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Event Identification in Hybrid Control Systems

M.D. Lemmon *, P.J. Antsaklis
Department of Electrical Engineering
University of Notre Dame
Notre Dame, IN 46556 USA

Abstract

Hybrid control systems can arise when a discrete event system is used to supervise the behaviour of continuous-state systems. In this context, the DES supervisor manipulates symbols which are representative of events occuring within the plant's state space. An important issue in the design of such control systems concerns conditions under which such symbolic manipulations are sufficient to adequately supervise the plant. This paper provides a method for identifying such supervisable events whose computational complexity is polynomial in plant complexity.

1 Introduction

Supervisory control systems use logical directives to control dynamical systems. When the system to be controlled evolves over a continuous-state space, then the entire system consists of two distinctly different types of systems; a discrete event system (DES) supervisor or controller and the continuous-state system (CSS) plant. This combination of two different types of systems is sometimes called a hybrid control system. Because the plant and supervisor are different types of systems, an *interface* is needed to connect the plant and supervisor. The interface has two functions. It must transform the plant's state vector into a symbol which is representative of some "event" occuring within the plant. The interface must also transform control symbols (directives) issued by the supervisor into a control vector which can be used by the plant. The interface is therefore responsible for identifying "events" occuring within the plant which can indeed be controlled by the control policies available to the system. This identification problem is referred to as event identification in this paper. The problem of event identification is, essentially, a problem of identifying an interface which insures that the plant is controllable in some appropriate sense.

The combined plant and interface can be treated as another discrete event system. This *equivalent plant DES* or *plant automaton* [Antsaklis 1994] is the entity which is actually controlled by the supervisor. The implicit assumption of such a supervisory control scheme is that controlling the plant automaton is sufficient to "acceptably" control the original plant. The sense in which DES controllability implies CSS controllability is therefore an important issue in hybrid system's theory. Addressing this issue requires a good understanding of the relationship between the CSS plant and its DES counterpart, a relationship which is largely determined by the type of interface used to connect the plant and supervisor.

There has been, to date, relatively little organized work attempting to study the relationship between DES and CSS controllability and the impact of this relationship on the system's interface. Most hybrid systems work has generally assumed a system interface with sufficient structure to render such questions secondary. The problem with this approach is that it lacks sufficient generality to provide a deep understanding of the relationship between continuous state system's and their derived symbolic behaviour. Some exceptions to this earlier trend will be found in [Ramadge 1990], which examined the symbolic be-

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haviour of smooth dynamical systems over state space partitions as well as [Nerode 1992] which examined the structural stability of hybrid systems. Additional work in this area will be found in [Stiver 1992] where the notion of quasi-determinism was introduced to handle this issue.

This paper discusses the "supervisability" of hybrid systems in a manner which relates hybrid system supervision to existing notions of CSS controllability. The principal result of this paper is a set of conditions on the hybrid system interface which is sufficient to guarantee the hybrid system's supervisability. These sufficient conditions provide a means for determining "supervisable" hybrid system interfaces. It is then shown that an interface satisfyng these sufficient conditions can be efficiently computed using the method of centers [Nemirovsky 83]. The method of centers is an approach to convex programming which is easily framed as a learning algorithm and which can be shown to converge in finite time that scales in a polynomial manner with plant complexity. These complexity results therefore indicate that the problem of event identification may be efficiently addressed in applications with a large number of available control policies.

2 Hybrid Dynamical Systems

Hybrid dynamical systems consist of a continuous-state plant interfaced to a discrete event supervisor. The system therefore consists of three components; the *plant*, *supervisor*, and *interface*. The interface can be decomposed into two subsystems known as the *actuator* and *generator*. This section formally discusses the form of the hybrid system components assumed in the remainder of this paper. The following formulation is an extension of earlier work in [Stiver 1992]. It was also influenced by other hybrid system models in [Nerode 1992] and [Peleties 1989].

