Model-Based Control with Intermittent Feedback

Tomas Estrada

Hai Lin

Panos J. Antsaklis

Abstract— In this paper we apply the concept of Intermittent Feedback to a class of networked control systems known as Model-Based Networked Control Systems (MB-NCS). MB-NCS use an explicit model of the plant in order to reduce the network traffic while attempting to prevent performance degradation. In the previous body of work regarding MB-NCS, updates of the plant were given instantaneously; however, in this paper we consider the case where the loop remains closed for a finite length fixed interval before the control system returns to openloop. We provide a full description of the output, as well as a necessary and sufficient condition for stability of the system. We also provide examples in order to illustrate the behavior indicated by the theory, and we show the advantages of the approach. Finally, we conclude the paper with discussions on possible future extensions.

I. INTRODUCTION

A networked control system (NCS) is a control system in which a data network is used as feedback media. NCS is an important area in control, see for example [19], [17], [20], and [22]. The use of networks as media to interconnect the different components of an industrial system is rapidly increasing. However, the use of NCSs poses some challenges. One of the main problems to be addressed when considering an NCS is the size of the bandwidth required by each subsystem. Clearly, the bandwidth required by the communication network is a major concern. Recently, modeling, analysis and control of networked control systems with limited communication capability has emerged as a topic of significant interest to control community, see for example [23], [6], [8], [24], [10], [2], and recent special issue [3]. An efficient way to address this is reducing the rate at which packets are transmitted.

A particular class of NCSs is model-based networked control systems (MB-NCS), introduced by Montestruque and Antsaklis [13]. The MB-NCS architecture makes explicit use of the knowledge of the plant dynamics to enhance the performance of the system, and it is an efficient way to address the issue of reducing packet rate. In this paper we extend the work done in MB-NCS by taking advantage of intermittent feedback. In the previous work done in MB-NCS, the state updates given to the model of the plant were for a time instant only, but with intermittent feedback the system remains in closed loop control for more extended intervals. This notion makes sense as it is motivated by human motor control observations. For example, while driving a car, when approaching a curve or hilly terrain, we pay attention to the road for a longer time, which is equivalent to staying in closed-loop mode, and we only reduce our attention -switch to open loop control- when the road is once again straight. While intermittent control is a very intuitive notion, its combination with the MB-NCS architecture allows for obtaining important results and opening new paths in controlling NCSs effectively.

With the finite bit-rate constraints, quantization has to be taken into consideration in NCSs. Therefore, quantization and limited bit rate issues have attracted many researchers' attention with the aim to identify the minimum bit rate required to stabilize a NCS, see for example [7], [6], [8], [21], [18]. In [7] it is shown that asymptotic stability cannot be achieved by (static) quantization. In [6] Brockett and Liberzon proposed a dynamic quantization scheme, so called "zoom-in, zoom-out" approach, to asymptotically stabilize linear systems. The idea behind the "zoom-in, zoom-out" scheme is to provide more detailed information when the states come closer to the origin through finer quantization (zoom-in), while only coarser quantization (zoom-out) is sufficient for states farther away from the origin. As an interesting observation of a person's response in face of changing environment, one usually tends to pay longer attention to objects of concern, instead of paying closer attention. This motivates us to use intermittent feedback in NCSs.

The rest of the paper is organized as follows. In Section II, we describe the problem formulation in detail. In Section III, we derive the complete description of the output of such a system. In Section IV, we present a necessary and sufficient condition for the stability of the system. An example is provided in Section V. Finally, we provide conclusions and propose future work.

II. PROBLEM FORMULATION

The basic setup for MB-NCS with intermittent feedback is quite similar to that proposed in the literature for traditional MB-NCS. Please see references [11] through [16] for more results on MB-NCS.

Consider the control of a continuous linear plant where the state sensor is connected to a linear controller/actuator via a network. In this case, the controller uses an explicit model of the plant that approximates the plant dynamics and makes possible the stabilization of the plant even under slow network conditions.

The main idea here is to perform the feedback by updating the model's state using the actual state of the plant that is provided by the sensor. The rest of the time the control actions is based on a plant model that is incorporated in

T. Estrada and P.J. Antsaklis are with the the Department of Electrical Engineering, University of Notre Dame, Notre Dame, IN 46556, USA {testrada,antsaklis.1}@nd.edu

H. Lin is with the Department of Electrical & Computer Engineering, National University of Singapore, Singapore 117576 hlin28@gmail.com



Fig. 1. Basic MB-NCS architecture.

the controller/actuator and is running open loop for a period of h seconds.

