Asset Pricing with Distorted Beliefs: Are Equity Returns Too Good to Be True?

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We study a Lucas asset-pricing model that is standard in all respects, except that the representative agent's subjective beliefs about endowment growth are distorted. Using constant relative risk-aversion (CRRA) utility, with a CRRA coefficient below 10; fluctuating beliefs that exhibit, on average, excessive pessimism over expansions; and excessive optimism over contractions (both ending more quickly than the data suggest), our model is able to match the first and second moments of the equity premium and risk-free rate, as well as the persistence and predictability of excess returns found in the data. (*JEL* E44, G12)

Ever since Rajnish Mehra and Edward Prescott (1985) first articulated the equity premium puzzle, achieving an understanding of aggregate asset-price dynamics has occupied a central role in macroeconomic research. In their original statement of the problem, Mehra and Prescott asked whether the rational, complete markets asset-pricing model, where the representative investor has constant relative risk-aversion (CRRA) utility with a coefficient below 10 and a discount factor between 0 and 1, could account for the approximately 8-percent per annum sample mean return on the Standard and Poor's index and the approximately 2-percent per annum sample mean return on relatively risk-free short-term bonds. Their answer was a resounding no, and the inability of that model to explain the first moment of asset-returns data has come to be known as the equity premium puzzle. The list of challenges taken up by aggregate asset-pricing theory has grown substantially since Mehra and Prescott's original investigation. In addition to the mean equity premium and risk-free rate, this paper seeks to understand the volatility, persistence, and long-horizon predictability of asset returns as well as their relationship to the business cycle.

Because the power of models with common knowledge and fully rational agents to explain these aspects of asset-return dynamics has been poor [see Narayana R. Kocherlakota (1996) for a survey of this research], we take an alternative approach that allows for small departures from rationality in an otherwise standard Robert E. Lucas, Jr. (1978) representative-agent asset-pricing model of an endowment economy. The agents in our model satisfy the Mehra-Prescott criteria of having CRRA utility with a relative risk-aversion coefficient below 10 and a discount factor below 1. Agents observe that the actual endowment process shifts stochastically between high- and low-growth states, but their beliefs about the transition probabilities that govern this switching deviate from the true probabilities.

These belief distortions enter along two empirically plausible dimensions. First, we introduce systematic deviations of the subjective probabilities of transitions between high- and low-growth states. We motivate these distortions by showing how agents who use simple rules of thumb to estimate the transition probabilities governing endowment growth will form

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subjective probabilities that deviate from maximum-likelihood estimates (MLE) obtained from U.S. per capita consumption growth data. Rule-of-thumb estimates that imply relatively pessimistic beliefs about the persistence of the expansion state (with the subjective probability of continuation being lower than the MLE) and relatively optimistic beliefs about the persistence of the contraction state (with the subjective probability of continuation being lower than the MLE), allow us to match the mean equity premium and risk-free rate found in the data and solve the equity premium puzzle.

Unfortunately, systematic distortions alone are not sufficient to explain the volatility of asset returns or the pattern of serial correlation and predictability exhibited in the data. To go beyond an explanation of the first moments of asset returns, we introduce a second distortion in which beliefs about the transition probabilities fluctuate randomly about their subjective (distorted) mean values. This randomization is our way of modeling the effects of the type of *noise* first suggested by Fischer Black (1986). Because the economic environment is complex and noisy, Black argued that individuals are not always able to distinguish noise from information. For example, investors may misinterpret the guarded statements from the chairman of the Federal Reserve System or the rise in the popularity of a particular political candidate as information, when it is not. These pseudosignals become determinants of beliefs and of asset prices because people occasionally mistake them for news about fundamentals. An important implication of our analysis is that randomization of a particular type is required to match the data—not randomization per se. Specifically, it is random fluctuations in beliefs about the persistence of the low endowment growth state that generate volatility and predictability in asset returns. Furthermore, it is necessary for the subjective transition probabilities themselves to be quite persistent.

An important ingredient in our formulation of the asset-pricing problem is that investors are incapable of learning that their distorted beliefs are distorted. With 104 years of data, Bayesian learners, for example, would almost surely have learned how to be rational by now. But, as is well known, the model performs poorly under rational expectations with CRRA utility, and so the introduction of optimal learning is unlikely to aid much in understanding asset-returns data. If there is learning, it would have to be very gradual and explicit modeling of slow learning, although interesting in its own right, would introduce substantial complexity, while contributing only modest insight into the questions at hand.

Instead, our work is part of a growing literature that studies the implications of local (and persistent) departures from full rationality. For example, John H. Cochrane (1989) demonstrates that adoption of rule-of-thumb behavior instead of optimal decision rules entails only trivial economic costs, even when agents know the objective probability law governing the driving processes. Robert Barsky and J. Bradford DeLong (1993) find that long swings in stock prices can be explained by the present-value model if people believe that dividend *growth*, which standard statistical analysis suggests is stationary, contains a unit root. Lars P. Hansen et al. (1999) show that in their linear-quadratic specification, the combination of habit formation and distorted beliefs allows them to mimic the behavior of the market price of risk.

Introducing belief distortions contrasts sharply with the more common approach in which full rationality is preserved, and asset-price puzzles are resolved by introducing increasingly complex preference structures. For example, John Y. Campbell and Cochrane (1999) are able to provide a fully rational, unified explanation for the same asset-pricing anomalies we seek to explain by using a utility function that displays a sophisticated form of habit persistence. Instead, we retain the simplicity of time-separable CRRA utility but allow for what we believe to be reasonable departures from full rationality.

The remainder of the paper is organized as follows. The next section reports the stylized facts for equity and bond returns that we seek to explain. Section II presents the fully rational Lucas asset-pricing model that serves as a benchmark for our analysis. In Section III, we introduce distorted beliefs and discuss how the model is solved. Section IV reports on the model's solution to the many empirical puzzles. Section V contains some concluding remarks.

I. Stylized Facts of Asset Returns

Table 1 presents the list of anomalies around which we organize our investigation. The data on

TABLE 1—STYLIZED FACTS OF EQUITY AND SHORT-TERM BOND RETURNS USING ANNUAL OBSERVATIONS FROM 1871–1993

A. *First and second moments as a percentage*		
Mean equity premium	(μ_{eq})	5.75
Mean risk-free rate	(μ_f)	2.66
Standard deviation		
Equity premium	(σ_{eq})	19.02
Risk-free rate	(σ_f)	5.13
Correlation	$(\rho_{eq,f})$	−0.24

B. *Predictability and persistence*

Horizon	Regression slope	R^2	Variance ratio
1	0.148	0.043	1.000
2	0.295	0.081	1.038
3	0.370	0.096	0.921
5	0.662	0.191	0.879
8	0.945	0.278	0.766

C. *Cyclicality of equity premium*

Average of years beginning and ending in		Average of years ending in	
Expansions	Contractions	Expansions	Contractions
10.54	−14.63	13.66	−11.24

Notes: The data and methods are described in the text. The regression slope and R^2 are for regressions of the k-year ($k = 1, 2, 3, 5, 8$) ahead equity premium on the current log dividend–price ratio. The variance ratio is the variance of the k-year equity premium divided by k times the variance of the one-year equity premium. Cyclicality is computed as the mean return for years that begin and end, or those that simply end, in either expansions or contractions. The stage of the business cycle at the beginning and end of years is determined by the NBER reference chronology.

equity and short-term bond returns is updated from Campbell and Robert J. Shiller (1988b). The equity data are the Standard and Poor's 500 Price Index and Dividends. The nominally risk-free rate is the return from six-month commercial paper bought in January and rolled over in July. Using the Producer Price Index, we measure expected inflation as the fitted value from a regression of inflation on two lags each of inflation, the nominally risk-free rate, and nominal equity returns. The real risk-free rate is then constructed by subtracting expected inflation from the nominally risk-free rate.

The first two lines of panel A of Table 1 report our estimates of a mean level of the equity premium somewhat above 5.5 percent, and the mean level of the *ex ante* risk-free rate a bit over 2.5 percent. Next, we report estimates for the volatility of the equity premium and the risk-free rate, and the correlation between these returns. As has been noted before, these facts pose a volatility puzzle, because it is difficult to match simultaneously the standard deviation of the equity premium, which is nearly 20 percent per year, with the standard deviation of the risk-free rate, which is only 5 percent per year.[1]

Third, we report variance ratio statistics for the equity premium in the fourth column of panel B of Table 1.[2] These provide a measure of the serial correlation properties of the data. That the variance ratio statistics are generally less than 1 and fall with the horizon suggests that excess equity returns are negatively serially correlated, or that asset prices are mean reverting.[3]

Fourth, we study the phenomenon that the log(dividend yield) predicts long-horizon realized excess returns. This predictability puzzle, first described in work by Campbell and Shiller (1988a) and Eugene F. Fama and Kenneth R. French (1988), can be characterized by regressions of the k-period realized excess return on the log(dividend–price ratio). As can be seen from the second and third columns of panel B of Table 1, the predictive regressions run on our data display the familiar pattern of slope coefficients and R^2's that increase with the return horizon.

