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Hybrid Systems and Intelligent Control

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Abstract

This paper introduces a finite-time algorithm which allows a hybrid system to determine whether or not a specified symbolic behaviour can be realized by the system. The proposed algorithm is an inductive inference protocol based on the ellipsoid method. It can be viewed as a means by which the hybrid system can autonomously set its achievable goal behaviour. This ability for goal self-determination is argued as an important attribute of intelligent control systems.

1 Introduction

Hybrid control systems have begun to attract considerable interest from the research community [1] [2] [3] [5] [6] [7] [8] [9] [10] [11]. Such systems arise when a discrete event system is used to supervise the behaviour of a continuous-state plant through the issuance of logical directives. The hybrid control system therefore consists of two distinctly different types of system. The supervisor is a discrete event system (DES) evolving over a finite symbolic state space. The plant is a continuous-state dynamical system (CSS) evolving over an infinite vector space which is distinctly nonsymbolic in nature.

From the standpoint of the supervisor, the original plant appears to be another discrete event system which this paper refers to as the *plant automaton*. The DES supervisor, therefore, attempts to control the CSS plant by controlling the behaviour of the equivalent plant automaton. An important issue concerns conditions under which controlling the plant automaton yields acceptable control of the original plant. This paper addresses that issue by determining a set of sufficient conditions to insure such controllability. These conditions are used in an inductive learning algorithm which permits the system to decide on the controllability of a given plant automaton after observing the system's response to finite number of control actions.

The remainder of the paper is organized as follows. Section 2 presents the hybrid system model used in this paper. Section 3 discusses plant controllability, the plant automaton, and derives sufficient conditions for the existence of the plant automaton. Section 4 uses these conditions to formulate the inductive learning algorithm. This section also derives bounds on the algorithm's convergence time. Section 5 then discusses the impact which the proposed algorithm has on intelligent control systems.

2 Hybrid Systems

Hybrid dynamical systems consist of a continuous-state plant interfaced to a discrete event supervisor. The system therefore consists of three components; the *plant*, *supervisor*, and *interface*. The interface can be decomposed into two subsystems known as the *actuator* and *generator*. The following section formally discusses the form of these architectural components (plant, supervisor, and interface) which is assumed in the remainder of this paper.

The system to be controlled is called the *plant*. It is modeled as a time-invariant continuous-state system. For the purposes of this paper, however, it will be convenient to focus on a specific class of plants which are affine in the control vector. The plant's differential equations can therefore be

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written as

$$\frac{d\bar{x}}{dt} = f_0(\bar{x}) + \sum_{i=1}^{m} r_i f_i(\bar{x}) \qquad (1)$$

where f_i are assumed to be Lipshitz continuous, $\bar{x} \in \Re^n$ is the state vector, and $\bar{r} \in \Re^m$ is the control vector. The vector fields associated with functions f_i for $i = 1, \ldots, m$ are controlled by the components of the control vector, $\bar{r}$. In this regard, the control vector can be thought of as "coordinating" a set of available control policies, f_i.

The plant is controlled through logical directives issued by a *supervisor*. The supervisor is modeled as a discrete event system. Such a discrete event system can take on a variety of forms including deterministic automatons , Petri nets, recursively enumerable processes, or directed acyclic graphs. The only assumptions on the supervisor invoked by this paper concern the input to and output from the supervisor. In particular, the supervisor's inputs are symbols $\tilde{x}$ drawn from a finite alphabet $\tilde{X}$ and the supervisor's outputs are symbols $\tilde{r}$ drawn from a finite alphabet $\tilde{R}$. For $n = 0, 1, \ldots, \infty$, the sequence of input symbols will be denoted as $\tilde{x}[n]$ and will be called the *plant symbol sequence.* The sequence of output symbols will be denoted as $\tilde{r}[n]$ and will be called the *control symbol sequence.*

The *generator* transforms the plant's state trajectory, $\bar{x}(t)$, into a plant symbol sequence $\tilde{x}[n]$. This sequence is obtained from the following equation

$$\tilde{x}[n] = \gamma(\bar{x}(\tau_e[n]), \bar{x}(\tau_e[n-1])) \qquad (2)$$

where $\gamma : \Re^n \times \Re^n \rightarrow \tilde{X}$ is a surjective mapping and where $\tau_e[n]$ is a sequence of *control instants* representing the times (measured with respect to the plant's clock) when the generator issues a plant symbol.