As mentioned before, the main difference between modelbased networked control systems as have been studied previously, and the case with intermittent feedback, which we are introducing here, is that in the literature, the loop is closed instantaneously, and the rest of the time the system is running open loop. Here, we start with the same basic idea, but the loop remains closed for intervals of time which are different from zero. Intuitively, we should be able to achieve much better results the longer the loop is closed, since the level of degradation of performance increases the longer the system is running open loop. So intermittent feedback should yield better results than those for traditional MB-NCS.

In dealing with intermittent feedback, we have two key time parameters: how frequently we want to close the loop, which we shall denote by h, and how long we wish the loop to remain closed, which we shall denote by τ . Naturally, in the more general cases both h and τ can be time-varying. For the purposes of this paper, however, we will deal only with the case where both h and τ are fixed.

We consider then a system such that the loop is closed periodically, every *h* seconds, and where each time the loop is closed, it remains so for a time of $\tau < h$ seconds. The loop is closed at times t_k , for $k = 1, 2, \cdots$. Thus, there are two very clear modes of operation: closed loop and open loop. The system will be operating in closed loop mode for the intervals $[t_k, t_k + \tau)$ and in open loop for the intervals $[t_k + \tau, t_{k+1})$. When the loop is closed, the control decision is based directly on the information of the state of the plant, but we will keep track of the error nonetheless.

The plant is given by $\dot{x} = Ax + Bu$, the plant model by $\dot{x} = \hat{A}\hat{x} + \hat{B}u$, and the controller by $u = K\hat{x}$. The state error is defined as $e = x - \hat{x}$ and represents the difference between plant state and the model state. The modeling error matrices $\tilde{A} = A - \hat{A}$ and $\tilde{B} = B - \hat{B}$ represent the plant and the model. We also define the vector $z = [x \ e]^T$.

In the next section we will derive a complete description of the response of the system.

III. STATE RESPONSE OF THE SYSTEM

We will now proceed to derive the response in a direct way. To this effect, let us separately investigate what happens when the system is operating under closed and open loop conditions.

During the open loop case, that is, when $t \in [t_k + \tau, t_{k+1})$, we have that

$$u = K\hat{x} \tag{1}$$

so

$$\begin{bmatrix} \dot{x} \\ \dot{\hat{x}} \end{bmatrix} = \begin{bmatrix} A & BK \\ 0 & \hat{A} + \hat{B}K \end{bmatrix} \begin{bmatrix} x(t) \\ \hat{x}(t) \end{bmatrix}$$
(2)

with initial conditions $\hat{x}(t_k + \tau) = x(t_k + \tau)$.

Rewriting in terms of x and e, that is, of the vector z:

$$\dot{z}(t) = \begin{bmatrix} \dot{x}(t) \\ \dot{e}(t) \end{bmatrix} = \begin{bmatrix} A + BK & -BK \\ \tilde{A} + \tilde{B}K & \hat{A} - \tilde{B}K \end{bmatrix} \begin{bmatrix} x(t) \\ e(t) \end{bmatrix}$$
$$z(t_k + \tau) = \begin{bmatrix} x(t_k + \tau) \\ e(t_k + \tau) \end{bmatrix} = \begin{bmatrix} x(t_k + \tau^{-1}) \\ 0 \end{bmatrix}$$

for all $t \in [t_k + \tau, t_{k+1})$.

Thus, we have

$$\dot{z} = \Lambda_o z$$
, where $\Lambda_o = \begin{bmatrix} A + BK & -BK \\ \tilde{A} + \tilde{B}K & \hat{A} - \tilde{B}K \end{bmatrix}$ (3)

for all $t \in [t_k + \tau, t_{k+1})$.

For the closed loop case, that is for $t \in [t_k, t_k + \tau)$, similarly we obtain

$$\dot{z} = \Lambda_c z$$
, where $\Lambda_c = \begin{bmatrix} A + BK & -BK \\ 0 & 0 \end{bmatrix}$ (4)

for $t \in [t_k, t_k + \tau)$. This is due to the error being zero, while the state progresses in the same way as before.

