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# STABILITY AND $\mathcal{H}_\infty$ PERFORMANCE PRESERVING SCHEDULING POLICY FOR NETWORKED CONTROL SYSTEMS

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Abstract: In this paper, the exponential stability and the $\mathcal{L}_2$ induced gain performance are investigated for a collection of plants whose feedback control loops are closed via a shared network link. Due to a limited communication capacity, the network link can only close one feedback control loop at a time, while the other control loops are assumed to be open-loop. Therefore, it is necessary to carefully allocate the communication resources in order to guarantee exponential stability and achieve desired $\mathcal{H}_\infty$ performance of the whole networked control systems. In this paper, we derive a condition for scheduling the network so that all the plants achieve the exponential stability and some reasonable $\mathcal{H}_\infty$ disturbance attenuation levels. The proof is constructive. A time-division based scheduling policy is proposed to guarantee the exponential stability and a weighted $\mathcal{H}_\infty$ performance. The techniques used in this paper are based on the average dwell time approach incorporated with piecewise quadratic Lyapunov-like functions.


Keywords: Networks, Switches, Lyapunov methods, Fault tolerance, Sampled data control.

## 1. INTRODUCTION

By Networked Control Systems (NCSs), we mean feedback control systems where networks, typically digital band-limited serial communication channels, are used for connecting spatially distributed system components like sensors to controllers, and controllers to actuators. These channels are usually shared by a number of feedback control loops. In the traditional feedback control systems these connections are established via point-to-point cables. Compared with the point-to-point cables, the introduction of digital communication networks has many advantages, such as high system testability and resource utilization, as well as low weight, space, power and wiring requirements and easy system diagnosis and maintenance (Ishii and Francis, 2002). These advantages make control over networks more and more popular in a wide variety of applications, including traffic control, satellite clusters, mobile robotics, etc. Recently, modeling, analysis and control of networked control systems with limited communication capability has emerged as a topic of significant interest to control community, see for example (Brockett and Liberzon, 2000; Nair and Evans, 2000; Elia and Mitter, 2001; Zhang *et al.*, 2001).

If a communication channel is shared by many control loops, which is common in practice, then one problem inherent to such NCSs is how to

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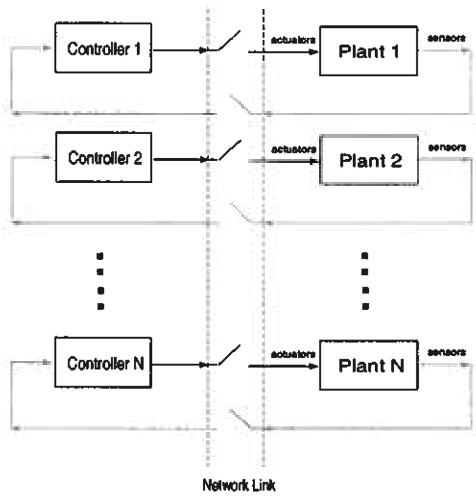


Fig. 1. A collection of networked control systems shared by a communication link.

efficiently allocate the communication resources so that the stability and performance of all the control loops are guaranteed. This is referred to as the scheduling problem of NCSs, which has been considered in, for example, (Hristu, 2001; Hristu and Kumar, 2002; Walsh *et al.*, 2002; Branicky *et al.*, 2002). In (Hristu, 2001), a static time-division stability preserving network scheduling policy was derived by employing a common Lyapunov-like function, and the static token-type scheduling policy was improved by introducing a interrupt-based strategy in (Hristu and Kumar, 2002). In (Walsh *et al.*, 2002), a scheduling policy among multiple sensors and actuators, called Try-Once-Discard (TOD), was proposed for the exponential stability of a MIMO networked controlled LTI system. In (Branicky *et al.*, 2002), the Rate Monotonic (RM) scheduling algorithm was applied to a collection of network controlled LTI systems to obtain a stability preserving scheduling policy for the NCSs. Note that only the stability problem was studied in the existing NCSs scheduling literature. This paper will investigate scheduling problems for both the stability and the $\mathcal{L}_2$ induced gain performance of NCSs.

