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Chapter 15: Decentralized Formation Tracking of Multi-Vehicle Systems with Consensus-Based Controllers[1]

In the problem of formation tracking, multiple unmanned vehicles are required to follow spatial trajectories while keeping a desired inter-vehicle formation pattern in time. This Chapter considers vehicles with nonlinear dynamics that follow very general trajectories generated by some reference vehicles. Formations are specified using vectors of relative positions of neighboring vehicles and using consensus-based controllers in the context of decentralized formation tracking control. The key idea is to combine consensus-based controllers with the cascaded approach to tracking control, resulting in a group of linearly coupled dynamical systems. Two types of tracking controllers are proposed under different information flow topologies. Their stability properties are examined by using nonlinear synchronization theory. Simulation results are presented to illustrate the proposed method. The major advantage of the approach is that it is applicable to both unmanned ground vehicles, as well as aerial vehicles flying at a certain altitude. As such, the Chapter refers to 'unmanned mobile vehicles' in general.

15.1 Introduction

Control problems involving unmanned mobile vehicles have attracted considerable attention in the control community during the past decade. One of the basic motion tasks assigned to a mobile vehicle may be formulated as following a given trajectory [13] [25]. The trajectory tracking problem was globally solved in [20] by using a time-varying continuous feedback

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law, and in [2] [12] [16] through the use of dynamic feedback linearization. The backstepping technique for trajectory tracking of nonholonomic systems in chained form was developed in [6] [10]. In the special case when the vehicle model has a *cascaded structure*, the higher dimensional problem can be decomposed into several lower dimensional problems that are easier to solve [17].

An extension to the traditional trajectory tracking problem is that of *coordinated tracking* or *formation tracking* as shown in Figure 15.1. The problem is often formulated as to find a coordinated control scheme for multiple unmanned vehicles that forces them to maintain some given, possibly time-varying, formation while executing a given task as a group. The possible tasks could range from exploration of unknown environments where an increase in numbers could potentially reduce the exploration time, navigation in hostile environments where multiple vehicles make the system redundant and thus robust, to coordinated path following. Detailed information may be found in recent survey papers [1] [21].

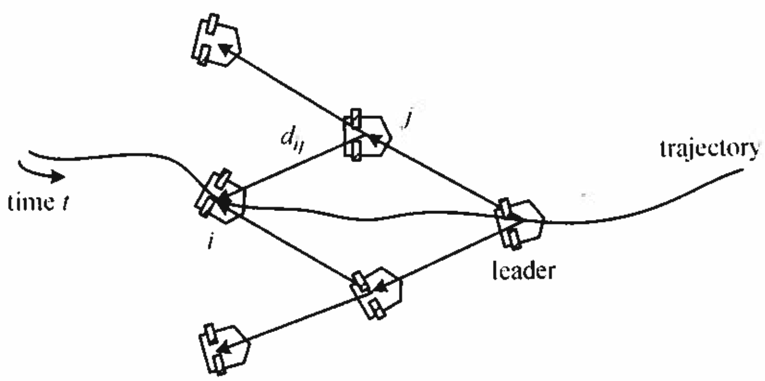

Fig. 15.1. Six unmanned vehicles perform a formation tracking task.

In formation control of multi-vehicle systems, different control topologies can be adopted depending on applications. There may be one or more leaders in the group, with other vehicles following one or more leaders in a specified way. In many scenarios, vehicles have limited communication ability. Since global information is often not available to each vehicle, distributed controllers using only local information are desirable. One approach to distributed formation control is to represent formations using the vectors of relative positions of neighboring vehicles and the use of consensus-based controllers with input bias [3] [11].

