PPP Strikes Out: The effect of common factor shocks on the real exchange rate

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Background and motivation

- Quest for ‘realistic’ (i.e., conform to our priors) half-life measurement

$$q = \ln\left(\frac{SP^*}{P}\right)$$

$$\begin{aligned} q_t &= \rho q_{t-1} + e_t \\ h &= \frac{-\ln(2)}{\ln(\rho)} \end{aligned}$$

- Univariate analyses of PPP over post 1973 era cannot reject the unit root.
  - Implies half-life of infinity
  - Example: Real $/DM rate 1973Q2 to 2007Q3.

	Augmented	Dickey-Fuller	t-statistic	P-value
Test	critical	values:	-2.544	0.107

- Is this because sample is too small? If so, get more from the cross-section, do panel data work.

- Panel Data Analysis. AR(1) example with fixed effects γ_i

$$q_{it} = \gamma_i + \rho q_{it-1} + e_{it}$$

  - Econometric Issues
    * $\hat{\rho}$ is biased down. Kendall (Nickell) bias from constant (fixed effect).
    * Errors are not independent. They are cross-sectionally correlated.

$$e_{it} = \delta_i F_t + v_{it}$$

      F_t is a common factor. δ_i is a 'factor loading.' v_{it} is the idiosyncratic shock.
  - Panel data studies (e.g., Papell, Murray and Papell, Choi, Mark and Sul) account for these issues. They estimate the half-life to be between 3 and 5 years. Less than infinity, but still not good enough.

- Imbs, Mumtaz, Ravn, Rey (IMRR) use sectoral price index data. They say one should not constrain ρ to be the same across sectors (goods). Doing so creates an upward aggregation bias in the half-life estimate. Prices from 19 sectors and 11 countries.

  – The log relative price of good i between the US and country c

$$q_{ict} = \ln\left(\frac{S_{ct}P_{ict}}{P_{0ct}}\right)$$

  S_{ct} is the nominal US dollar price of a unit of country c's money and P_{ict} is the country c currency price of good i in country c.

– IMRR specification. Take cross-sectional average of the panel as common factor

$$\bar{q}_t = \frac{1}{CI} \sum_{i=1}^{I} \sum_{c=1}^{C} q_{ict},$$

Run this regression

$$q_{ict} = \underbrace{\sum_{j=1}^{p} \rho_{icj} q_{ict-j}}_{\rho q_{it-1}} + \underbrace{\gamma_{ic} + \underbrace{\sum_{h=0}^{H} \lambda_{ich} \bar{q}_{t-h}}_{\delta_i F_t} + \epsilon_{ict}}_{\text{Error components model}} \tag{1}$$

γ_{ic} good-and-country fixed effect, ϵ_{ict} idiosyncratic shock. Require lags of $\bar{q}_t$ because common factor is persistent.

- PPP convergence applies to the real exchange rate between the U.S. and country c,

$$Q_{ct} = \sum_{i=1}^{I} \omega_{ic} q_{ict}, \quad \text{where} \quad \sum_{i=1}^{I} \omega_{ic} = 1,$$

- Link deviations from LOOP to PPP with Peseran's mean group common correlated elements (MG-CCE) estimator of the AR coefficients for Q_{ct}. MG-CCE for $k-$th autoregressive lag

$$\hat{\rho}_k = \frac{1}{IC} \sum_{i=1}^{I} \sum_{c=1}^{C} \hat{\rho}_{ick}$$

- IMRR punchline: Half-life estimate is less than 1 year$\rightarrow$PPP has struck back. Claim: Different goods have different speeds of adjustment. Aggregation bias caused long half life estimates due to contamination of heterogeneous speeds of adjustment. Note: Crucini and Shintani, find similar results with more disaggregated price data.

What's wrong with that?

$$q_{it} = \underbrace{\rho q_{it-1}}_{\text{Transitory}} + \gamma_i + \underbrace{\underbrace{\delta_i F_t + v_{it}}_{e_{it}}}_{\text{Permanent}}$$

- Factor structure treated as nuisance parameters one estimates so as to get 'good' estimates of ρ.

- Forgot that common factor also drives exchange rate dynamics.

- Critique leveled not just at IMRR, but to (almost?) all panel data studies on PPP.

PPP strikes out

- We use IMRR's data which we obtained from their website. Ask these questions:
  - How does the common factor behave?
  - How important is the factor in deviation from LOOP dynamics?
  - Revisit tests for PPP

- Recast dynamics of relative prices as two-component model

$$q_{ict} = e_{ict} + \lambda_{ic} F_t \tag{2}$$

with e_{ict} orthogonal to F_t, and each component is a p-th order autoregression,

$$\begin{aligned} e_{ict} &= \gamma_{ic} + \sum_{j=1}^{p} \rho_{icj} e_{ict-j} + \epsilon_{ict} & (3) \\ F_t &= \sum_{j=1}^{p} \phi_j F_{t-j} + v_t & (4) \end{aligned}$$

is observationally equivalent to IMRR's specification (2) with $F_t = \bar{q}_t$.

