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ACTUATOR AND SENSOR FAILURE ACCOMMODATION WITH INCOMPLETE INFORMATION

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Abstract: The problem of actuator and sensor failure accommodation is treated here when only incomplete information about failure is available. The main contribution of this paper is in the formulation of the above problems. Due to space limitations, only a brief discussion of the methods used in each case is given.

I. Introduction

The Reconfigurable Control Problem (RCP) has drawn substantial attention from researchers because of its potential impact on practical applications, particularly in aircraft control systems. The goal is to design a control system that has the so called "fault-tolerant", or, "self-repairing" capability. In case of severe system failures, such control systems will automatically reconfigure the control law to maintain system stability as well as performance. Although modern control theory has advanced significantly, it can not be used to deal with such problems directly due to its limitations.

The majority of the research on RCP focuses on the idea of controlling the impaired system so that it is "close", in some sense, to the nominal system. Proposed methods include, but are not limited to, linear-quadratic (LQ) control methodology [10,12,13]; adaptive control systems [3,14,17]; eigenstructure assignment [7,15], knowledge-based systems [8,9], and standard linear model following approach (LMF) [6]. Another method is the Pseudo-Inverse Method (PIM) [2,5,11,16,18,19], which is a special case of the LMF approach.

In this paper, we'll consider different failure scenarios in which the control reconfiguration might have to be executed with partial failure information. Under such circumstances, a major concern is system stability. Because the actions that should be taken very much depend on the type and severeness of the failures, the reconfiguration problems must be studied according to the nature of the failures. Here we limit ourselves to certain types of actuator and sensor failures which are problems of common concern in flight control systems.

Note that the accommodation of actuator failures is a challenging problem even when all the information is available. Here in control reconfiguration with incomplete information, we do not expect perfect solutions for the problem. Instead, we are trying to develop practical control redesign methods that generates quick solutions to stabilize the system. The mathematical model of the impaired system is given as follows: Let the open-loop nominal plant be given by

$$\dot{x} = Ax + Bu, \; y = Cx \tag{1}$$

where $A \in R^{nxn}$, $B \in R^{nxm}$ and $C \in R^{pxn}$. Assume the nominal closed-loop system is designed by using the state feedback $u = Kx$, $K \in R^{mxn}$, then the closed-loop system is

$$\dot{x} = (A+BK)x, \; y = Cx \tag{2}$$

where K is the state feedback gain. The model of the impaired system is given by

$$\dot{x}_f = A_f x_f + B_f u_f + Dw, \; y_f = C_f x_f \tag{3}$$

where $A_f \in R^{lxl}$, $B_f \in R^{lxr}$, $C_f \in R^{qxl}$, $D \in R^{lx(m-r)}$, and w contains the disturbances due to actuator failures.

II. Actuator Failure Accommodation with Incomplete Information

In literature, a common assumption is that an actuator failure corresponds to one of the columns in the B matrix becoming a zero column. Physically it means that the impaired actuator simply ceases functioning and does not affect the system anymore. It is felt, however, that real world problems might be much more complicated. In case of failure, the electronic signal, u, sent to actuators can no longer produce the corresponding actuation to the plant. Instead, the actual output of the actuators is $\hat{u}$, which is different from u. For example, in the case of a actuator being stuck, one of the elements in $\hat{u}$ becomes a constant regardless of u; while for actuator degradation, one element in $\hat{u}$ may be only 50% of its nominal value. Note that $\hat{u}$ takes on different values, including zero, depending on the type and severity of the failures. To accommodate the actuator failures, the input to the actuators, u, will be manipulated so that the output of the actuator, $\hat{u}$, is as close to the desired one as possible.

<u>Actuator Degrading</u>

This is the type of failure where one of the actuators loses some of its effectiveness, and it is called an "actuator degrading" failure. An example of this kind of failure could be an elevator or a flap on an aircraft losing part of its surface. Here it is assumed that we only know which actuator is faulty and the its range. Under this assumption, the *i*th impaired element in the output of the actuators, which is the input to the plant, can be modeled as

$$\hat{u}_i = u_i(1+\delta) \tag{4}$$

where δ is an unknown constant that indicates how much an actuator has degraded. It is assumed that a bound on the range of δ is known, i.e.

$$\delta \in [\alpha, \beta]. \tag{5}$$

For this type of failures, the system equation (3) can be rewritten as:

$$\dot{x}_f = Ax_f + \hat{B}_f u_f, \; y_f = Cx_f, \; u_f = K_f x_f \tag{6}$$

with $\hat{B}_f = [b_1 \ldots b_{i-1} \; b_i(1+\delta) \; b_{i+1} \ldots b_m]$, $K_f = [k_{f1}' \ldots k_{fi}' \ldots k_{fm}']'$, where b_i $i = 1, \ldots, m$ are column vectors of the B matrix in the nominal system, k_{fi} are the row vectors of the new feedback K_f to be found.

