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STABILIZING SUPERVISORY CONTROL OF HYBRID SYSTEMS BASED ON PIECEWISE LINEAR LYAPUNOV FUNCTIONS[1]

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Abstract. In this paper, the stability of discrete-time piecewise linear hybrid systems is investigated using piecewise linear Lyapunov functions. In particular, we consider switched discrete-time linear systems and we identify classes of switching sequences that result in stable trajectories. Given a switched linear system, we present a systematic methodology for computing switching laws that guarantee stability based on the matrices of the system. In the proposed approach, we assume that each individual subsystem is stable and admits a piecewise linear Lyapunov function. Based on these Lyapunov functions, we compose "global" Lyapunov functions that guarantee stability of the switched linear system. A large class of stabilizing switching sequences for switched linear systems is characterized by computing conic partitions of the state space.

Key Words. Stability, switched linear systems, piecewise linear Lyapunov functions, partitions of the state space.

1. INTRODUCTION

In this paper, we study the stability of piecewise linear hybrid systems using piecewise linear Lyapunov functions. In particular, we consider discrete-time switched linear systems. The control problem considered is to *identify classes of switching sequences that result in stable trajectories*. The main motivation behind this problem is that it is often easier to find switching controllers than to find a simple continuous controller. For example, in the case when we have multiple control objectives, we may design a continuous controller for each control objective, and control the behavior of the plant by switching among different controllers.

In order to investigate the stability properties of practical hybrid systems, there is an important need to characterize partitions of the state space that lead to stable trajectories based on the structural properties of the switched system. In our approach, we characterize a large class of switching signals that result in stable trajectories. Given a switched linear system, we present a systematic methodology for computing switching laws that guarantee stability based on the matrices of the system. We assume that each individual subsystem is stable and admits a piecewise linear Lyapunov function. Based on these Lyapunov functions, we compose "global" Lyapunov functions that guarantee stability of the switched linear system. The main contribution of the paper is that using piecewise linear Lyapunov function we construct a conic partition of the state space that is used to characterize a large class of switching laws that result in stable trajectories.

Stability of switched systems has been studied extensively in the literature; see for example [7, 13, 15] and the references therein. Analysis tools for switched and hybrid systems based on multiple Lyapunov functions are presented in [4]. The application of the theoretical results to practical hybrid systems may be accomplished using a linear matrix inequality (LMI) problem formulation for constructing a set of quadratic Lyapunov-like functions [9, 18].

The stability analysis presented in this paper is based on piecewise linear Lyapunov functions. Piecewise linear Lyapunov functions have been used extensively for the analysis of dynamical systems. The problem of constructing piecewise linear Lyapunov functions and their application to nonlinear and large scale systems has been considered in [5, 6, 16, 17]. More recently, positively invariant polyhedral sets for discrete-time dynamical systems have been studied in [2]. Lyapunov functions described by the infinity norm have been also investigated in [10, 19, 20].

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The paper is organized as follows. In Section 2, a mathematical model for discrete-time switched linear systems is introduced and the problem of identifying stabilizing switching sequences is stated. Section 3 presents the necessary background for piecewise linear Lyapunov function. The emphasis is put on computational methods for constructing such Lyapunov functions. The technical results for the characterization of stabilizing switching sequences are presented in Section 4, and the approach is illustrated with a numerical example.

2. PROBLEM STATEMENT

In this section, we consider switched discrete-time linear systems described by

$$x(t+1) = A_q x(t), \quad q \in Q = \{1, \dots, N\} \tag{1}$$

where $x(t) \in \mathbb{R}^n$ and $A_q \in \mathbb{R}^{n \times n}$. The mathematical model described by Equation (1) represents the continuous portion of piecewise linear hybrid dynamical systems. The particular mode q at any given time instant may be selected by a decision-making process. We represent such a decision-making process by a switching law of the form

$$q(t+1) = \delta(q(t), x(t)). \tag{2}$$

Given $x(t)$, the next state is computed by $x(t+1) = A_{q(t)}x(t)$. The function $\delta : Q \times \mathbb{R}^n \to \mathbb{R}^n$ is discontinuous with respect to x. Such a switching law is usually defined using a partition of the state space.