Finally, we measure the cyclicality of the equity premium in two ways, both based on the NBER reference cycle chronology. In the first, we compute the mean equity premium for years that both begin and end in expansions, and for those years beginning and ending in contractions. We also report the mean equity premium computed from years that end in expansions and those that end in contractions, regardless of state of the business cycle at the beginning of the year. In both cases the equity premium is higher in expansions than that in contractions.

[1] For analyses of the second moments, see George M. Constantinides (1990) and Cecchetti et al. (1993).

[2] The variance ratio statistic, popularized by Cochrane (1988), is the variance of the k-year excess return divided by k times the variance of the one-year excess return.

[3] Cecchetti et al. (1990) and Shmuel Kandel and Robert F. Stambaugh (1991) study the theory's implications for mean-reverting equity returns.

TABLE 2—MAXIMUM-LIKELIHOOD ESTIMATES OF THE ENDOWMENT PROCESS

	p	q	$\alpha(1)$	$\alpha(0)$	σ
Estimate	0.978	0.516	2.251	−6.785	3.127
(t ratio)	(50.94)	(1.95)	(6.87)	(−3.60)	(13.00)

Note: Per capita consumption growth rate as a percentage, 1890 to 1994.

II. The Rational Economy

Our main purpose is to examine the implications of departures from rationality. To provide a benchmark for our investigation, we begin by summarizing the properties of the fully rational model—a variant of Lucas's (1978) representative agent endowment economy—that has served as the workhorse in aggregate asset-pricing studies. Let P be the price of the equity, which is a claim to the future stream of the nonstorable endowment called dividends (D). One perfectly divisible share of the equity trades in a competitive market. The first-order condition that must hold if the agent behaves optimally is

$$(1) \qquad P = \mathcal{E}([M'(P'+D')]|I),$$

where primes denote next-period values, M is the intertemporal marginal rate of substitution, and $\mathcal{E}(\,\cdot\,|I)$ is the representative individual's subjective expectation conditioned on currently available information (I). Under time-separable CRRA utility defined over consumption (C), $M' = \beta(C'/C)^{-\gamma}$, where $\gamma > 0$ is the coefficient of relative risk aversion and $0 < \beta < 1$ is the subjective discount factor. Following the standard practice of modeling per capita consumption as the endowment, and assuming that it is consumed in equilibrium ($D = C$), equation (1) becomes

$$(2) \quad P = \beta \mathcal{E}\left(\left[\left(\frac{C'}{C}\right)^{-\gamma}(P'+C')\right]\bigg| I\right).$$

Now define $\omega \equiv P/C$ as the price–consumption ratio and let $c \equiv \ln(C)$. Dividing equation (2) by C yields the stochastic difference equation in ω,

$$(3) \qquad \omega = \beta \mathcal{E}([e^{(1-\gamma)\Delta c'}(\omega'+1)]|I),$$

where Δ is the first difference operator.

A. *The Endowment Process*

We follow Cecchetti et al. (1990, 1993), who assume that Δc evolves according to the following version of James D. Hamilton's (1989) Markov switching process:

$$(4) \qquad \Delta c = \alpha(S) + \varepsilon,$$

where ε is independently and identically distributed (i.i.d.) as $N(0, \sigma^2)$ and S is an underlying two-point Markov state variable that assumes values of 0 or 1. We normalize $S = 1$ to be the good (expansion) state of high-consumption growth and $S = 0$ to be the bad (contraction) state of low-consumption growth, so that $\alpha(1) > \alpha(0)$. S evolves according to the four transition probabilities

$$(5) \qquad \Pr(S' = 1|S = 1) = p,$$
$$\Pr(S' = 0|S = 1) = 1 - p,$$
$$\Pr(S' = 0|S = 0) = q,$$
$$\Pr(S' = 1|S = 0) = 1 - q,$$

where p is the probability of remaining in an expansion if currently in one and $1 - p$ is the transition probability for moving from an expansion to a contraction. Similarly, q is the probability of remaining in a contraction if currently in a contraction, whereas $1 - q$ is the transition probability of moving from a contraction to an expansion.

Table 2 reports the maximum-likelihood estimates of this Markov switching model using U.S. per capita consumption data extending from 1890 to 1994.[4] In our subse-

[4] These data are updated from Cecchetti et al. (1990), who also perform a number of diagnostic tests on the

quent analysis, we calibrate the endowment by setting its parameter values equal to the maximum-likelihood estimates to ensure that the economy's endowment growth evolves according to the objectively true process.

Several features of the "truth" are worth noting. First, expansions are highly persistent. Given that the economy is in an expansion state, the probability that the expansion will end is less than 0.03. Moreover, the economy spends most of the time in the expansionary state. The unconditional probability of being in an expansion is $\Pr(S = 1) = (1 - q)/(2 - p - q) = 0.96$. Second, contractions are moderately persistent, with slightly more than an even chance of continuing once they start. Finally, the bad state is very bad, as mean per capita consumption growth in the contractionary state is -6.79 percent.

B. *Solution and Properties of the Rational Economy*

In the benchmark model, the subjective expectations of individuals $\mathcal{E}(\cdot)$ coincide with the mathematical expectations $E(\cdot)$, taken with respect to the truth. This formulation gives rise to a particular complete markets rational economy that is closely related to the environment studied by Cecchetti et al. (1990), Kandel and Stambaugh (1990), and Mehra and Prescott (1985).

To solve for equilibrium returns, we first note that because the shock ε is independent of S, we can write the stochastic difference equation for the price–consumption ratio, ω in equation (3), in terms of the single state variable S,

$$\omega(S) = \beta e^{(1-\gamma)^2(\sigma^2/2)} \times E([1 + \omega(S')]e^{(1-\gamma)\alpha(S')}|S). \tag{6}$$

Because S can take on only the values 0 or 1, equation (6) is a system of two linear equations in $\omega(0)$ and $\omega(1)$

$$\begin{bmatrix} q\hat{\beta}(0) + (1-q)\hat{\beta}(1) \\ p\hat{\beta}(1) + (1-p)\hat{\beta}(0) \end{bmatrix} = \begin{bmatrix} 1 - q\hat{\beta}(0) & -(1-q)\hat{\beta}(1) \\ -(1-p)\hat{\beta}(0) & 1 - p\hat{\beta}(1) \end{bmatrix} \times \begin{bmatrix} \omega(0) \\ \omega(1) \end{bmatrix}, \tag{7}$$

where $\hat{\beta}(S) \equiv \beta e^{(1-\gamma)^2(\sigma^2/2)+(1-\gamma)\alpha(S)}$, for $S = 0, 1$. Solving the system (7) yields

$$\omega(0) = \frac{q\hat{\beta}(0) + (1-q)\hat{\beta}(1) - (p+q-1)\hat{\beta}(0)\hat{\beta}(1)}{\Delta} \tag{8}$$

and

$$\omega(1) = \frac{p\hat{\beta}(1) + (1-p)\hat{\beta}(0) - (p+q-1)\hat{\beta}(0)\hat{\beta}(1)}{\Delta}, \tag{9}$$

where $\Delta = 1 - q\hat{\beta}(0) - p\hat{\beta}(1) + pq\hat{\beta}(0)\hat{\beta}(1)$.[5]

The next step is to characterize the solution for one-period returns. Gross equity returns R^e are given by

$$R^e(S', S) = \left[\frac{\omega(S') + 1}{\omega(S)}\right] e^{\alpha(S')+\varepsilon'}. \tag{10}$$

Since the price of a one-period risk-free asset P^f is the expected intertemporal marginal rate of substitution

$$P^f(S) = \beta e^{\gamma^2(\sigma^2/2)} E[e^{-\gamma\alpha(S')}|S], \tag{11}$$

the implied gross risk-free rate R^f is,

$$R^f(S) = \frac{1}{P^f(S)}. \tag{12}$$

specification and conclude that this estimated Markov-switching model is a plausible representation of the data-generating process.

[5] The information structure here corresponds to Cecchetti et al. (1993) and Andrew B. Abel (1994), in that the information set, on which asset prices are based, includes S_{t-i} and ε_{t-i}, for $i = 0, 1, 2, \ldots$.

