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Reconfigurable control system design via perfect model following

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A novel model-following approach is developed to design reconfigurable control systems. The conventional state-space linear model-following approach to control is first re-examined with emphasis on the conditions for perfect model following and its application to reconfigurable control system design. New frequency domain necessary and sufficient conditions for perfect model following are then obtained and they are used to gain insight into the selection of the reference model and to develop a new design approach. This novel design approach yields fewer constraints on the reference model than before, and provides much greater flexibility in specifying the state trajectories of the impaired system.

1. Introduction

Reconfigurable control systems (RCSs) are control systems that possess the ability to accommodate system failures automatically based upon *a priori* assumed conditions. The research in this area is largely motivated by control problems encountered in aircraft control system design. In that case, the ideal goal is to achieve so-called 'fault-tolerant' or 'self-repairing' capability in the flight control systems, so that unanticipated failures in the system can be accommodated and the airplane can be, at least, landed safely whenever possible. Owing to the time constraints in many failure scenarios, the control law redesign process must be automated and the algorithms used should be as numerically efficient as possible.

In the context of RCS, the idea of controlling the impaired system so that it is 'close', in some sense, to the nominal system has been explored in numerous publications. Proposed methods include, but are not limited to, linear-quadratic (LQ) control methodology (Looze *et al.* 1985, Joshi 1987, Huang and Stengel 1990, etc.), adaptive control systems (Morse and Ossman 1990, Pogoda and Maybeck 1989, Dittmar 1988, etc.), eigenstructure assignment (Gavito and Collins 1987, Napolitano and Swaim 1991, etc.), knowledge-based systems (Huang 1989, Handelman and Stengel 1989, etc.), among others. Furthermore, it will be shown in this paper that a widely used design approach for RCS, the pseudo-inverse method (PIM) (Caglayan *et al.* 1988, Huber and McCulloch 1984, Ostroff 1985, Raza and Silverthorn 1985, Rattan 1985) is in fact a special case of classical linear model-following control (LMF).

The PIM is attractive because of its simplicity in computation and implementation. The objective in this method is to adjust the constant feedback gain, assuming that such gain is used in the nominal system, so that the reconfigured system approximates the nominal system in some sense. A measure of closeness

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between the systems before and after the failure is the Frobenius norm of the difference in closed-loop 'A' matrices. It was shown by Gao (1990) and Gao and Antsaklis (1989, 1991) that by minimizing this norm the bound on the variations of the closed-loop eigenvalues due to failures is minimized. Note that a drawback of this method is that the stability of the impaired system is not guaranteed and this may lead to unacceptable behaviour in certain failure scenarios. A modified version of PIM (MPIM) was proposed to address the stability issue, by which the difference of the closed-loop 'A' matrices is minimized subject to some stability constraints (Gao 1990, Gao and Antsaklis 1991).

The LMF control system has been studied for over a decade and can be found in many text books on control (see, for example, Landau 1979). The objective of such systems is to make the trajectories of the output or state of a physical plant close to that of a reference model that exhibits desired behaviour. The design process of such systems is straightforward and the resulting control systems possess simple structures with only the constant gains to be implemented in the feedback and feedforward path. This type of control configuration has been widely used in model-reference adaptive control systems. Note that the LMF approach has been mainly studied in the state-space domain (Erzberger 1968, Chen 1973, Landau 1979).

There is a similarity between the design objectives of the LMF and the PIM in terms of making one system, the plant or the impaired system, imitate the reference model (in LMF) or the nominal system (in PIM). In this paper, the conventional LMF approach in control system design will be examined in the context of RCS. An important characteristic of the LMF control system design is whether the plant can follow the reference model exactly, which is referred to as perfect model following (PMF). Obviously, PMF is desirable since it enables us to specify the behaviour of a system completely. Without achieving PMF, the LMF approach cannot specify in advance how close the trajectory of the plant to that of the reference model should be and this arbitrariness may not be acceptable in certain control applications such as the RCS. On the other hand, in conventional LMF the conditions for PMF, known as Erzberger's conditions, put severe constraints on the reference model and therefore make it in many cases impracticable to obtain the PMF.

In the following sections, new conditions are derived in the frequency domain for the PMF that are necessary and sufficient, as opposed to Erzberger's conditions which are only sufficient. These new conditions give much insight and intuition on how to choose the reference model. They also show the limitations of the conventional LMF approach. Furthermore, based on these new conditions of the PMF in the frequency domain, a new design approach is developed to achieve PMF with many fewer constraints on the reference model and the plant. In this approach, the compensators designed could be either static or dynamic depending on the reference model chosen. The stability robustness of such a system is also studied.

In § 2, the standard state-space LMF approaches are discussed and the conditions for PMF are analysed. In § 3, a new design approach is developed to achieve PMF with fewer constraints on the reference model. This new approach is shown to provide better performance for the reconfigured system. Examples are included for illustration. Finally, concluding remarks are given.

2. Standard linear model following methods

The linear model-following approach to control is a state-space design methodology by which a control system is designed to make the output of the plant follow the output of a model system with the desired behaviour. In this approach the design objectives are incorporated into the reference model and both feedback and feedforward controllers are used, which are typically of zero order. By using a reference model to specify the design objectives, a difficulty in control system design is avoided, namely that the design specifications should be expressed directly in terms of the controller parameters.