Our objective is to investigate the stability of the switched linear system (1) under the switching law (2). Note that the origin $x_e = 0$ is an equilibrium for the system (1). Furthermore, for a fixed switching law, the switched system (1) can be viewed as a special case of a time-varying linear system, and therefore the usual definitions of stability can be used; see for example [1]. The control problem considered in this chapter is to *identify classes of switching sequences that result in stable trajectories*. It is assumed that all the individual subsystems are stable and therefore constant switching signals of the form $q(t) = i \in Q$ for every t result in stable trajectories. The problem considered here can be partially solved using “slow switching signals”; see for example [13] and the references therein. Here we follow a different approach in order to develop a systematic methodology to compute regions of the state space where switchings are allowed to occur. First, we compute a partition of the state space into conic regions based on the matrices A_q of the system. Then, we characterize a large class of stabilizing switching signals by requiring the switchings will occur in certain regions of the state space.

3. PIECEWISE LINEAR LYAPUNOV FUNCTIONS

In this section, we briefly present some background material necessary for the stability analysis of switched linear systems presented later in this chapter.

3.1. Set-induced Lyapunov functions

We consider the discrete-time linear system $x(t+1) = Ax(t)$ where $x(t) \in \mathbb{R}^n$ and $A \in \mathbb{R}^{n \times n}$.

Definition 1 A nonempty set $P \subset \mathbb{R}^n$ is said to be (*positively*) *invariant* for the system $x(t+1) = Ax(t)$ if $x(0) \in P$ implies that $x(t) \in P$ for every $t \in \mathbb{Z}$ ($\mathbb{Z}^+$).

In the case when the system admits a positively invariant polyhedral set P containing the origin, a Lyapunov function can be constructed by considering the *Minkowski functional* (*gauge function*) of P; see for example [17, 3]. Consider a polytope $P \subset \mathbb{R}^n$ and assume that $0 \in \text{int}(P)$. The Minkowski functional of P is defined by $V(x) = \inf\{\rho > 0 | x \in \rho P\}$ where $\rho P = \{\rho x | x \in P\}$.

Let F_i be a face of a polytope and consider the corresponding hyperplane H_i as shown in Fig. 1. The hyperplane can be described (perhaps after normalization) by $H_i = \{x \in \mathbb{R}^n : \langle x, w_i \rangle = 1\}$.

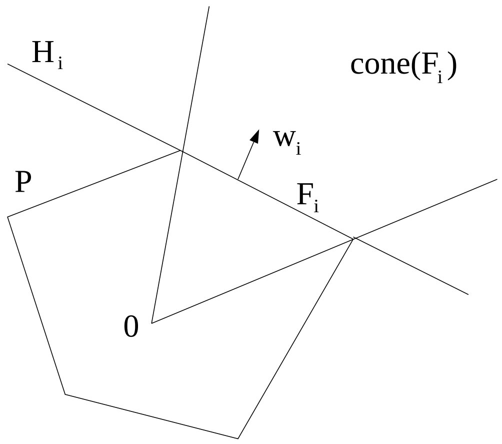

Figure 1: A polytope P, a face F_i and its corresponding hyperplane H_i.

Since the set P includes an open neighborhood of the origin, $\mathbb{R}^n$ can be partitioned into a finite number of cones defined as follows. Each face F of the polytope can be described as the convex hull of its extreme points $f_j \in \mathbb{R}^n$, $j = 1, \dots, r$. A finitely generated cone can be defined for the face F . Consider a particular face F_i and the corresponding cone. Since $F_i \in \partial P$ there exist unique $\rho > 0$ and $\hat{x} \in F_i$ such that $x = \rho\hat{x}$ and the Minkowski functional can be computed by

$$V(x) = \frac{\|x\|_2}{\|\hat{x}\|_2} = \rho = \rho\langle\hat{x}, w_i\rangle = \langle x, w_i\rangle \tag{3}$$

since $\langle\hat{x}, w_i\rangle = 1$. Therefore, for $x \in \text{cone}(F_i)$, the Lyapunov function induced by the set P can be written as $V(x) = \langle x, w_i \rangle$. Consequently, the Lyapunov function can be computed for $x \in \mathbb{R}^n$ by