In this Chapter, the formation tracking problem for a group of vehicles is studied using the consensus-based controllers combined with the cascade approach [17]. The idea is to specify a reference path for a given, nonphysical point. Then a multiple vehicle formation, defined with respect to the real vehicles as well as to the nonphysical virtual leader, should be maintained at the same time as the virtual leader tracks its reference trajectory. The vehicles exchange information according to a communication digraph, G. Similar to the tracking controller in [17], the controller for each vehicle can be decomposed to two 'sub-controllers', one for positioning and one for orientation. Different from the traditional single vehicle tracking case, each vehicle uses information from its neighbors in the communication digraph to determine the reference velocities and stay at their designation in the formation. Based on nonlinear synchronization results [27], it is proven that consensus-based formation tracking can be achieved as long as the formation graph had a spanning tree and the controller parameters are large enough; they can be lower-bounded by a quantity determined by the formation graph.

Related work includes [4] [5] [9] [19] [22]. In [9], the vehicle dynamics were assumed to be linear and formation control design was based on algebraic graph theory. In [19], output feedback linearization control was combined with a second-order (linear) consensus controller to coordinate the movement of multiple mobile vehicles. The problem of vehicles moving in a formation along constant or periodic trajectories was formulated as a nonlinear output regulation (servomechanism) problem in [4]. The solutions adopted in [5] [22] for coordinated path following control of multiple marine vessels or wheeled vehicles built on Lyapunov techniques, where path following and inter-vehicle coordination were decoupled. Detailed information on consensus problems in networked systems may be found in [15] [18].

The proposed approach offers two key contributions: i) The consensus-based formation tracking controller for nonlinear vehicles is novel and its stability properties are examined using cascaded systems theory and nonlinear synchronization theory; ii) Global results allow one to consider a large class of trajectories with arbitrary (rigid) formation patterns and initial conditions.

Further, a novelty of this research that should not be overlooked is that the formation tracking in a 2-D setting studied in this Chapter includes hovercraft coordinating on a flat surface [7] or UAV flying at a constant altitude. Thus, the methodology proposed is easily extended and applied to UAV formation tracking in more general settings.

15.2 Preliminaries

15.2.1 Tracking Control of Unmanned Mobile Vehicles

A kinematics model of a hovercraft with two degrees of freedom is given by the following equations:

$$\dot{x} = v\cos\theta,\ \dot{y} = v\sin\theta,\ \dot{\theta} = \omega \tag{15.1}$$

where the forward velocity v and the angular velocity ω are considered as inputs, (x, y) is the center of the rear axis of the vehicle, and θ is the angle between heading direction and x-axis as shown in Figure 15.2.

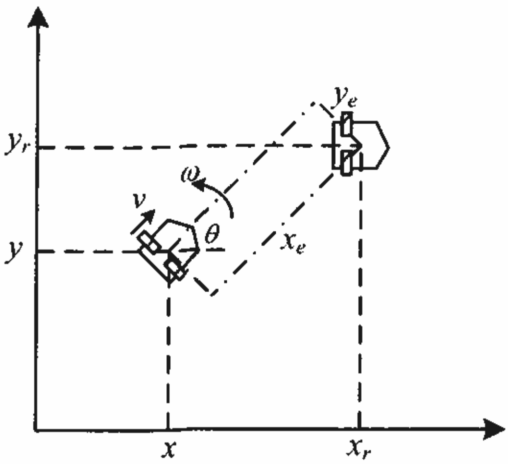

Fig. 15.2. Mobile hovercrafts and the error dynamics.

For time-varying reference trajectory tracking, the reference trajectory must be selected to satisfy the nonholonomic constraint. The reference trajectory is hence generated using a virtual reference hovercraft [8] which moves according to the model:

$$\dot{x}_r = v_r\cos\theta_r,\ \dot{y}_r = v_r\sin\theta_r,\ \dot{\theta}_r = \omega_r \tag{15.2}$$

where $[x_r\ y_r\ \theta_r]$ is the reference posture obtained from the virtual vehicle. Following [8] the error coordinates are defined as (Figure 15.2):

$$p_e = \begin{bmatrix} x_e \\ y_e \\ \theta_e \end{bmatrix} = \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_r - x \\ y_r - y \\ \theta_r - \theta \end{bmatrix} \tag{15.3}$$