From this, it should be quite clear that given an initial condition $z(t = 0) = z_0$, then at time $t \in [0, \tau)$, the solution of the trajectory of the vector is given by

$$z(t) = e^{\Lambda_c(t)} z_0, \ t \in [0, \tau).$$
(5)

In particular, at time τ , $z(\tau) = e^{\Lambda_c(\tau)} z_0$.

Once the loop is opened, the open loop behavior takes over, so that

$$z(t) = e^{\Lambda_o(t-\tau)} z(\tau) = e^{\Lambda_o(t-\tau)} e^{\Lambda_c(\tau)} z_0, \ t \in [\tau, t_1).$$
(6)

In particular, when the time comes to close the loop again, that is, after time h, then $z(t_1^-) = e^{\Lambda_c(h-\tau)} e^{\Lambda_c(\tau)} z_0$.

Notice, however, that at this instant when we close the loop again, we are also resetting the error to zero, so that we must pre-multiply by $\begin{bmatrix} I & 0 \\ 0 & 0 \end{bmatrix}$ before we analyze the closed loop trajectory for the next cycle. Because we wish to always start with an error that is set to zero, we should actually multiply by $\begin{bmatrix} I & 0 \\ 0 & 0 \end{bmatrix}$ at the beginning of each cycle.

So after k cycles, this analysis yields the solution

$$z(t_k) = \left(\begin{bmatrix} I & 0 \\ 0 & 0 \end{bmatrix} e^{\Lambda_o(h-\tau)} e^{\Lambda_c(\tau)} \begin{bmatrix} I & 0 \\ 0 & 0 \end{bmatrix} \right)^k z_0$$

= $\Sigma^k z_0$,
where $\Sigma = \begin{bmatrix} I & 0 \\ 0 & 0 \end{bmatrix} e^{\Lambda_o(h-\tau)} e^{\Lambda_c(\tau)} \begin{bmatrix} I & 0 \\ 0 & 0 \end{bmatrix}$.
The final step is to consider the last (partial) cycle

The final step is to consider the last (partial) cycle that the system goes through, that is, the time $t \in [t_k, t_{k+1})$. If the system is in closed loop, that is, $t \in [t_k, t_k + \tau)$, then the solution can be achieved merely by pre-multiplying $z(t_k)$ by $e^{\Lambda_c(t-t_k)}$. In the case of the system being in open loop, that is, $t \in [t_k + \tau, t_{k+1})$, then clearly we must pre-multiply by $e^{\Lambda_o(t-(t_k+\tau))}e^{\Lambda_c(\tau)}$.

The results can thus be summarized in the following proposition.

Proposition 1: The system described by (3) and (4) with initial conditions $z(t_0) = \begin{bmatrix} x(t_0) \\ 0 \end{bmatrix} = z_0$ has the following response:

$$z(t) = \begin{cases} e^{\Lambda_{c}(t-t_{k})} \left(\begin{bmatrix} I & 0 \\ 0 & 0 \end{bmatrix} \Sigma \begin{bmatrix} I & 0 \\ 0 & 0 \end{bmatrix} \right)^{k} z_{0} \\ \text{for } t \in [t_{k}, t_{k} + \tau) \end{cases}$$
$$e^{\Lambda_{o}(t-(t_{k}+\tau))} e^{\Lambda_{c}(\tau)} \left(\begin{bmatrix} I & 0 \\ 0 & 0 \end{bmatrix} \Sigma \begin{bmatrix} I & 0 \\ 0 & 0 \end{bmatrix} \right)^{k} z_{0} \\ \text{for } t \in [t_{k} + \tau, t_{k+1}) \end{cases}$$

where

$$\Sigma = e^{\Lambda_o(h- au)} e^{\Lambda_c(au)}, \ \Lambda_o = \left[egin{array}{c} A+BK & -BK \ ilde{A}+ ilde{B}K & \hat{A}- ilde{B}K \end{array}
ight], \ \Lambda_c = \left[egin{array}{c} A+BK & -BK \ 0 & 0 \end{array}
ight],$$

and $t_{k+1} - t_k = h$.

In the next section we will present a necessary and sufficient condition for the stability of the system.