$$V(x) = \max_{1 \le i \le m} \langle x, w_i \rangle. \tag{4}$$

3.2 Lyapunov Functions Defined by the Infinity Norm

A special case of piecewise linear Lyapunov function arise when the set P is centrally symmetric. In this case, the Lyapunov function $V(x)$ can be represented using the infinity norm. Furthermore, there exists a class of linear systems for which such a Lyapunov function can be computed very efficiently. Consider the following Lyapunov function candidate $V(x) = \|Wx\|_\infty$ where $W \in \mathbb{R}^{m \times n}$ and $\|\cdot\|_\infty$ denotes the infinity norm defined by $\|x\|_\infty = \max_{1 \le i \le n} |x_i|$.

Theorem 1 ([2]) *$V(x) = \|Wx\|_\infty$ is a Lyapunov function for the system $x(t+1) = Ax(t)$ if and only if there exist a matrix $Q \in \mathbb{R}^{m \times m}$ such that $WA - QW = 0$ and $\|Q\|_\infty < 1$.*

It should be noted a generalization of the above theorem for every normed space that satisfies the *self-extension property* has been presented in [14].

Corollary 1 ([2]) *If $V(x) = \|Wx\|_\infty$ is a Lyapunov function for the system $x(t+1) = Ax(t)$ then the polyhedral set $P = \{x \in \mathbb{R}^n : \|Wx\|_\infty \le 1\}$ is positively invariant. In addition, the set ρP for every real $\rho > 0$ is also positively invariant.*

The set P is a centrally symmetric polyhedron. In the case when $\mathrm{rank} W = n$ $(m \ge n)$ then P is bounded. The number of vertices of the polyhedron P rises with the number of rows m. If $W \in \mathbb{R}^{n \times n}$ then we obtain a centrally symmetric polyhedron with 2^n vertices. Note that in the case when $rank W < n$, then $V(x)$ is positive semidefinite and cannot be a Lyapunov function for the system. However if $DV = V[x(t+1)] - V[x(t)] < 0$ the set $P = \{x \in \mathbb{R}^n : \|Wx\|_\infty < \rho\}$ is a positively invariant set (for any $\rho > 0$), but is not always a domain of stability since it can be unbounded (expanding infinitely into $n - \mathrm{rank} W$ dimensions).

In order to study the stability properties of the switched linear system (1) we assume that each individual subsystem admits a piecewise linear Lyapunov function. Therefore, the efficient computation of each Lyapunov function is very important for the application of the methodology to practical hybrid systems. A class of linear systems for which such a Lyapunov function can be computed very efficiently is presented in [2].

Corollary 2 ([2]) *If all the eigenvalues $\lambda_i = \mu_i \pm j\sigma_i$ of the n^{th} order linear system $x(t+1) = Ax(t)$ are in the open square $|\mu_i| + |\sigma_i| < 1$, then there exists a matrix $W \in \mathbb{R}^{n \times n}$ with $\mathrm{rank} W = n$ such that the polyhedral set $P = \{x \in \mathbb{R}^n : \|Wx\|_\infty < 1\}$ is a positively invariant set for the system.*

Remark The condition $|\mu_i| + |\sigma_i| < 1$ can be replaced by $|\mu_i| + |\sigma_i| \le 1$ with the additional hypothesis that to each eigenvalue λ_i such that $|\mu_i| + |\sigma_i| = 1$ with multiplicity ν_i there correspond ν_i linearly independent eigenvectors.

The matrix W can be computed as the solution to the matrix equation $WA - QW = 0$ with the condition $\|Qx\|_\infty < 1$. It is well known [8] that if the matrices A and Q do not have common eigenvalues then the only solution is $W = 0$. The important assumption in the Corollary 2 is that $W \in \mathbb{R}^{n \times n}$ with $\mathrm{rank} W = n$. In this case the matrix W can be computed as the similarity transformation matrix by which A is transformed to the *Real Jordan Canonical Form* [12].