We consider the NCSs consisting of a collection of continuous-time LTI plants whose feedback control loops are closed via a shared network link, as illustrated in Figure 1. The $i$-th plant $(i = 1, \cdots, N)$ is given by

$$\begin{cases} \dot{x}_i(t) = A_i x_i(t) + B_i u_i(t) + E_i d_i(t) \\ z_i(t) = C_i x_i(t) \end{cases} \tag{1}$$

where the time $t \in \mathbb{R}^+$ (nonnegative real numbers), the state $x_i(t) \in \mathbb{R}^{n_i}$, the control input $u_i(t) \in \mathbb{R}^{p_i}$, the disturbance input $d_i(t) \in \mathbb{R}^{r_i}$, and the controlled output $z_i(t) \in \mathbb{R}^{m_i}$. We assume that each open-loop plant is unstable, but can be exponentially stabilized by a linear static state feedback gain $K_i$ with the $\mathcal{L}_2$ induced gain from $d_i(t)$ to $z_i(t)$ being bounded by $\gamma_{s,i}$. Each system communicates with its remote controller that occasionally transmits control signals over the shared network, according to the static state feedback law $u_i(t) = K_i x_i(t)$. Here we omit the time delays and errors in the communication link.

With finite bit-rate constraints, quantization effects need to be considered in the NCSs. The quantization problem has been studied extensively in the literature, see for example (Brockett and Liberzon, 2000; Nair and Evans, 2000; Elia and Mitter, 2001; Hespanha *et al.*, 2002; Liberzon, 2003). It has been known that an exponential data representation scheme is most efficient for certain cases (Brockett and Liberzon, 2000; Elia and Mitter, 2001). In this paper, we will consider the floating point representation, which is an exponential data representation scheme and has been widely used in practice. It is known that floating point quantization can be viewed as a nonlinear operation described by a time variant sector gain, i.e., $Q(x) = \kappa x$, $\kappa \in [1-\epsilon, 1]$, with $\epsilon$ depending on the mantissa length. Now the closed-loop plant can be modeled as the following parametric uncertain system

$$\begin{cases} \dot{x}_i(t) = (A_i + \kappa B_i K_i) x_i(t) + E_i d_i(t) \\ z_i(t) = C_i x_i(t) \end{cases} \tag{2}$$

where $\kappa \in [1-\epsilon, 1]$. On the other hand, it was shown in (Fu and Xie, 2004) that many quantized feedback design problems might be converted to the robust control problems with sector bound uncertainties.

Due to a limited communication capacity, not all the control loops in the NCSs can be addressed at the same time. It is assumed that the network link can only feedback one control loop at a time, while the other control loops are assumed to be open-loop. In order to preserve the stability and robust performance, a plant should be attended to with an adequate frequency and length in time. Otherwise, some plants may exhibit undesirable behaviors without properly allocating the communication resources. Therefore, it is necessary to carefully schedule the NCSs in order to guarantee the stability and robust performance of each plant. In particular, the exponential stability and the $\mathcal{H}_\infty$ disturbance attenuation performance are considered here.

The rest of the paper is organized as follows. First, a condition for preserving the exponential stability and a reasonable $\mathcal{H}_\infty$ disturbance attenuation performance for a single control loop is derived in Section 2, which gives bounds on how frequent and how long a control loop should be taken care of by the controller. Then, a sufficient condition for the existence of a feasible scheduling policy is derived in Section 3, and a network scheduling policy is

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proposed to guarantee that each NCS sharing a common network link preserves the exponential stability and achieves a weighted $\mathcal{H}_\infty$ disturbance attenuation level. The techniques used in this paper are based on the average dwell time approach (Hespanha and Morse, 1999) incorporated with piecewise quadratic Lyapunov-like functions.

## 2. SINGLE CONTROL LOOP ANALYSIS

We suppose that the control system is open-loop for some time because the shared network link is occupied by another network user. The following definition on the attention rate of the controller plays a crucial role in the sequel.