IV. STABILITY CONDITION

We now present a necessary and sufficient condition for the stability of the model-based networked control system with intermittent feedback. We use the following definition for global exponential stability. [1]

Definition 2: The equilibrium z = 0 of a system described by $\dot{z} = f(t, z)$ with initial condition $z(t_0) = z_0$ is exponentially stable at large (or globally) if there exists $\alpha > 0$ and for any $\beta > 0$, there exists $k(\beta) > 0$ such that the solution

$$\|\phi(t, t_0, z_0)\| \le k(\beta) \|z_0\| e^{-\alpha(t-t_0)}, \,\forall t \ge t_0$$
(7)

whenever $||z_0|| < \beta$.

With this definition of stability, we state the following theorem characterizing the necessary and sufficient conditions for the system described in the previous section to have globally exponential stability around the solution z = 0. The norm used here is the 2-norm, but any other consistent norm can also be used.

Theorem 3: The system described by Equation (3)-(4) is globally exponentially stable around the origin if and only if the eigenvalues of $\begin{bmatrix} I & 0 \\ 0 & 0 \end{bmatrix} \Sigma \begin{bmatrix} I & 0 \\ 0 & 0 \end{bmatrix}$ are strictly inside the unit circle, where $\Sigma = e^{\Lambda_o(h-\tau)}e^{\Lambda_c(\tau)}$.

Proof: Sufficiency. Taking the norm of the solution described in Proposition 1:

$$\begin{aligned} \|z(t)\| \\ &= \left\| e^{\Lambda_{c}(t-t_{k})} \left(\begin{bmatrix} I & 0 \\ 0 & 0 \end{bmatrix} \Sigma \begin{bmatrix} I & 0 \\ 0 & 0 \end{bmatrix} \right)^{k} z_{0} \right\| \\ &\leq \|e^{\Lambda_{c}(t-t_{k})}\| \left\| \left(\begin{bmatrix} I & 0 \\ 0 & 0 \end{bmatrix} \Sigma \begin{bmatrix} I & 0 \\ 0 & 0 \end{bmatrix} \right)^{k} \right\| \|z_{0}\| \qquad (8) \end{aligned}$$

Notice we are only doing this part for the case when $t \in [t_k, t_k + \tau)$, but the process is exactly the same for the intervals where $t \in [t_k + \tau, t_k + 1)$. Analyzing the first term on the right hand side:

$$\left\| e^{\Lambda_c(t-t_k)} \right\| \leq 1 + (t-t_k) \,\bar{\sigma} \left(\Lambda_c\right) + \frac{(t-t_k)^2}{2!} \cdots$$
$$= e^{\bar{\sigma}(\Lambda_c)(t-t_k)} \leq e^{\bar{\sigma}(\Lambda_c)(\tau)} = K_1$$
(9)

where $\bar{\sigma}(\Lambda_c)$ is the largest singular value of Λ_c . In general this term can always be bounded as the time difference $t - t_k$ is always smaller than τ . That is, even when Λ_c has eigenvalues with positive real parts, $\|e^{\Lambda_c(t-t_k)}\|$ can only grow a certain amount. This growth is completely independent of k.

We now study the term

$$\left(\left[\begin{array}{cc} I & 0 \\ 0 & 0 \end{array} \right] e^{\Lambda_o(h-\tau)} e^{\Lambda_c(\tau)} \left[\begin{array}{cc} I & 0 \\ 0 & 0 \end{array} \right] \right)^k \right\|.$$

It is clear that this term will be bounded if and only if the eigenvalues of $\begin{bmatrix} I & 0 \\ 0 & 0 \end{bmatrix} e^{\Lambda_o(h-\tau)} e^{\Lambda_c(\tau)} \begin{bmatrix} I & 0 \\ 0 & 0 \end{bmatrix}$ lie inside the unit circle:

$$\left\| \left(\begin{bmatrix} I & 0 \\ 0 & 0 \end{bmatrix} e^{\Lambda_o(h-\tau)} e^{\Lambda_c(\tau)} \begin{bmatrix} I & 0 \\ 0 & 0 \end{bmatrix} \right)^k \right\| \le K_2 e^{-\alpha_1 k} \quad (10)$$

with $K_2, \alpha_1 > 0$.

Since k is a function of time we can bound the right term of the previous inequality in terms of t:

$$K_2 e^{-\alpha_1 k} < K_2 e^{-\alpha_1 \frac{t-1}{h}} = K_2 e^{\frac{\alpha_1}{h}} e^{-\frac{\alpha_1}{h}t} = K_3 e^{-\alpha t}$$
(11)

with $K_{3,\alpha} > 0$.

