate goods firms  $f \in [0, 1]$
- $c_{i,j,t}$  consumed in  $i$ , made in  $j$

$$c_{i,j,t} = \left[ \int_0^1 c_{i,j,t}(f)^{\frac{\sigma-1}{\sigma}} df \right]^{\frac{\sigma}{\sigma-1}},$$

$$P_{i,j,t} = \left[ \int_0^1 p_{i,j,t}(f)^{1-\sigma} df \right]^{\frac{1}{1-\sigma}}$$

$$c_{i,t} = \left( d^{\frac{1}{\mu}} c_{i,i,t}^{\frac{\mu-1}{\mu}} + (1-d)^{\frac{1}{\mu}} c_{i,j,t}^{\frac{\mu-1}{\mu}} \right)^{\frac{\mu}{\mu-1}}$$

$$P_{i,t} = \left[ d P_{i,i,t}^{1-\mu} + (1-d) P_{i,j,t}^{1-\mu} \right]^{\frac{1}{1-\mu}}$$

# Production

Production fn:  $y_{i,t}(f) = A_{i,t}\ell_{i,t}(f)$

Total costs:  $\frac{W_{i,t}}{P_{i,t}}\ell_{it}(f)$

Demand determined output:  $y_{i,t}(f) = c_{i,i,t}(f) + c_{i,j,t}(f)$

# LCP Price Setting

Choose  $p_{i,i,t}$  and  $p_{j,i,t}$  to maximize

$$E_t \sum_{s=0}^{\infty} (\alpha_c \beta)^s M_{i,t,t+s} \left[ \begin{array}{l} \frac{e^{-s\bar{\pi}_i} p_{i,i,t}(f)}{P_{i,t+s}} c_{i,i,t+s}(f) + \frac{Q_{i,j,t+s} p_{j,i,t}(f) e^{-s\bar{\pi}_j}}{P_{j,t+s}} c_{j,i,t+s}(f) \\ - \frac{W_{i,t+s}}{P_{i,t+s}} \ell_{i,t+s}(f) \end{array} \right],$$

# Monetary Policy with Interest Rate Smoothing

Potential (log) GDP

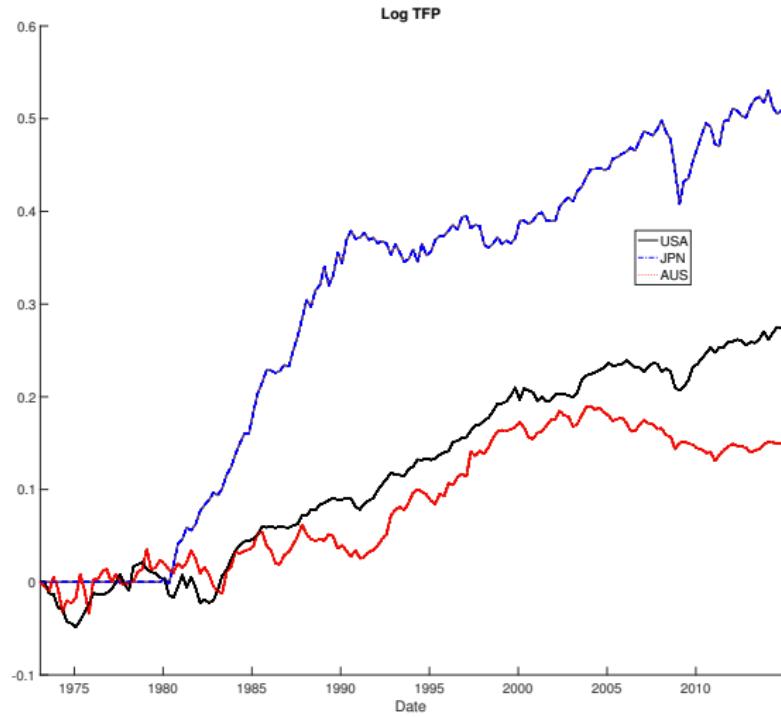
$$\ln(\bar{y}_{j,t}) = \rho_y \ln(\bar{y}_{j,t-1}) + (1 - \rho_y) \ln(y_{j,t})$$