It can be verified that in these coordinates the error dynamics become:

$$\dot{p}_e = \begin{bmatrix} \dot{x}_e \\ \dot{y}_e \\ \dot{\theta}_e \end{bmatrix} = \begin{bmatrix} \omega y_e - v + v_r \cos\theta_e \\ -\omega x_e + v_r \sin\theta_e \\ \omega_r - \omega \end{bmatrix}. \tag{15.4}$$

The aim of (single hovercraft) trajectory tracking is to find appropriate velocity control laws v and ω of the form:

$$\begin{aligned} v &= v(t, x_e, y_e, \theta_e) \\ \omega &= \omega(t, x_e, y_e, \theta_e) \end{aligned} \tag{15.5}$$

such that the closed-loop trajectories of (15.4) and (15.5) are stable in some sense (e.g., uniform globally asymptotically stable). As discussed in Section 15.1, there are numerous solutions to this problem in the continuous time domain. Here, the cascaded approach proposed in [17] is revisited. As a starting point, the notion of globally K-exponential stability is introduced.

Definition 15.1: A continuous function α: $[0, a) \rightarrow [0, \infty)$ is said to belong to class K if it is strictly increasing and $\alpha(0) = 0$.

Definition 15.2: A continuous function β: $[0, a)\times[0,\infty\ \infty) \rightarrow [0,\infty\ \infty)$ is said to belong to class KL if for each fixed s the mapping $\beta(r, s)$ belongs to class K with respect to r, and for each fixed r the mapping $\beta(r, s)$ is decreasing with respect to s and $\beta(r, s) \rightarrow 0$ as $s \rightarrow \infty$.

Definition 15.3: Consider the system:

$$\dot{x} = g(t, x),\ g(t, 0) = 0\ \forall t \geq 0 \tag{15.6}$$

where $g(t, x)$ is piecewise continuous in t and locally Lipschitz in x.

The system (15.6) is called *globally K-exponentially stable* if there exist $\xi > 0$ and a class K function $k(\cdot)$ such that:

$$\|x(t)\| \leq k(\|x(t)\|) e^{-\xi(t-t_0)}.$$

Theorem 15.1 ([17]): Consider the system (15.4) in closed-loop with the controller:

$$\begin{aligned} v &= v_r + c_2 x_e, \\ \omega &= \omega_r + c_1 \theta_e, \end{aligned} \tag{15.7}$$

where $c_1 > 0$ $c_2 > 0$. If $\omega_r(t)$, $\dot{\omega}_r(t)$, and $v_r(t)$ are bounded and there exist δ and k such that:

$$\int_t^{t+\delta} \omega_r(\tau)^2\, d\tau \geq k,\ \forall t \geq t_0 \tag{15.8}$$

then the closed-loop system (15.4) and (15.7), written compactly as:

$$\dot{p}_e = h(x_e, y_e, \theta_e)\Big|_{v_r,\omega_r} = h(p_e)\Big|_{v_r,\omega_r} \tag{15.9}$$

is globally K-exponentially stable.

In the above, the subscriptions for $h(\cdot)\big|_{v_r,\omega_r}$ mean that the error dynamics are defined relative to reference velocities v_r and ω_r. The tracking condition (15.8) implies that the reference trajectories should not converge to a point (or straight line).

This also relates to the well-known persistence-of-excitation condition in adaptive control theory. Note that control laws in (15.7) are linear with respect to x_e and θ_e. This is critical in designing consensus-based controller for multiple vehicle formation tracking as we shall see below.

15.2.2 Formation Graphs

Formations are considered that can be represented by acyclic directed graphs. In these graphs, the agents involved are identified by vertices and the leader-following relationships by (directed) edges. The orientation of each edge distinguishes the leader from the follower. Follower controllers implement static state feedback-control laws that depend on the state of the particular follower and the states of its leaders.

Definition 15.4 ([24]): A formation control graph $G = (V, E, D)$ is a directed acyclic graph consisting of the following.

