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Verification of Safe Inter-Event Behaviour in Supervisory Hybrid Dynamical Systems

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Extended Abstract

A dynamical system can be characterized by the ordered tuple, (X, Φ, T) where X is the *state space*, T is a semigroup (usually taken to be the set of integers or the real line), and $\Phi : X \times T \to X$ is a mapping satisfying the usual semigroup and identity properties associated with a transition or flow operator. In the event that X and/or T can be represented as the Cartesian product of sets with distinctly different topologies, then we obtain a *hybrid dynamical system.* The most common example of a hybrid system is a sampled-data system in which $T = \Re \times N$. Such systems occur when a computer is used to synchronously sample the behaviour of a continuous (or smooth) dynamical system. More recently, considerable attention has focused on hybrid systems with asynchronous sampling. Such systems have the form $(\Re^n \times \tilde{X}, \Phi, \Re)$ where the state space is the hybrid combination of a $\Re^n$ and a discrete set, $\tilde{X}$, of events. This type of hybrid systems occur when a continuous-state plant is supervised by a discrete event system.

Hybrid systems arising in the supervisory control of continuous-state plants have been previously discussed in [7]. In [7], controller design is accomplished by determining the optimal DES controller for a DES equivalent of the continuous-state plant. Such an approach to controller design focuses on the high-level or purely symbolic behaviour of the system. The *inter-event* behaviour of the continuous-state plant, however, is also important in controller design. Inter-event behaviour refers to the behaviour of the continuous-state plant between the system's generation of discrete events. Inter-event behaviours which satisfy specified constraints with respect to a signal norm, or with respect to the triggering of specified forbidden events will be said to be *safe.*

The methods used in [7] do not address the safety of a system's inter-event behaviour. In particular, this paper is concerned with the following problem.

> Given the continuous-state system's initial state, a set of controllers, a collection of forbidden events and a desired terminating event, does there exist an admissible control law which generates a safe system trajectory?

This paper discusses two methods for verifying the safety of inter-event behaviour. The first method uses a Fliess functional series to form a system of linear inequalities. The feasibility of this inequality system is sufficient to ensure the existence of a switched control law generating a safe state trajectory. This method has been outlined in [6]. The second method uses gain-scheduling on linear parameter varying (LPV) systems. This proposed method results in a set of Linear Matrix Inequalities (LMI) whose feasibility is sufficient to ensure the existence of a switched set of linear controllers generating safe inter-event behaviour. The primary objective of this paper is to discuss and compare these two methods for safety verification.

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The paper will be organized as follows. Section 1 formally discusses the safety problem considered. Section 2 discusses the Fliess series approach to safety verification. Section 3 discusses the gain-scheduled approach to safety verification. An example illustrating the performance of both methods will be presented in section 4 with final conclusions being presented in section 5

1 Safety of Inter-Event Behaviour

An important class of hybrid systems arise when computers are used to autonomously supervise the behaviour of continuous-state systems or plants. In this case, the computer issues a high-level supervisory command to the plant. This "discrete" command is then used to select a controller for the continuous-state system. The resulting continuous-state trajectory then triggers new "events" which are used by the computer to decide how best to supervise the plant. Such systems, therefore, contain a high-level or "decision" feedback path giving rise to the hybrid nature of this system. Such systems will be refered to as supervisory hybrid control systems.

When the computer issues a supervisory control command, it expects the plant to behave in an appropriate or *safe* manner. What constitutes safety can interpreted in a variety of ways. In this paper, however, safe behaviours occur when the commanded transition results in a state trajectory that reaches a specified *goal set* without crossing into any *forbidden sets*. The objective of the control is to transition the continuous-plant's state to a new setpoint, without violating a known set of forbidden conditions. When that goal set is reached, then a plant event is generated which can be used by the supervisor to select a new high-level command. Note that this paper is not concerned with the overall symbolic behaviour of the hybrid system. Rather, we are concerned with the behaviour of the system between the generation of plant events. In other words, this paper is concerned with *inter-event* behaviour of supervisory hybrid systems in the same way that discrete time control is concerned with inter-sample behaviour.

