X.Xu and P.J. Antsaklis, "On the Reachability of a Class of Second-Order Switched Systems, "Proc of the American Control Conference, San Diego, CA, June 2-4, 1999. A more detailled version appeared as Technical Report isis-99-003, Dept. of Electrical Engineering, University of Notre Dame, February 1999.

IFAC World Congress 1999

DESIGN OF STABILIZING CONTROL LAWS FOR SECOND-ORDER SWITCHED SYSTEMS

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Abstract: In this paper, we design asymptotically stabilizing switching control laws for switched systems consisting of several second-order LTI subsystems with unstable foci. Switching is needed for the stabilization of a system consisting of several subsystems if each of its subsystems is unstable. We first study switched systems consisting of two subsystems with unstable foci and conic switching stabilizing control laws are derived. Then the result is extended to several subsystems. In particular, necessary and sufficient conditions for the asymptotic stabilizability of a switched system are derived. If the switched system is asymptotically stabilizable, an asymptotically stabilizing switching law can also be obtained. Finally, we briefly compare the method derived here to a matrix inequality based design method.

Keywords: Switching theory; Hybrid; Stabilization; Control system synthesis;

1. INTRODUCTION

A switched system is a system that consists of several subsystems and a (switching) law that specifies which subsystem dynamics will be followed by the system trajectory at each instant of time. Switchings may be time-driven (i.e., a switching happens at specific time instants) or event-driven (i.e., a switching happens when some internal or external event takes place). One issue of major concern is the stability issue.

There are many papers on stability analysis and design of switched systems (Branicky, 1994*a*; Branicky, 1994*b*; Guckenheimer, 1995; Johansson and Rantzer, 1996; Peleties and DeCarlo, 1991). Several authors adopt multiple candidate Lyapunov functions for analysis (Branicky, 1994*a*; Peleties and DeCarlo, 1991). In (Pettersson and Lennartson, 1996; Johansson and Rantzer, 1996), Linear Matrix Inequality (LMI) problems are formulated for the stability analysis of switched systems.

In this paper, we focus on the design of stabilizing switching control laws. We establish necessary and sufficient conditions for the asymptotic stabilizability of a special kind of switched systems, namely, switched systems consisting of second-order LTI subsystems with unstable foci. We will design switching control laws for this kind of switched systems by studying the vector fields of the subsystems. The effectiveness of our approach for this kind of switched systems will be shown in examples.

The outline of the paper is as follows. In Section 2, we look into the asymptotic stabilizability problem for switched systems with two subsystems. In Section 3, the method in Section 2 is extended to the case of several subsystems. In Section 4, examples illustrate our method, which is then compared to a matrix inequality method. Section 5 concludes the paper.

2. TWO SUBSYSTEMS

In this section, we will design stabilizing switching laws for switched systems consisting of two second-order LTI subsystems with unstable foci. By a LTI subsystem with unstable focus, we mean that the origin of $\mathbb{R}^2$ plane is a unstable focus for the subsystem (Khalil, 1996).

We will study the vector fields of both subsystems and give an intuitive way of obtaining a stabilizing switching control law. We shall say that a subsystem is of clockwise(counterclockwise) direction if starting from any nonzero initial point in the phase plane its trajectory is a spiral around the origin in the clockwise(counterclockwise) direction. In the following, we will first consider switched systems with subsystems of the same direction and then consider switched systems with subsystems of opposite directions. For more information on the direction of second order LTI systems with focus, please refer to (Xu and Antsaklis, 1998).

2.1 *Two Subsystems of the Same Direction*

We will consider the following switched systems,

$$\dot{x}(t) = A_1 x(t),\ \dot{x}(t) = A_2 x(t), \qquad (1)$$

whose subsystems are both with unstable foci and are both of the clockwise direction. For switched systems whose subsystems both are of counterclockwise direction, all the following discussion can be applied in a completely analogous manner.

Let $x = (x_1, x_2)^T$ be a nonzero point on $\mathbb{R}^2$ plane, and let

$$f_1 = A_1 x = (a_1, a_2)^T,\ f_2 = A_2 x = (a_3, a_4)^T \quad (2)$$

We can view x, f_1 and f_2 as vectors in $\mathbb{R}^2$ and we denote $\theta_i, i = 1, 2$ to be the angle from x to f_i measured counterclockwise with respect to x (θ_i is confined to $-\pi \le \theta_i < \pi$). So in this case, $-\pi \le \theta_i \le 0$ (Figure 1(a)). Since if it were otherwise, assume that f_i (here we use f_1 for illustration) is on the other side of x as shown in Figure 1(b), then in sufficiently small elapsed time dt, the trajectory will travel in the counterclockwise direction, which contradicts our assumption.