*Definition 1.* For any $t > 0$, we denote by $\alpha_i(t)$ the total time interval that the $i$-th plant is closed-loop (attended by the controller) during $[0, t)$, and call the ratio $\frac{\alpha_i(t)}{t}$ the *attention rate* of the $i$-th plant.

In this section, we will focus on a single plant and derive the condition on the attention rate under which the stability and robust performance of this plant are preserved.

Assume that the $i$-th closed-loop dynamics is quadratically stable and the $\mathcal{L}_2$ induced gain from $d_i(t)$ to $z_i(t)$ is bounded by $\gamma_{s,i}$. Therefore, there exists a positive definite matrix $P_{s,i} > 0$ such that

$$(A_i + \kappa B_i K)^T P_{s,i} + P_{s,i}(A_i + \kappa B_i K) + \gamma_{s,i}^{-2} P_{s,i} E_i E_i^T P_{s,i} + C_i^T C_i < 0 \quad (3)$$

for all $\kappa \in [1-\epsilon, 1]$. Then, there always exists a positive scalar $\lambda_{s,i} > 0$, such that

$$(A_i + \kappa B_i K)^T P_{s,i} + P_{s,i}(A_i + \kappa B_i K) + \gamma_{s,i}^{-2} P_{s,i} E_i E_i^T P_{s,i} + C_i^T C_i + \lambda_{s,i} P_{s,i} < 0 \quad (4)$$

holds for all $\kappa \in [1-\epsilon, 1]$. Note that this condition is equivalent to the following two inequalities

$$\begin{cases} [A_i + (1-\epsilon)B_i K_i]^T P_{s,i} + P_{s,i}[A_i + (1-\epsilon)B_i K_i] \\ \qquad + \gamma_{s,i}^{-2} P_{s,i} E_i E_i^T P_{s,i} + C_i^T C_i + \lambda_{s,i} P_{s,i} < 0 \\ (A_i + B_i K_i)^T P_{s,i} + P_{s,i}(A_i + B_i K_i) \\ \qquad + \gamma_{s,i}^{-2} P_{s,i} E_i E_i^T P_{s,i} + C_i^T C_i + \lambda_{s,i} P_{s,i} < 0 \end{cases}$$

which can be converted into LMIs with respect to $P_{s,i}$ by Schur complement (Boyd *et al.*, 1994) and thus can be solved efficiently using the existing software.

On the other hand, for unstable open-loop system $\dot{x}(t) = A_i x(t) + E_i d(t)$, it can be shown that there exists a positive scalar $\lambda_{u,i}$, such that $A_i - \frac{\lambda_{u,i}}{2} I$ is stable. Hence, there exists a positive definite matrix $P_{u,i}$ such that

$$A_i^T P_{u,i} + P_{u,i} A_i + \gamma_{u,i}^{-2} P_{s,i} E_i E_i^T P_{s,i} + C_i^T C_i - \lambda_{u,i} P_{s,i} < 0 \quad (5)$$

holds for some scalar $\gamma_{u,i} > 0$. Similarly, the matrix $P_{u,i}$ can be determined by solving some LMIs.

In the sequel, the subscript $i$ is dropped for notational simplicity. Using the solutions $P_s$ and $P_u$, we define the following *piecewise Lyapunov-like function*

$$V(x(t)) = V_{\sigma(t)}(x(t)) = x(t)^T P_{\sigma(t)} x(t) \quad (6)$$

for the system. Here $P_{\sigma(t)}$ is a two-valued piecewise constant matrix function as

$$P_{\sigma(t)} = \begin{cases} P_s, & \text{if closed}-\text{loop}, \\ P_u, & \text{if open}-\text{loop} \end{cases} \quad (7)$$

and $V_{\sigma(t)}(x)$ is defined correspondingly. Then the following properties of $V(x)$ are obtained:

1) Both $V_s(x) = x^T P_s x$ and $V_u(x) = x^T P_u x$ are continuous and their derivatives along the solutions of the corresponding system satisfy

$$\begin{cases} \dot{V}_s \le -\lambda_s V_s - z^T z + \gamma_s^2 d^T d \\ \dot{V}_u \le \lambda_u V_u - z^T z + \gamma_u^2 d^T d \end{cases} \quad (8)$$

2) There exist constant scalars $a_2 \ge a_1 > 0$ such that $a_1\|x\|^2 \le V_s(x) \le a_2\|x\|^2$ and $a_1\|x\|^2 \le V_u(x) \le a_2\|x\|^2$ hold for any $x \in \mathbb{R}^n$;

3) There exists a constant scalar $\mu \ge 1$ such that $V_s(x) \le \mu V_u(x)$ and $V_u(x) \le \mu V_s(x)$ hold for any $x \in \mathbb{R}^n$.

The first property is a straightforward consequence of the inequalities (4) and (5), while the second and third properties hold, for example, with $a_1 = \min\{\lambda_m(P_s), \lambda_m(P_u)\}$, $a_2 = \max\{\lambda_M(P_s), \lambda_M(P_u)\}$, and $\mu = \frac{a_2}{a_1}$, respectively. Here, $\lambda_m(\cdot)$ ( $\lambda_M(\cdot)$ ) denotes the smallest (largest) eigenvalue of a symmetric matrix. Note that the eigenvalues of a positive symmetric matrix are all positive real numbers. Therefore, $a_1$, $a_2$, and $\mu$ are all positive real numbers.

Without loss of generality, we assume that the controller works during $[t_{2j}, t_{2j+1})$, and the plant is open-loop during $[t_{2j+1}, t_{2j+2})$, $j = 0, 1, \cdots$, where $t_0 = 0$. Then, using the differential inequality theory, we get for any $t > 0$ that

$$V(x(t)) \le e^{-\lambda_s(t-t_{2j})} V_s(x(t_{2j})) - \int_{t_{2j}}^{t} e^{-\lambda_s(t-\tau)} \Gamma(\tau) d\tau$$

if $t_{2j} \le t \le t_{2j+1}$, and

$$V(x(t)) \le e^{\lambda_u(t-t_{2j+1})} V_u(x(t_{2j+1})) - \int_{t_{2j+1}}^{t} e^{\lambda_u(t-\tau)} \Gamma(\tau) d\tau$$

for $t_{2j+1} \le t \le t_{2j+2}$. Here $\Gamma(\tau) = z^T(\tau) z(\tau) - \gamma_0^2 d^T(\tau) d(\tau)$ and $\gamma_0 = \max\{\gamma_s, \gamma_u\} > 0$.

Therefore, for any given time instant $t$, we get from (8) by induction that

$$\begin{aligned} &V(x(t)) + \int_0^t \mu^{2N(t)-2N(\tau)} e^{\delta(t,\tau)} z^T(\tau) z(\tau) d\tau \\ &\le \mu^{2N(t)} e^{\lambda_u(t-\alpha(t)) - \lambda_s \alpha(t)} V(x(0)) \\ &\qquad + \gamma_0^2 \int_0^t \mu^{2N(t)-2N(\tau)+1} e^{\delta(t,\tau)} d^T(\tau) d(\tau) d\tau \end{aligned}$$

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where $\delta(t,\tau) = \lambda_u(t-\tau-\alpha(t)+\alpha(\tau)) - \lambda_s(\alpha(t)-\alpha(\tau))$, and $N(t)$ denotes the total number of switchings from closed-loop to open-loop within the interval $[0,t)$, i.e., the attention frequency. Assume $\alpha_i(0) = 0$ for simplicity.