To go much further, we need to be more specific about the nature of our problem. Let's assume that the continuous-state plant is given by the following differential equation

$$\dot{x}_p = f_p(x, u) \tag{1}$$

where f_p is Lipshitz continuous in x and u. $x \in \Re^n$ is, of course, the continuous-state and $u \in \Re^m$ is the control vector.

Now define a set of functionals, $h_j : \Re^n \rightarrow \Re$ $(j = 0, \ldots, N)$ which *separate* the state space. Specifically, h_j is said to separate $\Re^n$ if and only if for all $x, y \in \Re^n$ such that $h_j(x) < 0$ and $h_j(y) > 0$, then there exists a $0 \leq \lambda \leq 1$ such that $h_j(\lambda x + (1 - \lambda)y) = 0$. The set

$$H_j = \{x \in \Re^n j \, : \, h_j(x) = 0\} \tag{2}$$

forms an $n-1$-dimensional manifold which we call a *trigger*. The trigger, H_0, will be the boundary for a subset of $\Re^n$ called the goal set. The other triggers, H_j, for $j = 1, \ldots, N$ will be called forbidden triggers.

Let t_0 be the time when a control directive is issued by the computer and let $x_0 = x(t_0)$ be the initial state of the continuous-state plant at t_0. Assume that $h_j(x_0) < 0$ for all j. Such an initial state will be said to be feasible. As noted before, the control directive selects a controller which generates the control signal u. The state trajectory generated by the control directive will be said to be *safe* if there exist finite times T_1 and T_2 $(T_1 < T_2)$ such that

- $h_j(x(t)) < 0$ for all $t_0 \leq t < T_2$ and for $j = 1, ldots, N$,
- $h_0(x(t)) < 0$ for all $t_0 \leq t < T_1$,

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- and $h_0(x(t)) > 0$ for all $T_1 < t < T_2$

The simple question addressed in this paper concerns sufficient tests for the existence of admissible controllers generating safe inter-event trajectories.

2 Verification via Fliess Series

One method suggested for verifying the safety of inter-event behaviours was outlined in [6]. In this section we outline the principal ideas behind this method and identify some of the underlying assumptions that are needed to ensure this method can be used.

In the first place, we assume the plant is described by differential equations of the form

$$\dot{x}_p = g_0(x_p) + \sum_{i=1}^{m} g_i(x_p)u_i \tag{3}$$

The forbidden triggers are described by the equations

$$y_j = h_j(x_p) \;\; 1 \leq j \leq M \tag{4}$$

and the goal trigger is given by

$$y_0 = h_0(x_p) \tag{5}$$

This system is defined over all $\Re^n$. The vector fields $g_0, g_1, \ldots, g_m$ are assumed to be analytic. In a similar manner h_j $(0 \leq j \leq M)$ are also analytic functions which "separate" the state space in the sense mentioned earlier.

The following discussion will require the iterated integral of a set of functions. Letting $(i_k \ldots i_0)$ be a sequence of $k+1$ elements of integers between 0 and M, inclusive, called a multi-index, then the corresponding iterated integral is denoted as

$$\int_0^t d\eta_{i_k} \ldots d\eta_{i_1} d\eta_{i_0} \tag{6}$$

where $\eta_0(t) = t$ and $\eta_i(t) = \int_0^t u_i(\tau)d\tau$ for $1 \leq i \leq M$. Given the initial state x_0, the outputs $y_j(t)$ can be represented by a formal power series, in particular a Fliess functional series provided certain conditions are satisfied. These conditions are summarized in the following theorem whose proof will be found in [4].