3.3 Computer Generated Lyapunov Functions

It should be noted that in our stability analysis for switched linear systems, it is not necessary for the individual invariant polyhedral sets to be centrally symmetric. Positively invariant polyhedral sets for stable discrete-time systems can be determined using *computer generated Lyapunov functions* [5]. The class of computer generated Lyapunov functions has been used for stability analysis of nonlinear systems in [5, 6, 16, 17]. The main idea is to construct a Lyapunov function that guarantees the stability of a set of matrices that is determined by applying Euler's discretization method to a nonlinear differential equation.

Our approach here is to use a computer generated Lyapunov function for each individual subsystem. Consider the matrix $A \in \mathbb{R}^{n \times n}$ and let $P_0 \subset \mathbb{R}^n$ be a bounded polyhedral region of the origin. We denote the convex hull of P by conv(P). Following [5] we define

$$P_k = \mathrm{conv}\left(\bigcup_{i=0}^{\infty} A^i P_{k-1} \right), \quad P^* = \bigcup_{i=0}^{\infty} P_i. \tag{5}$$

The following results can be derived from [5]: First, the matrix A is stable if and only if P^* is bounded. Second, if A is stable then each set P_k can be computed by P_{k-1} using finitely many iterations. Furthermore, it is shown in [6] that if there exists constant $K \in \mathbb{R}$ such that the eigenvalues of A satisfy the condition $|\lambda_i| \le K < 1$, then the set P^* is finitely computable. In this case the set P^* is polyhedral as the convex hull of finitely many points. Furthermore, P^* is a positively invariant polyhedral set of the system.

4. STABILIZING SWITCHING SEQUENCES

In this section, we present an approach based on multiple Lyapunov functions for the stability analysis of the switched system (1). The main contribution is an efficient characterization of a class of switching laws of the form (2) which guarantee the stability of the system.

We assume that each individual subsystem admits a positively invariant polyhedral set that contains the origin which is described by

$$P_q = \{x \in \mathbb{R}^n : W^q x < \bar{1}\}$$

where $W^q \in \mathbb{R}^{m_q \times n}$ and $\bar{1} = [1, \ldots, 1]^T \in \mathbb{R}^n$. We denote the rows of the matrix W^q by $w_i^q \in \mathbb{R}^n$, $i = 1, \ldots, m_q$. The Lyapunov function induced by the set P_q can be described by

$$V_q(x) = \max_{1 \le i \le m_q} \langle x, w_i^q \rangle. \tag{6}$$

Note that if P_q is centrally symmetric then there exists $W^q \in \mathbb{R}^{n \times n}$ and the corresponding Lyapunov function can be written as $V_q(x) = \|W^q x\|_\infty$.

We consider a class S of switching sequences of the form

$$s = (q_0, t_0), (q_1, t_1), \ldots, (q_j, t_j), \ldots, \quad x(t_0) = x_0. \tag{7}$$

It is assumed that if s is finite then $t_{j+1} = \infty$ and that $q_j \neq q_{j+1}$. Such a sequence can be generated by the switching law

$$q_j(t_j + 1) = \delta(q_{j-1}(t_j), x(t_j)), \; j = 1, 2, \ldots. \tag{8}$$

Proposition 1 *Consider a switching sequence $s \in S$. If $V_{q_j}[x(t_j+1)] \le V_{q_{j-1}}[x(t_j)]$, $j = 1, 2, \ldots$, then the switched system $x(t+1) = A_q x(t)$ is stable in the sense of Lyapunov.*

Proof Consider the multiple Lyapunov function defined by $V[x(t)] = V_{q_j}[x(t)]$, $t_j < t \le t_{j+1}$ then we have that for every $t > t_0, t \in \mathbb{Z}^+$, $DV(x) = V[x(t+1)] - V[x(t)] \le 0$. Note that the switched system for a fixed switching sequence s can be viewed as a time-varying system. Since $V(x)$ is positive definite and radially unbounded, and DV negative semidefinite, the system is stable in the sense of Lyapunov; see for example [1]. □

If the condition $V_{q_j}[x(t_j+1)] < V_{q_{j-1}}[x(t_j)]$ is used in the previous proposition, then the origin is asymptotically stable for the switched system.