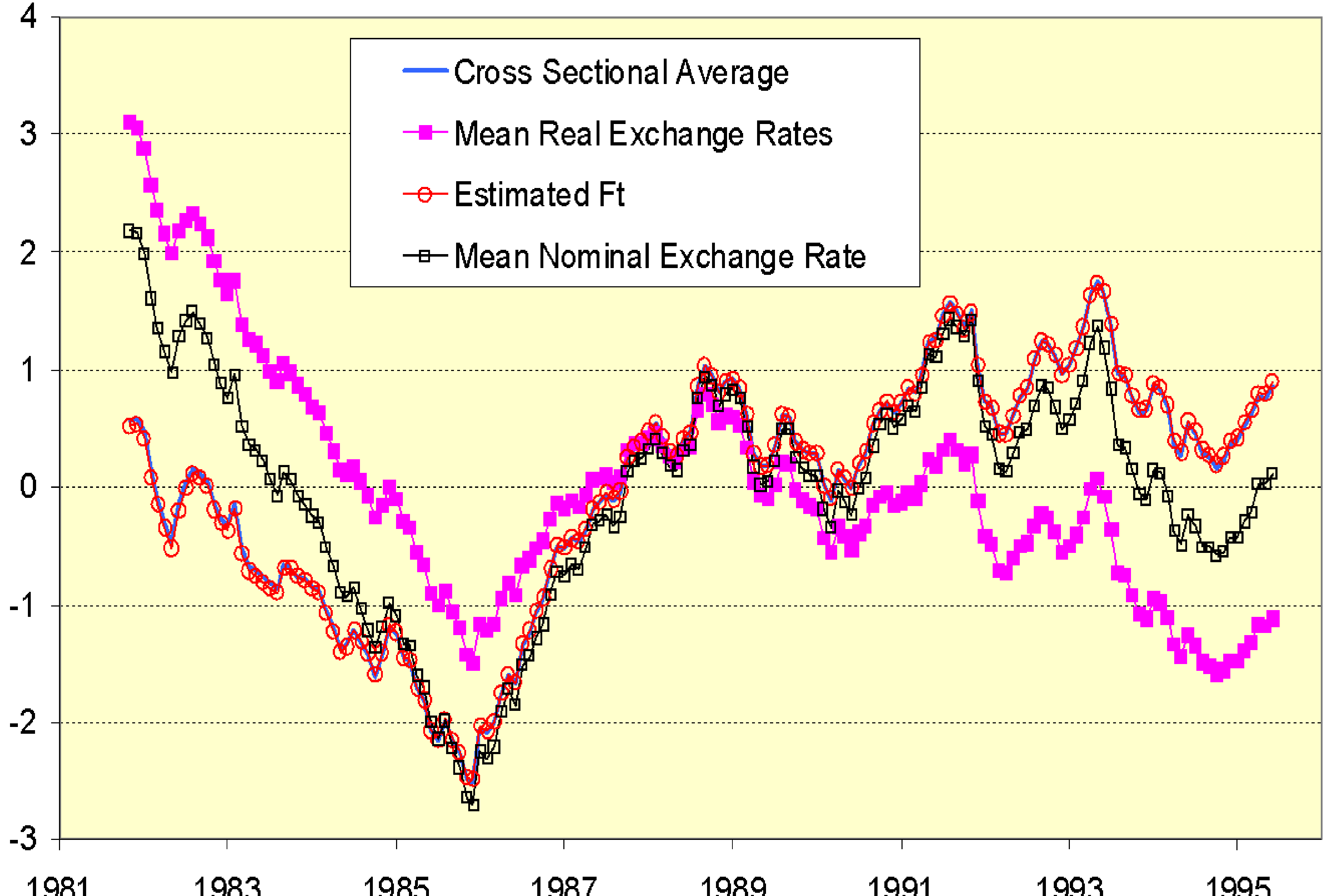
$$q_{ict} = \gamma_{ic} + \sum_{j=1}^{p} \rho_{icj} q_{ict-j} + \lambda_{ic} F_t - \sum_{j=1}^{p} \lambda_{ic} \rho_{icj} F_{t-j} + \epsilon_{ict}, \tag{5}$$

Let us examine measurements of F_t

- The first principle component of deviations from LOOP $\hat{F}_t$.

- Plus three others (of little concern here)

$\hat{F}_t$ is nearly identical to the cross-sectional average used by IMRR.
They look like unit root processes.