Assume that the plant and the reference model are of the same order. Let the reference model be given as

$$\left.\begin{aligned} \dot{x}_m &= A_m x_m + B_m u_m \\ y_m &= C_m x_m \end{aligned}\right\} \tag{1}$$

and the plant be represented by

$$\left.\begin{aligned} \dot{x}_p &= A_p x_p + B_p u_p \\ y_p &= C_p x_p \end{aligned}\right\} \tag{2}$$

where x_m, $x_p \in \mathbb{R}^n$, u_m, $u_p \in \mathbb{R}^m$, A_m, $A_p \in \mathbb{R}^{n\times n}$, B_m, $B_p \in \mathbb{R}^{n\times m}$, C_m, $C_p \in \mathbb{R}^{p\times n}$. The corresponding transfer function matrices of the reference model and the plant are

$$T_m(s) = C_m(sI - A_m)^{-1}B_m \tag{3}$$

$$P(s) = C_p(sI - A_p)^{-1}B_p \tag{4}$$

Let $e(t)$ represent the difference in the state variables

$$e(t) = x_m(t) - x_p(t) \tag{5}$$

To achieve the PMF, one must insure that for any u_m, piecewise continuous, and $e(0) = 0$, we shall have $e(t) \equiv 0$ for all $t > 0$.

Next, we discuss under what conditions the PMF is possible and how to find the feedback and feedforward controllers to achieve the PMF. In the cases when the PMF cannot be achieved, it is shown how the error can be minimized. These results have been described by Landau (1979).

2.1. *Implicit LMF*

In the control system configuration of implicit LMF, the reference model does not appear explicitly. Instead, the model is used to obtain the control parameters, k_u and k_p. From (1) and (2), by simple manipulation we have

$$\dot{e} = A_m e + (A_m - A_p)x_p + B_m u_m - B_p u_p \tag{6}$$

From the control configuration, the control input u_p has the form

$$u_p = k_p x_p + k_u u_m \tag{7}$$

The PMF is achieved if the control parameter k_u and k_p are chosen such that

$$\dot{e} = A_m e \tag{8}$$

or, equivalently,

$$(A_m - A_p)x_p + B_m u_m - B_p u_p \equiv 0 \tag{9}$$

Note that if a solution u_p of (9) exists, it will take the form

$$u_p = B_p^+(A_m - A_p)x_p + B_p^+ B_m u_m \tag{10}$$

where B_p^+ represents the pseudo-inverse of the matrix B_p. From (10) k_u and k_p can be found as $k_p = B_p^+(A_m - A_p)$, and $k_u = B_p^+ B_m$. By substituting (10) in (6), a sufficient condition for the existence of the solution of (9) is

$$\left.\begin{aligned}(I - B_p B_p^+)(A_m - A_p) &= 0\\ (I - B_p B_p^+)B_m &= 0\end{aligned}\right\} \tag{11}$$

Note that the equalities are known as Erzberger's conditions (Erzberger 1968). Clearly, these are rather restrictive conditions, since most systems have more states than inputs, $B_p B_p^+ \neq I$. Thus (11) can only be fulfilled when $(I - B_p B_p^+)$ is in both the left null spaces of $(A_m - A_p)$ and B_m. It seems that, for an arbitrary plant, it is rather difficult to find an appropriate reference model that represents the desired dynamics and at the same time satisfies (11).

It should be noted that even when the conditions for the PMF in (11) are not fulfilled, the solution in (10) still minimizes the 2-norm of the last three terms in the right side of (6), i.e. $\|(A_m - A_p)x_p + B_m u_m - B_p u_p\|_2 = \|\dot{e} - A_m e\|_2$.

This particular method of choosing u_p has the advantage of not involving x_m in the feedback thus eliminating the need for running the model on line. Therefore the complexity of the control system is relatively low. One of the disadvantages of this method is that when the PMF is not achievable, the trajectory of e may not be desirable since we do not have control over the location of the poles in the system. Another disadvantage with this approach is that, when the conditions in (11) are not satisfied, the solution (10) may result in an unstable system. This drawback will be further discussed in the next section when the relationship between the implicit LMF and the PIM method is explored.

2.2. *Explicit LMF*

A typical configuration of the explicit LMF is as illustrated in Figure 2. In the configuration of the explicit LMF, the reference model is actually implemented as part of the controller. To compare with the implicit LMF, let $e = x_m - x_p$, or equivalently, assume $C_m = I$ and $C_p = I$. By manipulating (1) and (2), $\dot{e}$ can also be written as

$$\dot{e} = A_p e + (A_m - A_p)x_m + B_m u_m - B_p u_p \tag{12}$$

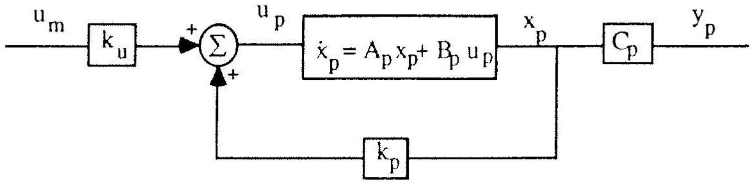

Figure 1. The control configuration of the implicit LMF.