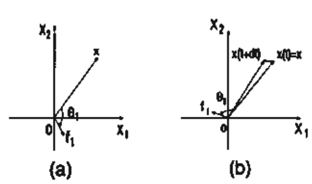

Fig. 1. (a) The angle θ_1 from x to f_1. (b) f_1 is on the other side of vector x.

Note: In the following, we will take $a_1x_1 + a_2x_2$ as the dot product of vectors f_1 and x.

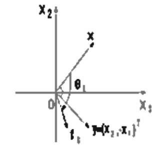

Fig. 2. Dot Product.

And we will also take $a_1x_2 - a_2x_1$ as the dot product of vectors f_1 and $y = (x_2, -x_1)^T$. In this way, we can provide a geometrical interpretation for the above expressions (refer to Figure 2), i.e., $a_1x_1 + a_2x_2 = x^T f_1 = \|x\|_2 \|f_1\|_2 \cos\theta_1$ and $a_1x_2 - a_2x_1 = y^T f_1 = \|y\|_2\|f_1\|_2 \cos(\theta_1 + \frac{\pi}{2}) = \|x\|_2\|f_1\|_2 \cos(\theta_1 + \frac{\pi}{2})$. Similar argument also applies to f_2.

Now we design the switching law that asymptotically stabilizes the switched system. In other words, by our switching law, we want to drive the trajectory closer and closer to the origin, i.e., $\|x(t)\|_2 \to 0$ as $t \to 0$. In the following, we try to design such a switching law by always choosing a subsystem which has the potential to drive the trajectory closer to the origin.

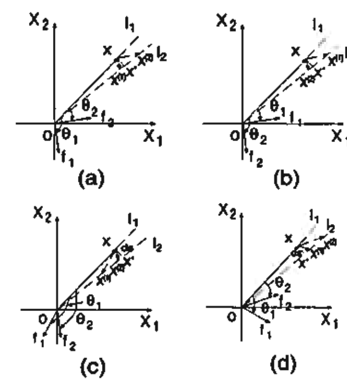

Fig. 3. The four different cases.

For a nonzero $x \in \mathbb{R}^2$, let l_1 be a ray determined by the vector x. Let l_2 be a ray sufficiently close to l_1 as shown in Figure 3(a). The trajectory will intersect l_2 at the point $x^{(1)}$ if it follows subsystem 1, it will intersect l_2 at $x^{(2)}$ if it follows subsystem 2. Let x^* be the point where l_2 is intersected by a line perpendicular to l_1. Let $\|x^{(1)}\|_2$, $\|x^{(2)}\|_2$ be the distances from $x^{(1)}$, $x^{(2)}$ to the origin, respectively. And let $ds = \|x^* - x\|_2$ (Figure 3). Also we define the following conic regions

$$\begin{aligned} E_{is} &= \{x \mid -\pi \le \theta_i(f_i) \le -\frac{\pi}{2} \text{ or } \frac{\pi}{2} \le \theta_i(f_i) < \pi\} \\ &= \{x \mid x^T f_i(x) = x^T A_i x \le 0\},\ i = 1, 2 \qquad (3) \\ E_{iu} &= \{x \mid -\frac{\pi}{2} \le \theta_i(f_i) \le \frac{\pi}{2}\} \\ &= \{x \mid x^T f_i(x) = x^T A_i x \ge 0\},\ i = 1, 2. \qquad (4) \end{aligned}$$

Now we try to associate a better subsystem to x according to the followings.

Case 1: Assume that $x \in E_{1s} \cap E_{2u}$. In this case, as shown in Figure 3(a), $\|x^{(1)}\|_2 \leq \|x^*\|_2 \leq \|x^{(2)}\|_2$ when ds is sufficiently small. So we can choose subsystem 1 in order to drive the trajectory closer to the origin. We denote the conic region $E_{1s} \cap E_{2u}$ as Ω_1.