Quantify the persistence

Fit AR(12) to IMRR measure,

$$\bar{q}_t = a + \sum_{k=1}^{12} \rho_k \bar{q}_{t-k} + \epsilon_t. \tag{6}$$

Do a Jackknife correction for bias and find

$$\sum_{k=1}^{12} \hat{\rho}_k = 1.028$$

Preliminary evidence says F_t is unit-root nonstationary.

How big is the unit root in the real exchange rate?

F_t is orthogonal to e_{it}. Therefore, short-run variance of the relative price of the $i-$th good q_i is

$$\mathsf{Var}\left(q_{ict}\right) = \mathsf{Var}\left(e_{ict}\right) + \mathsf{Var}\left(\lambda_{ic} F_t\right).$$

Table 1 uses the cross-sectional average $\bar{q}_t$ to measure F_t and regression estimates of the factor loading coefficients λ_{ic}.

Table 1: Decomposition of Var(q_i)

i	Total	Common	Common share
Fruits	2.816	0.651	0.231
Comm	3.182	1.175	0.369
Tobacco	2.706	1.165	0.430
Sound	4.251	3.189	0.750
Hotel	2.825	2.252	0.797
Pub.Trans.	3.309	2.741	0.828
Dairy	3.083	2.555	0.829
Fuel	2.926	2.433	0.832
Rents	4.255	3.609	0.848
Meat	2.278	1.932	0.848
Clothing	5.308	4.508	0.849
Footwear	4.968	4.221	0.850
Vehicles	3.743	3.182	0.850
Bread	3.242	2.803	0.864
Leisure	3.101	2.684	0.866
Book	4.123	3.581	0.868
Alcohol	4.841	4.246	0.877
Dom.Appl.	4.160	3.743	0.900
Furniture	5.120	4.631	0.904
Average	3.697	2.911	0.787

Statistical tests for international price convergence

- We should probably re-examine the evidence for PPP in panel data. But we've just identified a pitfall in panel unit root tests: Suppose dynamics are driven (in part) by a unit-root factor structure. Then a single unit root process drives dynamics of all the prices. The cross-sectional dimension creates no advantages for unit root tests.

A log-t test of law-of-one price convergence

- Log-t test suggested by Phillips and Sul (2007b) which they employed in growth context. Gives a test of relative convergence. Concept: Cross-sectional dispersion of USD price of a good is constant or shrinking over time.

$$\pi_{ict} = S_{ct} P_{ict}$$

  - Idea: Regress cross-sectional variance of prices on a trend. Is slope zero or negative?

  - DGP that underlies test is a two-component model

$$\ln \pi_{ict} = \delta_{ict} F_t + e_{ict} \tag{7}$$

  - There is relative convergence to the law-of-one price if

$$\delta_{ict} \to \delta_i \text{ as } t \to \infty$$

– To implement test, give time-varying factor loadings a parametric form

$$\delta_{ict} = \delta_i + \psi_{ict} \tag{8}$$

$$\psi_{ict} \overset{iid}{\sim} \left(0, \frac{(t)^{-2\alpha_i}}{\ln(t)}\sigma^2_{\psi ic}\right). \tag{9}$$

* $\alpha_i \geq 0 \Longrightarrow \text{Var}(\psi_{ict}) \to 0 \Longrightarrow \delta_{ict} \to \delta_i \Longrightarrow$Asymptotic convergence

* $\alpha_i < 0 \Longrightarrow \text{Var}(\psi_{ict}) \to \infty$ as $t \to \infty \Longrightarrow$Asymptotic divergence

* α_i is convergence rate to the 'asymptotic' law-of-one price. Can say $\ln(\pi_{ict})$ and $\ln(\pi_{irt})$ have a common (stochastic or deterministic) trend. If trend $\delta_i F_t$ is stochastic, then $\ln(\pi_{ict})$ and $\ln(\pi_{irt})$ are cointegrated with CI vector $(1, -1)$.