For this paper specific assumptions are placed on γ and $\tau_e[n]$. Let $\mathbf{X}$ denote an open covering of the plant's state space. This open covering will be called the hybrid system's *basis event covering* and is given by

$$\mathbf{X} = \{ \ \mathbf{x}_1 \ \ \cdots \ \ \mathbf{x}_q \ \} \qquad (3)$$

where $\mathbf{x}_i \subset \Re^n$ for $i = 1, \ldots, q$. Since $\mathbf{X}$ is an open covering, each of the subsets $\mathbf{x}_i$ is an open set and $\bigcup_i \mathbf{x}_i = \Re^n$. Each element of $\mathbf{X}$ will be referred to as a *basis event* for the hybrid system.

For an assumed state trajectory, $\bar{x}(t)$, let $I(t)$ be a subset of the integers (indexed by the time t) between 1 and q such that the integer $j \in I(t)$ if and only if $\bar{x}(t) \in \mathbf{x}_j$. This set $I(t)$ is called the *index set* for the plant trajectory, $\bar{x}(t)$, at time t. The index set is therefore a list of the basis events which contain the plant state at time t. Event instants represent times at which the index set for a plant trajectory changes. Let $\mathbf{T}$ denote the collection of open intervals on the postive real line, $\mathbf{t} = (\tau^+, \tau^-)$, such that $\tau^\pm$ is rational and such that $I(\tau^+) \neq I(\tau^-)$. The times just before boundary crossings are given by the inferior limit set of $\mathbf{T}$. The sequence of control instants $\tau_e[n]$ is therefore defined as a nondecreasing sequence drawn from the set $\liminf \mathbf{T}$.

The mapping γ is assumed to indicate whether the plant state has entered or exited a basis event. To realize this function, the mapping uses the plant state at $\tau_e[n]$ to determine which basis event boundary is being crossed and it uses the plant state at a time $\tau_e[n-1]$ to determine in which direction the boundary is being crossed. Let $\text{clo}(\mathbf{x})$ denote the closure of set $\mathbf{x}$, then output, $\tilde{x}[n]$, of the generator is $\tilde{x}_i^+$ if $\bar{x}(\tau_e[n]) \in \text{clo}(\mathbf{x}_i)$ and $\bar{x}(\tau_e[n-1]) \notin \mathbf{x}_i$). The output is $\tilde{x}_i^-$ if $\bar{x}(\tau_e[n]) \in \text{clo}(\mathbf{x}_i)$ and $\bar{x}(\tau_e[n-1]) \in \mathbf{x}_i$). The issuance of symbol $\tilde{x}_i^+$ therefore implies that the plant state has entered basis event $\mathbf{x}_i$. The issuance of symbol $\tilde{x}_i^-$ indicates that the plant state has exited basis event $\mathbf{x}_i$.

The *actuator* transforms the control symbol sequence, $\tilde{r}[n]$, into a control vector trajectory, $\bar{r}(t)$. This relationship is given by the following equation

$$\bar{r}(t) = \sum_{n=0}^{\infty} \alpha(\tilde{r}[n]) I(\tau_c[n], \tau_c[n-1]) \qquad (4)$$

where $I(t_1, t_2)$ is an indicator function taking on the value of unity over the interval $[t_1, t_2)$ and zero elsewhere, where $\alpha : \tilde{R} \rightarrow \Re^m$ is an injective function from the control symbols into a finite set of control vectors, and where $\tau_c[n]$ for $n = 0, 1, \ldots, \infty$ is a sequence of *control instants* representing the times when the supervisor issued the nth control symbol. The control instants are measured with respect to the CSS plant's clock.

Full characterization of the actuator is obtained once the mapping α and the mechanism for determining the sequence of control instants ,$\tau_c[n]$, has been specified. Due to causality considerations, it will be assumed that the nth control instant must be between the nth and $n+1$st event instants. This implies that $\tau_e[n] \leq \tau_c[n] \leq \tau_e[n+1]$ for all $n = 1, \ldots, \infty$. It is further assumed that $\tau_e[0] = \tau_c[0] = 0$. For this paper, it will be assumed that the control instant occurs "immediately" after the associated event instant.

3 Controllability

The combination of plant and interface forms another discrete event system. The hybrid control system, from the supervisor's perspective, can then be seen as two interconnected DES; the supervisor and an equivalent plant DES. Since the supervisor is directly connected to the plant DES, one approach to hybrid controller design is to synthesize a supervisor which effectively controls the equivalent DES model of the plant. The natural concern is whether or not control of the DES is sufficient to "acceptably" control the original plant.

The following definitions state precisely what is to be controlled within the plant.