- A finite set $V = \{v_1, \ldots, v_N\}$ of N vertices and a map assigning to each vertex a control system $\dot{x}_i = f_i(t, x_i, u_i)$ where $x_i \in \mathrm{R}^n$ and $u_i \in \mathrm{R}^m$.
- An edge set encoding leader-follower relationships between agents. The ordered pair $(v_i, v_j) \equiv e_{ij}$ belongs to E if u_j depends on the state of agent i, x_i.
- A collection $D = \{d_{ij}\}$ of edge specifications, defining control objectives (setpoints) for each j: $(v_i, v_j) \in E$ for some $v_i \in V$.

For agent j, the tails of all incoming edges to vertex represent leaders of j, and their set is denoted by $L_j \subset V$. Formation leaders (vertices of in-degree zero) regulate their behavior so that the formation may achieve some group objectives, such as navigation in obstacle environments or tracking reference paths.

Given a specification d_{kj} on edge $(v_k, v_j) \in E$, a set point for agent j can be expressed as $x_j^r = x_k - d_{kj}$. For agents with multiple leaders, the specification redundancy can be resolved by projecting the incoming edges specifications into orthogonal components:

$$x_j^r = \sum_{k \in L_j} S_{kj} \left(x_k - d_{kj} \right) \tag{15.10}$$

where S_{kj} are projection matrices with $\sum_k \text{rank}(S_{kj}) = n$. Then the error for the closed-loop system of vehicle j is defined to be the deviation from the prescribed set point $\tilde{x}_j = x_j^r - x_j$, and the formation error vector is constructed by stacking the errors of all followers:

$$\tilde{x} \equiv \left[\cdots \tilde{x} \cdots \right]^T, \; v_j \in V \setminus L_F .$$

15.2.3 Synchronization in Networks of Nonlinear Dynamical Systems

Definition 15.5: Given a matrix $V \in R^{n \times n}$, a function $f(y, t) : R^{n+1} \to R^n$ is V-uniformly decreasing if $(y-z)^T V \left(f(y,t) - f(z,t) \right) \le -\mu \|y-z\|^2$ for some $\mu > 0$ and all $y, z \in R^n$ and $t \in R$.

Note that a differentiable function $f(y, t)$ is V-uniformly decreasing if and only if $V(\partial f(y)/\partial y) + \delta I$ for some $\delta > 0$ and all y, t. Consider the following synchronization result for the coupled network of identical dynamical systems with state equations:

$$\dot{x} = \left(f(x_1,t), \ldots, f(x_n,t) \right)^T + \left(C(t) \otimes D(t) \right) x + u(t), \tag{15.11}$$

where $x = (x_1, \ldots, x_N)^T$, $u = (u_1, \ldots, u_N)^T$ and $C(t)$ is a zero sums matrix for each t. $C \otimes D$ is the Kronecker product of matrices C and D.

Theorem 15.2 ([27]): Let $Y(t)$ be an n by n time-varying matrix and V be an n by n symmetric positive definite matrix such that $f(x, t) + Y(t)x$ is V-uniformly decreasing. Then the network of coupled dynamical systems in (11) synchronizes in the sense that $\|x_i - x_j\| \to 0$ as $t \to \infty$ for all i, j if the following two conditions are satisfied:

- $\lim_{t \to \infty} \|u_i - u_j\| = 0$ for all i, j;
- There exists an N by N symmetric irreducible zero row sums matrix U with nonpositive off-diagonal elements such that:

$$\left(U \otimes V \right) \left(C(t) \otimes D(t) - I \otimes Y(t) \right) \le 0 \tag{15.12}$$

for all t.