Multiply both sides of the above inequality by $\mu^{-2N(t)}e^{-\lambda_u(t-\alpha(t))+\lambda_s\alpha(t)}$ to obtain

$$\mu^{-2N(t)}e^{-\delta(t,0)}V(x(t)) + \int_0^t \mu^{-2N(\tau)}e^{-\delta(\tau,0)}z^T z d\tau$$
$$\leq V(x(0)) + \gamma_0^2\mu \int_0^t \mu^{-2N(\tau)}e^{-\delta(\tau,0)}d^T(\tau)d(\tau)d\tau \quad (9)$$

If there exists a positive scalar $0 < \lambda^* < \lambda_s$ such that

$$\frac{\alpha(t)}{t} \geq \frac{\lambda_u + \lambda^*}{\lambda_u + \lambda_s}, \quad \forall t > 0 \quad (10)$$

which is a condition on the attention rate of the controller. Furthermore, if there exists a positive scalar $c$ and a positive scalar $\lambda$ satisfying $\lambda < \lambda^*$ such that

$$N(t) \leq N_0 + \frac{t}{\tau_a^*}, N_0 = \frac{\ln c}{2\ln\mu}, \tau_a^* = \frac{2\ln\mu}{\lambda^* - \lambda} \quad (11)$$

which is exactly an average dwell time scheme (Hespanha and Morse, 1999). Then, the left hand side of the inequality (9) is greater or equal to

$$c^{-1}e^{\lambda t}V(x(t)) + \int_0^t c^{-1}e^{\lambda\tau}z^T(\tau)z(\tau)d\tau \quad (12)$$

On the other hand the right hand side of the inequality (9) is less or equal to

$$V(x(0)) + \gamma_0^2\mu \int_0^t e^{\lambda_s\alpha(\tau)}d^T(\tau)d(\tau)d\tau \quad (13)$$

since $\mu^{-2N(\tau)} \leq 1$, $e^{-\lambda_u(\tau-\alpha(\tau))} \leq 1$, and $\alpha(\tau) \leq \tau$. Therefore, we obtain

$$c^{-1}e^{\lambda t}V(x(t)) + \int_0^t c^{-1}e^{\lambda\tau}z^T(\tau)z(\tau)d\tau$$
$$\leq V(x(0)) + \gamma_0^2\mu \int_0^t e^{\lambda_s\tau}d^T(\tau)d(\tau)d\tau$$

Multiply both side of the above inequality with $e^{-\lambda t}$, then

$$c^{-1}V(x(t)) + c^{-1}\int_0^t e^{-\lambda(t-\tau)}z^T(\tau)z(\tau)d\tau$$
$$\leq e^{-\lambda t}V(x(0)) + \gamma_0^2\mu \int_0^t e^{\lambda_s\tau-\lambda t}d^T(\tau)d(\tau)d\tau \quad (14)$$

Considering the fact that $V(x(t)) \geq 0$, (14) can be written as

$$c^{-1}\int_0^t e^{-\lambda(t-\tau)}z^T(\tau)z(\tau)d\tau \leq e^{-\lambda t}V(x(0))$$
$$+\gamma_0^2\mu \int_0^t e^{-\lambda(t-\tau)}e^{(\lambda_s-\lambda)\tau}d^T(\tau)d(\tau)d\tau$$

Integrating both sides of the above inequality from $t = 0$ to $t = \infty$ yields

$$\frac{1}{c\lambda}\int_0^\infty z^T(\tau)z(\tau)d\tau$$
$$\leq \frac{1}{\lambda}V(x(0)) + \frac{\gamma_0^2\mu}{\lambda}\int_0^\infty e^{(\lambda_s-\lambda)\tau}d^T(\tau)d(\tau)d\tau$$

and thus

$$\int_0^\infty z^T(\tau)z(\tau)d\tau$$
$$\leq cV(x(0)) + c\gamma_0^2\mu \int_0^\infty e^{(\lambda_s-\lambda)\tau}d^T(\tau)d(\tau)d\tau \quad (15)$$

which means that a weighted $\mathcal{H}_\infty$ disturbance attenuation level $\sqrt{c\mu}\gamma_0$ is achieved.