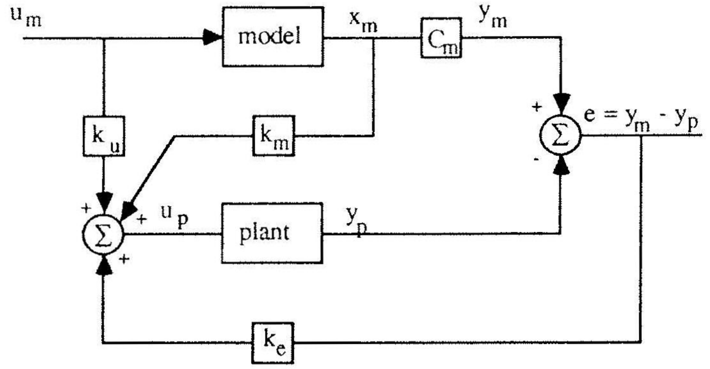

Figure 2. The control configuration of the explicit LMF.

Let the control input be

$$u_p = u_1 + u_2 = (k_e e) + (k_m x_m + k_u u_m) \tag{13 a}$$

where $u_1 = k_e e$ is the stabilizing gain, and $u_2 = k_m x_m + k_u u_m$ is to be determined to minimize $\|(A_m - A_p)x_p + B_m u_m - B_p u_2\|_2 = \|\dot{e} - (A_p - B_p k_e)e\|_2$. From (12), it can easily be shown that the sufficient conditions for the PMF are exactly the same as those given in (11) and the corresponding control gains are

$$\left.\begin{aligned} k_m &= B_p^+(A_m - A_p) \\ k_u &= B_p^+ B_m \end{aligned}\right\} \tag{13 b}$$

with k_e any stabilizing gain. Substituting (13 a) and (13 b) in (12), the equation of error is

$$\dot{e} = (A_p - B_p k_e)e + (I - B_p B_p^+)(A_m - A_p)x_m + (I - B_p B_p^+)B_m u_m \tag{14}$$

When the conditions in (11) are met, we have

$$\dot{e} = (A_p - B_p k_e)e \tag{15}$$

In this approach, if the plant is stabilizable we can guarantee the stability of the closed-loop system by choosing k_e appropriately, regardless of whether the conditions in (11) are met or not. This can be illustrated as follows. Since $\{A_p, B_p\}$ is stabilizable, k_e can be chosen such that $(A_p - B_p k_e)$ has all its eigenvalues in the left half-plane; furthermore, let

$$f(t) = (I - B_p B_p^+)(A_m - A_p)x_m(t) + (I - B_p B_p^+)B_m u_m(t)$$

then (14) can be expressed as $\dot{e} = (A_p - B_p k_e)e + f(t)$. Because the reference model in (1) is stable, x_m will be bounded and therefore $f(t)$ will be bounded for any bounded u_m. This implies that (14) is bounded-input bounded-output (BIBO) stable.

A challenging problem in the LMF approach is to choose the reference model appropriately. Not only must it reflect the desired system behaviour, but it also must be reasonably chosen so that the plant can follow its trajectory closely. Erzberger's conditions give indications on the constraints of the reference model for the PMF. It can be used to check whether the existing reference model satisfies the PMF conditions. However, as we can see in (11), it does not give much information on how to select (A_m, B_m, C_m). In the design process, what is needed is a guideline that can be used to select the reference model so that it will satisfy the Erzberger's condition. We shall look into the frequency domain for explanations of the PMF conditions to gain additional insight to the problem.

2.3. *Using the explicit LMF in RCS design*

The standard LMF control systems described above can be directly applied to the RCS design. Here, the explicit LMF approach seems to have significant advantages over the implicit one since it guarantees stability (if the plant is stabilizable). In the following example, this method is used to reconfigure a flight control system. Assume that the plant is the impaired open-loop system. To begin the design process, one needs to choose a reference model first. In general, the reference model represents the desired behaviour of the closed-loop system. For RCS, it is convenient to use the state-space model of the nominal closed-loop system as the reference model. That is, the goal of the control reconfiguration is to make the state of impaired system, x_p, as close to that of the nominal system, x_m, as possible. Once the reference model is chosen, there are three feedback and feedforward gains, k_u, k_m and k_e, to be determined (13).

Example 1: Let the nominal plant be

$$A = \begin{bmatrix} -0{\cdot}0507 & -3{\cdot}861 & 0{\cdot}0 & -32{\cdot}17 \\ -0{\cdot}0012 & -0{\cdot}5164 & 1{\cdot}0 & 0{\cdot}0 \\ -0{\cdot}0001 & 1{\cdot}4168 & -0{\cdot}4932 & 0{\cdot}0 \\ 0{\cdot}0 & 0{\cdot}0 & 1{\cdot}0 & 0{\cdot}0 \end{bmatrix}, \quad B = \begin{bmatrix} 0{\cdot}0 \\ -0{\cdot}0717 \\ -1{\cdot}645 \\ 0{\cdot}0 \end{bmatrix}$$

$$C = \begin{bmatrix} 0{\cdot}0 \\ 0{\cdot}0 \\ 1{\cdot}0 \\ 0{\cdot}0 \end{bmatrix}, \quad D = 0$$

and the closed-loop nominal system is $\{A - Bk, B, C, D\}$, where

$$k = [-0{\cdot}0043, -3{\cdot}872, -0{\cdot}7186, -0{\cdot}0988]$$

For a hypothetical impaired plant $\{A_f, B_f, C_f, D_f\}$, assume

$$A_f = A, \quad B_f = \begin{bmatrix} 0{\cdot}0 \\ -0{\cdot}0717 \\ -0{\cdot}1645 \\ 0{\cdot}0 \end{bmatrix}, \quad C_f = C, \quad D_f = D$$