Definition 1 *Let* $\mathbf{X}$ *be a finite open cover of* $\Re^n$ *consisting of elements* $\mathbf{x}_i$ *for* $i = 1, \ldots, q$. *The set of* conjunctive events, $\mathbf{C}$ *generated by* $\mathbf{X}$ *will consist of all subsets,* $\mathbf{c}$ *of* $\Re^n$ *which can be expressed as the intersection of elements in* $\mathbf{X}$,

$$\mathbf{c} = \bigcap_{i \in I} \mathbf{x}_i \tag{5}$$

where I *is some subset of the integers between* 1 *and* q. *The set,* I, *associated with* $\mathbf{c}$ *will be called the* index set *of the conjunctive event.*

The *symbolic* behaviour of the plant is described by the way in which the plant's state transitions between events. This behaviour is conveniently represented by the *plant automaton* associated with the hybrid system.

Definition 2 *Consider a hybrid system with a collection of conjunctive events,* $\mathbf{C}$, *generated by a covering collection,* $\mathbf{X}$. *Let* $\tilde{R}$ *be a finite alphabet of* control symbols, $V \subset \mathbf{C}$, *and* $A \subset V \times V \times \tilde{R}$. *The* plant automaton *associated with the hybrid system is a labeled directed graph,* (V, A) *where the ordered triple is in* A *if and only if the plant's state trajectory,* $\bar{x}(t)$ *generated by the system equations*

$$\frac{d\bar{x}}{dt} = f(\bar{x}, \alpha(\tilde{r})) \tag{6}$$

satsifies the following conditions;

- *there exists* T_o *such that* $\bar{x}(t) \in \mathrm{clo}(\mathbf{c}_o)$ *for* $0 \leq t \leq T_o$,
- *there exists* T_t *such that* $\bar{x}(t) \in \mathrm{clo}(\mathbf{c}_t)$ *for* $t = T_t$,
- *and* $\bar{x}(t) \notin \bigcup_i \mathbf{c}_i$ *for* $T_o < t < T_t$.

As a model of the plant's symbolic behaviour, the plant automaton provides the basis for determining a supervisor which can control the plant. In order for the resulting supervisor to effectively control the plant between conjunctive events, it is necessary that the plant automaton also be controllable. The following definition makes this idea precise.

Definition 3 *Consider the plant automaton,* (V, A) *associated with a hybrid control system and consider two events,* $\mathbf{c}_o$ *and* $\mathbf{c}_t$, *which are elements of* V. *Let* $\tilde{r}[n]$ *denote the control history for a directed walk from vertex* $\mathbf{c}_o$ *to* $\mathbf{c}_t$. *The event* $\mathbf{c}_t$ *is said to be* (V, A)-controllable *from* $\mathbf{c}_o$ *if and only if any other directed walk originating from* $\mathbf{c}_o$ *with control history* $\tilde{r}[n]$ *necessarily terminates at* $\mathbf{c}_t$.

The preceding definition contains two components. The first component is the standard notion of a control existing which transfers the plant state between events. The second component requires that this control be unique. The consequence of these definitions is that if (V, A) is a plant automaton for the hybrid system, then the controllability condition implies that the plant state can be directed by a sequence of control directives (symbols), $\tilde{r}[n]$, to some specified region of the state space. Once a "controllable" plant automaton is known to be associated with the hybrid system then there exists a variety of techniques for designing a DES supervisor to control the plant DES. In this case, controlling the plant DES means that the continuous state plant's state vector is controlled to specific regions of the state space.

The plant automaton can also be thought of as a specification for the plant's *desired* symbolic behaviour. This notion is related to linguistic notions of DES controllability, in which the plant's symbolic behaviour is constrained to generate some appropriate *controllable language*.

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One important problem associated with this particular use of the plant automaton is the determination of sufficient condtions which guarantee that the given plant automaton is indeed associated with the hybrid system. The following theorem formally states sufficient conditions guaranteeing that such an association can occur.

Proposition 1 *Consider a hybrid system with an associated basis event collection,* $\mathbf{X}$. *Let* (V, A) *be a given plant automaton. Let* $V_i : \Re^n \rightarrow \Re$ $(i = 1, \ldots, q)$ *be a family of* C^1 *functionals and let* $f_j : \Re^n \rightarrow \Re^n$ $(j = 0, \ldots, m)$ *be a family of Lipschitz continuous control policies. Let* $L_j V_i$ *be the Lie derivative of functional* V_i *with respect to vector field* f_j.