Collect pieces for log(t) test

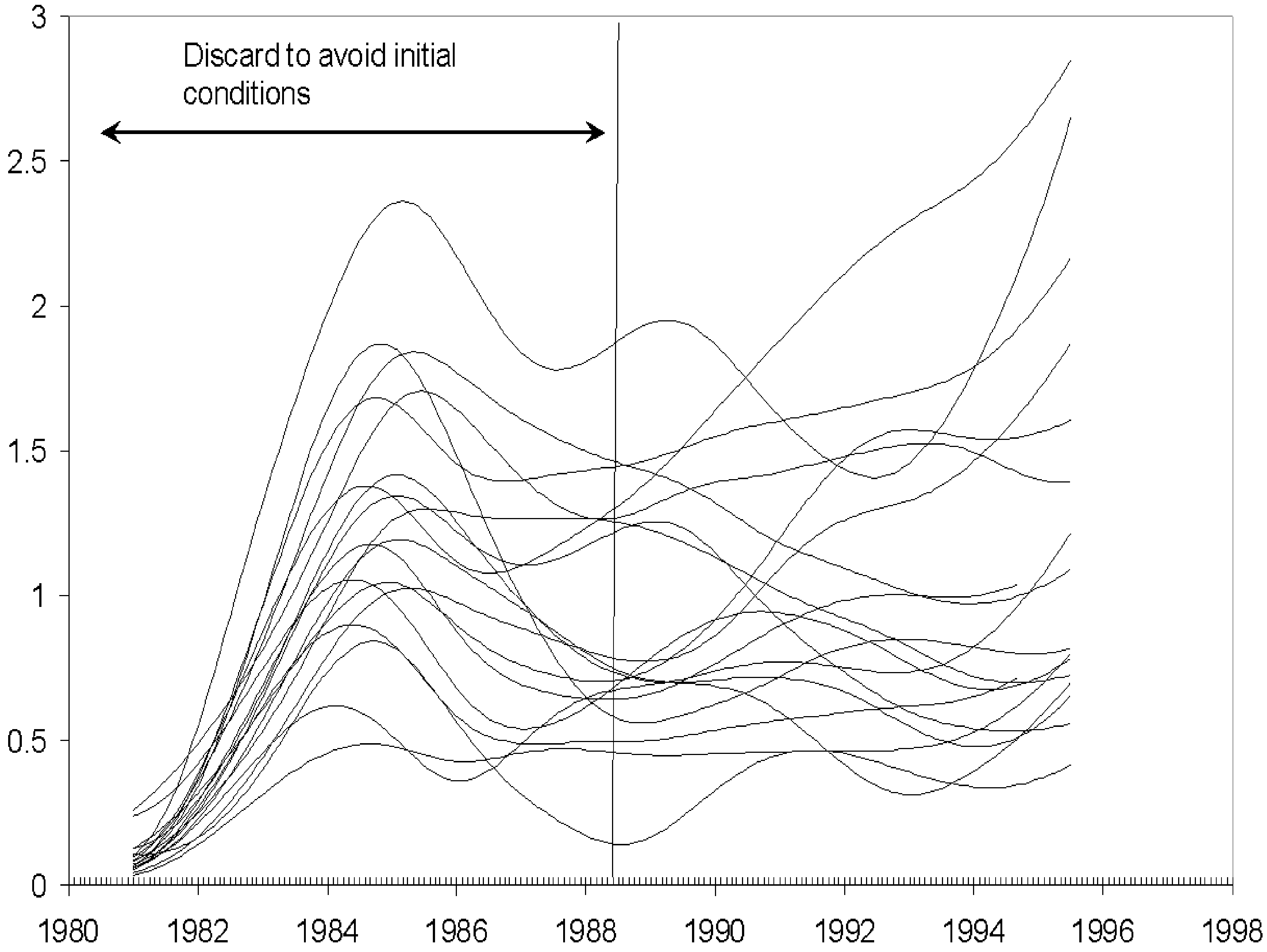
$$
\begin{aligned}
\underbrace{\ln\left(H_{i1}/H_{it}\right)}_{\text{Normalization}} - 2\left(\ln\left(\ln\left(t\right)\right)\right) &= a_i + 2\alpha_i \ln\left(t\right) + u_{it}, \\
\ln \pi_{ict} &= \delta_{ict} F_{it} + e_{ict}, \\
\delta_{ict} &= \delta_i + \psi_{ict}, \qquad \psi_{ict} \overset{iid}{\sim} \left(0, \frac{(t)^{-2\alpha_i}}{\ln\left(t\right)} \sigma^2_{\psi ic}\right) \\
h_{ict} &= \frac{\pi_{ict}}{\frac{1}{C}\sum_{k=1}^{C} \pi_{ikt}} \qquad \text{(relative to cross-sectional mean)} \\
H_{it} &= \frac{1}{C}\sum_{c=1}^{C} \left(h_{ict} - 1\right)^2 \qquad \text{(`Cross-sectional variance')} \\
\mathcal{H}_0 &: \alpha_{ic} \geq 0 \Longrightarrow \delta_{ict} \to \delta_i \quad \text{for all } c \in [1, 11], \\
\mathcal{H}_A &: \alpha_{ic} < 0 \Longrightarrow \delta_{ict} \nrightarrow \delta_i \quad \text{for some } c
\end{aligned}
$$

h_{ict}: price of good i in country c normalized by (controlling for) the common factor F_{it}, H_{it} has dimension of cross-sectional variance of π_{ict}

Behavior of the cross-sectional variances H_{it}.

- – Price data are **indices** not levels.
  - If base year is last observation, π_{ict} will all appear to converge.
  - Set base to first year. Prices will diverge from this point. See Figure 2: A view of the H_{it}

Plots of H_{it} (cross-sectional variance of good $i's$ price).