In failure accommodation, the reference model $\{A_m, B_m, C_m, D_m\}$ is chosen as the nominal closed-loop system, i.e. $A_m = A - Bk$, $B_m = B$, $C_m = C$ and $D_m = D$. Now the remaining task is to assign the feedback and feedforward gain matrices $\{k_e, k_u, k_m\}$. In this example the open-loop plant is unstable, thus the stabilizing gain, k_e, must be implemented. Such stabilizing gain was obtained using the LQR control design approach; where $k_e = [0{\cdot}2925\ -8{\cdot}83\ -13{\cdot}86\ -16{\cdot}74]$ is such stabilizing gain. The feedforward gain matrices, k_u and k_m are determined by $k_m = B_p^+(A_m - A_p)$ and $k_u = B_p^+ B_m$ as in (13) and they are $k_m = [0{\cdot}0367\ 33{\cdot}16\ 6{\cdot}15\ 0{\cdot}85]$, and $k_u = 8{\cdot}56$. To simulate the closed-loop system, its state-space description is derived having the form

$$\begin{bmatrix} \dot{x}_m \\ \dot{x}_p \end{bmatrix} = \begin{bmatrix} A_m & 0 \\ B_p(k_m + k_e) & A_p - B_p k_e \end{bmatrix} \begin{bmatrix} x_m \\ x_p \end{bmatrix} + \begin{bmatrix} B_m \\ B_p k_u \end{bmatrix} u_m$$

The closed-loop system is simulated using the initial conditions $x_p(0) = [0\ 0\ 0\ 0\ {\cdot}1]$ and $x_m(0) = [0\ 0\ 0\ 0\ {\cdot}1]$, and zero input. The outputs of the reference model and the plant are very close to each other, as shown in Figure 3. Note that the performance of the reconfigured system achieved here is much superior to that obtained by using the MPIM approach under exactly the same conditions (Gao and Antsaklis 1991). The PIM approach, in this example, results in an unstable closed-loop system. On the other hand, however, there is no guarantee that the explicit LMF can always achieve the performance as good as this one. In fact, it is shown in simulations for different failures that the system response is very much dependent on the types of failure that occurred.

It seems that the explicit LMF is more computationally efficient than the MPIM since the stability issue is resolved directly by using the stabilizing gain k_e. In contrast, to guarantee the stability of reconfigured systems using MPIM when the PIM solution cannot stabilize the system, a stabilizing gain must be determined first. Then, the stability bounds of the parameters in the gain matrix must be found, which is quite time consuming. Finally these bounds are used to adjust the feedback gains to obtain better performance. In general, it seems that the explicit LMF approach is well suited for the reconfigurable control problem.

2.4. *Properties of the LMF control systems*

The state-space model-following approaches shown above seems to have a simple control structure and their design philosophy seems to fit into the framework of reconfigurable control quite well. For these approaches, simple constant feedback controllers are used to regulate the state trajectories of the plant so that they follow the state of the reference model. To use it effectively in reconfigurable control, we need to gain better understanding of the LMF method. One of the vital properties of such systems is the system stability: is the

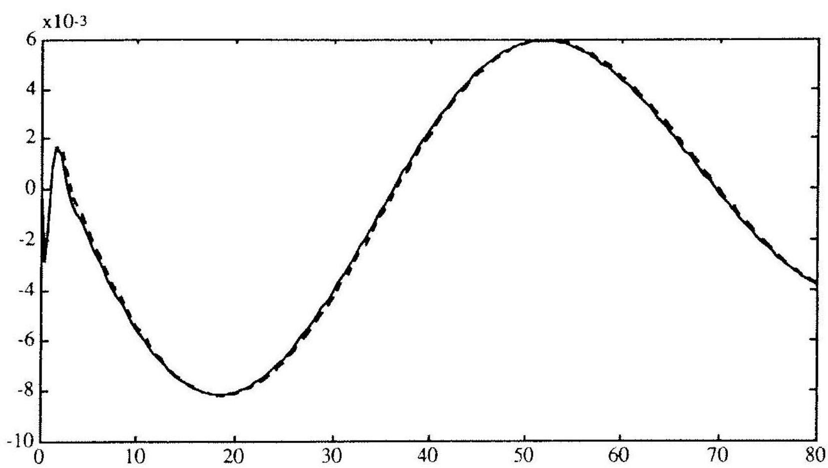

Figure 3. The outputs of the nominal and reconfigured closed-loop system using the explicit LMF approach.

system stability always guaranteed and does it have robust stability properties with respect to uncertainties in the plant model?

It is also of interest to see how the perfect match can be achieved between the plant and the reference model. The PMF is important because it enables us to specify the system behaviour completely. In this section, the frequency domain interpretation of the conditions of the PMF is studied. It is shown that, in contrast to Erzberger's conditions, the conditions in the frequency domain can be used directly in choosing the reference model in the design process. The stability robustness of the linear model-following system is also analysed.