A multiple Lyapunov function composed by piecewise linear Lyapunov functions of the individual subsystems offers a significant advantage. It allows the characterization of the switching sequences that satisfy the condition of Proposition 1 by computing a conic partition of the state space. Consider a pair of subsystems with matrices A_{q_1} and A_{q_2}. We want to compute the region

$$\Omega_{q_1}^{q_2} = \{x \in \mathbb{R}^n : V_{q_2}(x) \le V_{q_1}(x)\}. \tag{9}$$

Consider the faces $F_{i_1}^{q_1}$ and $F_{i_2}^{q_2}$ of the polytopes P_{q_1} and P_{q_2} respectively and assume that

$$C = \text{cone}(F_{i_1}^{q_1}) \cap \text{cone}(F_{i_2}^{q_2}) \neq \emptyset. \tag{10}$$

Next, we define the halfspace

$$H_{q_1}^{q_2} = \{x \in \mathbb{R}^n : \langle x, w_{i_2}^{q_2} - w_{i_1}^{q_1} \rangle \le 0\} \tag{11}$$

and the set

$$\Omega = C \cap H_{q_1}^{q_2}. \tag{12}$$

It is shown in the following lemma that the multiple Lyapunov function defined in Proposition 1 is decreasing if the system switches from q_1 to q_2 while $x \in \Omega$.

Lemma 1 *For every $x \in \Omega$ we have that $V_{q_2}(x) \le V_{q_1}(x)$.*

Proof For every $x \in C$ the Lyapunov functions for the subsystems are given by $V_{q_1}(x) = \langle x, w_{i_1}^{q_1} \rangle$ and $V_{q_2}(x) = \langle x, w_{i_2}^{q_2} \rangle$ respectively. If $x \in \Omega$ we have that $\langle x, w_{i_2}^{q_2} - w_{i_1}^{q_1} \rangle \le 0$ since $x \in H_{q_1}^{q_2}$, and therefore $V_{q_2}(x) \le V_{q_1}(x)$. □

Since $0 \in H_{q_1}^{q_2}$, the set Ω is a clearly a polyhedral cone as the intersection of cones with a common apex $(x = 0)$ as shown in Fig. 2. The set $\Omega_{q_1}^{q_2}$ can be computed as the union of polyhedral cones by repeating the above procedure for all the pairs $(F_{i_1}^{q_1}, F_{i_2}^{q_2})$ of $(n-1)-$dimensional faces of the polytope P as shown in the following algorithm.

Algorithm for the computation of $\Omega_{q_1}^{q_2}$

INPUT: W^{q_1}, W^{q_2};
for $i_1 = 1, \ldots, m_{q_1}$
    for $i_2 = 1, \ldots, m_{q_2}$
        $C = \text{cone}(F_{i_1}^{q_1}) \cap \text{cone}(F_{i_2}^{q_2})$;
        if $C \neq \emptyset$ then
            $H_{q_1}^{q_2} = \{x \in \mathbb{R}^n : \langle x, w_{i_2}^{q_2} - w_{i_1}^{q_1} \rangle \le 0\}$
            $\Omega = C \cap H_{q_1}^{q_2}$;
            $\Omega_{q_1}^{q_2} = \Omega_{q_1}^{q_2} \cup \Omega$;
        end
    end
end

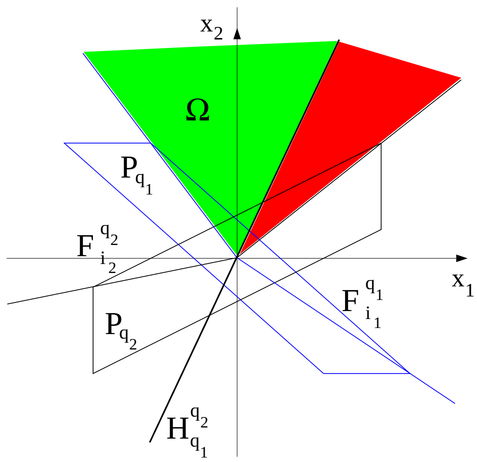

Figure 2: The conic partition of the state space.