The event $\mathbf{c}$ *in* $\mathbf{C}$ *with index set* I *will be controllable if*

- $V_i(\bar{x}) > 0$ *for all* $\bar{x} \notin \mathbf{c}_i$ *and* $V_i(\bar{x}) = 0$ *for al* $\bar{x} \in \mathbf{c}_i$,
- *for all* $i \in I$ *and for all* $\bar{x} \in \Re^n$,

$$0 > \left(L_0 V_i \quad L_1 V_i \quad \cdots L_m V_i \right) \begin{pmatrix} 1 \\ r_1 \\ \cdot \\ r_m \end{pmatrix} \tag{7}$$

- *and for all* $\bar{x}$ *on the boundary of events* $\mathbf{c}_o \neq \mathbf{c}$, *with index sets* I_o, *there exists an* $i \in I_o$ *such that* $V_i(\bar{x}) = 0$ *and*

$$0 < \left(L_0 V_i \quad L_1 V_i \quad \cdots \quad L_m V_i \right) \begin{pmatrix} 1 \\ r_1 \\ \cdot \\ r_m \end{pmatrix} \tag{8}$$

Proof(outline): In order for the event $\mathbf{c}$ to be controllable there must be an arc which goes from every event in V to $\mathbf{c}$ such that the state trajectory does not intersect any other event in V. One way to insure that this occurs is to require that $\mathbf{c}$ be a globally attracting invariant set of and that all other events are repellors of the plant controlled by $\alpha(\tilde{r})$. The above conditions are based in a straightforward manner on the LaSalle invariance principle. •

4 Plant Automaton Identification

The sufficient conditions obtained in the preceding section form a system of inequalities which are linear in the control vector, $\bar{r}$. Since the control vector is determined by the hybrid system's actuator mapping, these conditions also provide a method for determining an actuator mapping, α, which insures that the given automaton is realizable. Such α (or rather the $\bar{r}$ associated with the control symbol $\tilde{r}$) represent feasible points of the inequality system. There exist numerous numerical techniques for finding such feasible points. This section shows how one of these techniques, the ellipsoid method [4] can be used to realize an inductive inference algorithm which allows the system to *learn* the appropriate hybrid system interface by simply observing the system's behaviour.

Inductive inference is a machine learning algorithm in which a system learns by example. The algorithm begins with an initial hypothesis about the system's current status. The protocol then gathers "data" about the system's current state as part of a measurement experiment. The gathered data is then given to an algorithm commonly referred to as the oracle. The oracle is a Boolean functional which declares whether or not the gathered data is "consistent" with the current system hypothesis. The output of the oracle is a binary declaration indicating that the data is either consistent or inconsistent with the gathered data. If the oracle declares that the data is consistent, then nothing is done to the current hypothesis. If the oracle declares that the data is inconsistent, then an update algorithm is invoked to modify the current hypothesis so it is consistent with all prior data. This basic cycle of experiment, oracle query, and update is then repeated until no more inconsistencies are detected.

The algorithm consists of four distinct components. These components and how they work are itemized below.

- Hypothesis: The hypothesis assumes that the conjunctive event $\mathbf{c}$ is controllable by vector $\bar{r}_i = \alpha(\tilde{r}_i)$. This hypothesis is represented by an m by m positive definite symmetric matrix, Q_i and the vector $\bar{r}_i$. The specific hypothesis consists of two related assertions. The first assertion is that $\bar{r}_i$ will control the plant to $\mathbf{c}$ and the second assertion is that the set of all constant control vectors which supervise the plant to $\mathbf{c}$ will lie within the ellipsoid

$$E(Q_i, \bar{r}_i) = \{\bar{r} \in \Re^m : (\bar{r} - \bar{r}_i)' Q_i (\bar{r} - \bar{r}_i) \leq 1\} \tag{9}$$

The set $S \subset \Re^m$ denotes the set of all constant control vectors $\bar{r}$ which satisfy the linear inequalities of proposition 1.

- Experiment: The algorithm's next major component is an experiment for measuring or estimating the Lie derivatives needed in the evaluation of the inequality system.
- Query: The third component of the algorithm is an algorithm called the *oracle*. The oracle is a Boolean functional, $O_s : \Re^m \times \Re^{m+1} \rightarrow \{0,1\}$ which outputs 0 if the experimental data and the current control vector $\bar{r}_i$ satisfies the inequality system. The oracle outputs 1 otherwise.
- Update: If the oracle's response is 0, then nothing is done. If the oracle declares 1, then the current data is inconsistent with the hypothesis that $\bar{r}_i$ satisfies the inequality system. This means that the hypothesis has to be changed. The update algorithm which is used to effect this change is the central cut ellipsoid method [4].