15.3 Basic Formation Tracking Controller

The control objective is to solve a formation tacking problem for N unmanned vehicles. This implies that each unmanned vehicle must converge to and stay at their designation in the formation while the formation as a whole follows a virtual vehicle. Equipped with the results presented in the previous Section, at first one should construct a basic formation tracking controller (FTC) from (15.7). Let $d_{ri} = [d_{x_{ri}} \;\; d_{y_{ri}}]^T$ denote the formation specification on edge (v_r, v_i). In virtue of linear structures of (15.7), the following basic FTC is proposed for vehicle i:

$$\begin{cases} v_i = v_r + c_2 x_{e_i} \\ \omega_i = \omega_r + c_1 \theta_{e_i} \end{cases} \tag{15.13}$$

where $c_1 > 0$, $c_2 > 0$ and:

$$p_{e_i} = [x_{e_i} \;\; y_{e_i} \;\; \theta_{e_i}]^T = \begin{bmatrix} \cos\theta_i & \sin\theta_i & 0 \\ -\sin\theta_i & \cos\theta_i & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_r - x_i - d_{x_{ri}} \\ y_r - y_i - d_{y_{ri}} \\ \theta_r - \theta_i \end{bmatrix} \tag{15.14}$$

Remark 15.1: It is not required to have constraints for every pair of vehicles. We need only a sufficient number of constraints which uniquely determine the formation.

Theorem 15 3: The basic FTC (15.13) and (15.14) solves the formation tracking problem.

Proof: By Theorem 15.1, every vehicle i follows the virtual (or leader) vehicle, thus the desired trajectory, with a formation constraint d_{ri} on edge (v_r, v_i). Therefore, all vehicles track the reference trajectory while staying in formation, which is specified by formation constraints d_{ri}'s as shown in Figure 15.3.

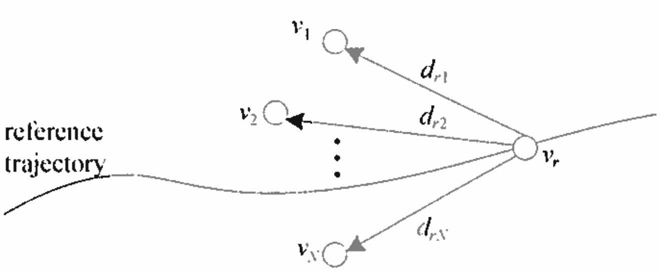

Fig. 15.3. Illustration of formation tracking using baseline FTC. The reference vehicle sends to vehicle i the formation specification d_{ri} as well as the reference velocities v_r and ω_r.

Corollary 15.1: Suppose only vehicle 1 follows the virtual vehicle. The composite system with inputs v_r and ω_r and states $\tilde{x}_1 = [x_{e_1}\ y_{e_1}\ \theta_{e_1}]$ is globally K-exponentially stable and therefore formation input-to-state stable (see Section 15.4).

Example 15.1-Basic FTC: Consider a system consisting of three vehicles, which are required to move in some predefined formation pattern. First, as in [4], consider the case of moving in a triangle formation along a circle. That is, the virtual (or reference) vehicle dynamics are given by:

$$x_r = v_r \cos(\omega r t) + x_{r0},\ y_r = v_r \sin(\omega r t) + y_{r0}$$

where v_r is the reference forward velocity, ω_r the reference angular velocity, and $[x_{r0}\ y_{r0}]^T$ the initial offsets.

Assume that that parameters have the following values: $v_r = 10$, $\omega_r = 0.2$, $[x_{r0}\ y_{r0}]^T = [-25\ 0]^T$. For simulation purposes, an isosceles right triangle was used with sides equal to $3\sqrt{2}$, $3\sqrt{2}$, and 6. Also fixed was the position of the virtual leader at the vertex with the right angle. Then, from the above constraints the required (fixed) formation specifications for the vehicles are given by $d_{r1} = [0\ \ 0]^T$, $d_{r2} = [3\ \ 3]^T$, $d_{r3} = [3\ \ -3]^T$.

For the basic FTC parameters were chosen as $c_1 = 0.3$ and $c_2 = 0.5$. Figure 15.4 shows the trajectories of the system for about 100 seconds. Initially the vehicles are not in the required formation; however, they form the formation quite fast (K-exponentially fast) while following the reference trajectory (solid line in the figure). Figure 15.5 shows the control signals v and ω for each vehicle.