So from (8), using (9) and (11) we conclude that:

$$\begin{aligned} &\|z(t)\| \\ &= \left\| e^{\Lambda_c(t-t_k)} \left(\left[\begin{array}{cc} I & 0 \\ 0 & 0 \end{array} \right] e^{\Lambda_o(h-\tau)} e^{\Lambda_c(\tau)} \left[\begin{array}{cc} I & 0 \\ 0 & 0 \end{array} \right] \right)^k z_0 \right\| \\ &\leq K_1 K_3 e^{-\alpha t} \|z_0\| . \end{aligned}$$

Necessity. We will now provide the necessity part of the theorem. We will do this by contradiction. Assume the system is stable and that $\begin{bmatrix} I & 0 \\ 0 & 0 \end{bmatrix} e^{\Lambda_o(h-\tau)} e^{\Lambda_c(\tau)} \begin{bmatrix} I & 0 \\ 0 & 0 \end{bmatrix}$ has at least one eigenvalue outside the unit circle. Let us

define $\Sigma(h) = e^{\Lambda_o(h-\tau)}e^{\Lambda_c(\tau)}$. Since the system is stable, a periodic sample of the response should converge to zero with time. We will take the samples at times t_{k+1}^- , that is, just before the loop is closed again. We will concentrate on a specific term: the state of the plant $x(t_{k+1}^-)$, which is the first element of $z(t_{k+1}^-)$. We will call $x(t_{k+1}^-)$, $\xi(k)$.

Now assume $\Sigma(\eta)$ has the following form:

$$\Sigma(oldsymbol{\eta}) = \left[egin{array}{cc} W(oldsymbol{\eta}) & X(oldsymbol{\eta})\ Y(oldsymbol{\eta}) & Z(oldsymbol{\eta}) \end{array}
ight]$$

Then we can express the solution z(t) as:

$$e^{\Lambda_{c}(t-t_{k})} \left(\begin{bmatrix} I & 0 \\ 0 & 0 \end{bmatrix} \Sigma(h) \begin{bmatrix} I & 0 \\ 0 & 0 \end{bmatrix} \right)^{k} z_{0}$$
(12)
$$= \begin{bmatrix} W(t-t_{k}) & X(t-t_{k}) \\ Y(t-t_{k}) & Z(t-t_{k}) \end{bmatrix} \begin{bmatrix} (W(h))^{k} & 0 \\ 0 & 0 \end{bmatrix} z_{0}$$

$$= \begin{bmatrix} W(t-t_{k}) (W(h))^{k} & 0 \\ Y(t-t_{k}) (W(h))^{k} & 0 \end{bmatrix} z_{0}.$$

Now, the values of the solution at times t_{k+1}^- , that is, just before the loop is closed again, are

$$\begin{aligned} z(t_{k+1}^{-}) &= & \left[\begin{array}{c} W(h) \left(W(h) \right)^{k} & 0 \\ Y(h) \left(W(h) \right)^{k} & 0 \end{array} \right] z_{0} \\ &= & \left[\begin{array}{c} (W(h))^{k+1} & 0 \\ Y(h) \left(W(h) \right)^{k} & 0 \end{array} \right] z_{0} \end{aligned}$$

We also know that

$$\left[\begin{array}{cc}I&0\\0&0\end{array}\right]e^{\Lambda_o(h-\tau)}e^{\Lambda_c(\tau)}\left[\begin{array}{cc}I&0\\0&0\end{array}\right]$$

has at least eigenvalue outside the unit circle, which means that those unstable eigenvalues must be in $W(\tau)$. This means that the first element of $z(t_{k+1}^-)$, which we call $\xi(k)$, will in general grow with k, if one select the initial condition z(0)along the direction of the eigenvector of the corresponding unstable eigenvalue. In other words we cannot ensure $\xi(k)$ will converge to zero for general initial condition x_0 .

$$\|x(t_{k+1}^{-})\| = \|\xi(k)\| = \|(W(h))^{k+1}x_0\| \to \infty$$
 (13)

as $k \to \infty$, which clearly means the system cannot be stable. Thus, we have a contradiction.

V. EXAMPLE

We ran simulations to verify the results suggested by the theory, which, in itself, is highly intuitive. Naturally, one would think that by using intermittent feedback as opposed to instantaneous closed loop control, there will be many things that will be gained in controlling the system.

