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COMPENSATOR STRUCTURE AND INTERNAL MODELS IN TRACKING AND REGULATION

by

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Introduction

To study the general Regulator Problem with Internal Stability (RPIS) consider the system description

$$y_m = -H_1 u + G_1 w$$
$$y_r = H_2 u + G_2 w \qquad (1)$$

where y_m and y_r are the measured and regulated outputs respectively, u is the control input and w describes the exogenous signal; w is considered to be the output of a known system R, that is w=Rd for a bounded vector d, and it is assumed that all the poles of R are in the unstable region of the s-plane (or z-plane). If the control law

$$u = Cy_m \qquad (2)$$

is used, the compensator C is to be determined so that, under internal stability, the effect of the exogenous (disturbance) signal w on the regulated output y_r diminishes asymptotically to zero. The latter condition is the regulation requirement and it is equivalent to making the transfer matrix between y_r and d, T_{rd}, stable.

To satisfy the internal stability requirement certain assumptions must be made to guarantee that all signals in the feedback loop will be bounded. In particular, let

$$H_1 = N_1 D_1^{-1} = \underline{D}_1^{-1}\underline{N}_1 \ , \quad C = \underline{D}_c^{-1}\underline{N}_c \qquad (3)$$

be prime polynomial matrix factorizations and assume that: $H_1 D_1$ and $\underline{D}_1 G_1$ are stable transfer matrices. Under these assumptions; the compensated system is internally stable if and only if $(\underline{D}_c D_1 + \underline{N}_c N_1)^{-1}$ is stable. Note that these assumptions have been shown in [10] to be equivalent to the ones in [9], while other equivalent stability conditions also appear in [11,17].

Compensator Structure and Internal Models

The special case, when the regulated and measured outputs are the same, $y_m=y_r$ $(-H_1=H_2,\ G_1=G_2)$, has been studied extensively; necessary and sufficient conditions have been reported, all solutions C have been characterized and their structure has been discussed [1-3]. The compensators C have been shown to be of the form

$$C = G_d^{-1}\hat{C} \qquad (4)$$

where G_d is a polynomial matrix and the rational $\hat{C}$ is chosen to arbitrarily assign the closed loop eigenvalue [4,5]. The poles of G_d^{-1} are those unstable exogenous signal modes which do not appear as poles of the plant (structure included). The role of G_d is to introduce into the loop information about the exogenous signal, so that the cascade connection of the plant and the compensator contains an internal model of R.

The concept of an Internal Model was introduced using state space methods in the context of robust RPIS in [6] and [7], where the case $y_m \neq y_r$ was treated. By robust RPIS it is understood that the properties of output regulation and internal stability will hold under small perturbations of the plant's and some of the compensator's parameters. In this setting it is shown that the compensator includes an appropriate internal model of the exogenous signal. Note that internal models have been also used implicity in [8].

In the frequency domain, the concept of internal models was defined for the special case $y_m=y_r$ in [1] without robustness considerations. Note that one way to define an internal model in this setting is as follows [1]:

Definition: Let $V_1 = \underline{Q}_1^{-1}\underline{P}_1$, $V_2 = \underline{Q}_2^{-1}\underline{P}_2$ be prime polynomial matrix factorization of the proper rational matrices V_1 and V_2; assume that V_1 and V_2 have the same number of rows. Then, V_1 contains an internal model of V_2 if and only if $\underline{Q}_2$ is a right divisor of $\underline{Q}_1$.

In this case, $y_m = y_r$, for regulation with internal stability, the cascade connection of the plant and the compensator $(H_1 C)$ must contain an internal model of the exogenous signal (R), which in turn implies that the compensator C must be as in (4).

For the general model, when $y_m \neq y_r$, the results reported are mainly necessary and sufficient conditions for the existence of C [9-15]. The structure of the compensator C in this case has been studied in [13,14] where it is shown that if a solution exists then $C=\underline{D}_c^{-1}\underline{N}_c$ must satisfy

$$\underline{D}_c D_1 + \underline{N}_c N_1 = \underline{D}_k \qquad (5a)$$
$$\underline{D}_c(-\underline{Y}_1) + \underline{N}_c \underline{X}_1 = \underline{N}_k \qquad (5b)$$

where $H_1 = N_1 D_1^{-1} = \underline{D}_1^{-1}\underline{N}_1$ prime polynomial matrix factorizations, $\underline{D}_1\underline{X}_1 + \underline{N}_1\underline{Y}_1 = I$, $\underline{D}_k^{-1}$ stable with poles the desired closed loop eigenvalues and $\underline{N}_k$ an appropriate polynomial matrix. Equation (5a) corresponds to the internal stability requirement while equation (5b) to regulation. It should be pointed out that the polynomial matrix $\underline{N}_k$ is not fixed, but it can be any member of a (large) class of matrices defined in [13] (note that if $\underline{N}_k$ were arbitrary, then equations (5) would guarantee just internal stability).