The above procedure can be repeated for every pair of subsystems to identify a class of stabilizing switching signals for the switched linear system. The class of switching sequences is characterized by the following result.

Theorem 2 *Consider the class of switching sequences S defined by $q_j(t_j+1) = \delta(q_{j-1}(t_j), x(t_j)),\ x(t_j) \in \Omega_{q_{j-1}}^{q_j} \neq \emptyset$ for $j = 1, 2, \ldots$. The switched linear system $x(t+1) = A_q x(t)$ is stable in the sense of Lyapunov for every switching sequence $s \in S$.*

Proof By induction, we have that if $s = (q_0, t_0)$ then the system is stable since A_{q_0} is stable. Assume that the switched system is stable for $s = (q_0, t_0), (q_1, t_1), \ldots, (q_{j-1}, t_{j-1})$ and consider the switching sequence $s' = (q_0, t_0), (q_1, t_1), \ldots, (q_{j-1}, t_{j-1}), (q_j, t_j)$. Since $x(t_j) \in \Omega_{q_{j-1}}^{q_j}$, we have that $V_{q_j}[x(t_j+1)] \leq V_{q_{j-1}}[x(t_j)]$. Therefore, the multiple Lyapunov function defined by $V[x(t)] = V_{q_j}[x(t)],\quad t_j < t \leq t_{j+1}$ is decreasing for every t and the system is stable in the sense of Lyapunov. □

We have presented a methodology for the partition of the state space into conic regions that are used to characterize a class of stabilizing switching sequences. The following example illustrates the approach.

Example Consider the switched discrete-time linear system $x(t+1) = A_q x(t),\quad q \in \{1, 2\}$ where

$$A_1 = \begin{bmatrix} 1.7 & 4 \\ -0.8 & -1.5 \end{bmatrix},\quad A_2 = \begin{bmatrix} 0.95 & -1.5 \\ 0.75 & -0.55 \end{bmatrix}.$$

The system with matrix A_1 has two complex conjugate eigenvalues $\lambda_{1,2} = 0.1 \pm j0.8$. The real Jordan canonical form can be computed by the similarity transformation

$$Q_1 = W^1 A_1 (W^1)^{-1} = \begin{bmatrix} 0.1 & 0.8 \\ -0.8 & 0.1 \end{bmatrix}$$

where

$$W^1 = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}.$$

We have that

$$\|Q_1\|_\infty = \max_{1 \leq i \leq n} \sum_{j=1}^{n} |q_{ij}| = 0.9 < 1$$

and therefore, $V_1(x) = \|W^1 x\|_\infty$ is a Lyapunov function for the system. Furthermore, the set

$$P_1 = \{x \in \mathbb{R}^2 :\ \|W^1 x\|_\infty \leq 1\}$$

shown in Fig. 3 is a positively invariant polyhedral set. The matrix A_2 has two complex conjugate eigenvalues $\lambda_{1,2} = 0.2 \pm i0.75$. A positively invariant polyhedral set P_2 is described by the Lyapunov function $V_2 = \|W^2 x\|_\infty$ where

$$W^2 = \begin{bmatrix} 1 & -2 \\ 1 & 0 \end{bmatrix}.$$

Consider the faces F^1 and F^2 of the polyhedral sets P_1 and P_2 respectively as shown in Fig. 3. For every $x \in \text{cone}(F^1) \cap \text{cone}(F^2)$ we have that $V_1(x) = \langle x, w^1 \rangle$ and $V_2(x) = \langle x, w^2 \rangle$ with $w^1 = [1, 2]$ and $w^2 = [1, -2]$ respectively. We consider the halfspace

$$\begin{aligned} H_1^2 &= \{x \in \mathbb{R}^2 :\ \langle x, w^2 - w^1 \rangle \leq 0\} \\ &= \{x \in \mathbb{R}^2 :\ x_2 \geq 0\}. \end{aligned}$$

Therefore, for every $x \in \Omega = \text{cone}(F^1) \cap \text{cone}(F^2) \cap H_1^2$ we have that $V_2(x) \leq V_1(x)$.