'Taylor Rule'

$$i_{j,t} = (1 - \delta_j) \bar{i} + \delta_j i_{j,t-1} + (1 - \delta_j) [\xi_j (\pi_{j,t} - \pi^*_{j,t}) + \zeta_j (\ln(y_{j,t}) - \ln(\bar{y}_{j,t}))]$$

# Productivity

Figure: Log TFP



# Productivity

Let  $a_{j,t} = \ln(A_{j,t})$

$$\Delta a_{1,t} = -\psi_1 (a_{1,t-1} - a_{2,t-1}) + \sigma_1 \epsilon_{1,t}$$

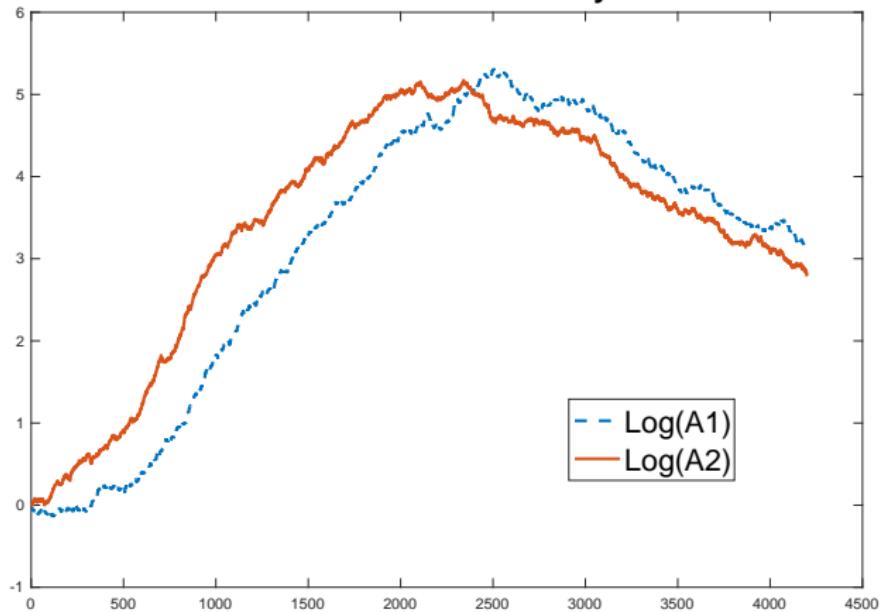
$$\Delta a_{2,t} = -\psi_2 (a_{1,t-1} - a_{2,t-1}) + \sigma_2 \epsilon_{2,t}$$

$$z_t = a_{1,t} - a_{2,t} = (1 + \psi_2 - \psi_1) z_{t-1} + (\sigma_1 \epsilon_{1,t} - \sigma_2 \epsilon_{2,t})$$

- $\epsilon_{i,t} \stackrel{i.i.d}{\sim} N(0, 1)$ ,
- $\sigma_i > 0$ , for  $i = \{1, 2\}$ ,
- $0 < \psi_2 < \psi_1 < 1$

# Productivity

Figure: Simulated Series of Log TFP  
**Simulated Productivity**



## Productivity and Parameter Values

$\alpha_c$	$d$	$\sigma$	$\mu$	$\chi$	$\beta$	$\psi_1$	$\psi_2$
0.8	0.85	10	1.5	3	0.99	0.003	0.0027

	$\delta_1$	$\delta_2$	$\xi_1$	$\xi_2$	$\zeta_1$	$\zeta_2$	$\rho_y$
symmetric	0.9	0.9	1.5	1.5	0.5	0.5	0.9
risky	0.9	0.9	1.2	1.5	0.5	0.0	0.9

$RRA : 10 \text{ to } 60$

$$\bar{\ell} = 1$$

## Forward premium bias (or puzzle?)

Table: Fama Regression–Forward Premium Puzzle/Bias in the Data

Country 2	$\alpha_0$	t-stat	$\beta_0$	t-stat	$R^2$
<b>Australia is Country 1</b>					
Canada	0.004	1.131	-0.507	-1.385	0.005
Great Britain	0.000	-0.012	0.047	0.100	0.000
Japan	0.005	1.074	0.278	1.088	0.003
Korea	-0.006	-0.927	-0.109	-0.203	0.000
Norway	-0.003	-0.622	1.318	1.850	0.021
New Zealand	-0.001	-0.301	-0.176	-0.347	0.001
Switzerland	0.010	1.030	-0.059	-0.071	0.000
Sweden	-0.004	-0.603	1.645	1.535	0.038
United States	0.005	1.035	-0.487	-1.160	0.005

The regression is  $\Delta \ln(S_{1,2,t+1}) = \alpha_0 + \beta_0(i_{1,t} - i_{2,t}) + \epsilon_{t+1}$ .