TABLE 3—IMPLICATIONS OF THE FULLY RATIONAL MODEL

Preferences		First and second moments		Predictability and persistence			
β	γ			Horizon	Slope	R^2	Variance ratio
0.976	0.0	μ_{eq}	0.0	1	0.209	0.032	1.000
		μ_f	2.5	2	0.276	0.034	0.987
		σ_{eq}	4.4	3	0.313	0.032	0.977
		σ_f	0.0	5	0.357	0.026	0.959
		$\rho_{eq,f}$	0.000	8	0.421	0.023	0.933
0.985	0.5	μ_{eq}	0.1	1	0.335	0.025	1.000
		μ_f	2.5	2	0.445	0.027	0.987
		σ_{eq}	4.0	3	0.504	0.026	0.975
		σ_f	0.5	5	0.573	0.023	0.956
		$\rho_{eq,f}$	−0.007	8	0.667	0.021	0.929
1.010	2.0	μ_{eq}	0.2	1	0.007	0.007	1.000
		μ_f	2.5	2	0.010	0.010	0.991
		σ_{eq}	3.2	3	0.011	0.012	0.982
		σ_f	1.9	5	0.012	0.014	0.965
		$\rho_{eq,f}$	0.002	8	0.013	0.016	0.939
1.026	3.0	μ_{eq}	0.2	1	0.077	0.018	1.000
		μ_f	2.5	2	0.105	0.020	0.977
		σ_{eq}	3.9	3	0.119	0.021	0.959
		σ_f	2.8	5	0.134	0.020	0.932
		$\rho_{eq,f}$	0.034	8	0.150	0.020	0.901

Note: See Table 1 for moment definitions.

It is now well known that many features of asset-returns data can be explained using the standard model by choosing both a large value of γ (e.g., 57) and a value of β in excess of 1.[6] The challenge, as articulated by Mehra and Prescott (1985) and Kocherlakota (1996), is to produce a model that explains the data with values of β between 0 and 1, and positive values of γ below 10. To establish the model's benchmark performance, we restrict our attention to values of γ less than 10 and associated values of β less than 1, which predict a mean risk-free rate of 2.5 percent, subject to satisfying the transversality condition of the model.[7] Table 3 displays the implied behavior of asset prices for selected values of the preference parameters when the representative agent is fully rational. The model is seen to fail badly: there is virtually no equity premium, the volatility of equity returns is far below its sample value, and excess returns have neither the persistence nor the predictability found in the data.

III. A Distorted Beliefs Economy

The implied behavior of asset returns is based on a model of preferences (M) and a model of beliefs ($\mathcal{E}$). In the benchmark model, preferences are CRRA and beliefs are rational. In considering departures from this basic formulation, one line of asset-pricing research has been aimed at retaining rational expectations, but enriching the model of preferences. For example, Larry G. Epstein and Stanley E. Zin (1989, 1991), Philippe Weil (1989), and Kandel and Stambaugh (1991) examine recursive nonexpected utility, whereas Abel (1990), Constantinides (1990), Wayne E. Ferson and Constantinides (1991), and John Heaton (1995) study versions of habit persistence. Recently, Campbell and Cochrane (1999) report success in accounting for many of the same features of the data that we consider by retaining rational expectations but by

[6] See Kocherlakota (1990), Kandel and Stambaugh (1991), and Cecchetti et al. (1993).

[7] The transversality condition requires that the power series for the price level implied by the difference equation (2) yield finite values. Practically, this requirement can be checked by verifying that implied values for $\omega(S)$ are always positive.

complicating preferences in a way that displays a particular form of habit persistence.

Our line of attack corresponds to that of Campbell and Cochrane (1999). The returns behavior they achieve by varying M for fixed $\mathcal{E}$ can also be obtained for a given M with the appropriate specification of $\mathcal{E}$—that is, by distorting beliefs. The interesting question, then, is what sorts of departures from rationality are necessary and how large do the departures need to be?[8] In the remainder of this section, we introduce our model with distorted beliefs and characterize the solution for asset returns. Section IV follows with empirical results.

A. *Modeling Distorted Beliefs*

Because the expected endowment growth rate $\mathcal{E}(\Delta c') = \mathcal{E}[\alpha(S')]$ depends on the state transition probabilities, variations in the subjective transition probabilities alone will induce changes in subjective endowment growth. To separate the effects of misperceived state persistence and misperceptions of endowment growth, we explicitly model both of these aspects of beliefs. We define all subjective belief parameters to be nonnegative and denote them with a tilde accent.

Agents observe the true state of the economy S, but not the true transition probabilities p and q or the true growth rates $\alpha(0)$ and $\alpha(1)$. When the endowment is in the expansionary state, their beliefs about p and $\alpha(1)$ are governed by

$$\tilde{p}(S_e) = \tilde{\mu}_{\tilde{p}} + \tilde{\delta}_p S_e \tag{13}$$

and

$$\tilde{\alpha}_e(S_e) = \tilde{\mu}_e + \tilde{\tau}_e S_e, \tag{14}$$

where S_e is an independent two-point Markov state variable that assumes values 1 or -1 with the symmetric transition probabilities

$$\begin{aligned}\tilde{\phi}_e &= \Pr[S_e' = 1 | S_e = 1] \\ &= \Pr[S_e' = -1 | S_e = -1]\end{aligned} \tag{15}$$

and

$$\begin{aligned}1 - \tilde{\phi}_e &= \Pr[S_e' = 1 | S_e = -1] \\ &= \Pr[S_e' = -1 | S_e = 1],\end{aligned} \tag{16}$$

where $\tilde{p}(S_e)$ is the subjective persistence of the expansionary state and has unconditional mean $\tilde{\mu}_{\tilde{p}}$ and $\tilde{\alpha}_e(S_e)$ is the subjective endowment growth during the expansion state with unconditional mean $\tilde{\mu}_e$. Subjective persistence and subjective endowment growth are perfectly correlated if $\tilde{\delta}_p$, $\tilde{\tau}_e > 0$. We say that agents display optimism over the expansion state when $S_e = 1$ because subjective persistence and subjective endowment growth exceed their mean values. Similarly, we say that agents display pessimism over the expansion state when $S_e = -1$. Beliefs will exhibit systematic distortion during the expansion state when $\tilde{\mu}_{\tilde{p}} \neq p = 0.98$ and $\tilde{\mu}_e \neq \alpha(1) = 2.25$.

When the economy is in the contractionary state, agents beliefs about q and $\alpha(0)$ are governed by

$$\tilde{q}(S_c) = \tilde{\mu}_{\tilde{q}} - \tilde{\delta}_q S_c \tag{17}$$

and

$$\tilde{\alpha}_c(S_c) = \tilde{\mu}_c + \tilde{\tau}_c S_c, \tag{18}$$

[8] In a linear-quadratic environment, Hansen et al. (1999) show that a model with rational expectations and recursive nonexpected utility combined with habit persistence is equivalent to a model with expected utility in conjunction with a particular form of distorted beliefs. Hansen et al. interpret these belief distortions as the way that agents deal with Knightian uncertainty—the situation where information is too imprecise to be summarized by probabilities—as studied by Epstein and Tan Wang (1994). Their representative consumer with quadratic expected utility can be thought of as facing a malevolent opponent, whose purpose is to minimize the welfare of the consumer by distorting the dynamics of the economy. From that perspective, they interpret the belief distortions as encoding the sophisticated attempts of the consumer to accommodate specification errors. The actions of the hypothetical malevolent opponent generates "the worst-case scenarios" in the Maxmin theory of Itzach Gilboa and David Schmeidler (1989).

where S_c is an independent two-point Markov state variable that assumes values 1 or -1 with the symmetric transition probabilities

$$(19) \qquad \tilde{\phi}_c = \Pr[S_c' = 1 | S_c = 1] = \Pr[S_c' = -1 | S_c = -1]$$

and

$$(20) \qquad 1 - \tilde{\phi}_c = \Pr[S_c' = 1 | S_c = -1] = \Pr[S_c' = -1 | S_c = 1],$$

where $\tilde{\mu}_{\tilde{q}}$ is the mean value of the subjective contractionary state persistence $\tilde{q}(S_c)$ and $\tilde{\mu}_c$ is the mean value of the subjective contractionary growth rate $\tilde{\alpha}_c(S_c)$. Beliefs exhibit contractionary state optimism when $S_c = 1$ because subjective persistence of the contractionary state lies below its mean value and subjective contractionary state endowment growth lies above its mean. Beliefs exhibit contractionary state pessimism when $S_c = -1$. Systematic belief distortions over the contraction state are present when $\tilde{\mu}_{\tilde{q}} \neq q = 0.52$ and $\tilde{\mu}_c \neq \alpha(0) = -6.79$.

To complete the specification, note that the state vector is now $\bar{S} = (S, S_e, S_c)$. Agents observe and condition on the true state S when they form their expectations. Accordingly, the subjective endowment growth rate is given by

$$(21) \qquad \tilde{\alpha}(\bar{S}) = \begin{cases} \tilde{\alpha}_e(S_e) & \text{if } S = 1 \\ \tilde{\alpha}_c(S_c) & \text{if } S = 0. \end{cases}$$

The fully rational model is nested in the distorted beliefs model, and is obtained by setting $\tilde{\delta}_p = \tilde{\delta}_q = \tilde{\tau}_e = \tilde{\tau}_c = 0$; $\tilde{\mu}_{\tilde{p}} = p = 0.98$; $\tilde{\mu}_{\tilde{q}} = q = 0.52$; $\tilde{\mu}_e = 2.25$; and $\tilde{\mu}_c = -6.79$.