- Attenuate effect of initial conditions.
  - Discard the first half of the sample. So from the original 175 time-series observations, we work with the 87 observations from October 1988 to December 1995.
  - From this subsample, we discard an additional fraction r of the sample.

Table 2: $t-$ratios for slope coefficients in
$\ln(H_{i1}/H_{it}) - 2(\ln(\ln(t))) = a_i + 2\alpha_i \ln(t) + u_{it}$

	First Observation 1990.6
Clothing	-287
Vehicles	-120
Fruits	-41.5
Comm.	-23.8
Dom.Appl.	-21.9
Footwear	-19.7
Meat	-14.6
Sound	-12.5
Furniture	-12.1
Bread	-9.36
Tobacco	-8.47
Rent	-1.67

$\ln(H_{i1}/H_{it}) - 2(\ln(\ln(t))) = a_i$

Pub.Trans.	0.91
Leisure	**2.75**
Hotel	**2.49**
Book	**5.25**
Alcohol	**8.86**
Dairy	**14.7**
Fuel	**15.1**

(i) Strongest evidence against: clothing t-statistic is -287. (ii) Nontraded goods prices (public transportation and hotels) show evidence of law-of-one price convergence. (iii) Relative convergence found for only 1/3 of the goods.

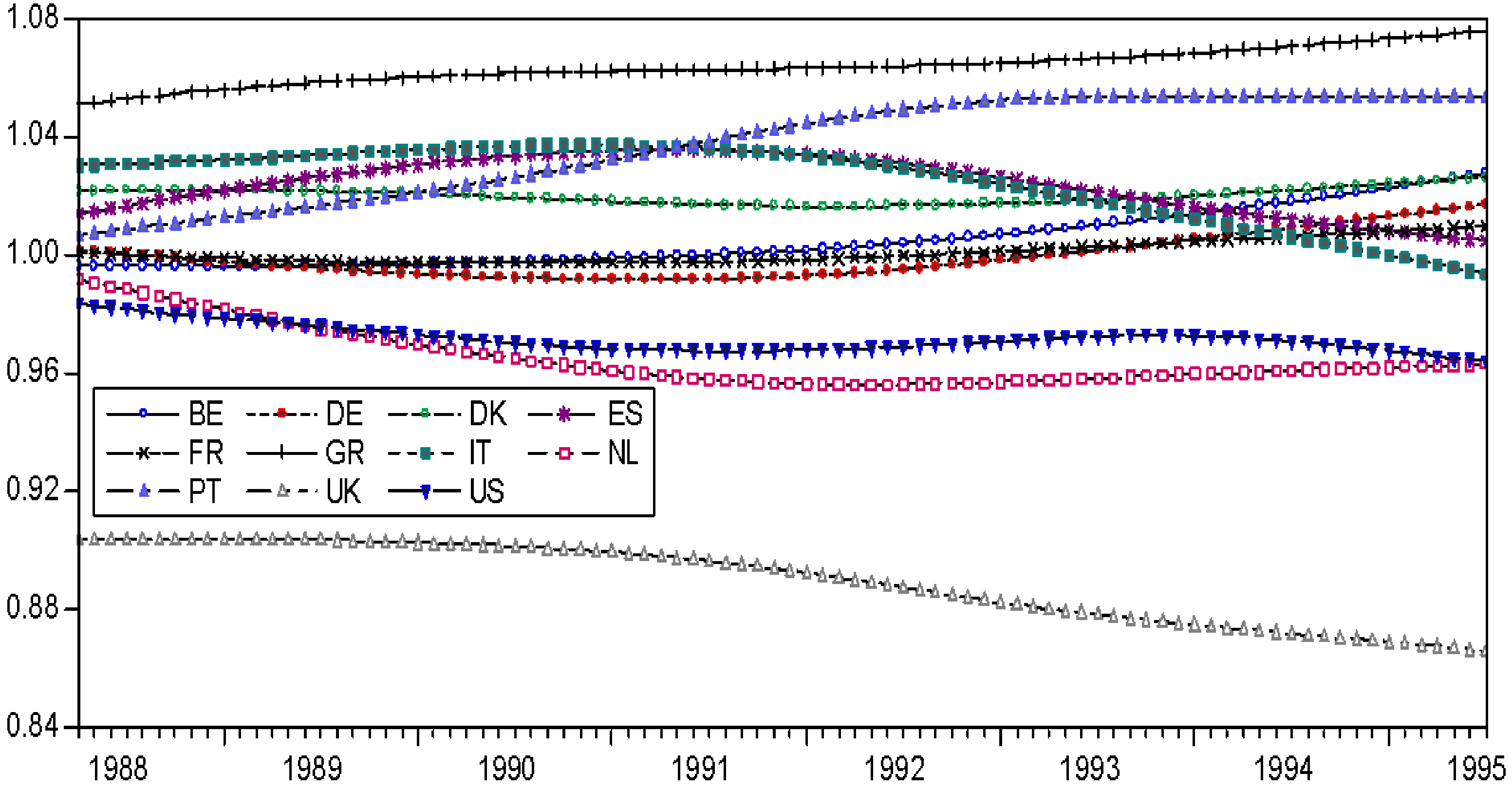


Figure 3: h_{ict} for the U.S. dollar price of clothing for each of 11 countries

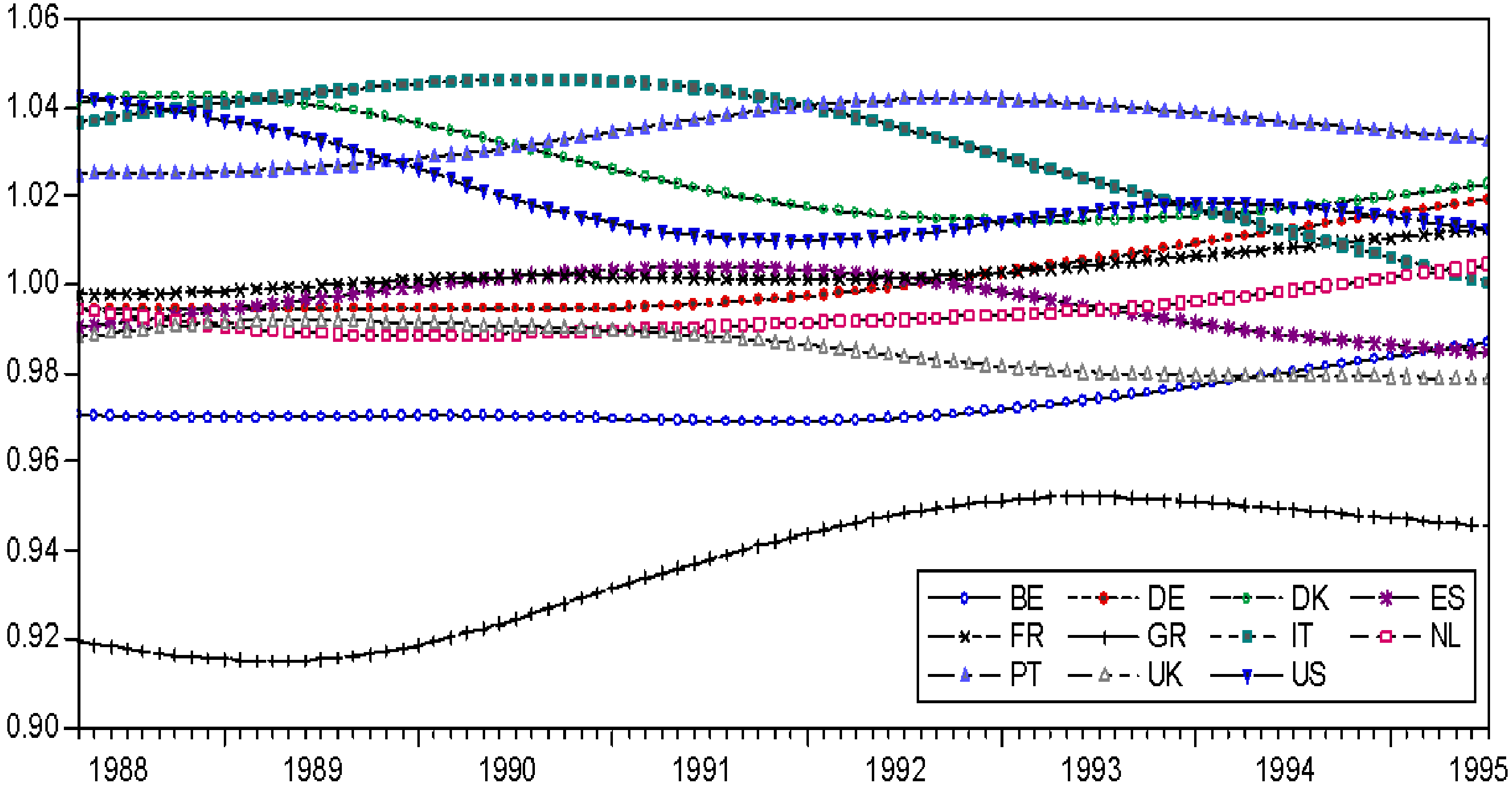


Figure 4: h_{ict} for the U.S. dollar price of Fuel for each of 11 countries

Split up the cross-sectional variances (the H_{it}) among the convergent and nonconvergent goods