Theorem 1 *Suppose there exists $K > 0$ and $M > 0$ such that*

$$\left| L_{g_{i_0}} \ldots L_{g_{i_k}} h_j(x_0) \right| < K(k+1)! M^{k+1} \tag{7}$$

for all $k \geq 0$ and all multi-indices $(i_0, \ldots, i_k)$. Then there exists a real number $T > 0$ such that for all t where $0 \leq t \leq T$ and each set of piecewise continuous functions, $u_1, \ldots, u_m$ defined on $[0, T]$ subject to

$$\max_{0 \leq t \leq T} |u_i(t)| < 1 \tag{8}$$

Then the jth output of the system may be expanded as

$$y_j(t) = h_j(x_0) + \sum_{k=1}^{\infty} \sum_{i_0 \ldots, i_k} L_{g_{i_0}} \ldots L_{g_{i_k}} h_j(x_0) \int_0^t d\eta_{i_k} \ldots d\eta_{i_0} \tag{9}$$

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Remark: The preceding series is infinite unless the plant's relative degree vector is finite. In most practical cases, we expect this to be the case.

Let's assume that we want to guarantee that the event transition is completed in time T. Divide the time interval $[t_0, T]$ into subintervals, $[t_0 + (\ell - 1)\delta, t_0 + \ell\delta]$ where $\delta = T/L$. Assume that the controls, $u_i(t)$ are piecewise constant over these subintervals and let the value of the control in the ℓth subinterval be written as w_ℓ. We can then rewrite the above series as

$$y_j(t) = h_j(x_0) + \sum_{k=1}^{\infty} \sum_{i_0,\ldots,i_k} L_{g_{i_0}} \ldots L_{g_{i_k}} h_j(x_0) \sum_{\ell=1}^{L} \alpha_\ell(t) w_\ell \tag{10}$$

where the functions $\alpha_\ell(t)$ are of the form $t^n/n!$. In this case it is clear that the output $y_j(t)$ can be expressed in the form

$$y_j(t) = b_j^T(t) w \tag{11}$$

where w is an L-vector consisting of the control signals w_ℓ, and $b_j(t)$ is an L-vector whose elements are known polynomials of the time t.

The preceding expression for $y_j(t)$, can now be used to form a sequence of inequality constraints of the form

$$\begin{array}{rcl}
0 & < & b_0^T(t_0+\delta)w \\
0 & < & b_0^T(t_0+2\delta)w \\
 & & \ldots \\
0 & < & b_0^T(T)w \\
0 & > & b_0^T(T+\delta)w \\
 & & \\
0 & < & b_1^T(t_0+\delta)w \\
0 & < & b_1^T(t_0+2\delta)w \\
 & & \ldots \\
0 & < & b_1^T(T+\delta)w \\
 & & \\
0 & < & b_L^T(t_0+\delta)w \\
0 & < & b_L^T(t_0+2\delta)w \\
 & & \ldots \\
0 & < & b_L^T(T+\delta)w
\end{array} \tag{12}$$

The feasibility of this inequality system subject to the constraints that $|w_m| \leq 1$ provides a sufficient condition for the safety of the commanded transition at the time instants $t_0 + \ell\delta$. If δ is chosen small enough then it will be shown that the system outputs for times between these instants also satisfy the safety constraints.

The preceding system of linear inequalities can also be used as the basis for synthesizing an "optimal" controller in which the problem is framed as a linear program. The synthesis can be framed in a variety of ways. One specific approach which appears to be useful in dealing with systems of unit relative degree is found in [3]. This paper will not comment on the synthesis problem, other than to note the relationship between verification and synthesis.

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3 Verification as Gain-Scheduling

Another approach that verifies the safety of inter-event plant behaviours is presented in [5]. In this case, the nonlinear plant is linearized about a "safe" reference trajectory and traditional gain-scheduling approaches for linear parameter varying systems are employed. This is basically the problem of determining whether or not there exist a set of switched linear controllers that can guide the plant's inter-event behaviour in a safe manner.

The approach taken in [5] assumes that the desired "safe" trajectories (i.e., the specified behaviour) can be represented by a nonlinear reference model of the form

$$\dot{x}_m = f_m(x_m) \tag{13}$$

where f_m is a Lipschitz continuous mapping. The reference state therefore evolves in a continuous manner over the state space. We assume that the model state reaches the terminal set in finite time T without entering any of the forbidden regions. The objective is to find a family of linear controllers such that expedient switching between controllers ensures that actual plant trajectory x_p is within a distance γ of the safe reference trajectory.