Lemma 1: *Assume $C_m = I$, $C_p = I$. A necessary and sufficient condition for the PMF is that the reference model T_m be obtained from the model of the plant using constant state feedback $u_p = fx_p + gr$.*

Proof: In the configuration of the explicit LMF, it is straightforward to find that the transfer function matrix between u_m and e is:

$$e = (I + Pk_e)^{-1}(T_m - P(k_m T_m + k_u))u_m \tag{16}$$

where T_m and P are defined in (3) and (4). This is derived as follows:

$$e = x_m - x_p = T_m u_m - P(k_e e + k_m T_m u_m + k_u u_m)$$

therefore

$$\begin{aligned}(I + Pk_e)e &= T_m u_m - P(k_m T_m u_m + k u_m)\\ &= (T_m - P(k_m T_m + k_u))u_m\end{aligned}$$

thus giving (16).

From (16), clearly $e \equiv 0$ for all u_m if and only if

$$T_m - P(k_m T_m + k_u) = 0 \tag{17}$$

which implies

$$T_m = (I - Pk_m)^{-1} Pk_u \tag{18}$$

and we have $f = k_m$ and $g = k_u$. □

Compare Lemma 1 to Erzberger's conditions (11), Lemma 1 gives necessary and sufficient conditions for the PMF which show exactly how to select the reference model. It makes good practical sense in that if a plant is designed to follow an artificial system exactly via constant state feedback, the artificial system must have the same basic structure as that of the plant. In fact, the Erzberger conditions can be seen as a special case of Lemma 1 since they are simply the sufficient conditions for $A_m = A_p + B_p k_p$, and $B_m = B_p k_u$. It also shows the limitation of this state-space approach in that the reference models a plant can follow exactly in this configuration are those which have the same zero structures as those of the plant. This is because the system zeros cannot be changed by state feedback unless they are cancelled by the closed-loop poles.

For a successful implementation of PMF, it is important that k_e provides robust stability with respect to plant parameter uncertainty. Note that k_e is the only design parameter that affects the stability robustness as shown below. Here, robust stability means that if the real plant is $\bar{P}$ instead of P, where $\bar{P} = P + \Delta P$ for some small ΔP, the closed-loop system should still be stable. In the following, the stability robustness is examined for the closed-loop system designed by using the explicit LMF to achieve the PMF.

Lemma 2: *For the control gains* $\{k_e, k_m, k_u\}$ *obtained by using the explicit LMF described in* (13), *the closed-loop system from* u_m *to* e *is stable if, for the real plant* $\bar{P} = P + \Delta P$, $(I - \bar{P}k_e)^{-1}\Delta P$ *is stable.*

Proof:

$$\begin{aligned} e &= (I + \bar{P}k_e)^{-1}(T_m - \bar{P}(k_m T_m + k_u))u_m \\ &= (I + \bar{P}k_e)^{-1}((I - \bar{P}k_m)T_m - \bar{P}k_u))u_m \\ &= (I + \bar{P}k_e)^{-1}((I - Pk_m)T_m - \Delta P k_m T_m - \bar{P}k_u))u_m \\ &= (I + \bar{P}k_e)^{-1}(Pk_u - \Delta P k_m T_m - \bar{P}k_u))u_m \\ &= (I + \bar{P}k_e)^{-1}(-\Delta P)(k_m T_m + k_u)u_m \end{aligned} \tag{19}$$

If $(I + \bar{P}k_e)^{-1}\Delta P$ is stable, then the transfer function matrix from u_m to e is stable. □

Note that when there is no uncertainty in the plant, i.e. when $\Delta P = 0$, (19) shows that the transfer matrix from u_m to e is zero, which agrees with the original PMF design objective. When there is an uncertainty in the model of the plant, Lemma 2 shows that, although the PMF is no longer valid, the system stability will be maintained if k_e provides robust stability. This implies that for bounded u_m, the error e will always be bounded; if u_m is a constant, then it will go to zero. Equation (19) also shows that to minimize the effect of system uncertainty, the error $(I + \bar{P}k_e)^{-1}\Delta P$ should be made small.

When the Erzberger condition is not met, an extra term will be added to $\dot{e}$ besides the homogeneous part. Since we have two different expressions for $\dot{e}$ and u_p in the derivation of the implicit and explicit LMF, $\dot{e}$ takes two different forms. For the implicit LMF:

$$\dot{e} = A_m e + g(t) \tag{20}$$

where $g(t) = (I - B_p B_p^+)(A_m - A_p)x_p + (I - B_p B_p^+)B_m u_m$ and for explicit LMF:

$$\dot{e} = (A_p - B_p k_e)e + f(t) \tag{21}$$

where $f(t) = (I - B_p B_p^+)(A_m - A_p)x_m + (I - B_p B_p^+)B_m u_m$.

Clearly, when Erzberger's conditions are satisfied, $f(t) = g(t) = 0$, and we have perfect model following. When the condition is not met, $f(t)$ and $g(t)$ are nevertheless minimized in terms of the Frobenius norm and e will diminish in the steady state if the system is stable. The key difference between these two formulations is that, for the closed-loop system to be stable, it is required that the plant be stable in the implicit LMF, while in the explicit LMF it only requires the plant to be stabilizable and k_e to be the stabilizing gain. This is because for e to be bounded for a bounded input u_m under the conditions that A_m and $(A_p - B_p k_e)$ are stable matrices, $g(t)$ and $f(t)$ must be bounded. Since the reference model is a stable system, x_m is always bounded for a bounded input u_m; therefore $f(t)$ is always bounded. On the other hand, $g(t)$ is bounded only when x_p is bounded, which requires that the plant be stable. It has also been shown (Chen 1973) that the upper bound on absolute value of e, $|e|$, is minimized by the explicit LMF when Erzberger's conditions are not fulfilled. It seems that the explicit LMF has a clear advantage over the implicit LMF in this sense. In the following it is shown that the PIM is only a special case of the implicit LMF.