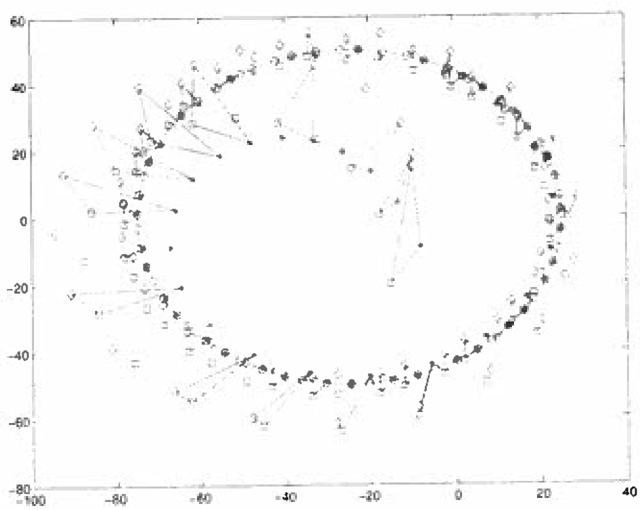

Fig. 15.4. Circular motion of three vehicles with a triangle formation. Initial vehicle postures are: $[-8\ -9\ 3\pi/5]^T$ for vehicle 1 (denoted as *); $[-15\ -20\ \pi/2]^T$ for vehicle 2 (□ - square); $[-10\ -15\ \pi/3]^T$ for vehicle 3 (◊ - diamond).

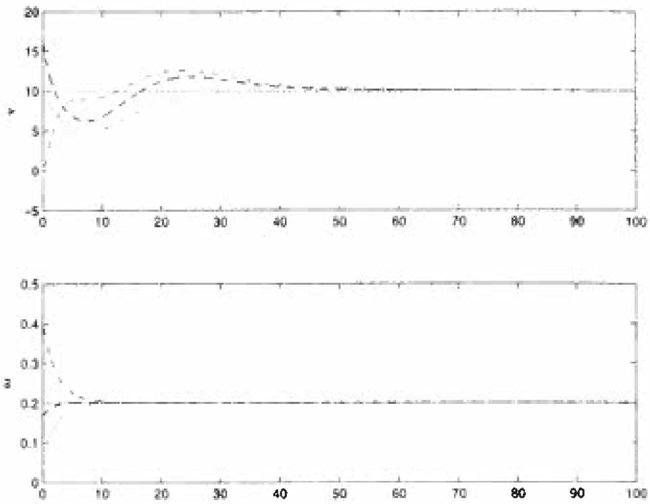

Fig. 15.5. Control signals v and ω for virtual vehicle: solid line; vehicle 1: dotted line; vehicle 2: dashed line; and vehicle 3: dot-dash line.

15.4 Consensus-Based Formation Tracking Controller

The basic FTC has the advantage that it is simple and leads to globally stabilizing controllers. A disadvantage, however, is that it requires every vehicle to get access to the reference velocities v_r and ω_r. This further implies that the reference vehicle needs to establish direct communication links with all other vehicles in the group, which may not be practical in some applications.

In a more general setting, one may assume that only a subset of vehicles (leaders) have direct access to the reference velocities. Other vehicles (followers) use their neighboring leaders' information to accomplish the formation tracking task. In this case, formation tracking controllers operate in a decentralized fashion since only neighboring leaders' information has been used.

Therefore, the consensus-based FTC for vehicle i is defined as follows:

$$\begin{cases} v_i = v_{r_i} + c_2 x_{e_i} + \sum_{j \in L_i} a_{ij}(x_{e_i} - x_{e_j}), \\ \omega_i = \omega_{r_i} + c_1 \theta_{e_i} + \sum_{j \in L_i} a_{ij}(\theta_{e_i} - \theta_{e_j}), \\ \dot{v}_{r_i} = \sum_{j \in L_i} a_{ij}(v_{r_j} - v_{r_i}), \\ \dot{\omega}_{r_i} = \sum_{j \in L_i} a_{ij}(\omega_{r_j} - \omega_{r_i}) \end{cases} \tag{15.15}$$

where:

$$p_{e_i} = \begin{bmatrix} x_{e_i} \\ y_{e_i} \\ \theta_{e_i} \end{bmatrix} = \begin{bmatrix} \cos\theta_i & \sin\theta_i & 0 \\ -\sin\theta_i & \cos\theta_i & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_i^r - x_i \\ y_i^r - y_i \\ \theta_i^r - \theta_i \end{bmatrix}.$$

and a_{ij} represents relative confidence of agent i in the information state of agent j.