B. *Discussion*

To summarize our strategy, we assume that investors believe that the underlying structure of the economy is more complex than the truth. They form overactive subjective beliefs concerning expansion and contraction growth and state persistence, but are rational in every other way. That is, given their distorted beliefs about the endowment process, they use the correct structure to solve for asset prices. Retaining this element of rationality provides a natural reduction in the number of admissible models to consider and imposes a certain discipline on our investigation.

Investors are susceptible to two types of distortions in their beliefs about the evolution of the endowment process that determines asset returns. First, their subjective beliefs about the state transition probabilities differ systematically from the truth. A plausible explanation for these systematic distortions is that individuals find it too costly to acquire the skills to do maximum-likelihood estimation or to perform integration with respect to multivariate densities when making decisions about everyday life. Instead, they respond by using rules of thumb that give approximately the right answer. The following simple rule-of-thumb calculations guide our modeling of the systematic distortion in the subjective transition probabilities.

Suppose that agents realize that consumption growth is governed by a two-state Markov process, but don't know the transition probabilities. A sensible person might pursue the following strategy for "estimating" p and q. Begin by obtaining a copy of the NBER reference cycle chronology, and convert it to an annual frequency.[9] Next, use this to estimate q as the proportion of years in which contractions are followed by contractions, and then estimate the unconditional probability of being in an expansion $[(1 - q)/(2 - p - q)]$ as the proportion of years in expansion. An estimate of p then follows. This simple exercise yields an estimate for q equal to 0.273 and an estimate for p equal to 0.667, both well below the maximum-likelihood estimates reported in Table 2.[10] We take these rule-of-thumb cal-

[9] This information is available on the NBER's home page at http://www.nber.org. In making the conversion from a monthly to an annual frequency, years in which a majority of the months are a contraction are assumed as being in the low-growth state.

[10] A second, and equally simple, method of generating rule-of-thumb estimates for p and q, uses per capita consumption growth data and yields similar results. Begin by assigning all years in which per capita consumption contracted to the bad state and the remainder, those with positive per capita consumption growth, to the good state. Then use the same simple technique based on the persistence of

culations to suggest that we should concentrate on subjective beliefs that exhibit excessive pessimism about expansion states $\tilde{p} < p$ and excessive optimism about contraction states $\tilde{q} < q$.

We also allow beliefs to be distorted along a second dimension: the subjective transition probabilities move randomly about their subjective means. That is, $\tilde{p}$ and $\tilde{q}$ fluctuate based on pseudosignals S_e and S_c, which are pure noise, unrelated to fundamentals, in the sense of Black (1986).[11]

It bears emphasizing that we are permitting distortions only to subjective beliefs of the endowment and not to the actual endowment—which continues to evolve according to the truth. This is a subtle but important point that distinguishes our approach from that of Thomas Rietz (1988), in which the actual endowment evolves according to a distorted process.

Finally, we note that it is also possible to interpret our model in the context of the "peso problem" literature, in which it is the agents who are rational and the econometrician who errs in believing that endowment growth follows the two-state Markov process of equations (4) and (5). In this interpretation, the true data-generating process for the endowment is presumed to conform to the probability model of equations (13) through (20), which we label as distorted. The econometrician is led to the wrong model (which we label as the truth) by attempting estimation and inference on a sample of insufficient size. Because of our limited historical experience with consumption growth, realizations are absent in regions of the sample space necessary to identify and to estimate the true model.

Problems of limited experience and the associated difficulties associated with drawing inference from small samples form the foundation for both interpretations. Whether the error is attributed to the economic agents (distorted beliefs) or to the econometrician (peso problem) is a philosophical issue that we cannot resolve here. We simply note that some readers may be more comfortable interpreting our model as one of a peso problem faced by an econometrician rather than one of agents with distorted beliefs.[12]

C. *Solving the Distorted Beliefs Model*

Returning to the analytical task, we need to characterize the behavior of asset prices in the distorted beliefs model. As was the case in the rational model, the solution centers on solving for the dynamics of the price–consumption ratio ω, which is a function of the state vector governing the endowment process. In the fully rational model, the state vector can take on only two values (S equals 0 or 1). The state vector in the distorted beliefs model includes the pseudosignals S_e and S_c, both of which can each take on values of -1 and 1.

We define the distorted beliefs state vector as $\bar{\mathbf{S}} \equiv (S, S_e, S_c)$, which can take on eight possible values. The price–consumption ratio $\omega(\bar{\mathbf{S}})$ now evolves according to the stochastic difference equation

$$(22)\quad \omega(\bar{\mathbf{S}}) = \beta e^{(1-\gamma^2)(\sigma^2/2)} \times \mathcal{E}([1+\omega(\bar{\mathbf{S}}')]e^{(1-\gamma)\tilde{\alpha}(\bar{\mathbf{S}}')}|\bar{\mathbf{S}}),$$

which is the analog to equation (6).

The next step in finding the solution is to define $\tilde{\mathbf{P}}$, the subjective transition matrix whose ijth element ($i, j = 1, \ldots, 8$) is $\tilde{\mathbf{P}}_{ij} = \Pr[\bar{\mathbf{S}}' = \bar{\mathbf{S}}_j|\bar{\mathbf{S}} = \bar{\mathbf{S}}_i]$. (A full description of $\tilde{\mathbf{P}}$ and $\bar{\mathbf{S}}$ is given in the Appendix.) Then equation (22) can be written as

$$(23)\quad \omega(\bar{\mathbf{S}}_i) = \beta e^{(1-\gamma)^2(\sigma^2/2)} \times \sum_{j=1}^{8} \tilde{\mathbf{P}}_{ij}[1 + \omega(\bar{\mathbf{S}}_j)]e^{(1-\gamma)\tilde{\alpha}(\bar{\mathbf{S}}_j)}, \quad i = 1, \ldots, 8.$$

contractions and the unconditional probability of expansions to estimate the transition probabilities. The result is an estimate for p equal to 0.760 and an estimate for q equal to 0.269.

[11] Abel (1997) models a related set of distortions. The systematic distortion in p and q loosely corresponds to "pessimism" in Abel's model, which describes the situation when subjective distributions are first order, dominated by the objective distribution. Randomization in p and q corresponds to "doubt" in Abel's model, which is said to occur when the subjective distribution is a mean-preserving spread over the objective distribution.

[12] In the original peso problem, the agent rationally attaches a nonzero probability to the event in which the monetary authorities would devalue the currency, even though the monetary history contained no such devaluations. Martin D. D. Evans (1998) recently investigated peso problem implications for stock returns.

Rewriting equation (23) in vector notation, the analog to equation (7) is

$$(24) \qquad \omega = \mathbf{G}\omega + \mathbf{f} = (\mathbf{I} - \mathbf{G})^{-1}\mathbf{f},$$

where $\mathbf{I}$ is an eight-dimensional identity matrix, $\omega_i = \omega(\bar{\mathbf{S}}_i)$, $\mathbf{G}_{ij} = \beta e^{[(1-\gamma)^2(\sigma^2/2)+(1-\gamma)\tilde{\alpha}(\bar{\mathbf{S}}_j)]}\tilde{\mathbf{P}}_{ij}$, and $\mathbf{f}_i = \beta e^{[(1-\gamma)^2(\sigma^2/2)]} \sum_{j=1}^{8} \tilde{\mathbf{P}}_{ij}\, e^{[(1-\gamma)\tilde{\alpha}(\bar{\mathbf{S}}_j)]}$, for $i, j = 1, \ldots, 8$. These definitions allow us to write the implied one-period gross equity return, the analog to equation (10), as

$$(25) \quad R^e(\bar{\mathbf{S}}', \bar{\mathbf{S}}) = \left[\frac{\omega(\bar{\mathbf{S}}') + 1}{\omega(\bar{\mathbf{S}})}\right] e^{[\alpha(\bar{\mathbf{S}}')+\varepsilon']}.$$

The final task is to determine the return on a one-period risk-free bond, and the equity premium. This begins with a computation of the price of the bond, which is a function of the state vector $\bar{\mathbf{S}}$. When the state takes on any of its eight possibilities, denoted by $\bar{\mathbf{S}}_i$, then the price of the one-period risk-free bond can be written as

$$(26) \quad P^f(\bar{\mathbf{S}}_i) = \beta e^{(\gamma^2\sigma^2/2)} \sum_{j=1}^{8} \tilde{\mathbf{P}}_{ij} e^{-\gamma\tilde{\alpha}(\bar{\mathbf{S}}_j)}.$$

The gross risk-free rate of return follows immediately as the inverse of this price $R^f(\bar{\mathbf{S}}) = 1/P^f(\bar{\mathbf{S}})$, and the excess return on equity is then just $R^p(\bar{\mathbf{S}}', \bar{\mathbf{S}}) = R^e(\bar{\mathbf{S}}', \bar{\mathbf{S}}) - R^f(\bar{\mathbf{S}})$. The moments of the equity premium and the risk-free rate are then computed with respect to the objective probability distribution of $\bar{\mathbf{S}}$.