## Forward premium bias (or puzzle?)

Table: Fama Regression–Forward Premium Puzzle/Bias in the Data

Country 2	$\alpha_0$	t-stat	$\beta_0$	t-stat	$R^2$
<b>Japan is Country 1</b>					
Australia	-0.005	-1.074	0.278	1.088	0.003
Canada	-0.006	-0.960	0.056	0.138	0.000
Great Britain	-0.008	-1.340	0.040	0.115	0.000
Korea	0.006	0.413	0.697	1.106	0.011
Norway	0.021	1.935	2.081	1.884	0.037
New Zealand	-0.003	-0.255	0.422	0.699	0.004
Switzerland	0.002	0.330	-0.059	-0.176	0.000
Sweden	0.020	1.252	2.533	1.363	0.046
United States	-0.007	-1.244	-0.171	-0.411	0.001

The regression is  $\Delta \ln(S_{1,2,t+1}) = \alpha_0 + \beta_0(i_{1,t} - i_{2,t}) + \epsilon_{t+1}$ .

## Forward premium bias in the model

Table: Implied Slope in Fama Regression–Forward Premium Puzzle/Bias

A. Symmetric Benchmark Monetary Policies				
Risk Aversion	10	20	30	60
Complete	1.046	0.965	0.880	0.705
Incomplete	1.232	1.162	1.069	0.839
B. Alternative Monetary Policies				
Policy Parameters				
$\xi_1$	1.5	1.5	2.0	2.0
$\xi_2$	0.5	0.0	0.5	0.9
$\zeta_1$	1.2	1.2	1.2	1.2
$\zeta_2$	0.5	0.5	0.5	0.5
<u>Risk Aversion is 60</u>				
Complete	0.794	0.834	0.775	0.761
Incomplete	0.646	0.643	0.563	0.659

# The Carry

**Table:** Monthly Currency Excess Return Summary Statistics (1973.04–2014.12): Developed Countries

	$P_1$	$P_2$	$P_3$	$P_4$	$P_5$	$P_6$
Mean Currency Excess Return	-1.188	-0.482	1.311	0.828	3.263	3.849
Mean Interest Rate Differential	-2.904	-1.297	0.024	1.144	2.590	6.736
Mean Exchange Rate Return	1.716	0.816	1.287	-0.316	0.674	-2.886

Notes: This table is taken from Berg and Mark (2017a). Developed countries include Australia, Austria, Belgium, Canada, Denmark, Finland, France, Germany, Greece, Hungary, Iceland, Ireland, Israel, Italy, Japan, Netherlands, New Zealand, Norway, Poland, Portugal, Singapore, South Korea, Spain, Sweden, Switzerland, Taiwan, United Kingdom, and United States.

A. Benchmark Monetary Policies				
	Complete Markets			
Risk Aversion	10	20	30	60
Gross Carry	-0.213	-0.533	-0.853	-1.763
Incomplete Markets				
Risk Aversion	10	20	30	60
Gross Carry	0.091	-0.164	-0.420	-1.191
Net Carry 1	-0.104	-0.579	-1.054	-2.480
Net Carry 2	-0.087	-0.537	-0.989	-2.351

# Model Implied Carry

Table: Carry Trade Returns

B. Alternative Monetary Policies Risk Aversion is 60				
	Policy Parameters			
$\xi_1$	1.5	1.5	2.0	2.0
$\xi_2$	0.5	0.0	0.5	0.9
$\zeta_1$	1.2	1.2	1.2	1.2
$\zeta_2$	0.5	0.5	0.5	0.5
Complete Markets				
Gross Carry	2.531	2.388	2.770	2.876
Incomplete Markets				
Gross Carry	2.515	2.393	2.594	2.698
Net Carry 1	1.063	0.938	1.142	1.240
Net Carry 2	1.248	1.297	1.163	1.217