Case 2: Assume that $x \in E_{1u} \cap E_{2s}$. In this case, as shown in Figure 3(b), we can choose subsystem 2 by a similar argument as in Case 1. We denote the conic region $E_{1u} \cap E_{2s}$ as Ω_2.

Case 3: Assume that $x \in E_{1s} \cap E_{2s}$. In this case, as shown in Figure 3(c), consider $\frac{\|x\|_2 - \|x^{(1)}\|_2}{\|x\|_2 - \|x^{(2)}\|_2}$ as $ds \to 0$. Let dt_1 and dt_2 be the time the system would take to intersect l_2 along the vector f_1 and f_2, respectively. When ds is sufficiently small, it is not difficult to see that we approximately have

$$dt_1 = \frac{ds}{\|f_1\|_2 \cos(\theta_1 + \frac{\pi}{2})} = \frac{ds}{\frac{(a_1x_2 - a_2x_1)}{\sqrt{x_2^2 + x_1^2}}}, \tag{5}$$

$$dt_2 = \frac{ds}{\|f_2\|_2 \cos(\theta_2 + \frac{\pi}{2})} = \frac{ds}{\frac{(a_3x_2 - a_4x_1)}{\sqrt{x_2^2 + x_1^2}}}. \tag{6}$$

By (5) and (6) and using L'Hospital Rule, we obtain

$$\begin{aligned}
&\lim_{ds \to 0} \frac{\|x\|_2 - \|x^{(1)}\|_2}{\|x\|_2 - \|x^{(2)}\|_2} \\
&= \lim_{ds \to 0} \frac{\sqrt{x_1^2 + x_2^2} - \sqrt{(x_1 + a_1dt_1)^2 + (x_2 + a_2dt_1)^2}}{\sqrt{x_1^2 + x_2^2} - \sqrt{(x_1 + a_3dt_2)^2 + (x_2 + a_4dt_2)^2}} \\
&= \frac{(a_1x_1 + a_2x_2)(a_3x_2 - a_4x_1)}{(a_1x_2 - a_2x_1)(a_3x_1 + a_4x_2)} \qquad (7) \\
&= \frac{\cos\theta_1 \cos(\theta_2 + \frac{\pi}{2})}{\cos(\theta_1 + \frac{\pi}{2}) \cos\theta_2} \qquad (8)
\end{aligned}$$

In view of the above, we see that if

$$\lim_{ds \to 0} \frac{\|x\|_2 - \|x^{(1)}\|_2}{\|x\|_2 - \|x^{(2)}\|_2} \geq 1, \tag{9}$$

or, by simplification using (7) (notice $(a_1x_2 - a_2x_1)(a_3x_1 + a_4x_2) < 0$ in this case), if $a_2a_3 - a_1a_4 \leq 0$, then we will have $\|x^{(1)}\|_2 \leq \|x^{(2)}\|_2$ for sufficiently small ds. In this case, we can choose subsystem 1. And we denote the conic region $E_{1s} \cap E_{2s} \cap \{x | a_2a_3 - a_1a_4 \leq 0\}$ as Ω_3.

Using (8), (9) can also be translated into the condition $\sin(\theta_1 - \theta_2) \leq 0$, in other words, $\theta_1 \leq \theta_2$. So in this case, if $\theta_1 \leq \theta_2$, we choose subsystem 1. Similarly, if $a_2a_3 - a_1a_4 \geq 0$, i.e., $\theta_1 \geq \theta_2$, we choose subsystem 2 and we denote the conic region $E_{1s} \cap E_{2s} \cap \{x | a_2a_3 - a_1a_4 \geq 0\}$ as Ω_4.

Case 4: Assume that $x \in E_{1u} \cap E_{2u}$. In this case, as shown in Figure 3(d), consider $\frac{\|x^{(1)}\|_2 - \|x\|_2}{\|x^{(2)}\|_2 - \|x\|_2}$ as $ds \to 0$. In a manner analogous to the derivation of (8) above, we obtain

$$\begin{aligned}
&\lim_{ds \to 0} \frac{\|x^{(1)}\|_2 - \|x\|_2}{\|x^{(2)}\|_2 - \|x\|_2} \\
&= \frac{(a_1x_1 + a_2x_2)(a_3x_2 - a_4x_1)}{(a_1x_2 - a_2x_1)(a_3x_1 + a_4x_2)} \qquad (10) \\
&= \frac{\cos\theta_1 \cos(\theta_2 + \frac{\pi}{2})}{\cos(\theta_1 + \frac{\pi}{2}) \cos\theta_2} \qquad (11)
\end{aligned}$$