Linear model following and its relation to the pseudo-inverse method (PIM)

The PIM is a method that can be used to accommodate system failures that are formulated as follows. Let the nominal plant be

$$\left.\begin{aligned} \dot{x} &= Ax + Bu \\ y &= Cx \end{aligned}\right\} \tag{22}$$

Assume that the nominal closed-loop system is designed by using the state feedback $u = kx$, and the closed-loop system is

$$\left.\begin{aligned} \dot{x} &= (A + Bk)x \\ y &= Cx \end{aligned}\right\} \tag{23}$$

where k is the state feedback gain. Suppose that the model of the system, in which failures have occurred, is given as

$$\left.\begin{aligned} \dot{x}_{\mathrm{f}} &= A_{\mathrm{f}}x_{\mathrm{f}} + B_{\mathrm{f}}u_{\mathrm{f}} \\ y_{\mathrm{f}} &= C_{\mathrm{f}}x_{\mathrm{f}} \end{aligned}\right\} \tag{24}$$

and the new closed-loop system is

$$\left.\begin{aligned} \dot{x}_{\mathrm{f}} &= (A_{\mathrm{f}} + B_{\mathrm{f}}k_{\mathrm{f}})x_{\mathrm{f}} \\ y_{\mathrm{f}} &= C_{\mathrm{f}}x_{\mathrm{f}} \end{aligned}\right\} \tag{25}$$

where k_{f} is the new feedback gain to be determined. In the PIM, the objective is to find a k_{f} so that the system A-matrix in (25) approximates in some sense that in (21). For this, $A + Bk$ is equated to $A_{\mathrm{f}} + B_{\mathrm{f}}k_{\mathrm{f}}$ and in approximate solution for k_{f} is given by

$$k_{\mathrm{f}} = B_{\mathrm{f}}^{+}(A - A_{\mathrm{f}} + Bk) \tag{26}$$

where B_{f}^{+} denotes the pseudo-inverse of B_{f}, and the resulting input to the impaired plant is

$$u_{\mathrm{f}} = B_{\mathrm{f}}^{+}(A - A_{\mathrm{f}} + Bk)x_{\mathrm{f}} \tag{27}$$

Clearly, this is just a special case of (10) in the implicit LMF where u_{m} is set to zero.

Since the PIM and the implicit LMF are essentially the same, our main interests are in the case of the explicit LMF. From the above discussion, the explicit LMF has the advantages of guaranteed stability and pre-specified error trajectory. Note that in terms of control reconfiguration the reference model can be selected as the nominal closed-loop system while the plant is the impaired open-loop system. Once the failures are identified and the new model obtained, the control gains $\{k_{\mathrm{e}}, k_{\mathrm{m}}, k_{\mathrm{u}}\}$ can be immediately reconfigured according to the new model of the plant and the explicit LMF algorithm. The stabilizing gain k_{e}

can be found via many available computer algorithms such as the pole placement or the LQ routine; the only non-trivial part in the determination of k_u and k_m is the calculation of the pseudo-inverse of B_p which can be found via various methods in the numerical analysis literature.

In general, it is felt that the explicit LMF is a practical approach that can be utilized in control reconfiguration. It seems that explicit LMF offers better tracking than implicit LMF or PIM, since it is the difference of the states that are being minimized. A disadvantage is that the closed-loop system is more complex since the reference model must be implemented on-line.

A common shortcoming among many RCS design methods, including the PIM and the LMF approaches, is the severe constraints needing to be satisfied for a perfect match between the nominal system and the impaired system. Although the output of the reconfigured system can be made very close to that of the nominal system as shown in Example 1, there are always cases, at least mathematically, in which the explicit LMF cannot achieve satisfactory performance for the reconfigured system. This is because the feedforward gains, k_u and k_m are determined using a pseudo-inverse type approach to minimize a cost function, $f(t) = (A_m - A_p)x_m + B_m u_m - B_p u_2$. The cost function $f(t)$ will be made small if $(I - B_p B_p^+)$ is close to the zero matrix, as shown in (14), or $B_p B_p^+$ is close to an identity matrix. Therefore, we cannot prespecify how close the nominal and the impaired systems should be because we do not have any control over B_p, which is part of the model of the plant.

3. Explicit LMF with dynamic compensators

In RCS, the ideal goal is to develop a control system that is able to accommodate a large class of different system impairments so that the reconfigured systems behave exactly as prespecified. The explicit LMF approach described above is an approach that makes the output of a plant follow that of a reference model to a certain extent using constant feedback and feedforward gains. The exact match only happens for a particular class of the reference models that have been chosen to satisfy the severe constraints illustrated in Lemma 1. In the control reconfiguration to accommodate system failures, these constraints seem to be too restrictive. In this section we investigate the use of dynamic compensators, instead of constant ones, to loosen the restrictions on the reference models.