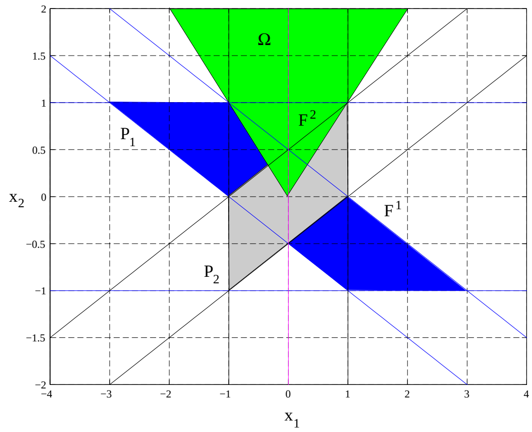

Figure 3: The region Ω.

By repeating the procedure for all the pairs of faces for the polytopes P_1 and P_2 we compute the region

$$\begin{aligned} \Omega_{q_1}^{q_2} &= \{x \in \mathbb{R}^2 :\ V_{q_2}(x) \leq V_{q_1}(x)\} \\ &= \{x \in \mathbb{R}^2 :\ x_2 \geq 0\}. \end{aligned}$$

Similarly we have that

$$\begin{aligned} \Omega_{q_2}^{q_1} &= \{x \in \mathbb{R}^2 :\ V_{q_1}(x) \leq V_{q_2}(x)\} \\ &= \{x \in \mathbb{R}^2 :\ x_2 \leq 0\}. \end{aligned}$$

Therefore, for any switching sequence s given by the switching law $q_2(t+1) = \delta(q_1(t), x(t)),$ only if $x(t) \in \Omega_{q_1}^{q_2}$ and $q_1(t+1) = \delta(q_2(t), x(t)),$ only if $x(t) \in \Omega_{q_2}^{q_1}$ the switched system is stable. A stable trajectory is shown in Fig. 4.

The characterization of the stabilizing switching sequences is based on sufficient conditions. Therefore, for a switching sequence s that does not satisfy the formulated conditions, the switched system is not necessarily unstable. However, the switched system of the example can generate unstable trajectories as shown in Fig. 5. □

5. CONCLUSIONS

In this paper, the stability of piecewise linear hybrid systems using piecewise linear Lyapunov functions is investigated. In the proposed approach, we assume that each individual subsystem is stable and admits a piecewise linear Lyapunov function. Based on these Lyapunov functions, we compose "global" Lyapunov functions that guarantee stability of the switched linear system. These multiple Lyapunov functions correspond to conic partitions of the state space which are effi-

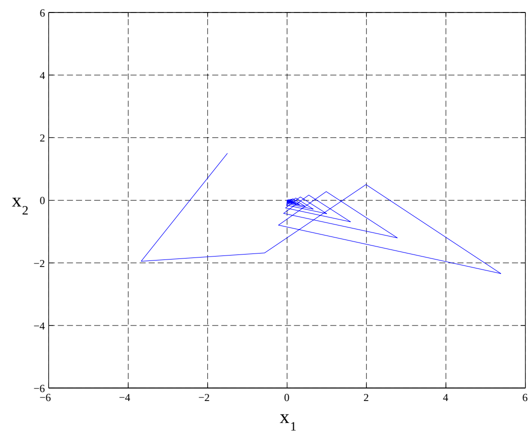

Figure 4: A stabilizing switching sequence for the discrete-time switched system.

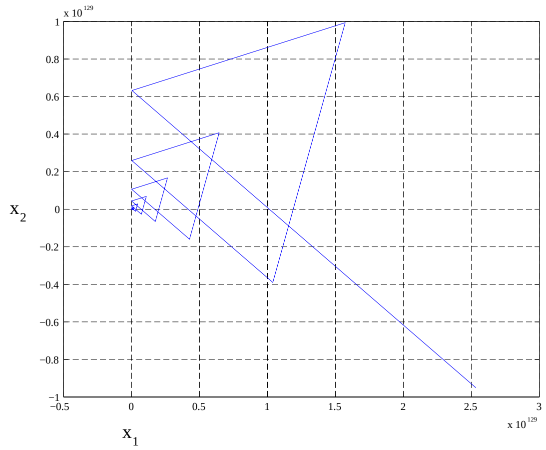

Figure 5: An unstable trajectory of the discrete-time switched system.

ciently computed using the developed algorithms. The main advantage of the approach is that the methodology for computing switching laws that guarantee stability is based on the matrices of the system. Therefore, the proposed approach can be used very efficiently to investigate the stability properties of practical hybrid systems. The methodology has been applied also to continuous-time switched linear systems; details can be found in [11].