The system to be controlled is called the *plant*. The plant is represented by the following differential equations

$$\frac{d\bar{x}}{dt} = f_0(\bar{x}) + \sum_{i=1}^{m} r_i f_i(\bar{x}) \qquad (1)$$

where $\bar{x} \in \Re^n$ is the state vector and $\bar{r} \in \Re^m$ is the control (reference) vector. The functions $f_i : \Re^n \rightarrow \Re^n$ for $i = 0, \ldots, m$ are assumed to be Lipshitz continuous with respect to the state vector to insure uniqueness of state trajectories, $\bar{x}(t)$. The vector fields f_i represent control policies which are coordinated through the control vector $\bar{r}$.

The plant is controlled through logical directives issued by a *supervisor*. The supervisor is a discrete event dynamical system. Such a discrete event system can take on a variety of forms including deterministic automatons, Petri nets, recursively enumerable processes, or directed acyclic graphs. The only assumptions on the supervisor invoked by this paper concern the input to and output from the supervisor. In particular, the supervisor's inputs are symbols $\tilde{x}$ drawn from a finite alphabet $\tilde{X}$ and the supervisor's outputs are symbols $\tilde{r}$ drawn from a finite alphabet $\tilde{R}$. For $n = 0, 1, \ldots, \infty$, the sequence of input symbols will be denoted as $\tilde{x}[n]$ and will be called the *plant symbol sequence*. The sequence of output symbols will be denoted as $\tilde{r}[n]$ and will be called the *control symbol sequence*.

The *generator* transforms the plant's state trajectory, $\bar{x}(t)$ into a plant symbol sequence, $\tilde{x}[n]$. This sequence is obtained from the following equation

$$\tilde{x}[n] = \gamma(\bar{x}(\tau_p[n])) \qquad (2)$$

where $\gamma : \tilde{R}^n \rightarrow \tilde{X}$ is a surjective mapping and where $\tau_p[n]$ is a sequence of *plant instants* representing the times (measured with respect to the plant's clock) when the generator issues a plant symbol.

For this paper, γ and $\tau_p[n]$ are defined with respect to an open covering of the plant's state space. Let $\mathbf{X}$ denote an open covering of the plant's state space. This open covering will be called the hybrid system's *basis event covering* and is given by

$$\mathbf{X} = \{ \mathbf{x}_1 \quad \cdots \quad \mathbf{x}_q \} \qquad (3)$$

where $\mathbf{x}_i \subset \Re^n$ for $i = 1, \ldots, q$. Since $\mathbf{X}$ is an open covering, each of the subsets $\mathbf{x}_i$ is an open set and $\bigcup_i \mathbf{x}_i = \Re^n$. Each element of $\mathbf{X}$ will be referred to as a *basis event* for the hybrid system.

The generator mapping and plant instants are defined with respect to the basis event covering. Let $\tilde{X}$, the alphabet of plant symbols consist of symbols labeling each one of the basis events in $\mathbf{X}$. If the plant state $\bar{x}(t)$ is on the boundary, $\Gamma(\mathbf{x})$, of a basis event $\mathbf{x}$, the generator mapping γ will output the symbol

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associated with that basis event. In the event that the state is not on a boundary, then the generator mapping's output is NULL.

$$\gamma(\mathbf{x}) = \begin{cases} \tilde{x}_i & \text{if } \bar{x} \in \Gamma(\mathbf{x}_i) \\ \text{NULL} & \text{otherwise} \end{cases} \tag{4}$$

The plant instants are defined as those times when the plant state is on the boundary of a basis event. A more formal definition of plant instants can be obtained by taking the inferior limit of the set of open time intervals over which the plant state crosses a boundary.

The *actuator* transforms the control symbol sequence, $\tilde{r}[n]$, into a control vector trajectory. This relationship is given by the following equation

$$\bar{r}(t) = \sum_{n=0}^{\infty} \alpha(\tilde{r}[n]) I(\tau_c[n], \tau_c[n-1]) \tag{5}$$

where $I(t_1, t_2)$ is an indicator function taking on the value of unity over the interval $[t_1, t_2)$ and zero elsewhere, where $\alpha : \tilde{R} \rightarrow \bar{R}^m$ is an injective mapping from the control symbols into a finite set of control vectors, and where $\tau_c[n]$ for $n = 0, 1, \ldots, \infty$ is a sequence of *control instants* representing the times when the supervisor issued the nth control symbol. The control instants are measured with respect to the plant's clock.