Then as in Case 3, we see that if

$$\lim_{ds \to 0} \frac{\|x^{(1)}\|_2 - \|x\|_2}{\|x^{(2)}\|_2 - \|x\|_2} \leq 1, \tag{12}$$

or, by simplification (notice $(a_1x_2 - a_2x_1)(a_3x_1 + a_4x_2) > 0$ in this case), if $a_2a_3 - a_1a_4 \leq 0$, then we will have $\|x^{(1)}\|_2 \leq \|x^{(2)}\|_2$ for sufficiently small ds, so we can choose subsystem 1. And we denote the conic region $E_{1u} \cap E_{2u} \cap \{x | a_2a_3 - a_1a_4 \leq 0\}$ as Ω_5.

Similarly to Case 3, this condition can also be translated into the condition $\theta_1 \leq \theta_2$. So in this case, if $\theta_1 \leq \theta_2$, we choose subsystem 1. Similarly, if $a_2a_3 - a_1a_4 \geq 0$, i.e., $\theta_1 \geq \theta_2$, we choose subsystem 2 and we denote the conic region $E_{1u} \cap E_{2u} \cap \{x | a_2a_3 - a_1a_4 \geq 0\}$ as Ω_6. □

Notice that Ω_i, $i = 1, 2, \cdots, 6$, are all conic regions that partition the $\mathbb{R}^2$ plane. We can associate with each region a subsystem according to the above discussion. *In essence, we associate with each point on $\mathbb{R}^2$ a subsystem i whose θ_i is smaller.* This partition of the $\mathbb{R}^2$ plane is of particular importance here. In the following, we call this switching law (that uses the partition and the associated subsystems) the **conic switching law**. We will show that by using the conic switching law, we can decide whether the system (1) is asymptotically stabilizable or not. If (1) is asymptotically stabilizable, the partition and associated subsystems also provide a stabilizing switching control law.

Note: If at a point x and $\theta_1 = \theta_2$, then we associate with x the subsystem whose valid conic region is to be entered.

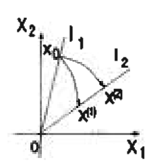

Fig. 4. The two rays and related points.

Consider the system (1). Let l_1 and l_2 be two rays that go through the origin and are in the same conic region of one Ω_i. Suppose l_2 is on the clockwise side of l_1 in the conic region and the conic region with angle from l_1 to l_2 is inside Ω_i. Suppose x_0 is on l_1. Let $x^{(1)}$ be the point on l_2 where the trajectory of the system intersects l_2 for the first time if the system evolves according to the subsystem associated with Ω_i in the conic switching law. Let $x^{(2)}$ be the point on l_2 where the trajectory of the system intersects l_2 for the first time if the system evolves according to the other subsystem. Figure 4 shows the two rays and corresponding points. For the above-mentioned rays and points, we have the following lemma.

Lemma 1. If l_1 and l_2 are in the same conic region Ω_i, $i = 1, 2, \cdots, 6$, then $\|x^{(1)}\|_2 \leq \|x^{(2)}\|_2$.

Proof: See (Xu and Antsaklis, 1998). □

In view of the above lemma, we can prove the following theorem (Xu and Antsaklis, 1998).

Theorem 2. Let l_1 be a ray that goes through the origin. Let x_0 be on l_1. If x^* is the point on l_1 where the trajectory intersects l_1 for the first time after leaving x_0, when the switched system evolves according to the conic switching law, and if x_1 is the point on l_1 where the trajectory intersects l_1 for the first time after leaving x_0, when the switched system evolves according some arbitrary switching law, then we have $\|x^*\|_2 \leq \|x_1\|_2$.

Theorem 2 implies the following theorem for the necessary and sufficient conditions of the asymptotic stabilizability of switched system (1) with subsystems of the same direction.

Theorem 3. The switched system (1) with subsystems of the same direction is asymptotically stabilizable if and only if $\|x^*\|_2 < \|x_0\|_2$ by the conic switching law, where x^* and x_0 are the same as in Theorem 2.