## Long-Run Risk Reversal: Engel (2016)

- Let  $r_2 > r_1$ . Currency 2 should be strong, strength implies safety, safety means negative premium. paid  
Forward premium bias says 2 is risky.
- Ex post excess return on country 2

$$\rho_{t+1} = r_{2,t} - r_{1,t} + \Delta \ln(Q_{1,2,t+1})$$

- Forward premium bias says

$$E_S \equiv \text{Corr}((E_t \rho_{t+1}), (r_{2,t} - r_{1,t})) > 0.$$

- Long-run country 2 risk premium:

$$\sum_{j=0}^{\infty} E_t(\rho_{t+1+j}) = E_t\left(\sum_{j=0}^{\infty} (r_{2,t+j} - r_{1,t+j}) + \ln(Q_{1,2,\infty}) - \ln(Q_{1,2,t})\right).$$

Must be negatively correlated with real interest rate differential,

$$E_L \equiv \text{Corr}\left(E_t\left(\sum_{j=0}^{\infty} \rho_{t+1+j}\right), (r_{2,t} - r_{1,t})\right) < 0.$$

## Engel's Evidence

- Estimated VECM on  $\ln(S_{1,2,t})$ ,  $(i_{1,t} - i_{2,t})$ ,  $(p_{1,t} - p_{2,t})$  generates expectations,  $E_S, E_L$ , for G7 with USD as base currency.
- Finds  $E_L < 0$  in every instance.
- Log-linearized LRR models under complete markets are consistent with FP bias  $E_S > 0$ , but not the risk-reversal.

# Model Implied Long-Run Risk Reversal

Table: Risk Reversal

A. Benchmark Monetary Policies				
	Complete Markets			
Risk Aversion	10	20	30	60
$E_S$	0.182	0.199	0.241	0.401
$E_L$	-0.099	-0.152	-0.210	-0.387
	Incomplete Markets			
Risk Aversion	10	20	30	60
$E_S$	-0.141	-0.115	-0.032	0.173
$E_L$	0.178	0.080	-0.032	-0.250

# Model Implied Risk Reversal

Table: Risk Reversal

B. Heterogeneous Monetary Policies (Risk Aversion is 60)				
	Policy Parameters			
$\xi_1$	1.5	1.5	2.0	2.0
$\xi_2$	0.5	0.0	0.5	0.9
$\zeta_1$	1.2	1.2	1.2	1.2
$\zeta_2$	0.5	0.5	0.5	0.5
Complete Markets				
$E_S$	0.299	0.272	0.230	0.220
$E_L$	-0.306	-0.275	-0.241	-0.235
Incomplete Markets				
$E_S$	0.454	0.498	0.428	0.407
$E_L$	-0.464	-0.508	-0.439	-0.417

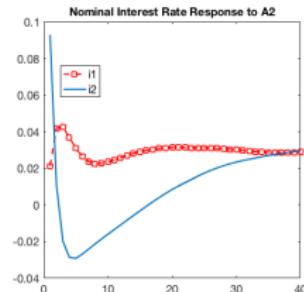
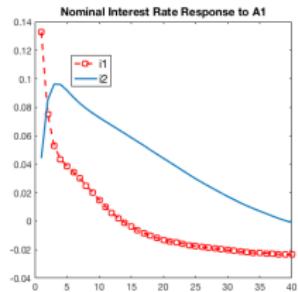
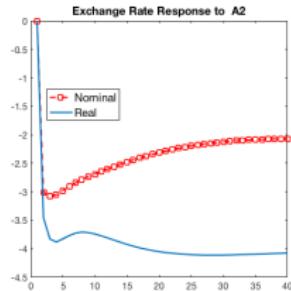
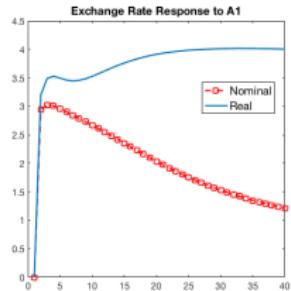
# Stationary Symmetric Productivity