Full characterization of the actuator is obtained once the mapping α and the mechanism for determining the sequence of control instants ,$\tau_c[n]$, has been specified. Due to causality considerations, it will be assumed that the nth control instant will be between the nth and $n+1$st plant instants. This therefore implies that

$$\tau_p[n] \leq \tau_c[n] \leq \tau_p[n+1] \tag{6}$$

for all $n = 1, \ldots, \infty$. It is further assumed that $\tau_p[0] = \tau_c[0] = 0$. For this paper, it will be basically assumed that the control instant occurs "immediately" after the associated plant instant.

3 Supervisability

The combination of plant and interface forms another discrete event system. The hybrid control system, from the supervisor's perspective, can then be seen as two interconnected DES; the supervisor and an equivalent plant DES. Since the supervisor is directly connected to the plant DES, one approach to hybrid controller design is to synthesize a supervisor which effectively controls the equivalent DES model of the plant. The natural concern is whether or not control of the DES is sufficient to "acceptably" control the original plant.

The following discussion indicates precisely what is being controlled by the supervisor. Let $\mathbf{X}$ be a finite open cover of $\Re^n$ consisting of elements $\mathbf{x}_i$, for $i = 1, \ldots, q$. The set of *conjunctive events*, $\mathbf{C}$, generated by $\mathbf{X}$ will consist of all subsets, $\mathbf{c}$, of $\Re^n$ which can be expressed as the intersection of elements in $\mathbf{X}$. In other words, for any conjunctive event $\mathbf{c}$ there exists a collection, I, of integers between 1 and q such that

$$\mathbf{c} = \bigcap_{i \in I} \mathbf{x}_i \tag{7}$$

The set I will be called the index set of the conjunctive event.

The *symbolic* behaviour of the plant is described by the way in which the plant's state transitions between events. Consider a hybrid system with a collection, $\mathbf{C}$, of conjunctive events generated by a finite cover, $\mathbf{X}$, of the state space. Let $\tilde{R}$ be a finite alphabet of control symbols, let $V \subset \mathbf{C}$, and let $A \subset V \times V \times \tilde{R}$. The *plant automaton* associated with the hybrid system is a labeled directed graph, (V, A), where an ordered triple of $(\mathbf{c}_o, \mathbf{c}_t, \tilde{r})$ is in A if and only if the plant's state trajectory, $\bar{x}(t)$ generated by the system equations

$$\frac{d\bar{x}}{dt} = f(\bar{x}, \alpha(\tilde{r})) \tag{8}$$

satisfies the following conditions.

- there exists T_o such that $\bar{x}(t)$ is in the closure of $\mathbf{c}_o$ for $0 \leq t \leq T_o$.
- there exists T_t such that $\bar{x}(t)$ is in the closure of $\mathbf{c}_t$ for $t = T_t$.
- and $\bar{x}(t)$ is not in the closure of $\bigcup_i \mathbf{c}_i$ for $T_o < t < T_t$.

As a model of the plant's symbolic behaviour, the plant automaton provides the basis for determining a supervisor which can control the plant. In order

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for the resulting supervisor to effectively control the plant between conjunctive events, it is necessary that the plant automaton also be supervisable. The following makes this notion of supervisability more precise. Consider the plant automaton, (V, A) associated with a hybrid control system and consider two events, $\mathbf{c}_o$ and $\mathbf{c}_t$, which are elements of V. Let $\bar{r}[n]$ denote the control history for a directed walk from vertex $\mathbf{c}_o$ to $\mathbf{c}_t$. The event $\mathbf{c}_t$ is said to be *supervisable from* $\mathbf{c}_o$ if and only if any other directed walk originating from $\mathbf{c}_o$ with control history $\bar{r}[n]$ necessarily terminates at $\mathbf{c}_t$.