IV. Properties of the Distorted Beliefs Model

We assess the value added of the separate components of our model by proceeding in a series of steps. First, we investigate the role of systematic distortions in subjective transition probabilities by fixing $\tilde{p}$ and $\tilde{q}$ at values that deviate from p and q. Next, we randomize beliefs by allowing the subjective transition probabilities $\tilde{p}$ and $\tilde{q}$ to evolve as i.i.d. processes. Third, we introduce persistence in the stochastic subjective transition probabilities by allowing the pseudosignals $\tilde{\phi}_e$ and $\tilde{\phi}_c$ to deviate from 0.5. Throughout these first three experiments, subjective endowment growth is nonstochastic and undistorted, with $\tilde{\alpha}(\bar{\mathbf{S}}) = \alpha(S)$. In a fourth experiment, we relax this restriction and allow for randomized subjective endowment growth, in addition to randomized conditional subjective transition probabilities. In the fifth, and final, experiment we relax the assumption that the persistence of the pseudosignals $\tilde{\phi}_e$ and $\tilde{\phi}_c$ is itself undistorted and allow investors to irrationally forecast their own irrational beliefs.

TABLE 4—FIXED DISTORTED BELIEFS: IMPLIED SECOND MOMENTS FOR PARAMETER VALUES THAT SOLVE THE EQUITY PREMIUM PUZZLE

Beliefs		Preferences		Second moments		
$\tilde{p}$	$\tilde{q}$	β	γ	σ_{eq}	σ_f	$\rho_{eq,f}$
0.5	0.2	0.933	1.831	4.3	1.1	−0.42
0.5	0.4	0.918	2.371	4.1	0.5	−0.31
0.5	0.8	0.965	0.429	4.0	0.2	0.11
0.6	0.2	0.923	3.159	4.5	1.3	−0.41
0.6	0.4	0.825	6.985	3.9	0.0	0.00
0.6	0.9	0.969	0.411	4.2	0.4	−0.01
0.7	0.1	0.930	4.691	4.9	2.1	−0.47
0.7	0.2	0.909	5.795	4.5	1.3	−0.38
0.7	0.9	0.975	0.150	4.9	0.2	−0.12
0.8	0.1	0.933	8.300	5.0	2.1	−0.44

A. *Systematic Distortions in Subjective Transition Probabilities*

To what extent can the asset price puzzles set forth in Section I be resolved by the introduction of perceptions that are systematically distorted? We begin to answer this question by setting beliefs over $(\tilde{p}, \tilde{q})$ to fixed values such that $\tilde{p} = \tilde{\mu}_{\tilde{p}} \neq p = 0.98$ and $\tilde{q} = \tilde{\mu}_{\tilde{q}} \neq q = 0.52$, with $\tilde{\delta}_p = \tilde{\delta}_q = \tilde{\tau}_e = \tilde{\tau}_c = 0$. To keep conditional subjective endowment growth undistorted, we set $\tilde{\alpha}_e = \tilde{\mu}_e = \alpha(1) = 2.25$ and $\tilde{\alpha}_c = \tilde{\mu}_c = \alpha(0) = -6.79$.

Table 4 reports parameter values for preferences and beliefs that solve the equity premium puzzle nearly exactly. These are combinations of $(\tilde{p}, \tilde{q}, \beta, \gamma)$ for which the mean equity premium and mean risk-free rate predicted by the model are 5.5 and 2.5 percent, respectively. The region of the parameter space that accomplishes this feat is quite large, as it includes values of $\tilde{p}$ ranging from 0.5 to 0.8, and values of $\tilde{q}$ varying between 0.1 to 0.9. Overall, we note that relatively mild degrees of relative risk aversion are required to match the first moments of returns.

The systematic belief distortions presented in Table 4 fall into two categories. The first are those in which moderately risk-averse agents display pessimism about expansion persistence ($\tilde{p} < p$), believing that expansions are shorter than the data indicate, but display optimism about contraction persistence ($\tilde{q} < q$), thinking they are shorter than suggested by the MLE truth. For example, when $(\tilde{p}, \tilde{q}) = (0.7, 0.2)$, roughly our rule-of-thumb values, $(\beta, \gamma) = (0.91, 5.8)$ solve the equity premium puzzle. The second category of systematically distorted beliefs that allows us to match the first moments are those in which nearly risk-neutral individuals are uniformly pessimistic in thinking that both expansions will be shorter ($\tilde{p} < p$) and contractions longer ($\tilde{q} > q$) than the data justify. For example, with $(\tilde{p}, \tilde{q}) = (0.6, 0.9)$, the mean value of returns can be matched with $(\beta, \gamma) = (0.97, 0.41)$.

The job of matching the first moments of the equity premium and the risk-free rate is accomplished primarily through a judicious choice of two parameters—$\tilde{p}$ and γ. To see why, note first that lowering $\tilde{p}$ generates expansionary state pessimism. This strengthens the motive for precautionary saving and reduces R^f. This means that for a given $(\beta, \gamma, \tilde{q})$–triple, we can choose $\tilde{p}$ to match the mean risk-free rate. Second, note that as we increase γ, the increase in required compensation for covariance risk will force up the equilibrium equity return and reduce the agent's willingness to substitute intertemporally. In other words, a higher γ increases both the equity return and the risk-free rate. In the fully rational model, it is possible to match the equity premium only for values of R^f that are too high, creating a risk-free rate puzzle, but here, we can reduce $\tilde{p}$ to offset the impact of higher γ and match both of the first moments.

We now turn our attention to the implied volatility of the equity premium and the risk-free rate, and the correlation between them, reported in the last three columns of Table 4. Although the model can account for the fact that the equity premium is substantially more volatile than the risk-free rate, and the ratio of the standard deviations is not far from the 3.7 in Table 1, the levels are much too low. Asset-return volatility in the data is between two- and fourfold what we can produce using the fixed beliefs model. Evidently, to match the volatilities and correlation, systematic belief distortions are not enough, and so we now proceed to introduce time variation in distorted beliefs.[13]

B. *Stochastically Independent Beliefs of State Persistence*

The next step is to assume that agents' beliefs about regime persistence are random but independent. By setting $\tilde{\delta}_p$ and $\tilde{\delta}_q$ to nonzero values and $\tilde{\phi}_c = \tilde{\phi}_e = 0.5$, the subjective transition probabilities $\tilde{p}$ and $\tilde{q}$ will evolve as i.i.d. processes. In this section, subjective growth rates remain undistorted, with $\tilde{\alpha}_e = \alpha(1) = 2.25$, and $\tilde{\alpha}_c = \alpha(0) = -6.79$.

Table 5 reports the mean and volatility of the equity premium and the risk-free rate, and the correlation between these returns, predicted by the i.i.d. subjective transition probability model. A nontrivial region of the admissible parameter space allows the model to match mean returns and the volatility of the risk-free rate, in addition to producing a small negative correlation between the equity premium and risk-free rate. But, as is clear from the column labeled "σ_{eq}," i.i.d. variations in $\tilde{p}$ and $\tilde{q}$ do not raise the implied volatility of the equity premium by a sufficient magnitude, as it is still only one-half to one-third the level of the standard deviation in the sample.

To investigate why the model with i.i.d.

[13] The literature suggests that introducing leverage might help the fixed distorted beliefs model to match the volatility in returns. In Simon Benninga and Aris Protopapadakis (1990) the standard model is shown to match the first two moments of asset returns with relative risk aversion coefficient near 10 and a debt to market value ratio of 0.6, but they also require negative time discounting. Abel (1999) combines a similar degree of leverage with a "catching up with the Joneses" preference structure and matches the volatility of asset returns with positive time preference. We have examined the impact of leverage, of a magnitude found in the actual data, and find that with CRRA utility it does increase volatility, but not by enough to match the data. Furthermore, we suspect that a simple model with leverage would be incapable of matching the persistence and predictability of asset returns.