2.2 *Two Subsystems of Opposite Directions*

We will consider the following switched system,

$$\dot{x}(t) = A_1 x(t),\ \dot{x}(t) = A_2 x(t), \tag{13}$$

where both subsystems are with unstable foci, and subsystem 1 is of clockwise direction while subsystem 2 is of counterclockwise direction. Figure 5 shows the angle θ_1 ($-\pi \leq \theta_1 \leq 0$) and θ_2 ($0 \leq \theta_2 < \pi$) in this case.

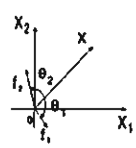

Fig. 5. The angle θ_i from x to f_i, $i = 1, 2$.

We would like to consider a switching law that asymptotically stabilizes the switched system. By our switching law, we want to lead the trajectory closer and closer to the origin, i.e., $\|x(t)\|_2 \to 0$ as $t \to 0$. In the following, we will design a switching law by choosing a subsystem which has the potential to drive the trajectory closer to the origin. We first define the following conic regions

$$\Omega_1 = E_{1s} \cap E_{2s}, \tag{14}$$
$$\Omega_2 = E_{1u} \cap E_{2u}, \tag{15}$$
$$\Omega_3 = E_{1s} \cap E_{2u} \cap \{x | a_2 a_3 - a_1 a_4 \geq 0\}, \tag{16}$$
$$\Omega_4 = E_{1s} \cap E_{2u} \cap \{x | a_2 a_3 - a_1 a_4 \leq 0\}, \tag{17}$$
$$\Omega_5 = E_{1u} \cap E_{2s} \cap \{x | a_2 a_3 - a_1 a_4 \geq 0\}, \tag{18}$$
$$\Omega_6 = E_{1u} \cap E_{2s} \cap \{x | a_2 a_3 - a_1 a_4 \leq 0\}, \tag{19}$$

where E_{is}, E_{iu} are as defined in (3) and (4). Notice, some Ω_i's in this case are different from Ω_i's in the previous subsection.

Case 1: If $Int(\Omega_1) \neq \emptyset$, it is clear that any trajectory of the switched system that is totally inside Ω_1 is stable (Figure 6(a)). Now if the initial point of the switched system is outside Ω_1, we can follow the trajectory of any one subsystem to go into Ω_1. Then we can use the **conic switching law**, i.e., *switch to another subsystem upon intersecting the boundary of* Ω_1, as this will give us a trajectory that is totally inside Ω_1 and it is bounded. If there exists $x \in \Omega_1$, such that $|\theta_i(f_i(x))| \neq \frac{\pi}{2}$ for some $i = 1$, or 2, then the switched system is asymptotically stablizable by the conic switching law.

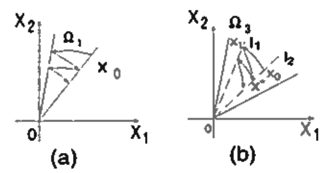

Fig. 6. (a) Case 1:$Int(\Omega_1) \neq \emptyset$. (b) Case 2:$Int(\Omega_3) \neq \emptyset$.

Case 2: Let $Int(\Omega_3) \neq \emptyset$. In this case, assume l_1 and l_2 be two rays that go through the origin and are in the same conic region Ω_3, and that the conic region with angle from l_1 to l_2 is inside Ω_3 (Figure 6(b)). Suppose l_2 is on the clockwise side of l_1 in the conic region and suppose x_0 is on l_2. Then we have the following Lemma.

Lemma 4. Let the switched system follow subsystem 2 until the trajectory intersects l_1 for the first time at x_1 and then let the system switch to subsystem 1 and evolve according to subsystem 1. Suppose x^* is a point on l_2 where the trajectory intersects l_2 for the first time after the switching. Then we have $\|x^*\|_2 \leq \|x_0\|_2$.

Proof: See (Xu and Antsaklis, 1998). □

According to Lemma 4, the **conic switching law** (i.e., *switch to another subsystem upon intersecting the boundary of* Ω_3 so as to keep the trajectory inside Ω_3 (Figure 6(b))) will give us a trajectory that is totally inside Ω_3 so it is bounded. If there exists $x \in \Omega_3$, such that $|\theta_1(f_1)| + |\theta_2(f_2)| \neq \pi$ at x, then the switched system is asymptotically stablizable by the conic switching law.