It has been shown in [13] that the regulation conditions (5) imply that C can be written as

$$C = G_d^{-1}\hat{C}\,G_n \qquad (6)$$

where G_d, G_n are polynomial matrices with the property that $|G_d||G_n|$ divides the polynomial $|Q|$ the roots of which are those unstable exogenous modes which do not appear in H_1. Notice that when $y_r = y_m$ then $G_n = I$ and (6) reduces to (4), where $\hat{C}$ is chosen to arbitrarily assign the closed loop eigenvalues. In the general case however, the numerator and the denominator of C must satisfy structural requirements in addition to (6), that is $\hat{C}$ in (6) must be chosen not only to arbitrarily assign the closed loop eigenvalues, but also to satisfy additional conditions. In other words, (6) is not always a substitute for the regulation equation (5b); this happens only when [13]

$$|G_d||G_n| = \alpha|Q| \ , \quad \alpha \in R \qquad (7)$$

In general, $\underline{N}_c$ and $\underline{D}_c$ need to satisfy both relations in (5) for regulation with internal stability. Regulation imposes structural requirements on the numerator and denominator of C via (5b) which, under certain conditions, are equivalent to exogenous poles appearing in G_d^{-1} and G_n (see (6)). This is in contrast to the situation in the special case $y_m = y_r$ where the regulation requirement is always equivalent to exogenous poles appearing only in the denominator of C, in G_d^{-1} (see (4)).

While G_d^{-1} in (4) can be easily seen to indicate the existence of an internal model of the exogenous signal in the cascade connection H_1C (see above definition), in the general setting of RPIS the necessity of such an internal model for regulation is not clear. If, of course, $G_n = I$ and (7) is satisfied in which case the regulation requirement is equivalent to having $C = G_d^{-1} \hat{C}$ as in (4) (see example), then, using the above definition of an internal model it can be shown that regulation is equivalent to having an internal model of the exogenous signal in the cascade connection H_1C. This however happens only under certain conditions; in general, (5b) is satisfied and might not imply any particular factors for the numerator and denominator of C. So: In the general RPIS without robustness considerations, an internal model of the exogenous signal as defined above, does not necessarily exist in the cascade connection of the plant and the compensator. The following example clarifies the issue:

Example 1 [16] Consider the system

$$y_m = [a/(s+b)]u + w$$
$$y_r = u + [e/(s+f)]w$$
$$u = C\, y_m \;,\; C = n_c/d_c$$

where the exogenous signal w is a step of the form $w=(1/s)d$. To simplify the analysis assume that $a,e \neq 0$, $b>0$ and $f>0$. The objective is to find C so that the effect of w on y_r is decreased asymptotically to zero.

Solution to this problem does exist. The compensator C must satisfy conditions (5a) and (5b) which in this case imply:

(a) $(s+b)d_c + (-a)n_c = D_k$, D_k^{-1} stable,
(b) $(ae-bf)n_c(0) - be\, d_c(0) = 0$, $ae-bf \neq 0$;
when $ae-bf = 0$ then, $G_d = s$ and $G_n = 1$.

In [10] this example was used with $a=1$, $b=2$, $e=-2$ and $f=1$ to show that the RPIS of (1) could be solved without an internal model. Observe that in this case $ae-bf = -4$ and it follows from (b) that the compensator needs to satisfy the structural relation $n_c(0)-d_c(0)=0$ in addition to (a). If $D_k = s+1$ is the desired closed loop characteristic polynomial, an error feedback compensator $C=1$ will regulate and internally stabilize this system. However, if the gain $e=2$ then, $C=-1/s$ would regulate the system and give two closed loop eigenvalues at -1.●

It is clear from the above that the unstable modes of the exogenous signal which are not poles of the plant do not necessarily appear as poles of C for regulation; that is the existence of an internal model, in the above sense, is not necessary for regulation. This result agrees with the general comments made in [10,18]. It is of course clear that in special cases an internal model does exist. Such is the case when $y_r=Q_m y_m$ with Q_m stable rational; note that this case has been studied with robustness considerations in [16]. In [12] a sufficiency condition for the existence of internal models has also been derived using state space models and geometric methods.

Perhaps one could better understand the necessity for (or the lack of) an internal model of the exogenous signal in the feedback loop by studying the role of the compensator $u=Cy_m$ in regulation. The regulation condition (5b) on C guarantees that the control signal will be able to eliminate the unstable modes due to w on y_r. This implies that C will make those exogenous modes unobservable from the output y_r. In input-output operator terms this also implies that C will create multivariable zeros in T_{rw} to counterbalance the effect of the unstable exogenous modes. For this to happen, it suffices, in the special case $y_r=y_m$, that the compensator be of the form (4) where the unstable exogenous modes which are not poles of the plant appear as poles of C (in G_d^{-1}). In the general case, however, for the control signal u to eliminate the unstable modes due to w on y_r, it is not generally sufficient for these modes to appear as poles of C as before. Certain structural relations on C, in particular (5b), must be satisfied for this to happen. (5b) implies that certain unstable modes of w could appear in either the denominator or numerator of C (see (6)) but this is not in general, sufficient for regulation. Thus, regulation in the general RPIS can take place without an internal model as defined above.

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