In this case the plant is represented by the vector differential equation

$$\dot{x}_p = f_p(x_p, u) \tag{14}$$

where f_p is Lipschitz continuous. The obvious approach involves developing a linear parameter varying representation of the error dynamics. Let the error be denoted as

$$x = x_m - x_p \tag{15}$$

Linear error models parameterized by the plant and reference model states can be derived in a number of ways. In our case, we linearize the plant dynamics around the reference model states. One advantage of this approach is that zero error in the linear model corresponds to zero error in the nonlinear model.

Linearizing the plant around the operating point $(x_p, u) = (x_m, 0)$ yields the following LPV system

$$\dot{x} = A(\theta)x + A_u(\theta)u + B_w(\theta)w \tag{16}$$

where $w = 1$ is introduced as a fictitious disturbance input with the parameter $\theta = S(x_m, x, u)$ being functionally dependent on the model/plant state and control. It is assumed that the θ changes in a bounded manner, i.e. $|\dot{\theta}| \leq \dot{\theta}_{\max}$. It is also assumed that A, A_u, and B_w are affine in θ. The term $B_w(\theta)$ represents the mismatch between the reference model and the plant dynamics as well as the nonlinearities present in the dynamical error model. The introduction of the fictitious disturbance implies that a control robust to disturbance through the input vector $B_w(\theta)$ will also be robust to the nonlinearities and model mismatch.

As noted above, we attempt to ensure safe performance by requiring that the controlled plant's trajectory follows the reference trajectory within a distance of γ. This means that

$$\sup_{0 \leq t \leq T} \|x(t)\| \leq \gamma \tag{17}$$

This is accomplished by using linear controllers designed for linearized plants about N_K setpoints. These setpoints are chosen along the parameter trajectory $\theta(t) = S(x_m(t), 0, 0)$. Let $I_K = \{1, \ldots, N_K\}$ be a set of indices. Associated with each setpoint is an index $i \in I_K$. For each $i \in I_K$ there is the tuple $(\theta^{(i)}, K^{(i)}, P^{(i)}, \Sigma_{cl}^{(i)})$ where

- $\theta^{(i)}$ is the setpoint $S(x_m^{(i)}, 0, 0)$,

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- $K^{(i)}$ is a constant gain feedback controller designed for the linearized plant at setpoint $\theta^{(i)}$,
- $P^{(i)}$ is a positive definite matrix characterizing desired invariant subsets of the state space. These invariant sets are ellipsoids of the form

$$\mathcal{E}_r(P^{(i)}) = \left\{ x \in \Re^n \, : \, x' P^{(i)} x \le r \right\} \tag{18}$$

- Σ^i_{cl} is a state-space model of the ith LPV system obtained at setpoint $\theta^{(i)}$. This model is of the form

$$\begin{bmatrix} \dot{x} \\ q \end{bmatrix} = \begin{bmatrix} A^{(i)} & B_p^{(i)} & B_w^{(i)} \\ C_q^{(i)} & 0 & D_{qw}^{(i)} \end{bmatrix} \begin{bmatrix} x \\ p \\ w \end{bmatrix} \tag{19}$$

  where $p = \text{diag}(\hat{\theta}_1 I, \ldots, \hat{\theta}_s I)q$ where $\hat{\theta} = \theta - \theta^{(i)}$ and I is an identity matrix of appropriate dimension. $A^{(i)} = A(\theta^{(i)}) + A_u(\theta^{(i)})K^{(i)}$ and $B_w^{(i)} = B_w(\theta^{(i)})$. The other matrices in this model are obtained using standard methods [8].

The continuous-state's system is controlled by switching between the different control configurations represented by the tuple, $(\theta^{(i)}, K^{(i)}, P^{(i)}, \Sigma_{cl}^{(i)})$. Controller $K^{(i)}$ is switched into feedback when the condition,

$$\|\hat{\theta}^{(i)}\|_\infty \ge \vartheta_{\text{out}} \tag{20}$$

is detected. ϑ_{out} represents the level of θ variation that the control system can tolerate. One way ϑ_{out} can be chosen is by using the μ-norm of the closed loop system.