6. REFERENCES

[1] P. Antsaklis and A. Michel. *Linear Systems*. McGraw-Hill, 1997.

[2] G. Bitsoris. Positively invariant polyhedral sets of discrete-time linear systems. *International Journal of Control*, 47(6):1713–1726, 1988.

[3] F. Blanchini. Nonquadratic Lyapunov functions for robust control. *Automatica*, 31(3):451–461, 1995.

[4] M. Branicky. Multiple Lyapunov functions and other analysis tools for switched and hybrid systems. *IEEE Transactions on Automatic Control*, 43(4):475–482, 1998.

[5] R. Brayton and C. Tong. Stability of dynamical systems: A constructive approach. *IEEE Transactions on Circuits and Systems*, CAS-26(4):224–234, 1979.

[6] R. Brayton and C. Tong. Constructive stability and asymptotic stability of dynamical systems. *IEEE Transactions on Circuits and Systems*, CAS-27(11):1121–1130, 1980.

[7] R. DeCarlo, M. Branicky, S. Pettersson, and B. Lennartson. Perspectives and results on the stability and stabilizability of hybrid systems. *Proceedings of IEEE, Special Issue on Hybrid Systems*, July 2000. To appear.

[8] F. Gantmacher. *Matrix Theory*. Chelsea, 1959.

[9] M. Johansson and A. Rantzer. Computation of piecewise quadratic Lyapunov functions for hybrid systems. *IEEE Transactions on Automatic Control*, 43(1):31–45, 1998.

[10] H. Kiendl, J. Adamy, and P. Stelzner. Vector norms as Lyapunov functions for linear systems. *IEEE Transactions on Automatic Control*, 37(6):839–1842, 1992.

[11] X. Koutsoukos. *Analysis and Design of Piecewise Linear Hybrid Dynamical Systems.* PhD thesis, Department of Electrical Engineering, University of Notre Dame, Notre Dame, IN, 2000.

[12] P. Lancaster and M. Tismenetsky. *The Theory of Matrices*. Academic Press, 1985.

[13] D. Liberzon and A. Morse. Basic problems in stability and design of switched systems. *IEEE Control Systems Magazine*, 19(5):59–70, October 1999.

[14] K. Loskot, A. Polanski, and R. Rudnicki. Further comments on "Vector norms as Lyapunov functions for linear systems". *IEEE Transactions on Automatic Control*, 43(2):289–291, 1998.

[15] A. Michel. Recent trends in the stability analysis of hybrid dynamical systems. *IEEE Transactions on Circuits and Systems I*, 46(1):120–134, 1999.

[16] A. Michel, B. Nam, and V. Vittal. Computer generated Lyapunov functions for interconnected systems: Improved results with applications to power systems. *IEEE Transactions on Circuits and Systems*, CAS-31(2):189–198, 1984.

[17] Y. Ohta, H. Imanishi, L. Gong, and H. Haneda. Computer generated Lyapunov functions for a class of nonlinear systems. *IEEE Transactions on Circuits and Systems-I: Fundamental Theory and Applications*, 40(5):428–433, 1993.

[18] S. Pettersson and B. Lennartson. Stability and robustness of hybrid systems. In *Proceedings of the 35th IEEE Conference on Decision and Control*, pages 1202–1207, Kobe, Japan, December 1996.

[19] A. Polanski. On infinity norms as Lyapunov functions for linear systems. *IEEE Transactions on Automatic Control*, 40(7):1270–1273, 1995.

[20] A. Polanski. Lyapunov function construction by linear programming. *IEEE Transactions on Automatic Control*, 42(7):1013–1016, 1997.