To accommodate such failures, we propose a two-step approach:

1) Stability Analysis: check if the nominal system will remain stable for any δ satisfying (5).

2) If the answer for 1) is negative, use the LQ approach to desensitize the system with respect to the uncertain parameter δ.

In the first step, a recent result on stability bound in [4] can be used to provide a stability region with regard to the uncertain parameter δ. To be more efficient, these stability conditions can be calculated off-line for every actuator and the data stored in a flight control computer. When failures occur, the stability conditions associated with the particular failures can be obtained from the computer quickly and a decision can be reached as to whether or not a control reconfiguration is needed.

The method to desensitize the system stability with respect to δ is now described as follows. Assuming the ith actuator is degraded by a factor of δ, the closed-loop system in (5) can be expressed as:

$$\dot{x}_f = (A_c + \delta E)x_f \tag{7}$$

with $A_c = A + BK_f$ and $E = b_i k_{fi}$. For $K_f = K$, the nominal feedback gain, if the system (7) is stable, then there is no need for an immediate control reconfiguration. In other words, we can wait for the FDI system to provide more accurate information before any action is taken. If the system in (7) is unstable for $K_f = K$, then a new stabilizing gain K_f must be found which not only makes A_c stable but also makes the extra term δE in (7) insignificant. This can achieve by using the LQ approach and choosing the weighting matrices properly so that k_{fi} is small. The desirable gain margin of the LQ design also makes this approach appealing.

Actuator Stuck:

Actuator stuck is the type of failure where at least one of the actuators is "frozen" at a constant value. It is different from the actuator degrading in that once an actuator is stuck, its output becomes a constant. This kind of actuator failures will change, in general, the number of useful control inputs in the system. If the ith actuator is stuck at a constant, say α, the closed-loop system takes the form of

$$\dot{x}_f = A_f x_f + B_f u_f + b_i\alpha\ ,\ y_f = C_f x_f,\ u_f = K_f x_f \tag{8}$$

where the number of elements in the new input vector, u_f, as well as the number of columns in B_f are both reduced by one. In this case $\bar{u} = [u_1, u_2, ..., u_{i-1}, \alpha, u_{i+1}, ...]'$, and B_f and u_f are the same as B and u except they do not have the ith column and row, respectively. Assuming that the actuator dynamics are independent of the plant, therefore, the changes in the system equations caused by the failure can be restricted to the B matrix and the input vector.

From (8), the actuator failure can be viewed as an input disturbance. For a stuck actuator, if the value α is known, then a disturbance cancellation technique [20] can be used. Let

$$u_f = u_{fc} + u_{fw} \tag{9}$$

where u_{fw} satisfies

$$B_f u_{fw} + b_i\alpha = 0 \tag{10}$$

and u_{fc} is designed to achieve desired system performance. Since B_f usually has more rows that columns, (10) is over-determined and does not have exact solutions. An approximate solution can be found as

$$u_{fw} = B_f^+ b_i \alpha\ , \tag{11}$$

where B_f^+ is the pseudo-inverse of B_f. It has been shown that the u_{fw} obtained in equation (11) minimizes the norm $\|B_f u_{fw} + b_i\alpha\|$ and therefore the impact of this actuator failure on the overall system is reduced. To use this technique, it is assumed that the value α is known. This is a reasonable assumption since the FDI system can usually provide the actual value given enough time. At the beginning, however, the value α will not be known exactly. Instead, an interval in which α resides is given, $[\alpha_{min}, \alpha_{max}]$ and a u_{fw} can be determined by $u_{fw} = B_f^+ b_i \alpha_{avg}$, (12)

where $\alpha_{avg} = (\alpha_{max} - \alpha_{min})/2$. As time passes, FDI will provide more accurate information and the interval $[\alpha_{min}, \alpha_{max}]$ will become smaller. In this way, the solution in (12) will approach the solution in (11).