Remark 15.2: As one can see from (15.15), the communication between vehicles is local and distributed, in the sense that each vehicle receives the posture and velocity information only from its neighboring leaders.

The following theorem is proven regarding the stability of the consensus-based FTC.

Theorem 15.4: The consensus-based FTC (15.15) solves the formation tracking problem if the formation graph G has a spanning tree and the controller parameters c_1, $c_2 > 0$ are large enough. Lower bounds for c_1 and c_2 are related to the Laplacian matrix for G.

Proof: Let L_G be the Laplacian matrix induced by the formation graph G and it is defined by:

$$(L_G)_{ij} = \begin{cases} \sum_{k=1,k\neq i}^{N} a_{ik}, & j = i \\ -a_{ij}, & j \neq i \end{cases}$$

with $P_e = [p_{e_1}, \ldots, p_{e_N}]^T \in R^{3N}$, $[V_r \quad \Omega_r]^T = [v_{r_1}, \ldots, v_{r_N}, \omega_{r_1}, \ldots, \omega_{r_N}]^T \in R^{2N}$. The closed loop system (15.15) - (15.4) for all vehicles can be expressed in a compact form as:

$$\dot{P}_e = \begin{bmatrix} h(p_{e_1})\big|v_{r_1}, \omega_{r_1} \\ \vdots \\ h(p_{e_N})\big|v_{r_N}, \omega_{r_N} \end{bmatrix} + (-L_G \otimes D)P_e, \tag{15.16}$$

$$\begin{bmatrix} \dot{V}_r \\ \dot{\Omega}_r \end{bmatrix} = (-L_G \otimes I_2)\begin{bmatrix} V_r \\ \Omega_r \end{bmatrix}, \tag{15.17}$$

where:

$$D = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{bmatrix} \tag{15.18}$$

describes the specific coupling between two vehicles.

It can be seen that (15.17) is in the form of linear consensus algorithms. Since the formation graph has a rooted spanning tree (with the root corresponding to the virtual vehicle), the reference velocities (coordination variables) $v_{r_i}(t)$ and $\omega_{r_i}(t)$ for any vehicle i in the group will approach $v_r(t)$ and $\omega_r(t)$, respectively, but with bounded tracking errors [14]. For an easy exposition, one may consider the tracking errors to be zero in this proof, and defer the discussion of its implication to the end of this Section.

Therefore, (15.16) may be re-written as:

$$\dot{P}_e = \begin{bmatrix} h(p_{e_1})|v_r, \omega_r \\ \vdots \\ h(p_{e_N})|v_r, \omega_r \end{bmatrix} + (-L_G \otimes D)P_e + \begin{bmatrix} \phi_1(t) \\ \vdots \\ \phi_N(t) \end{bmatrix} \tag{15.19}$$

and $\phi_i(t) \to 0$ as $t \to \infty$. The functions ϕ_i can be considered as residual errors that occurred when replacing v_{r_i} and ω_{r_i} in (15.16) with v_r and ω_r, respectively. Now (15.19) is in the same form with (15.12). Further, set $Y = \alpha D$ so that $h(p_e) + \alpha D p_e$ is V-uniformly decreasing (see Lemma 11 in [26]) provided that $c_1 - \alpha > 0$ and $c_2 - \alpha > 0$. Theorem 15.2 states that (15.19) synchronizes if there exists a symmetric zero row sums matrix U with non-positive off-diagonal elements such that $(U \otimes V)(-L_G \otimes D - I \otimes Y) \le 0$. Since $VD \le 0$ and $Y = \alpha D$, this is equivalent to:

$$U(-L_G - \alpha I) \ge 0\,. \tag{15.20}$$

Let $\mu(-L_G)$ be the supremum of all real numbers such that $U(-L_G - \alpha I) \ge 0$. It was shown in [28] that $\mu(-L_G)$ exists for constant row sum matrices and can be computed by a sequence of semi-definite programming problems. Choose c_1 and c_2 to be large enough such that:

$$\min\{c_1, c_2\} > -\mu(-L_G) \tag{15.21}$$

and the proof is complete.