TABLE 5—MEAN, STANDARD DEVIATIONS, AND CORRELATION WITH I.I.D. SUBJECTIVE TRANSITION PROBABILITIES

Beliefs and preferences						First and second moments				
$\tilde{\mu}_{\tilde{p}}$	$\tilde{\mu}_{\tilde{q}}$	β	γ	$\tilde{\delta}_p$	$\tilde{\delta}_q$	μ_{eq}	μ_f	σ_{eq}	σ_f	$\rho_{eq,f}$
0.5	0.2	0.933	1.831	0.33	0.0	5.25	2.78	5.80	5.5	−0.631
				0.33	0.1	5.25	2.78	5.80	5.5	−0.632
0.5	0.4	0.918	2.371	0.26	0.0	5.29	2.79	5.54	5.5	−0.452
				0.25	0.1	5.29	2.79	5.54	5.5	−0.452
0.6	0.2	0.923	3.159	0.18	0.0	5.32	2.78	5.80	5.5	−0.431
				0.18	0.1	5.32	2.78	5.80	5.5	−0.433
0.6	0.4	0.825	6.985	0.08	0.0	5.43	2.79	6.13	5.5	−0.085
				0.08	0.1	5.43	2.79	6.12	5.5	−0.090
0.7	0.1	0.930	4.691	0.11	0.0	5.36	2.75	6.12	5.5	−0.390
				0.11	0.1	5.36	2.75	6.12	5.5	−0.399
0.7	0.2	0.909	5.795	0.09	0.0	5.38	2.78	6.10	5.5	−0.251
				0.09	0.1	5.38	2.78	6.08	5.5	−0.260
0.8	0.1	0.933	8.300	0.06	0.0	5.40	2.75	6.39	5.5	−0.271
				0.05	0.1	5.40	2.74	6.30	5.5	−0.307

TABLE 6—RESPONSE OF ASSET RETURNS AND PRICE–CONSUMPTION RATIO TO CHANGES IN $\tilde{p}$

β	γ	$\tilde{q}$	$\tilde{p}$	ω	Mean equity return	Mean risk-free rate	Mean equity premium
0.933	1.831	0.2	0.826	16.513	10.260	8.088	2.172
			0.174	17.155	6.137	−2.963	9.099
0.918	2.371	0.4	0.756	16.338	11.399	8.319	3.080
			0.244	17.312	5.133	−2.903	8.036
0.923	3.159	0.2	0.783	16.347	11.502	7.994	3.508
			0.417	17.371	4.933	−2.967	7.900
0.825	6.985	0.4	0.684	16.023	13.525	8.433	5.092
			0.516	17.598	3.360	−2.817	6.177
0.930	4.691	0.1	0.812	16.356	11.690	7.536	4.154
			0.588	17.458	4.635	−2.906	7.542
0.909	5.795	0.2	0.794	16.209	12.533	8.009	4.524
			0.606	17.541	3.991	−2.967	6.957
0.933	8.300	0.1	0.856	16.274	12.379	7.533	4.846
			0.744	17.589	3.978	−2.906	6.884

Notes: The conditional expectations used to compute ω and the risk-free rate are based on $\tilde{p}$ and $\tilde{q}$, whereas the evaluation of the mean equity return and mean equity premium are based on this value of ω, along with the true values of the transition probabilities p and q. All calculations take the current state as expansionary.

randomization of subjective persistence fails to match the second moments, we examine the behavior of the price–consumption ratio and mean asset returns during an expansion. Specifically, we consider a case in which $\tilde{q}$ is fixed and $\tilde{p}$ fluctuates between high and low values by an amount that allows us to match the first moments of asset returns and the volatility of the risk-free rate. This exercise mimics the one in Table 5 for cases with $\delta_{\tilde{q}} = 0$. The results of this exercise are reported in Table 6. As is evident from the last column of the table, variation in the mean equity premium induced by shocks to $\tilde{p}$ fall far short of the 19-percent

TABLE 7—IMPLIED FIRST AND SECOND MOMENTS WITH PERSISTENT BUT RANDOM $\tilde{p}$ AND $\tilde{q}$

Beliefs and preferences							First and second moments					
$\tilde{\phi}_e = \tilde{\phi}_c$	$\tilde{\mu}_{\tilde{p}}$	$\tilde{\delta}_p$	$\tilde{\mu}_{\tilde{q}}$	$\tilde{\delta}_q$	β	γ	μ_{eq}	μ_f	σ_{eq}	σ_f	$\rho_{eq,f}$
0.91	0.71	0.023	0.24	0.181	0.823	9.886	5.5	2.5	19.0	5.5	−0.107
0.94	0.71	0.040	0.24	0.163	0.841	9.118	5.5	2.5	19.0	5.5	−0.119
0.97	0.71	0.052	0.24	0.140	0.859	8.472	5.5	2.5	19.0	5.5	−0.128

standard deviation of the equity premium measured in the data. The price–consumption ratio ω is increasing in expansion state pessimism; but since a switch to pessimism is transitory, the price–consumption ratio subsequently reverts to its mean, implying a future capital loss. Thus, low mean equity returns are associated with episodes of expansion state pessimism. At the same time, precautionary saving increases so that pessimistic episodes are also associated with a low risk-free rate. The upshot is that randomized but serially independent movements in $\tilde{p}$ generate substantial volatility in gross returns, but not in excess returns. Although we have focused our discussion on fluctuations in $\tilde{p}$ with fixed $\tilde{q}$, the price–consumption ratio and mean asset returns exhibit essentially the same behavior during contractions when $\tilde{q}$ switches between high and low values with fixed $\tilde{p}$.

C. *Persistently Evolving Beliefs of State Persistence*

We now relax the restriction that $\tilde{\phi}_e = \tilde{\phi}_c = 0.5$, and allow for persistence in the subjective state transition probabilities. We continue to assume that the shocks are equally persistent ($\tilde{\phi}_e = \tilde{\phi}_c$) and that subjective endowment growth is undistorted ($\tilde{\alpha}_e = \alpha(1) = 2.25$, $\tilde{\alpha}_c = \alpha(0) = -6.79$). We search for combinations of $\tilde{\mu}_{\tilde{p}}$, $\tilde{\mu}_{\tilde{q}}$, $\tilde{\delta}_p$, $\tilde{\delta}_q$, $\tilde{\phi}_e$, and $\tilde{\phi}_c$ that match the mean and volatility of the risk-free rate and equity premium. To narrow the scope of the search, we limited our evaluation to values of $\tilde{\mu}_{\tilde{p}} = 0.71$ and $\tilde{\mu}_{\tilde{q}} = 0.24$, the means of the subjective transition probabilities that are in the neighborhood of our rule-of-thumb calculations.

Volatility of Asset Returns.—Table 7 reports three cases in which the model is able to match all five characteristics of the data with values of the discount factor that are less than 1 and risk-aversion coefficient that are less than 10. In the first line of the table, the representative agent discounts the future at a rate of 17 percent and has a relative risk aversion coefficient of 9.89. The subjective probability that an expansion will continue, given that the economy is currently in an expansion, fluctuates between 0.73 and 0.69, whereas the subjective probability about the persistence of the next contraction fluctuates between 0.06 and 0.42. Individuals display pessimism over expansions, since even the high value of $\tilde{p}$ lies below p. Similarly, since the high value of $\tilde{q}$ lies below q, agents display optimism about contractions. Agents' subjective beliefs about the persistence of the states are themselves persistent. Once agents adopt a view, the probability that these views will change is $1 - \tilde{\phi}_c = 0.09$.

The effect of changes in subjective contraction persistence is large. Consider a simple example where contraction sentiment changes, where agents display mild expansion optimism (relative to their subjective mean of $\tilde{\mu}_{\tilde{p}} = 0.71$, with $\tilde{p} = 0.73$) but become pessimistic about the next contraction, with $\tilde{q}$ shifting from 0.06 to 0.42. This causes the price consumption ratio ω to increase from 16.80 to 28.02, resulting in a sharp jump in equity prices.[14] The risk-free rate, on the other hand, is unaffected by the change so long as the economy is in the expansion state.

Why do serially correlated subjective transition probabilities generate sufficiently volatile excess returns, without an attendant increase in the volatility of risk-free returns? The key lies in the fact that the risk-free rate depends only on

[14] Similarly, had agents initially displayed expansion pessimism ($\tilde{p} = 0.69$), and switched from contraction optimism to contraction pessimism, the price–consumption ratio rises from 18.83 to 31.84.

TABLE 8—PERSISTENCE AND PREDICTABILITY OF EXCESS RETURNS

A. *Beliefs and preferences*

Case	$\tilde{\phi}_e = \tilde{\phi}_c$	$\tilde{\mu}_{\bar{p}}$	$\tilde{\delta}_p$	$\tilde{\mu}_{\bar{q}}$	$\tilde{\delta}_q$	β	γ
I	0.91	0.71	0.023	0.24	0.181	0.823	9.886
II	0.94	0.71	0.040	0.24	0.163	0.841	9.188
III	0.97	0.71	0.052	0.24	0.140	0.859	8.472

B. *Consumption–price ratio summary statistics*

	Data	Case I	Case II	Case III
Mean	21.902	23.895	24.394	26.009
Standard deviation	5.782	6.439	7.950	11.686
ρ_1	0.783	0.814	0.875	0.936
ρ_2	0.590	0.664	0.768	0.877
ρ_3	0.542	0.543	0.674	0.821
ρ_5	0.360	0.365	0.520	0.722

C. *Predictability*

Configuration	Horizon	Slope	R^2	Variance ratio
Data	1	0.148	0.043	1.000
	2	0.295	0.081	1.038
	3	0.370	0.096	0.920
	5	0.662	0.191	0.878
	8	0.945	0.278	0.765
Case I	1	0.192	0.077	1.000
	2	0.352	0.138	0.975
	3	0.488	0.186	0.943
	5	0.692	0.251	0.874
	8	0.881	0.297	0.778
Case II	1	0.127	0.047	1.000
	2	0.242	0.087	1.009
	3	0.346	0.120	1.009
	5	0.520	0.171	0.991
	8	0.710	0.216	0.949
Case III	1	0.070	0.028	1.000
	2	0.138	0.052	1.036
	3	0.203	0.074	1.062
	5	0.321	0.110	1.094
	8	0.472	0.152	1.116

what happens between the current period and the next, whereas the equity return depends on expectations of events over the infinite future. Since the economy is normally in an expansionary state, changes in $\tilde{q}$ reflect changes in beliefs about a state of the world that the economy experiences only infrequently. Changes in $\tilde{q}$ will therefore affect the risk-free rate only on those relatively rare occasions when the economy is actually in a bad state. By contrast, given that the economy is currently in the expansionary state, persistent changes in $\tilde{q}$ affect equity returns through expected marginal utility and expected dividends in the distant, as well as in the immediate, future.