  Assume that $E(Q_i, \bar{r}_i)$ contains S, the set of all constant controls which control the plant to c. Assume that the experiment provides a data collection represented by vectors

  $$\bar{d}_j' = \begin{pmatrix} L_1 v_j & L_2 v_j & \cdots & L_m v_j \end{pmatrix} \quad (10)$$

  where $j \in I$ and for which the oracle gives a 1 declaration. This means that the following inequality holds

  $$\bar{d}_j' \bar{r}_i \geq L_0 v_j \quad (11)$$

  where $\bar{r}_i = (r_1, \cdots, r_m)'$. From the above inequality, it is clear that any $\bar{r} \in \Re^m$ such that

  $$\bar{d}_j' \bar{r} \geq \bar{d}_j' \bar{r}_i \quad (12)$$

  can not be in S. Therefore the convex body formed by intersection of the halfplane complementing the above inequality and the ellipsoid will contain S. There exists a unique minimal volume ellipsoid which contains this convex body. This ellipsoid is computed by the following set of equations [4]

  $$\bar{b} = \frac{Q_i \bar{d}_j}{\sqrt{\bar{d}_j' Q_i \bar{d}_j}} \quad (13)$$

  $$\bar{r}_i = \bar{r}_i - \frac{m^2}{m+1} \bar{b} \quad (14)$$

  $$Q_i = \frac{m^2}{m^2 - 1} \left(Q_i - \frac{2}{m+1} \bar{b}\bar{b}' \right) \quad (15)$$

The following result [4] has proven useful in establishing the complexity results.

Proposition 2 *Let $E(Q_{i+1}, \bar{r}_{i+1})$ be an ellipsoid computed from an ellipsoid $E(Q_i, \bar{r}_i)$ using the above algorithm in equation 13, 14, and 15. Then the quotient of ellipsoid volumes is bounded as*

$$\frac{\mathrm{vol} E(Q_{i+1}, \bar{r}_{i+1})}{\mathrm{vol} E(Q_i, \bar{r}_i)} \leq e^{-1/2m} \quad (16)$$

The preceding proposition allows the statement and proof of the following propositions The significance of this result when applied to the event identification algorithm proposed above is stated in the following corollary

Proposition 3 *Let S be the set of all control vectors for which the inequality system holds and assume that S is enclosed in an m-d ellipsoid of unit volume. The plant identification algorithm will determine a control vector $\bar{r}$ in S after no more than $2m \ln \epsilon^{-1}$ updates where ϵ is the volume of an ellipsoid contained completely within the set, S.*

proof: The proof of this proposition is a direct consequence of the bound in proposition 2. After L updates (oracle declarations of failure), the volume of the bounding ellipsoid will be given by $e^{-L/2m}$. Since this cannot be smaller than ϵ, the upper bound shown above results immediately. •

Proposition 4 *Under the hypothesis of corollary 3, assume that the smallest ellipsoid which can be specified has a radius of $\sqrt{\gamma}$. Then the plant identification algorithm will determine a control vector, $\bar{r}$, in S after no more than $m^2(2\ln(m) + \ln \gamma^{-1})$ failed oracle queries.*

proof: The proof of this proposition is obtained by recognizing that the volume of an m-dimensional ellipoid of radius $\sqrt{\gamma}$ is bounded below by $\gamma^{m/2} m^{-m}$. Inserting this into the bound of proposition 3 yields the desired result. •

5 Is this intelligent?

Intelligent control generally refers to control systems which involve goal oriented behaviour. This notion of intelligent control, however, is incomplete without a discussion of how goals are determined. For example, a simple thermostat may

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be seen as a goal oriented control system. The *intelligence* of this type of controller,however, arises from the way that goal is set. If the goal is set by a human operator, then the entire system must not only include the thermostat, it must include the operator who determined that set point. From this perspective it is the human operator who endows the system with "intelligent" behaviour. The conclusion to be drawn from the preceding observation is that in addition to possessing an ability to act on specified goals, *intelligent systems must possess the capability to internally determine what those goals should be.*

The adaptive hybrid system presented in the preceding section is a simple example of a system capable of internal goal determination. Consider a hybrid system with an associated plant automaton, (V, A). Cause/effect relationships are represented by the automaton's arcs. These relationships connect the various "goals" (automaton vertices) of the system. The preceding section's adaptation algorithm allows the system to identify those cause/effect relations of a specified automaton which are indeed achievable with the hybrid system's fixed set of control policies. By determining the achievable arcs of a specified plant automaton, the system is also determining which goals (vertices) of the original automaton are controllable. The preceding observations imply that the proposed inductive inference algorithm is allowing the system to autonomously determine which goals of the specified plant automaton can be achieved by the system. Since this determination is based on an internally implemented oracle function, goal determination is internal to the system and it can be argued that this system exhibits one key attribute of intelligent systems, namely the attribute of goal self-determination.

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