III. Sensor Failure Accommodations with Incomplete Information

In case of sensor failures, the u_f can be represented by

$$u_f = K_f \tilde{x}_f\ , \tag{13}$$

where $K_f \in R^{rxs}$ is the new feedback gain to be found, $\tilde{x}_f$ is the output of the sensors and has the form

$$\tilde{x}_f = Lx_f \tag{14}$$

where $\tilde{x}_f$ is different from x_f reflecting the sensor failure. Assume, for the time being, that there is only one impaired sensor and the failure is characterized as follows:

$$\tilde{x}_f = [x_1, x_2, ..., (1+\alpha)x_i, ..., x_n]' \tag{15}$$

where $x = [x_1, x_2, ..., x_i, ..., x_n]'$ and α is an uncertain parameter, some real number typically unknown. In this case, L is an nxn diagonal matrix with the ith element on the diagonal equal to $(1+\alpha)$, and all the rest of the elements on the diagonal equal to 1. We call this kind of failure sensor degrading. Here, our interest is in the cases where the information on the failures is incomplete, (i.e. α is unknown). However, the range on which α may reside is available, (i.e. a and b are given such that $\alpha \in [a, b]$). The emphasis here is to analyze the stability of such systems to determine how much uncertainty can be tolerated by the nominal system and how to stabilize such systems when the uncertainty exceeds the stability bound.

The approach to analyze system stability is very similar to the ones used in actuator failure analysis. To determine how much variation in α can be allowed without destabilizing the system, the feedback gain matrix is kept unchanged, i.e. $K_f = K$. Let $K = [k_1, k_2, ..., k_m]$, where k_i, $i = 1, ..., m$, are column vectors in K. Then

$$u_f = K\tilde{x}_f = Kx_f + \alpha[0, ..., 0, k_i, 0, ..., 0]x_f \tag{16}$$

Therefore, the closed-loop matrix A_c is given by

$$A_c = A + BKL = (A + BK) + E \tag{17}$$

where $E = \alpha[0, ..., 0, Bk_i, 0, ..., 0]$. Note that this formulation can be easily extended to include the multiple sensor failures, where E takes the form

$$E = \sum_{i=1}^{m} \alpha_i E_i, \tag{18}$$

$$\text{where } E_i = [0, ..., 0, Bk_i, 0, ..., 0]. \tag{19}$$

Now the problem is to find bounds on α_i, for all i, such that the closed-loop matrix A_c in (17) is stable. This problem can be solved by the new approach proposed in [4].

Assuming that, with the partial information, the upper and lower bounds on each α_i is known, that is

$$\alpha_i \in [a_i, b_i] \qquad \forall\ i, \tag{20}$$

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where $b_i > a_i$. When the intervals $[a_i, b_i]$ are such that they are not completely contained within stability bound derived in [4], the feedback must be adjusted to maintain stability. Let the new feedback gain be K_f, $K_f = [k_{f1}, k_{f2}, ..., k_{fm}]$, where k_{fj} are column vectors, and let $c_i = (a_i+b_i)/2$. If c_i is not in the neighborhood of -1, in particular, if c_i satisfies

$$c_i < -2, \text{ or } c_i > 0 \tag{21}$$

K_f is assigned as follows:

$$k_{fj} = \begin{cases} k_j & j \neq i \\ k_j/(1+c_i) & j = i \end{cases} \qquad j = 1, ..., m \tag{22}$$

Note that K_f in (22) reduces the size of the interval $[a_i, b_i]$ when c_i satisfies (21) and therefore enhances the stability robustness in the presence of uncertain sensor failures. For simplicity, this is illustrated in the following example of a single sensor failure. Assume that the sensor associated with x_i is impaired and the failure can be described by (16) and the uncertain parameter, α, satisfies $\alpha \in [a, b]$. Let the feedback gain matrix K_f be assigned in (22). Then, the closed-loop matrix A_c of the reconfigured system is

$$A_c = A + BK_fL = (A + BK) + E, \tag{23}$$

where $E = \bar{\alpha}[0, ..., 0, Bk_i, 0, ..., 0]$ and

$$\bar{\alpha} = (\alpha-c)/(1+c). \tag{24}$$

The closed-loop 'A' matrix of the reconfigured system in (23) is the same as that of the nominal system with uncertainties in (17) except for the uncertain parameters. The uncertain parameters before and after the reconfiguration are related in the form of (24). Note that the size of the uncertainty is reduced since the length of the interval is $l = b-a$ for α and $\bar{l} = l/|1+c|$ for $\bar{\alpha}$. For $c = c_1$ satisfies (21), $|1+c| > 1$, therefore $l < \bar{l}$.

Another characteristic of this reconfiguration approach is that it is practical and easy to implement. This is because the approach is very simple and there is hardly any complex computation involved. For each sensor failure, only one column in the feedback gain matrix is to be changed and the adjustment procedure can be set up in advance to make it more efficient for on-line operation.

It is felt that the assumption in (21), that c_i are not in the neighborhood of -1, is reasonable. When $\alpha_i = -1$, it means the corresponding sensor is completely disabled with its output fixed at zero. In this case there is no information whatsoever contained in the output of that sensor and it cannot be used in the control reconfiguration. Obviously, in the cases when α_i is close to -1, the corresponding sensor is of little use also since it provides too little information to be useful.

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