The following proposition provides a sufficient condition for testing whether or not an arc of a plant automaton is indeed supervisable

Proposition 1 *Consider a hybrid system with an associated basis event collection,* $\mathbf{X}$. *Let* (V, A) *be a given plant automaton. Let* $V_i : \Re^n \to \Re$ *(i=1,...,q) be a family of* C^1 *functionals and let* $f_j : \Re^n \to \Re^n$ $(j = 0, \ldots, m)$ *be a family of Lipschitz continuous control policies. Let* $L_j V_i$ *be the Lie derivative of the functional* V_i *with respect to vector field* f_j. *The event* $\mathbf{c}$ *in* $\mathbf{C}$ *with index set* I *will be supervisable if*

- $V_i(\bar{x}) > 0$ *for all* $\bar{x} \notin \mathbf{c}_i$ *and* $V_i(\bar{x}) = 0$ *for all* $\bar{x} \in \mathbf{c}_i$.
- *for all* $i \in I$ *and for all* $\bar{x} \in \Re^n$,

$$0 > \begin{pmatrix} L_0 V_i & L_1 V_i & \cdots & L_m V_i \end{pmatrix} \begin{pmatrix} 1 \\ r_1 \\ \cdot \\ r_m \end{pmatrix} \tag{9}$$

- *and for all* $\bar{x}$ *on the boundary of events* $\mathbf{c}_o \neq \mathbf{c}$, *with index sets* I_o, *there exists an* $i \in I_o$ *such that* $V_i(\bar{x}) = 0$ *and*

$$0 < \begin{pmatrix} L_0 V_i & L_1 V_i & \cdots & L_m V_i \end{pmatrix} \begin{pmatrix} 1 \\ r_1 \\ \cdot \\ r_m \end{pmatrix} \tag{10}$$

Proof: (outline) In order for event $\mathbf{c}$ to be supervisable there must be an arc which goes from every event in V to $\mathbf{c}$ such that the state trajectory does not intersect any other event in V. One way to insure that this occurs is to require that $\mathbf{c}$ be a globally attracting invariant set and that all other events are repellors of the plant controlled by $\alpha(\bar{r})$. The above conditions are based in a straightforward manner on the LaSalle invariance principle. •

Remark: If the set of conjunctive events form a full partition of the plant's state space, then supervisability (determinism) is too restrictive to be useful. Insuring determinism over full state partitions will require that the partition boundaries be integral manifolds of the system; a condition which will probably be impossible to satisfy exactly in practice. The formulation stated above, however, does not require that events fully partition the state space. Therefore it is no longer necessary that event boundaries be integral manifolds. Consequently, there is significant flexibility in the way we can assign the control vectors $\bar{r}$ to insure determinism in the plant automaton.

Remark: The inequalities used to test the plant automaton's supervisability require that the Lie derivatives of the dynamic system be known. If there exist prior models for the plant, then these Lie derivatives can be estimated directly from state estimates. If no such models exist however, then these derivatives must be estimated by direct observation of the system. In this respect, the preceding inequalities indicate precisely what type of information must be made available to the system so that it can determine whether or not its plant automaton is indeed supervisable.

Remark: The conditions stated above are global statements. In many situations, however, satisfaction of these conditions on a local basis may be extremely useful. For example, if the plant's state trajectory is known to be contained within a given region of the state space, then it is only necessary that the above inequalities be satisfied in that region (rather than the entire state space) to insure the supervisability of the resulting plant automaton.

4 Event Identification

The sufficient conditions obtained in the preceding section form a system of inequalities which are linear in the control vector, $\bar{r}$. Since the control vector is determined by the hybrid system's actuator mapping, these conditions also provide a method for determining an actuator mapping, α, which insures that

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the given automaton will be implementable. Such α (or rather the $\bar{r}$ associated with the control symbol $\tilde{r}$) represent feasible points of the inequality system. The identification of those $\bar{r}$ which yield a supervisable plant automaton can be viewed as an event identification problem. In this case, the event is the arc between two conjunctive events. The identification algorithm determines whether or not such an arc can be actually realized by the system.

The event identification problem is actually a problem in determining feasible points for inequality systems. There exist good numerical techniques for finding such points. One class of algorithms for determining such points uses the so-called method of centers to update a hypothesized feasible point. The method of centers computes a sequence of convex bodies and their centers in such a way that the computed centers converge to the feasible point. Depending on the analytic form of the convex bodies and the centers, different types of algorithms are obtained. Examples of such algorithms are the so-called ellipsoid method [Bland 1981] and interior point methods based on logarithmic barrier functions [Gonzaga 1992].