In the proof of Lemma 1, it is shown that for PMF a necessary and sufficient condition is that the transfer function matrix from u_m to e is zero. Lemma 1 applies only to the reference models and plants where $C_m = C_p = I$. In the following, the transfer function matrix is derived for the general case where C_m and C_p are not necessarily identities.

$$
\begin{aligned}
e &= y_m - y_p \\
&= T_m u_m - P u_p \\
&= T_m u_m - P(k_e e + k_m x_m + k_u u_m) \\
&= -P k_e e + T_m u_m - P[k_m (sI - A_m)^{-1} B_m u_m + k_u u_m] \\
&= (I + P k_e)^{-1} [T_m - P[k_m (sI - A_m)^{-1} B_m + k_u]] u_m \qquad (28)
\end{aligned}
$$

Clearly, a necessary and sufficient condition for the PMF is that

$$[T_{\mathrm{m}} - P[k_{\mathrm{m}}(sI - A_{\mathrm{m}})^{-1}B_{\mathrm{m}} + k_{\mathrm{u}}]] = 0 \tag{29}$$

Now it remains to solve (29) with respect to k_{m} and k_{u}. Note that there are many solutions of (29). A simple solution is

$$[k_{\mathrm{m}}, k_{\mathrm{u}}] = [0, P_{\mathrm{ri}}T_{\mathrm{m}}] \tag{30}$$

where P_{ri} is defined as the right inverse of P, i.e. $PP_{\mathrm{ri}} = I$, assuming it exits. It is known that the conditions for k_{u} to be proper and stable is that the reference model T_{m} is chosen such that it is 'more proper' than the plant and it has as its zeros all the RHP zeros of P together with their zero structures (see, for example, Gao and Antsaklis 1989). It is also known that the right-inverse of P can be calculated using a state-space algorithm which has good numerical properties (Patel 1982). Note that the complexity of the compensator k_{u} is dependent on how different the reference model T_{m} is from the open-loop plant, P. This can be seen clearly from (30), where $k_{\mathrm{u}} = P_{\mathrm{ri}}T_{\mathrm{m}}$. For example, if only a pair of open-loop poles are undesirable, i.e. the poles and zeros of P and T_{m} are the same except for one pair of poles, then k_{u} is a second-order compensator since all the poles and zeros of P_{ri} and T_{m} are cancelled except one pair of zeros of P_{ri} and one pair of poles of T_{m}. In case of failures, perhaps all open-loop zeros and poles will be shifted. However, only the unstable poles and dominant poles are of major concern in surviving the failures since their locations dominate how the system will behave in general. Therefore, in order to produce a fast and simple solution to keep the system running, T_{m} should be chosen close to the impaired plant P with the exception of only a few critical poles.

The main advantage of this approach is that there are fewer restrictions on the reference model than before. The only restrictions are on the zeros of the reference model, which are much more manageable than before. If the plant does not have RHP zeros, or its RHP zeros are unchanged after the failure, then the reference model is almost arbitrary except that it should be at least as 'proper' as P so that the compensator k_{u} is proper. The disadvantage of this approach is the increased complexity of the control system due to the higher-order compensators required. This is a trade-off between the performance and complexity of the control system.

Note that there are many control configurations, other than that of the explicit LMF, that can be used for dynamic compensators. Here the same configuration is used for both constant and dynamic compensators because it is felt that the dynamic compensator can be used in conjunction with the constant compensators for reconfigurable control purposes. As mentioned earlier, there are two steps in the accommodation of failures. First, the impaired system must be stabilized. In the explicit LMF approach, this is accomplished via the implementation of the stabilizing gain k_{e}. This must be executed quickly to prevent catastrophic results from happening. Once the system is stabilized, it gives time to the control reconfiguration mechanism to manipulate the compensators to obtain better system performance. Assuming, by this time, that the model of the impaired system is available, a reference model should be chosen that has the desired behaviour for the system under the specific system failure. Once the reference model is chosen, either the constant or the dynamic compensators can be computed and implemented as explained above. The choice of the types of the compensator depends on the performance requirements and the limitations on the complexity of the compensators.

Next, the stability robustness is analysed. Let the real plant be $\bar{P}$ and the nominal transfer matrix of the plant be P, where $\bar{P} = P + \Delta P$ for some small ΔP.

Lemma 3: *For the control gains* $\{k_e, k_m, k_u\}$ *obtained by using the explicit LMF with the dynamic compensator described above from* (28) *to* (30), *the closed-loop system from* u_m *to* e *is stable if, for the real plant* $\bar{P} = P + \Delta P$, $(I + \bar{P}k_e)^{-1}\Delta P$ *is stable.*

Proof: From (28) and (30)

$$e = (I + \bar{P}k_e)^{-1}(-\Delta P)k_u u_m \tag{31}$$

If $(I + \bar{P}k_e)^{-1}\Delta P$ is stable, then the transfer function matrix from u_m to e is stable. □

Note that when there is no uncertainty in the plant, i.e. $\Delta P = 0$, (30) shows that the transfer matrix from u_m to e is zero, which agrees with the original design objective. When there is an uncertainty in the model of the plant, Lemma 3 shows that the system stability will be maintained if k_e provides robust stability. This is because, by design, k_u is a stable compensator; therefore the closed-loop system is stable if $(I - \bar{P}k_e)^{-1}\Delta P$ is stable.