Case 3: Let $Int(\Omega_5) \neq \emptyset$. Then similar argument as in Case 2 can be applied and we find that the system can be stabilized by conic switching law to keep the trajectory inside Ω_5. □

The following theorem shows us that the system can only be stabilized for the above-mentioned three cases.

Theorem 5. The switched system (13) with two subsystems of opposite directions is asymptotically stabilizable if and only if $Int(\Omega_1) \cup Int(\Omega_3) \cup Int(\Omega_5) \neq \emptyset$ and there exists $x \in Int(\Omega_1) \cup Int(\Omega_3) \cup Int(\Omega_5)$ such that $|\theta_1(f_1)| + |\theta_2(f_2)| \neq \pi$ at x.

Proof: See (Xu and Antsaklis, 1998). □

3. SEVERAL SUBSYSTEMS

Now we study stabilizing switch laws for switched systems with several second-order LTI subsystems with unstable foci

$$\dot{x}(t) = A_i x(t), \; i = 1, 2, \cdots, N. \tag{20}$$

3.1 *All Subsystems of the Same Direction*

Assume that all the N subsystems are of the clockwise direction. Notice that from the discussion in subsection 2.1, we find the conic switching law by associating with each point on $\mathbb{R}^2$ a subsystem i with smaller θ_i. As an extension of the conic switching law there, we obtain the **conic switching law** by associating with each point on $\mathbb{R}^2$ a subsystem i where θ_i is the smallest.

Similar argument as in section 2.1 can be made for several subsystems to show the following theorem.

Theorem 6. Let l_1 be a ray that goes through the origin. Let x_0 be on l_1 and consider the conic switching law. Let x^* on l_1 be the point where the trajectory intersects l_1 for the first time after leaving x_0. The switched system (20) is asymptotically stabilizable if and only if $\|x^*\|_2 < \|x_0\|_2$.

3.2 *Not All Subsystems of the Same Direction*

Assume that $K(K > 0)$ subsystems are of clockwise direction and $M(M > 0)$ subsystems are of counterclockwise direction ($K + M = N$). Combining the previous results, the following theorem can be obtained.

Theorem 7. The switched system (20) with $K(K > 0)$ subsystems $S_1^-, \cdots, S_K^-$ of clockwise direction and $M(M > 0)$ subsystems $S_1^+, \cdots, S_M^+$ of counterclockwise direction is asymptotically stabilizable if and only if at least one of the following three conditions holds:

(1) The switched system consisting of $S_1^-, \cdots, S_K^-$ is asymptotically stabilizable.
(2) The switched system consisting of $S_1^+, \cdots, S_M^+$ is asymptotically stabilizable.
(3) There exist i and j with $1 \leq i \leq K$ and $1 \leq j \leq M$ such that the switched system consisting of two subsystems S_i^- and S_j^+ is asymptotically stabilizable.

4. EXAMPLES AND COMPARISON TO MATRIX INEQUALITY APPROACH

4.1 *Examples*

In the following examples, the conic switching laws are used.

Example 8. Consider the switched system (1) with two subsystems of the same direction with

$$A_1 = \begin{bmatrix} 1 & 13 \\ -2 & 3 \end{bmatrix}, \; A_2 = \begin{bmatrix} -1 & 2 \\ -10 & 3 \end{bmatrix}.$$

It can be shown that it is asymptotically stabilizable using the conic switching law, though each subsystem has unstable foci. The regions for subsystems are specified and a stabilizing control law is derived. Figure 7 shows the regions for subsystems, the trajectory and the time domain responses of the switched system.

Example 9. Consider the switched system (13) with two subsystems of different directions with

$$A_1 = \begin{bmatrix} -2 & 52 \\ -8 & 6 \end{bmatrix}, \; A_2 = \begin{bmatrix} 11 & -10 \\ 50 & -9 \end{bmatrix}.$$

It is asymptotically stabilizable since the region $Int(\Omega_1)$ is nonempty. Figure 8 shows the conic regions, the trajectory and the time domain responses of the switched system.