The method of centers can be viewed as an inductive inference algorithm. Inductive inference is a machine learning algorithm in which a system learns a Boolean function from examples[Angluin 1983]. The learning algorithm is outlined below.

- Hypothesis: The hypothesis assumes that the conjunctive event c is supervisable by vector $\bar{r}_i = \alpha(\tilde{r}_i)$. This hypothesis is represented by a convex body K_i in which $\bar{r}_i$ is a "center" for K_i. Prior work [Lemmon 1993b] [Lemmon 1993]used ellipsoidal convex bodies and their associated geometric centers. It is quite possible and probably desirable to also use analtyic centers defined by logarithmic barrier functions. This latter approach has proven useful in interior point linear programming algorithms [Gonzaga 1992].
- Experiment: The inequalities in the proposition indicate that specific information has to be gathered by the system before the sufficient conditions can be tested. The experimental component of the algorithm involves the measurement of these quantities. In particular, the system must be able to estimate or measure the Lie derivatives of the plant. As noted above, these derivatives can be estimated if there is a prior plant model. When no such model is available, there must be sensor capable of measuring or estimating these derivatives.
- Query: The third component of the algorithm is the oracle query. The oracle is a Boolean functional which outputs 0 if the experimental data is consistent with the inequality system and which outputs 1 if the data does not satisfy the inequality system.
- Update: If the oracle's response is 0, then nothing is done. If the oracle declares 1, then the current data is inconsistent with the hypothesis that $\bar{r}_i$ satisfies the inequality system. This means that the hypothesis has to be changed. An update algorithm is called to compute a new convex body K_i and its associated center. The center of the updated convex body is then taken to be the new control vector $\bar{r}_i$. Prior work examined the use of central cut ellipsoid algorithms [Lemmon 1993b] [Lemmon 1993] in performing this update step. More recent experimental work has implied that the use of analytic centers (rather than geometric centers) may provide a more efficient way of updating the hypothesis.

The advantage of using the method of centers in solving such problems is that these methods have provable polynomial complexity. There are well-known results on the convergence of such methods. For example, an idealized method of centers algorithm which uses the geometric center can be shown to converge in finite time [Nemirovsky 83]. It is further shown that this finite convergence time scales in a way which is independent of problem complexity. The problem with the idealized algorithm is that the computation of geometric centers for arbitrary convex bodies is impractical. The central-cut ellipsoid algorithm represents a more practical way of computing geometric centers. For this algorithm, it has been shown that the finite convergence time scales on the order of $o(n \ln L)$ where n is a measure of plant complexity (generally the number of control policies available to control the plant) and L is a measure of the volume of the feasible set that's being searched for [Groetshel 1988] [Lemmon 1993b].

The significance of the preceding discussion is that

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it suggests that the controllability of individual events in the plant automaton can be decided quickly and efficiently using the method of centers. The scaling results suggest that the $o(n \ln L)$ convergence rate for the ellipsoid method can be improved upon considerably and that from a theoretical perspective this rate will be bounded below in a manner which is independent of plant complexity. This immediately suggests that more efficient approaches than the ellipsoid method can be used for finding feasible points. This fact is already well-known and interior point algorithms using analytic centers based on so-called logarithmic barrier functions [Gonzaga 1992] represent an important class of update procedures which may provide even more efficient identification of supervisable automaton events.

5 Summary

This paper has provided a framework for discussing hybrid control systems in a way which clearly relates hybrid system *supervisability* back to classical notions of system controllability. It was seen that an interface insuring the supervisability of a hybrid system can be computed using convex programming algorithms based on the method of centers. It was seen that this approach for interface identification can be implemented in an on-line manner by framing the method of centers as an inductive learning protocol. Most important is the result that such learning procedures will converge in finite time and that this convergence time will scale in a polynomial manner with plant complexity. The results therefore suggest that the application of convex programming methodologies for validating (verifying) a specified DES controller or plant model can be done efficiently for large scale systems.

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