Indeed, one way to look at this, focusing in particular on the stability conditions derived above, is the following. Consider a control system with a certain plant model, then calculate the eigenvalues of the test matrix as h varies. This curve is very useful in that the stability of the system is determined by the maximum eigenvalue of its corresponding test matrix. So, by observing at which value of h the



Fig. 2. Maximum eigenvalue for traditional MB-NCS.

curve takes a maximum eigenvalue of 1, we are actually determining the range of h for stability.

The following figures illustrate the previous point. In every case, we took $A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$, $B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$, K = [-1, -1.5]. For our model we used $\hat{A} = \begin{bmatrix} 0 & 0.5 \\ 0 & 0 \end{bmatrix}$, $\hat{B} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$.

Figure 2 shows the behavior of the test matrix under traditional instantaneous feedback model-based control, and Figures 3-7 plot the same data for the case of of intermittent feedback.

Figure 3 shows the case where $\tau = 0.1h$. As we can see, the use of intermittent feedback is already extending the maximum *h* for stability from approximately 1.15 sec to 1.35 sec, in spite of the still relatively small τ .

As we increase the percentage of the time that the loop remains closed, the benefits become more obvious and dramatic. Figures 4 and 5 illustrate the cases where $\tau = 0.2h$ and $\tau = 0.3h$, respectively. In the latter case, the maximum h has been extended to 2.1 seconds, which represents an increase of over 80%.

Figure 6 depicts the case where $\tau = 0.5h$, that is, the time the system runs open loop is equal to the time it runs closed loop. At this stage, the benefits are incremented considerably, with the maximum *h* being extended to 5.0 seconds, which corresponds to a 334% percent increase.

Finally, in Figure 7, we can see what happens for $\tau = 0.7h$. The maximum *h* for stability in this case is of about 9.4 seconds, which corresponds to an increase of over 700% when compared to the traditional setup.

As we can see, using intermittent feedback provides valuable benefits, by dramatically increasing the sampling time h required for the system to remain stable. This naturally coincides with what our intuition suggested, as well as with the theoretical results developed in this paper.



Fig. 3. Maximum eigenvalue MB-NCS with Intermittent Feedback, $\tau = 0.1h$.



Fig. 4. Maximum eigenvalue MB-NCS with Intermittent Feedback, $\tau = 0.2h$.

VI. CONCLUSIONS AND FUTURE WORKS

A. Conclusions

We have introduced the concept of intermittent feedback for model-based networked control systems, provided the full description of the output of the system, as well as a necessary and sufficient condition for global exponential stability. We have also confirmed through examples the intuitive idea that intermittent feedback provides natural advantages over traditional model-based control in terms of stability.

The concept of intermittent feedback, while very intuitive, opens a wide variety of promising research paths which may lead to improved control of networked control systems and could be highly useful in practical applications. The work presented in this paper is but the first step in exploring the advantages that intermittent feedback may yield.



Fig. 5. Maximum eigenvalue MB-NCS with Intermittent Feedback, $\tau = 0.3h$.



Fig. 6. Maximum eigenvalue MB-NCS with Intermittent Feedback, $\tau = 0.5h$.

B. Future Works

For purposes of the current paper, we have begun the study of the case of non-instantaneous closed-loop times by restricting ourselves to fixed intervals τ and h. However, it would be very useful to extend the results for the cases where these values are not constant, that is, where the loop is closed at irregular intervals and remains closed for irregular intervals as well. Developing results for these cases would be especially important for practical applications, in which the attention the system must give to a certain control task is generally not periodic, but depends on external circumstances. The cases where h and τ behave according to certain probability distributions will be studied, and ultimately we hope to derive methods to develop scheduling policies for different goals. We propose to study these cases in a hybrid/switched system framework.



Fig. 7. Maximum eigenvalue MB-NCS with Intermittent Feedback, $\tau = 0.7h$.

Another natural extension of this work is to consider the case where the full state of the plant is not readily available, but there is a state observer involved, as well as cases where there is a delay in the network.

Also, while we have already introduced the advantages of intermittent feedback in terms of stability, we also plan to investigate the advantages that it brings in terms of performance.

Finally, we will seek to improve control of a networked control system by taking advantage of this intermittent feedback concept, by updating the model during the times when the system is running closed loop. Employing techniques like adaptation and model predictive control will enable the model to improve as time progresses, which should in turn enable the user to run the system closed loop for progressively shorter intervals.

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