$\rho_A$	0.90	0.96	0.98	0.99
Complete Markets				
Fama	0.986	0.987	0.985	0.979
Carry	0.014	0.040	0.085	0.155
$E_S$	-0.185	-0.229	-0.304	-0.403
$E_L$	-0.239	-0.196	-0.122	-0.037
Incomplete Markets				
Fama	0.997	0.999	0.999	0.990
Gross Carry	0.092	0.093	0.078	0.146
Net Carry 1	0.067	0.031	-0.057	-0.187
Net Carry 2	0.066	0.026	-0.060	-0.141
$E_S$	-0.196	-0.243	-0.311	-0.425
$E_L$	-0.104	-0.048	0.003	0.037

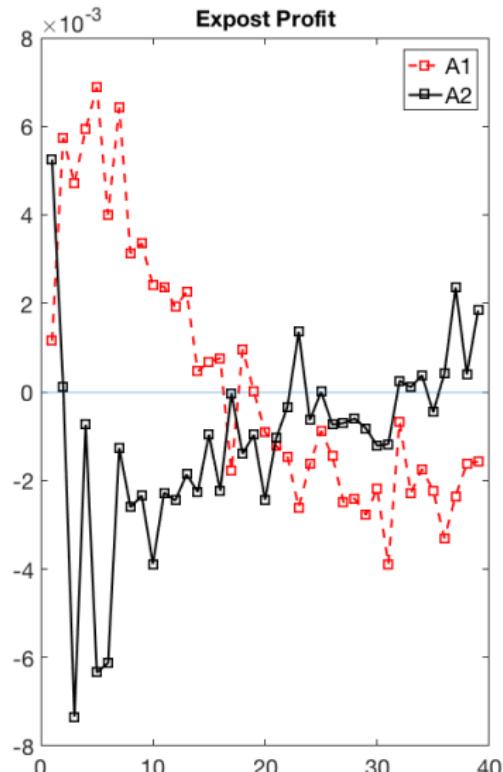
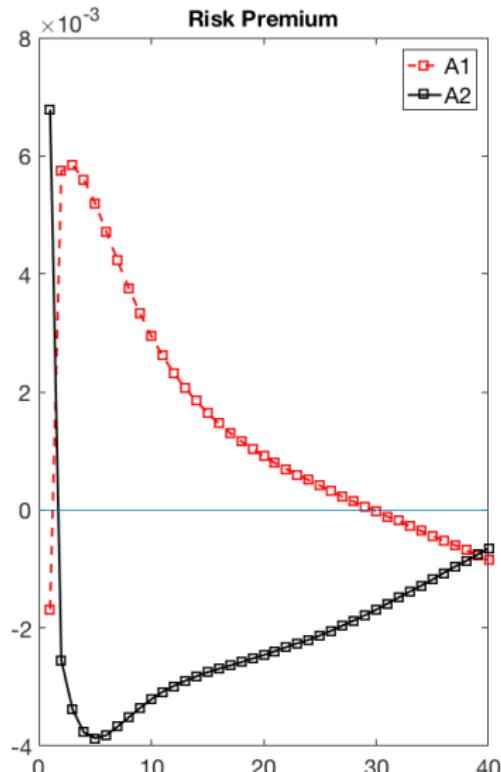
Note: Risk Aversion is 60. Monetary policy parameters are