In addition, when $d(t) = 0$, we get from (14) that

$$V(x(t)) \leq ce^{-\lambda t}V(x(0)).$$

Combining with the second property of the piecewise quadratic Lyapunov-like function $V(x(t))$, we obtain

$$a_1\|x(t)\|^2 \leq V(x(t)) \leq ce^{-\lambda t}V(x(0)) \leq ce^{-\lambda t}a_2\|x(0)\|^2$$

which implies

$$\|x(t)\| \leq \sqrt{\frac{ca_2}{a_1}}e^{-\frac{\lambda}{2}t}\|x(0)\|,$$

that is the exponential stability of the control loop.

*Theorem 2.* The system preserves exponential stability and achieves a weighted $\mathcal{H}_\infty$ disturbance attenuation level $\sqrt{c\mu}\gamma_0$ in the sense of (15), if the attention rate and the attention frequency $N(t)$ satisfy the following two conditions for all $t > 0$

(1) $\frac{\alpha(t)}{t} \geq \frac{\lambda_u+\lambda^*}{\lambda_u+\lambda_s}$ holds for some positive scalar $\lambda^*$, for $\lambda^* < \lambda_s$;
(2) $N(t) \leq N_0 + \frac{t}{\tau_a^*}$, where $N_0 = \frac{\ln c}{2\ln\mu}$ and $\tau_a^* = \frac{\ln\mu}{\lambda^*-\lambda}$. Here $c$ is an arbitrary positive scalar, and $\lambda$ is a positive scalar less than $\lambda^*$.

Similar results were previously derived in the switched systems and fault tolerance literature (Zhai, 2002). However, the introduction of the constant $N_0$ in the second condition of Theorem 2 makes it easier to be satisfied, since $N_0 = \frac{\ln c}{2\ln\mu}$ could be arbitrary large (but finite) by picking arbitrary large constant $\mu$. This advantage makes the design of a scheduling policy that satisfies the attention frequency condition possible as it is described in the next section.

Note that the attention rate and frequency conditions in the above theorem depend not only on the system dependent constants ($\lambda_s$, $\lambda_u$ and $\mu$), but also on some flexible constants, say $c$, $\lambda^*$ and $\lambda$. In the next section, we will show that the existence of a feasible scheduling policy only depends on the system constants, $\lambda_s$ and $\lambda_u$, which can be determined by solving some LMIs.

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## 3. STABILITY AND PERFORMANCE PRESERVING SCHEDULING

The question studied here is under what conditions there exists a scheduling policy such that all the control loops preserve exponential stability and attain certain weighted $\mathcal{H}_\infty$ disturbance attenuation level. If such policy exists, then design the stability and performance preserving scheduling policy for the given NCSs. The main result of the paper is stated in the following theorem.

*Theorem 3.* For a collection of networked control systems with a common shared network, if

$$\sum_{i=1}^{N} \frac{\lambda_{u,i}}{\lambda_{u,i}+\lambda_{s,i}} < 1, \tag{16}$$

then there exists a scheduling policy to guarantee the exponential stability and a weighted $\mathcal{H}_\infty$ disturbance attenuation level in the sense of (15) for all the control loops.

*Proof*: Because $\sum_{i=1}^{N} \frac{\lambda_{u,i}}{\lambda_{u,i}+\lambda_{s,i}} < 1$, there exists a positive scalar $\bar{\epsilon}$, for all $0 < \epsilon \leq \bar{\epsilon}$, the inequality

$$\sum_{i=1}^{N} \frac{\lambda_{u,i}}{\lambda_{u,i}+\lambda_{s,i}} + \epsilon \leq 1 \tag{17}$$

holds.Then

$$\sum_{i=1}^{N} \frac{\lambda_{u,i}}{\lambda_{u,i}+\lambda_{s,i}} + (\sum_{i=1}^{N} \frac{\lambda_{u,i}}{\lambda_{u,i}+\lambda_{s,i}})\epsilon < 1$$

since $\sum_{i=1}^{N} \frac{\lambda_{u,i}}{\lambda_{u,i}+\lambda_{s,i}} < 1$. Therefore,

$$\sum_{i=1}^{N} \frac{\lambda_{u,i}+\epsilon\lambda_{u,i}}{\lambda_{u,i}+\lambda_{s,i}} < 1,$$