4.2 *Comparison to a Matrix Inequality Approach*

In (Pettersson and Lennartson, 1996), Pettersson introduced a LMI based stability analysis method. Note that, by some modification of the LMI approach, the design of stabilizing control laws can be formulated as matrix inequality problems (Xu and Antsaklis, 1998). Yet because these problems typically are complicated and may be nonlinear in some variables, they are generally very difficult to solve. And if we cannot obtain a solution

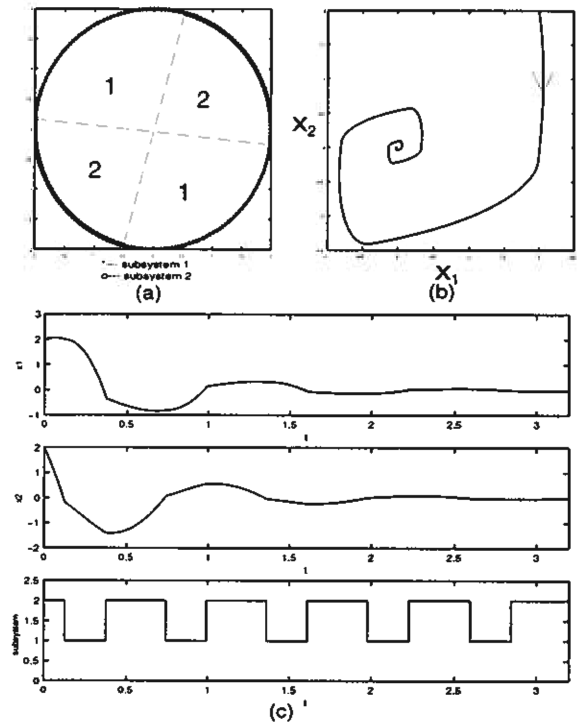

Fig. 7. Example 8: (a) The regions for subsystems. (b) The trajectory of the system. (c) Time domain responses. ($x_0 = (2, 2)^T$)

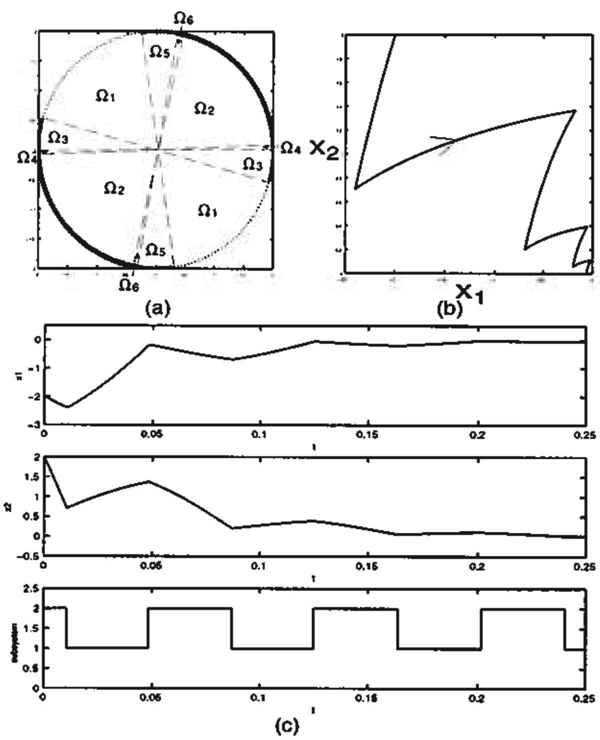

Fig. 8. Example 9: (a) The conic regions. (b) The trajectory of the system. (c) Time domain responses. ($x_0 = (-2, 2)^T$)

for the problem, we may not know whether the switched system is stabilizable or not. Even for second-order LTI switched systems discussed in this paper, the matrix inequality problem may not be easy to solve. One may need to associate multiple candidate Lyapunov functions for each subsystem and hope to find a solution.

Therefore, although the matrix inequality formulation for control design seems to be general for n-th order LTI switched systems, they are difficult to solve even for second-order switched systems. Our method in this paper can solve the design problem efficiently for second-order switched systems.

5. CONCLUSION

In this paper, we look into the problem of finding asymptotically stabilizing switching control laws for switched systems consisting of several second-order LTI subsystems with unstable foci. Necessary and sufficient conditions were given for the asymptotic stabilizability of such switched systems. Also a design procedure for deriving such stabilizing control laws was presented. For the general second-order LTI switched systems with subsystems having unstable nodes or saddle points instead of unstable foci, the design of stabilizing switching control laws poses further research questions and it is currently under investigation.

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