The controller, $K^{(i)}$ is chosen so that the following conditions are satisfied (for all $i \in I_K$)

- For $\|\hat{\theta}^{(i)}\|_\infty \le \vartheta_{\text{out}}$ $\mathcal{E}_1(P^{(i)})$ is invariant with respect to the controlled flow,
- and $x \in \mathcal{E}_1(P^{(i)})$ implies that $x'x \le \gamma^2$.

Using standard linear robust control methods [1], controllers satisfying the above assumptions can be synthesized.

The primary question is now whether or not the set of switched controllers introduced above yield a safe trajectory (i.e. a trajectory that follows the reference model within a distance of γ). This question can be answered by examining, in detail, the nature of the individual transitions between the ith and $(i+1)$st controllers. In verifying the safety of these individual transitions, we can then ensure the safety of the switching sequence.

The individual transitions are safe if the error's, $x_p - x_m$, magnitude is bounded above by γ. This is guaranteed in the following theorem. In this theorem let $\|\cdot\|$ and $\|\cdot\|_\infty$ denote vector 2-norms and sup-norms, respectively.

Theorem 2 *Let the ith controller be used at $t^{(i)}$ and let $x(t^{(i)}) \in \mathcal{E}_1(P^{(i)})$. If there exists T such that $\|\hat{\theta}^{(i)}(t)\|_\infty \le \vartheta_{\text{out}}$ for all $t^{(i)} \le t \le T$, then $\|x(t)\| \le \gamma$.*

Proof: From the assumptions on the ith controller, $\mathcal{E}_1(P^{(i)})$ is flow-invariant provided the conditions on $\hat{\theta}^{(i)}$ hold. If x starts in this ellipsoid, it must remain there and thus satisfy the bound stated above. •

Remark: This theorem provides two sufficient conditions to ensure that the error is bounded. The first condition is a constraint on the initial condition; namely that x_p must be close enough to x_m so that transients are sufficiently small. The second condition bounds the variation in θ that can be tolerated; namely a robust performance condition. To guarantee the safety of the individual transition, we need to determine when these conditions can be met at each switching instant.

We will determine conditions such that if $x(t^{(i)})$ lies in $\mathcal{E}_1(P^{(i)})$, then $x(t^{(i+1)})$ lies in the invariant ellipsoid, $\mathcal{E}_1(P^{(i+1)})$, at the next switching instant. This is done by finding an $r \leq 1$ such that the ellipsoid, $\mathcal{E}_r(P^{(i)})$, is completely contained within the invariant ellipsoid, $\mathcal{E}_1(P^{(i+1)})$, for the $(i+1)$st switching instant. This can be done by optimizing an appropriate linear matrix inequality [1]. With the disturbance, w, however, it is quite possible that $\mathcal{E}_r(P^{(i)})$ is not invariant unless the controller is appropriately designed. The invariance of $\mathcal{E}_r(P^{(i)})$ can be verified using the LMI in the following theorem. This theorem is proven using techniques out of [1].

Theorem 3 *Let $P = \frac{1}{r}P^{(i)}$ and consider the switched controller defined above at switching instant, i. If there exists a positive $\alpha^{(i)}$ and diagonal positive semidefinite matrices, Λ and Π, such that*

$$0 \geq \begin{bmatrix} A'P + PA + \alpha P + C_q'\Lambda C_q & PB_p & PB_w & PB_p \\ B_p'P & -\frac{1}{\vartheta_{\text{out}}^2}\Lambda & 0 & 0 \\ B_w'P & 0 & D_{qw}'\Pi D_{qw} - \alpha I & 0 \\ B_pP & 0 & 0 & -\frac{1}{\vartheta_{\text{out}}^2}\Pi \end{bmatrix} \tag{21}$$

then $\mathcal{E}_r(P^{(i)})$ is invariant with respect to the ith controller.