This same mechanism, where fluctuations in beliefs about $\tilde{q}$ during an expansion create movements in equity returns but not the risk-free rate, leads fluctuations in beliefs about $\tilde{p}$ during a contraction state to affect excess returns; however, because the economy is only rarely in the contractionary state, changes in $\tilde{p}$ are relatively unimportant. Indeed, the results in Table 7 are virtually unaffected when we fix $\tilde{\phi}_e = 0.5$, rather than allowing it to switch along with $\tilde{\phi}_c$.

Serial Correlation and Predictability of Returns.—We now move to an examination of the ability of our model to explain the two stylized facts about return persistence and predictability reported in Table 1: (1) variance ratio statistics that decline from above 1.0 at a two-year horizon to 0.8 at an eight-year horizon, and (2) the slope coefficients and R^2's of regressions of longhorizon excess returns on the current log(consumption–price ratio) that increase with horizon.

In Table 8 we present results for three cases, labelled I, II, and III, with parameter configurations reported. The preference and belief parameter configures are reported in panel A of the table (and are identical to those in Table 7). The dynamics of the price–consumption ratio ω, the model's analog to the price–dividend ratio, is key to understanding the evolution of equity returns. For this reason, we begin by presenting selected implied moments of ω in panel A of Table 8, along with sample values for comparison. As noted earlier, changes in ω are driven predominantly by changes in $\tilde{q}$. Consequently, the persistence of ω is governed largely by the persistence of $\tilde{q}$ and the serial correlation of ω for an economy with $\tilde{\phi}_e = \tilde{\phi}_c = 0.91$ roughly matches the persistence found in the data.

Panel B of Table 8 presents the model-implied values of the variance ratios, the regression slope coefficients, and the R^2's, at horizons of 1, 2, 3, 5, and 8 years. To account for the small-sample bias in these statistics, we generate them using a Monte Carlo experiment. In each trial of the experiment, an artificial sequence of 123 equity and risk-free returns and consumption–price ratio are generated from the model, corresponding to the 123 annual observations that we have in the data. A regression of the k-period excess return on the log(consumption–price ratio) $\log(1/\omega)$, for $k = 1, 2, 3, 5$, and 8, is then estimated to obtain a slope coefficient and an R^2. The slope coefficients and R^2's that we report are the mean values from 1 million trials.

The simulated data match the sample values nearly exactly. The performance of the model follows directly from the mean reversion of the price–consumption ratio, as excess returns inherit their stochastic properties from ω. When the process is away from its mean, the predicted deviation from its current level is increasing in the horizon. It is in this manner that a mean-reverting price–consumption ratio implies both predicted returns and predictability that increase with the horizon. Again, the economy with $\tilde{\phi}_e = \tilde{\phi}_c = 0.91$ provides the closest match to the data. When $\tilde{\phi}_e = \tilde{\phi}_c$ exceeds 0.91, the mean reversion in the price–consumption ratio is too slow to generate the short-horizon predictability observed in the data.

The results for the variance ratio statistics are nearly identical to those for the regressions. This is not all that surprising, as both are functions of the same underlying autocorrelations of the consumption–price ratio.

Cyclicality of the Equity Premium.—Next, we compare the implied cyclical pattern of the equity premium with that in the data. Using the parameter values from the configuration labeled case I in Table 8, and evaluating the mean equity premium under various states of endowment growth, the model produces a coarse correspondence between the predicted and actual cyclicality of the equity premium. The model predicts that the mean equity premium during transitions from expansion to expansion is 6.18 percent, whereas the sample counterpart is 10.54 percent (See panel C of Table 1). The predicted equity premium during transitions from contraction to contraction is -10.52 percent in the model, whereas the average value in the data is -14.63 percent. The model's predicted mean equity premium over periods ending in expansion is 6.10 percent, whereas the sample counterpart is 13.66 percent. Finally, the predicted mean equity premium over periods ending in contraction is -7.34 percent with sample counterpart -11.24 percent.[15] The cyclicality of the equity premium emerges because high endowment growth during expansions works to raise the return on equity, whereas the systematic distortions of expansion pessimism and contraction optimism contribute to a countercyclical risk-free rate.

Cyclicality of the Price–Dividend Ratio.—It is common for asset-pricing models based on CRRA preferences with $\gamma > 1$ to produce countercyclical price–dividend ratios. Although our model shares this counterfactual property, it is attenuated by the introduction of distorted beliefs.

[15] To compute the sample counterparts, we used the NBER monthly reference cycle dates to classify episodes of expansion and contraction.

Evaluating the model-implied price–dividend ratios for eight possible states, using the preference–belief configuration of case I in Table 8, we find that the implied mean price–dividend ratio is 23.87 over expansions and 24.36 over contractions—implying a mildly countercyclical price–dividend ratio. The extent of the cyclicality is neither quantitatively important, nor grossly at odds with the empirical values of 23.2 over expansions and 19.5 over contractions.

We note that there do exist scenarios in which the price–dividend ratio in our model falls during a contraction. For example, if investors display pessimism about p and optimism about q during the transition from expansion to contraction, the price–dividend ratio will drop by 18.4 percent, from 18.83 to 15.37.

D. *Distorted Subjective Endowment Growth*

We now switch our focus from distortions in the transition probabilities to the role of distortions in subjective endowment growth rates. We isolate this effect by setting $\tilde{\delta}_p = \tilde{\delta}_q = 0$ and fix $\tilde{p} = 0.71$ and $\tilde{q} = 0.24$, values that are close to the mean values of $\tilde{\mu}_{\tilde{p}}$ and $\tilde{\mu}_{\tilde{q}}$ of Section IV, subsection C. We restrict the state variables driving subjective growth during expansions and during contractions to be equally persistent ($\tilde{\phi}_e = \tilde{\phi}_c$), allow subjective growth rates to fluctuate about their objective mean values ($\tilde{\mu}_e = \alpha(1) = 2.25$, $\tilde{\mu}_c = \alpha(0) = -6.78$) and search over values of $\tilde{\tau}_e$, $\tilde{\tau}_c$, $\tilde{\phi}_e = \tilde{\phi}_c$, β, and γ in an attempt to mimic the behavior found in the data.

Although it is possible to match the first and second moments of the asset-return data when we allow subjective growth rates to fluctuate—with $\tilde{\phi}_e = \tilde{\phi}_c = 0.98$, $\tilde{\tau}_e = 0.53$, $\tilde{\tau}_c = 1.5$, $\beta = 0.88$, and $\gamma = 7.49$—distorting beliefs in this dimension fails to resolve the predictability puzzle. With this set of preference and belief parameters, the eight-year horizon regression slope coefficient is 0.08 with $R^2 = 0.08$, whereas the variance ratios for implied excess returns slightly exceed 1 for horizons of eight years and less. The failure to match this feature of the data is a result of the fact that switches in subjective growth induce correlated movements in equity returns and in the risk-free rate. A switch in subjective contractionary state growth causes fluctuations in both equity and risk-free returns during an expansion, whereas switches in subjective expansionary state growth lead to fluctuations in both returns during contractions. Subjective growth rates need to be very persistent to generate sufficiently high volatility in the equity premium; at the same time, however, these beliefs are associated with log(consumption–price ratio)s that are so persistent that they fail to predict future excess returns.

E. *Irrational Forecasts of Beliefs*

In our distorted beliefs model, agents exhibit irrationality only to the extent that they have the wrong model in mind. Within the context of their subjective model, they continue to behave rationally. They are irrational in responding to pseudosignals, but rational in conditioning their behavior on the fact that they do respond to the signals and know their true stochastic process. In this sense, agents rationally forecast their own irrationality. Our final task is to investigate the consequences of relaxing this assumption. We do this by creating a distinction between the agent's subjective transition probability $\tilde{\phi}_e = \tilde{\phi}_c$ and the "true" pseudosignal transition probabilities $\phi_e = \phi_c$.

We have considered three alternative values of the "truth," $\phi_e = \phi_c = 0.89$, 0.90, and 0.91, corresponding to the parameter configurations in case I of Table 8. For each of these values, we searched over the subjective beliefs of pseudosignal persistence $\tilde{\phi}_e = \tilde{\phi}_c$ and an associated preference–belief configuration that allowed us to match the unconditional moments, predictability, and mean reversion of asset returns. We are able to find values of $\tilde{\phi}_e = \tilde{\phi}_c$ that deviate from $\phi_e = \phi_c$ that accomplish this task.