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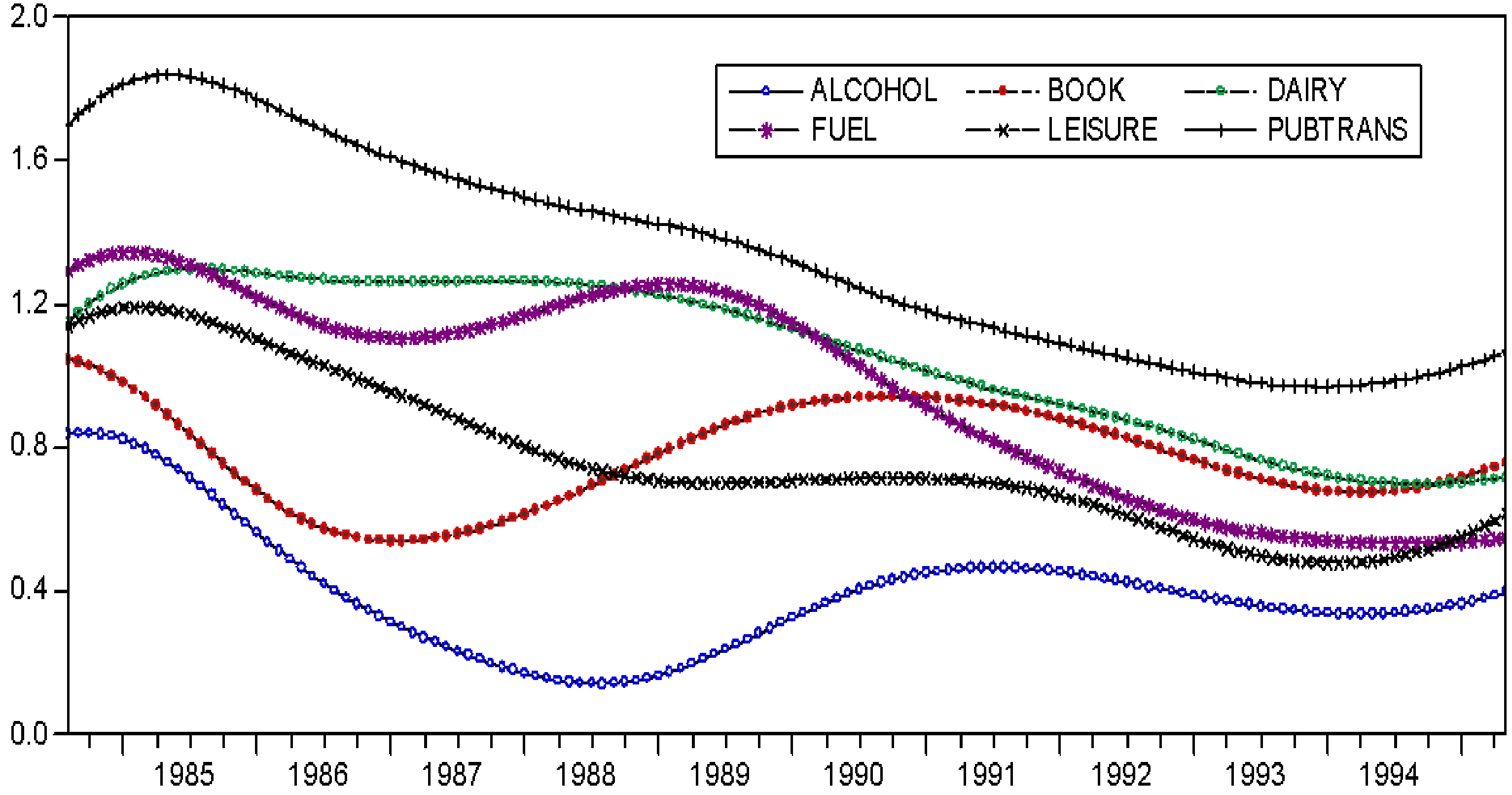

Figure 5: H_{it}: Cross-sectional variance of prices for convergent group of goods