Example 2: To show the effectiveness of the new approach, we use the same nominal system as in Example 1 except that the C matrix is changed to $[0{\cdot}0241 \;\; -135{\cdot}0 \;\; -103{\cdot}9 \;\; 21{\cdot}0]$. C is chosen as such for the convenience of the simulation, since now the output stabilizing gain is simply $k_e = 1$. The impaired plant $\{A_f, B_f, C_f, D_f)$ is as follows:

$$A_f = A, \quad B_f = \begin{bmatrix} 0{\cdot}0 \\ -0{\cdot}0717 \\ -0{\cdot}1645 \\ -1{\cdot}0 \end{bmatrix}, \quad C_f = C, \quad D_f = D$$

The dynamic compensator obtained from (30) is $k_m = 0$, and

$$k_u = 31{\cdot}2\frac{s^7 + 2{\cdot}7s^6 + 0{\cdot}6s^5 - 1{\cdot}9s^4 - 0{\cdot}09s^3 - 1{\cdot}4s^2 - 0{\cdot}0016s - 0{\cdot}0025}{s^7 + 6{\cdot}46s^6 + 21{\cdot}1s^5 + 39{\cdot}0s^4 + 35{\cdot}6s^3 + 11{\cdot}8s^2 + 0{\cdot}51s + 0{\cdot}1}$$

The impulse responses of both the reference model, which is the nominal closed-loop system, and the reconfigured system are shown. They match exactly, as expected. This is compared with the response of the system reconfigured with the standard explicit LMF method.

Here, the exact match is attained at the expense of having a seventh-order compensator. This compensator can be implemented, together with the reference model, in the real-time aircraft control environment via flight control computers. The complexity of these compensators may or may not be an issue in the implementation depending on the capacity of the computers. If it is, then the reference model has to be chosen close to the open-loop plant, as discussed above, so that the poles and zeros of P_{ri} and T_m cancel each other except for the critical poles. The standard explicit LMF has a very simple system structure

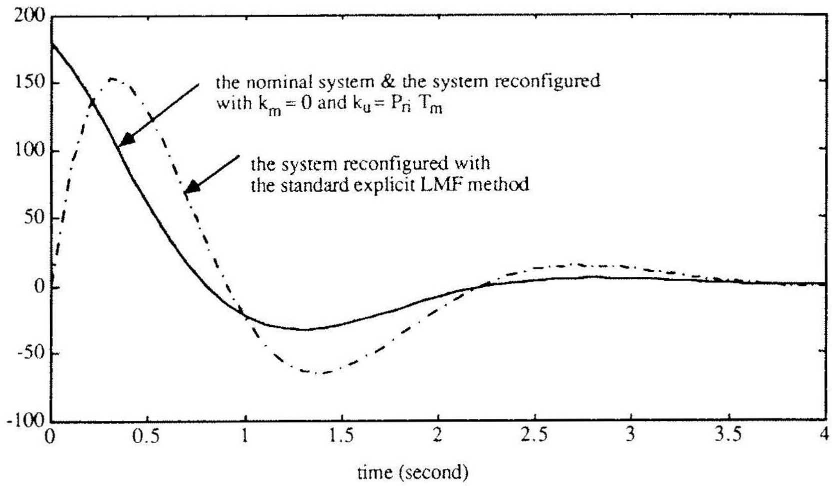

Figure 4. The impluse responses of the reference model and the system reconfigured using new and standard explicit LMF methods.

where only the constant gain matrices are to be adjusted for different failures. It should be used whenever the performance of the reconfigured system is acceptable. However, difficulty may arise when it does not provide satisfactory performance and the dynamic compensator cannot be used due to the limitations on the system complexity. Such problems will be investigated in future research.

4. Concluding remarks

The linear model-following methods in control system design were studied in the context of reconfigurable control. The necessary and sufficient conditions for perfect model-following were obtained using a transfer function approach, which yields simple and intuitive constraints on the reference model. A new approach was developed to design the reconfigurable control system so that the state trajectories of the reconfigured system follow those of the reference model exactly. This was achieved at the expense of increased system complexity due to the use of dynamic compensators.

The advantages of using the LMF control methodology in reconfigurable control system design can be summarized as follows. First, the widely used pseudo-inverse method is only a special case of the LMF. By examining the PIM in the context of LMF, it helped us understand the characteristics and the limitations of the PIM. Secondly, like the PIM, the LMF control system is simple in terms of design and implementation. More importantly, it guarantees the stability of the reconfigured system assuming the impaired plant is stabilizable. Thirdly, the new design approach proposed in this paper enables us to achieve the PMF, and thus completely specify the behaviour of the reconfigured system, with many fewer constraints on the performance specifications. Finally, since the adaptive model reference control systems have been developed based on LMF systems, it is possible to extend the results in this paper to adaptive control systems. This may be necessary in certain cases since it could make the reconfigurable control systems less dependent on the fault detection and identification systems.

For convenience, it was assumed that the input, output and state of the reference model and the plant, or the nominal and impaired systems, are of the same dimension. It can be shown quite easily that the approaches developed here are valid even if the dimension of the input changes due to actuator failures. In particular, the methods of determining the feedforward gains, (13 *b*) and (30), will not be affected by the change in the number of columns in B_p as long as B_p is a non-zero matrix, i.e. there is at least one actuator remaining effective. Moreover, the only constraint for the explicit LMF approach is that the states of the nominal and impaired systems have the same dimension. This is necessary since the trajectories of every state in the reconfigured system can be specified and manipulated only when the error is defined as $e = x_m - x_p$. Meanwhile, the only constant for the LMF with dynamic compensator is that the outputs of the nominal and impaired systems have the same dimension. For the reconfigured system to be stable, the impaired system must be stabilizable regardless of what method is to be used.

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