holds for all $0 < \epsilon \leq \bar{\epsilon}$. Let $\lambda_i^* = \epsilon\lambda_{u,i}$, and set $\lambda_i = \frac{\epsilon}{2}\lambda_{u,i}$. It is easy to verify that $\lambda_i < \lambda_i^* < \lambda_{s,i}$ for $i = 1, \cdots, N$. Denote $\beta_i = \frac{\lambda_{u,i}+\lambda_i^*}{\lambda_{u,i}+\lambda_{s,i}}$ for $i = 1, \cdots, N$.

Next, we propose a periodic scheduling policy for the NCSs.

First, choose $\mathcal{T} = \max_i\{\mathcal{T}_i\}$, where $\mathcal{T}_i$ is a positive time unit sufficiently large to satisfy the average dwell time condition for the $i$-th plant, for example set $\mathcal{T}_i$ to be the average dwell time $\tau_a^*$ of the $i$-th control system, where $\tau_a^* = \frac{\ln \mu_i}{\lambda_i^*-\lambda_i}$.

Attend each plant in order, and activate the $i$-th plant's controller for a time interval of length $\frac{\beta_i}{\sum_{j=1}^{N}\beta_j}\mathcal{T}$. Let us denote $\frac{\beta_i}{\sum_{j=1}^{N}\beta_j}$ as $\gamma_i$. Note that $0 < \gamma_i < 1$, $\sum_{i=1}^{N}\gamma_i = 1$, and $\gamma_i > \beta_i$ for all $1 \leq i \leq N$, since $\sum_{j=1}^{N}\beta_j < 1$.

In the following, we show that under the above scheduling policy all the plants are exponential stable and achieve weighted $\mathcal{L}_2$ gains.

First, consider the case of $i = 1$. For any $t > 0$, it can be written as $t = n \times \mathcal{T} + \Delta$, where $n$ is a nonnegative integer and $0 \leq \Delta < \mathcal{T}$ is a real number. There are two different cases to be considered.

- If $\Delta < \gamma_1\mathcal{T}$, then $\alpha_1(t) = n\gamma_1\mathcal{T} + \Delta$ and $N(t) = n$. Therefore,
$$\frac{\alpha_1(t)}{t} = \frac{n\gamma_1\mathcal{T}+\Delta}{n\mathcal{T}+\Delta} \geq \frac{n\gamma_1\mathcal{T}+\gamma_1\Delta}{n\mathcal{T}+\Delta} = \frac{\gamma_1(n\mathcal{T}+\Delta)}{n\mathcal{T}+\Delta} = \gamma_1 > \beta_1 = \frac{\lambda_{u,1}+\lambda_1^*}{\lambda_{u,1}+\lambda_{s,1}}$$
And
$$\frac{t}{\tau_a^*} \geq \frac{t}{\mathcal{T}} \geq n, \tag{18}$$
so $N(t) \leq N_0 + \frac{t}{\tau_a^*}$.
- If $\Delta \geq \gamma_1\mathcal{T}$, then $\alpha_1(t) = n\gamma_1\mathcal{T} + \gamma_1\mathcal{T}$ and $N(t) = n+1$. Therefore,
$$\frac{\alpha_1(t)}{t} = \frac{n\gamma_1\mathcal{T}+\gamma_1\mathcal{T}}{n\mathcal{T}+\Delta} \geq \frac{n\gamma_1\mathcal{T}+\gamma_1\mathcal{T}}{n\mathcal{T}+\mathcal{T}} = \frac{(n+1)\gamma_1\mathcal{T}}{(n+1)\mathcal{T}} = \gamma_1 > \beta_1 = \frac{\lambda_{u,1}+\lambda_1^*}{\lambda_{u,1}+\lambda_{s,1}}$$
If we set $N_0 \geq 1$, or equivalently
$$c \geq \mu^2,$$
then $N(t) = n+1 \leq N_0 + \frac{t}{\tau_a^*}$.