$$\xi_1 = 2.0, \xi_2 = 1.2, \zeta_1 = 0.9, \zeta_2 = 0.5.$$

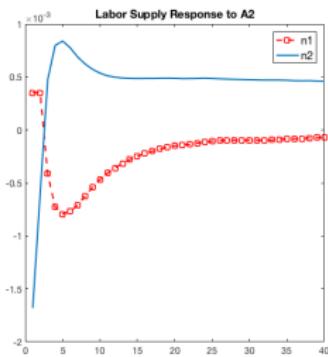
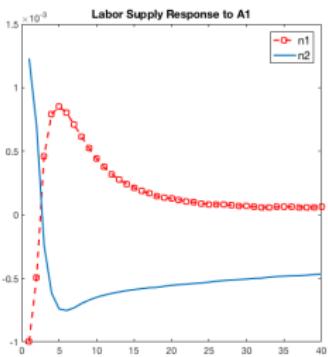
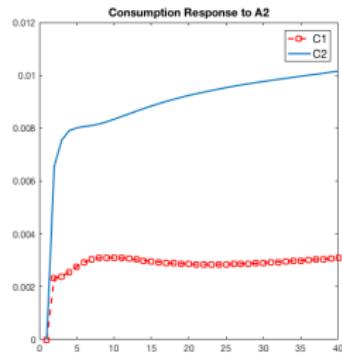
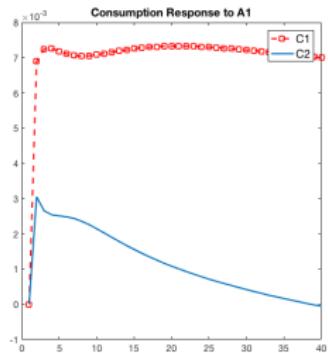
# A couple of IRFs



# A couple of IRFs



# A couple of IRFs



# Conclusion

What does the simplest general equilibrium macro model, with monetary policy, say about empirical patterns of international currency returns?

- Some sort of cross-country heterogeneity is essential
- A consistent, common source of risk
- TFP and monetary policy heterogeneity

# Complete Markets and Carry Premium

$$m_{t+1} = -\Delta c_{t+1} - \alpha (V_{t+1} - \mu_t (V_{t+1})) - \frac{\alpha^2 \sigma_t^2 (V_{t+1})}{2}$$
$$m_{t+1}^* = -\Delta c_{t+1}^* - \alpha (V_{t+1}^* - \mu_t (V_{t+1}^*)) - \frac{\alpha^2 \sigma_t^2 (V_{t+1}^*)}{2}$$

$$\begin{aligned} E_t (r_t^* - r_t + \Delta s_{t+1}) &= \frac{\sigma_t^2 (m_{t+1}) - \sigma_t^2 (m_{t+1}^*)}{2} \\ &= \frac{\sigma_t^2 (\Delta c_{t+1}) + \alpha^2 \sigma_t^2 (V_{t+1}) + 2\alpha \gamma_t (\Delta c_{t+1}, V_{t+1})}{2} \\ &\quad - \frac{\sigma_t^2 (\Delta c_{t+1}^*) + \alpha^2 \sigma_t^2 (V_{t+1}^*) + 2\alpha \gamma_t (\Delta c_{t+1}^*, V_{t+1}^*)}{2} \end{aligned} \quad (2)$$

# Complete Markets and FP Bias

$$\gamma_t(\Delta s_{t+1}, r_t - r_t^*) = E_t(m_{t+1}^* - m_{t+1}) \left( \mu_t(m_{t+1}^*) - \mu_t(m_{t+1}) + \frac{\sigma_t^2(m_{t+1}^*) - \sigma_t^2(m_{t+1})}{2} \right)$$

$$\begin{aligned}\gamma_t(\Delta s_{t+1}, r_t - r_t^*) &= E_t \left[ \begin{array}{l} (\Delta c_{t+1} - \Delta c_{t+1}^*) + \alpha(V_{t+1} - \mu_t(V_{t+1})) - \alpha(V_{t+1}^* - \mu_t(V_{t+1}^*)) \\ \quad + \frac{\alpha^2}{2} (\sigma_t^2(V_{t+1}) - \sigma_t^2(V_{t+1}^*)) \end{array} \right] \\ &\times \left[ \begin{array}{l} [\mu_t(\Delta c_{t+1}) - \mu_t(\Delta c_{t+1}^*)] + \frac{\alpha^2}{2} [\sigma_t^2(V_{t+1}) - \sigma_t^2(V_{t+1}^*)] \\ \quad + \overbrace{\frac{\sigma_t^2(\Delta c_{t+1}^*) - \sigma_t^2(\Delta c_{t+1}) + \alpha^2(\sigma_t^2(V_{t+1}^*) - \sigma_t^2(V_{t+1}))}{2}}^{**} + \overbrace{2\alpha(\gamma_t(\Delta c_{t+1}^*, V_{t+1}^*) - \gamma_t(\Delta c_{t+1}, V_{t+1}))}^{**} \end{array} \right]\end{aligned}$$