There are three main features of this exercise that are worth pointing out. First, the model fits the data with preferences that are (arguably) more reasonable than those in Table 8. For example, when $\phi_e = \phi_c = 0.91$ and $\tilde{\phi}_e = \tilde{\phi}_c = 0.934$, the model requires $\gamma = 7.22$ and $\beta = 0.88$, as compared with values of (9.886, 0.823) in Table 8. Second, the model fits the data when investors believe the pseudosignals are more persistent than the "truth" ($\tilde{\phi}_e = \tilde{\phi}_c > \phi_e = \phi_c$). The model does not match the predictability pattern when agents believe the signals to be *less* persistent than the truth ($\tilde{\phi}_e = \tilde{\phi}_c < \phi_e = \phi_c$). Third, the beliefs about the persistence of pseudosignals that fit the data could be quite significantly distorted. For example, the model fits the data when $\phi_e = \phi_c = 0.89$ and $\tilde{\phi}_e = \tilde{\phi}_c = 0.93$. This configuration implies that investors believe the expected duration of a signal is 14.3 years, whereas in "reality" it is only 9.1 years.

From this exercise we conclude that allowing investors to rationally forecast the evolution of their subjective beliefs is largely innocuous. To fit the data, the investors can be irrational in forecasting their own irrationality, so long as the irrationality is not too large.

V. Conclusion

We have shown that a simple aggregate asset-pricing model, in which agents have distorted beliefs about persistence of the state-transition probabilities of the endowment growth process, can replicate a number of features of observed asset-returns data. We are able to match the means, standard deviations, and correlation of the equity premium and the risk-free rate, as well as the persistence and predictability of long-horizon excess equity returns. In addition, the predicted cyclical patterns of the equity premium roughly correspond to those in the data. This is accomplished within the framework of a representative-agent endowment economy, with CRRA preferences, and a two-state Markov process governing per capita consumption growth. The simplicity of our approach provides an attractive alternative to the strategy of introducing either increasingly complex preference specifications, heterogeneity, or market incompleteness, to address the failures of the standard model.

Success in our task requires that subjective beliefs be distorted in two ways. First, agents must think that both expansions and contractions will be less persistent than what is implied by careful econometric analyses of the data. Second, agents' views of these transition probabilities must exhibit stochastic and persistent variation of a very specific type. Our formulation can be criticized for its inability or unwillingness to allow agents to learn over time. We suspect, however, that if agents were Bayesian learners, convergence would occur reasonably fast and that over the time span of our data set, they would have learned by now to be rational. Under CRRA utility, this provides little help in understanding asset returns. On the other hand, if the true endowment process is stationary, with the passage of sufficient time, any rule-of-thumb estimates of the transition probabilities should converge to the truth. Exactly how much time is required for convergence to occur is another issue. It is entirely likely that slow learning describes the real-world environment and that 104 years of data is still too short to overcome this small sample problem.

Appendix

The eight possible outcomes for the distorted beliefs state vector, $\bar{\mathbf{S}} \equiv (S, S_e, S_c)$ are, $\bar{\mathbf{S}}_1 = (1, 1, 1)$, $\bar{\mathbf{S}}_2 = (1, 1, -1)$, $\bar{\mathbf{S}}_3 = (1, -1, 1)$, $\bar{\mathbf{S}}_4 = (1, -1, -1)$, $\bar{\mathbf{S}}_5 = (0, 1, 1)$, $\bar{\mathbf{S}}_6 = (0, 1, -1)$, $\bar{\mathbf{S}}_7 = (0, -1, 1)$, $\bar{\mathbf{S}}_8 = (0, -1, -1)$.

Let $p_o = \tilde{\mu}_{\tilde{p}} + \tilde{\delta}_e$, $p_p = \tilde{\mu}_{\tilde{p}} - \tilde{\delta}_e$, $q_o = \tilde{\mu}_{\tilde{q}} - \tilde{\delta}_c$, $q_p = \tilde{\mu}_{\tilde{q}} + \tilde{\delta}_c$, $p_o^* = 1 - p_o$, $p_p^* = 1 - p_p$, $q_o^* = 1 - q_o$, $q_p^* = 1 - q_p$, $\tilde{\phi}_e^* = 1 - \tilde{\phi}_e$, $\tilde{\phi}_c^* = 1 - \tilde{\phi}_c$. Then the transition matrix for the state vector $\bar{\mathbf{S}}$ is

$$
\tilde{\mathbf{P}} = \begin{bmatrix}
p_o\tilde{\phi}_e\tilde{\phi}_c & p_o\tilde{\phi}_e\tilde{\phi}_c^* & p_o\tilde{\phi}_e^*\tilde{\phi}_c & p_o\tilde{\phi}_e^*\tilde{\phi}_c^* & p_o^*\tilde{\phi}_e\tilde{\phi}_c & p_o^*\tilde{\phi}_e\tilde{\phi}_c^* & p_o^*\tilde{\phi}_e^*\tilde{\phi}_c & p_o^*\tilde{\phi}_e^*\tilde{\phi}_c^* \\
p_o\tilde{\phi}_e\tilde{\phi}_c^* & p_o\tilde{\phi}_e\tilde{\phi}_c & p_o\tilde{\phi}_e^*\tilde{\phi}_c^* & p_o\tilde{\phi}_e^*\tilde{\phi}_c & p_o^*\tilde{\phi}_e\tilde{\phi}_c^* & p_o^*\tilde{\phi}_e\tilde{\phi}_c & p_o^*\tilde{\phi}_e^*\tilde{\phi}_c^* & p_o^*\tilde{\phi}_e^*\tilde{\phi}_c \\
p_p\tilde{\phi}_e^*\tilde{\phi}_c & p_p\tilde{\phi}_e^*\tilde{\phi}_c^* & p_p\tilde{\phi}_e\tilde{\phi}_c & p_p\tilde{\phi}_e\tilde{\phi}_c^* & p_p^*\tilde{\phi}_e^*\tilde{\phi}_c & p_p^*\tilde{\phi}_e^*\tilde{\phi}_c^* & p_p^*\tilde{\phi}_e\tilde{\phi}_c & p_p^*\tilde{\phi}_e\tilde{\phi}_c^* \\
p_p\tilde{\phi}_e^*\tilde{\phi}_c^* & p_p\tilde{\phi}_e^*\tilde{\phi}_c & p_p\tilde{\phi}_e\tilde{\phi}_c^* & p_p\tilde{\phi}_e\tilde{\phi}_c & p_p^*\tilde{\phi}_e^*\tilde{\phi}_c^* & p_p^*\tilde{\phi}_e^*\tilde{\phi}_c & p_p^*\tilde{\phi}_e\tilde{\phi}_c^* & p_p^*\tilde{\phi}_e\tilde{\phi}_c \\
q_o^*\tilde{\phi}_e\tilde{\phi}_c & q_o^*\tilde{\phi}_e\tilde{\phi}_c^* & q_o^*\tilde{\phi}_e^*\tilde{\phi}_c & q_o^*\tilde{\phi}_e^*\tilde{\phi}_c^* & q_o\tilde{\phi}_e\tilde{\phi}_c & q_o\tilde{\phi}_e\tilde{\phi}_c^* & q_o\tilde{\phi}_e^*\tilde{\phi}_c & q_o\tilde{\phi}_e^*\tilde{\phi}_c^* \\
q_p^*\tilde{\phi}_e\tilde{\phi}_c^* & q_p^*\tilde{\phi}_e\tilde{\phi}_c & q_p^*\tilde{\phi}_e^*\tilde{\phi}_c^* & q_p^*\tilde{\phi}_e^*\tilde{\phi}_c & q_p\tilde{\phi}_e\tilde{\phi}_c^* & q_p\tilde{\phi}_e\tilde{\phi}_c & q_p\tilde{\phi}_e^*\tilde{\phi}_c^* & q_p\tilde{\phi}_e^*\tilde{\phi}_c \\
q_o^*\tilde{\phi}_e^*\tilde{\phi}_c & q_o^*\tilde{\phi}_e^*\tilde{\phi}_c^* & q_o^*\tilde{\phi}_e\tilde{\phi}_c & q_o^*\tilde{\phi}_e\tilde{\phi}_c^* & q_o\tilde{\phi}_e^*\tilde{\phi}_c & q_o\tilde{\phi}_e^*\tilde{\phi}_c^* & q_o\tilde{\phi}_e\tilde{\phi}_c & q_o\tilde{\phi}_e\tilde{\phi}_c^* \\
q_p^*\tilde{\phi}_e^*\tilde{\phi}_c^* & q_p^*\tilde{\phi}_e^*\tilde{\phi}_c & q_p^*\tilde{\phi}_e\tilde{\phi}_c^* & q_p^*\tilde{\phi}_e\tilde{\phi}_c & q_p\tilde{\phi}_e^*\tilde{\phi}_c^* & q_p\tilde{\phi}_e^*\tilde{\phi}_c & q_p\tilde{\phi}_e\tilde{\phi}_c^* & q_p\tilde{\phi}_e\tilde{\phi}_c
\end{bmatrix}.
$$

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