Note that in the preceding LMI the superscript (i) has been dropped for notational simplicity. When used together, the preceding results provide a set of tests whose feasibility is sufficient to ensure that there is a time when the system state will satisfy the first conditions imposed in theorem 2.

The feasibility problem introduced in theorem 3 can actually be posed as an optimization problem in which we maximize $\alpha^{(i)}$. This gives a bound on the rate at which the x decays into the ellipsoidal set $\mathcal{E}_r(P^{(i)})$. This rate, in turn, allows us to specify an upper bound on the time (also called *dwell time* $t_d^{(i)}$) before which x enters the invariant ellipsoid, $\mathcal{E}_1(P^{(i+1)})$, of the next switching instant. This dwell time is readily shown to have the form

$$t_d^{(i)} = -\frac{1}{2\alpha^{(i)}}\log r^{(i)} \tag{22}$$

where $\alpha^{(i)}$ satisfies the LMI in theorem 3.

The dwell time is important, for it provides a lower bound on the length of time that the ith controller must be used before the first condition of theorem 2 is satisfied at the next switching instant. The second condition in theorem 2 requires that the magnitude of the parameter variation be bounded by ϑ_{out} over the length of time the ith controller is used. This condition can be satisfied over the dwell time provided the parameter variation at the initial switch is bounded in the following manner

$$\|\theta(t^{(i)}) - \theta^{(i)}\|_\infty \leq \vartheta_{\text{out}} - \dot{\theta}_{\max} t_d^{(i)} \tag{23}$$

This condition must also hold at the $i+1$st switch. Ensuring that this condition holds at all switch points is, of course, a matter of appropriately selecting the setpoints. Formal characterization of the selected setpoints is currently being finalized.

The preceding discussion has then shown that assuming the error model satisfies the conditions in theorem 2 while controller i's is in use, that the next switching interval will also satisfy these conditions provided the LMI constraints are satisfied by the controller and the setpoints are appropriately selected to satisfy the parameter variations bounds. These conditions provide a method for computationally verifying the safety of a gain-scheduled implementation of the supervisory command issued by the computer in a hybrid dynamical system.

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4 Example

The following example from [2] is a process control application. The plant dynamics are given by

$$\dot{x}_{p1} = f_{p1}(x_p, u) = \frac{-x_{p1}}{1 + 0.5\sin 10u_1} + u_1 \tag{24}$$

$$\dot{x}_{p2} = f_{p2}(x_p, u) = -x_{p2} + (1 + x_{p1}^2)u_2 \tag{25}$$

The control objective is to move the plant form an initial state near the operating point $(0,0)$ to a point near $(2.5, 2)$ according to the reference dynamics

$$\dot{x}_{m1} = \frac{6 + (x_{m1} + 3)\sin 10(x_{m1} + 3)}{2 + \sin 10(x_{m1} + 3)} \tag{26}$$

$$\dot{x}_{m2} = 2 \tag{27}$$

The presented paper will apply the two verification methods to this example. Current work has nearly completed the gain-scheduling analysis for this particular example. Application of the Fliess-series verification method is currently under way and will be completed for the conference.

5 Summary

This paper contrasts two different approaches to verification with an eye toward the rigorous characterization of conditions under which both methods can be used. It is possible to draw some tentative conclusions from the preceding discussion. As presented above, both methods represent computational methods that can be used to reach a binary decision about the existence of safe controllers. It is readily apparent that these inequalities also provide insight into how to synthesize such controllers. The Fliess approach is restricted to plants which are affine in the controls. A synthesis based on Fliess series would involve solving a linear programming problem to yield a piecewise constant control rule which is, essentially, open loop in character. The gain-scheduled approach does not appear to be restricted to affine plants. There are, however, some stringent growth assumptions which commonly occur when gain-scheduling with linear controllers. The synthesis procedure yields a family of switched linear controllers which have a distinct feedback nature.

The results to be presented in the final paper will use the example to explore the strengths and weaknesses of both methods. By providing a formal development for both methods, it is hoped that the insight essential for the development of improved synthesis methods will emerge.

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