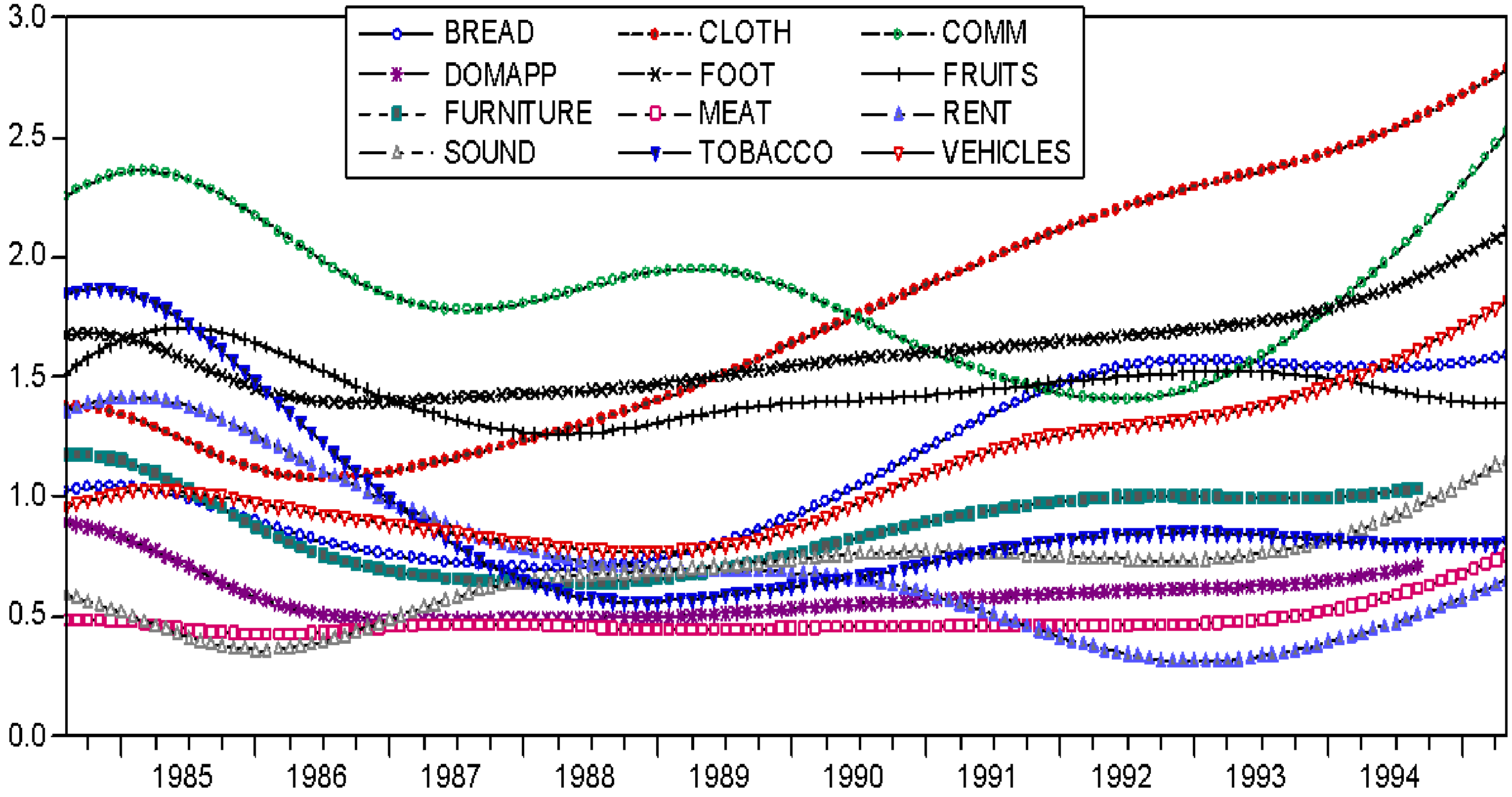

Figure 6: H_{it} :Cross-sectional variance of prices of divergent group of goods

Test for convergence to PPP.

Apply the convergence test to the overall price levels across countries measured in U.S. dollars. Looking for evidence that the cross-sectional dispersion in national price levels is decreasing over time. Regress

$$\ln\left(\frac{H_1}{H_t}\right) - 2\ln(\ln t) = a + 2\alpha \ln(t) + u_t$$

$$\begin{aligned} \hat{\alpha} &= -1.42 \\ t &= -81.4 \end{aligned}$$

No evidence of asymptotic convergence to PPP.

Conclusion

- Panel-data studies on PPP have left out the dynamics of a common factor, which is persistent and maybe unit-root nonstationary.

- Once this is taken into account, the evidence for PPP (and finite half-lives) goes away.

- We seem to be back where we started.

Convergence concepts

- - Absolute convergence: $\ln(\pi_{ict}) - \ln(\pi_{irt}) \to 0$.
  - Relative convergence: $\frac{\ln(\pi_{ict})}{\ln(\pi_{irt})} \to 1$.
  - What's the difference? Let e_{ict} be white noise and ignore it. Let

$$\delta_{ict} = 1 \quad \text{and} \quad \delta_{irt} = \left(1 - 1/\sqrt{t}\right).$$

    * Suppose common factor is deterministic time trend, $F_t = t$.
    * Relative convergence holds b/c $\ln\left(p^*_{ict}\right) / \ln\left(p^*_{irt}\right) = \left(1/\left(1 - 1/\sqrt{t}\right)\right) \to 1$.
    * Absolute convergence does not hold b/c $\ln\left(p^*_{ict}\right) - \ln\left(p^*_{irt}\right) = F_t/\sqrt{t} = \sqrt{t} \to \infty$.