Therefore, the two conditions in Theorem 2 are both satisfied for the first control loop. By Theorem 2, the first control loop preserves the exponential stability and a weighted $\mathcal{H}_\infty$ disturbance attenuation level in the sense of (15) under this scheduling policy.

For $i > 1$, we may simply shift the the initial time zero to $t_0 = \sum_{j=1}^{i}\gamma_j\mathcal{T}$ and adjust the initial state $x_0$ according to its open-loop dynamics correspondingly. Then it reduces to the case $i = 1$, and the two conditions in Theorem 2 are both satisfied for this shifted $i$-th subsystem. It is straightforward to show that the exponential stability and the $\mathcal{H}_\infty$ disturbance attenuation performance is equivalent between the time-shifted (by a finite constant) control system and the original system. Therefore, all the control loops in the NCSs preserve the exponential stability and achieve weighted $\mathcal{H}_\infty$ disturbance attenuation levels under this scheduling policy. □

It is worth pointing out that all the above results and scheduling policy can be easily adapted to the case that more than one plant can be attended at the same time. For example, if $M$, $1 \leq M < N$ control loops can share the link at the same time, then the NCSs may preserve the exponential stability and exhibit reasonable $\mathcal{H}_\infty$ performance if

$$\sum_{i=1}^{N} \frac{\lambda_{u,i}}{\lambda_{u,i}+\lambda_{s,i}} < M.$$

The proof is similar, and is omitted here.

Similar results for stability were obtained in (Hristu, 2001) by using a common quadratic

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Lyapunov-like function. However, the method developed here is based on piecewise quadratic Lyapunov-like functions and is less conservative than (Hristu, 2001). This claim is verified by the following example, which originally appeared (Hristu, 2001).

*Example 4.* Assume all the plants are of the same dynamics:

$$\dot{x} = Ax + Bu, \quad u = Kx,$$

with

$$A = \begin{bmatrix} 0 & 1 \\ 1.5 & -0.1 \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \quad K = [-35, -29].$$

Assume that only one plant can be attended at a time through the network link, i.e., $M = 1$. Using the presented method, we find $\lambda_s = 2.4014$, $\lambda_u = 2.3515$. And

$$\frac{\lambda_u}{\lambda_s + \lambda_u} = 0.4947.$$

Note that for $N = 2$, $2 \times 0.4947 = 0.9894 < 1 = M$. Based on Theorem 3, we may conclude that two ($N = 2$) of such control loops can share a common network link, which only takes care of one control loop at a time. However, if the common quadratic Lyapunov-like function in (Hristu, 2001) is used, we find $\lambda_s = 2.00$, $\lambda_u = 2.35$. Because

$$\frac{\lambda_u}{\lambda_s + \lambda_u} = 0.542,$$

and $2 \times 0.542 = 1.084 > 1$. It indicates that two of such control loops may fail to share a common network link, which is a more conservative conclusion than the one given by this paper.

The constructive proof of Theorem 3 also gives a systematic way to design such stability and performance preserving scheduling policy. The scheduling policy is given as a static time-division based scheduling policy, which is quite simple and can be easily implemented in a token-type field bus.

## 4. CONCLUDING REMARKS

In this paper, the $\mathcal{H}_\infty$ disturbance attenuation performance for a collection of plants whose feedback control loops are closed via a shared network link is investigated. Due to limited communication capacity, the network link can not handle all the feedback control loops at the same time. Therefore, it is necessary to carefully allocate the communication resources in order to preserve stability and desirable $\mathcal{H}_\infty$ disturbance attenuation levels for the whole NCSs. In this paper, we first derived a schedulibility condition, which only depends on the eigenvalues of the Lyapunov matrices for each control loop. Then, a time-division based scheduling policy was proposed to guarantee the stability and weighted $\mathcal{H}_\infty$ performance of the whole NCSs, which can be easily implemented as a token-type protocol network layer. The techniques used in this paper are based on the average dwell time approach incorporated with piecewise quadratic Lyapunov-like functions.

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