In particular, an upper bound for $\mu(-L_G)$ is given by $\mu_2(-L_G) = \min Re(\lambda)$ where $Re(\lambda)$ is the real part of λ, the eigenvalues of $-L_G$ that do not correspond to the eigenvector e. It suffices to make $\min\{c_1, c_2\} > \mu_2(-L_G)$.

Example 15.2: In this example, virtual vehicle dynamics are of a sinusoidal form: $(x_r(t), y_r(t)) = (t, \sin(t))$. The acyclic formation graph with formation specifications is shown in Figure 15.6. The (un-weighted) Laplacian matrix that corresponds to Figure 15.6 is given by:

$$L_G = \begin{bmatrix} 1 & -1 & 0 & 0 \\ 0 & 2 & -1 & -1 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 \end{bmatrix}. \tag{15.22}$$

Since $\mu_2(-L_G) = -2$, consensus-based FTC (15.15) was used with positive c_1, c_2, say $c_1 = 0.3$ and $c_2 = 0.5$. As shown in Figure 15.7, successful formation tracking with a desired triangle formation is achieved. Vehicle control signals v_i's and ω_i's are shown in Figure 15.8.

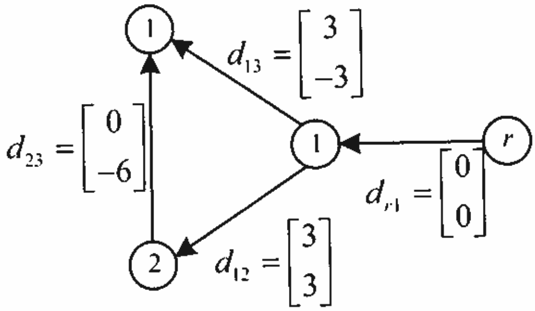

Fig. 15.6. A formation graph with formation specifications on edges: $d_{r1} = [0\ 0]^T$, $d_{12} = [3\ 3]^T$, $d_{13} = [3\ \text{-}3]^T$, $d_{23} = [0\ \text{-}6]^T$.

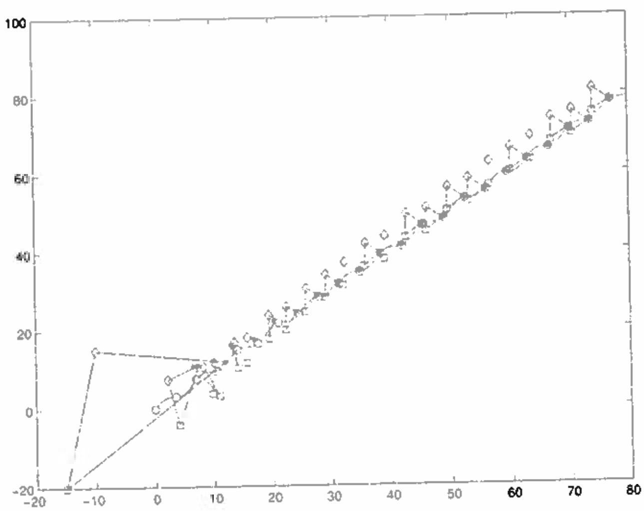

Fig. 15.7. Tracking a sinusoidal trajectory in a triangle formation. Initial vehicle postures are: $[12\ 12\ 0]^T$ for vehicle 1 (denoted as *); $[-15 - 20\ \pi/4]^T$ for vehicle 2 (□ - square); $[-10\ 15 - \pi/4]^T$ for vehicle 3 (◊ - diamond).

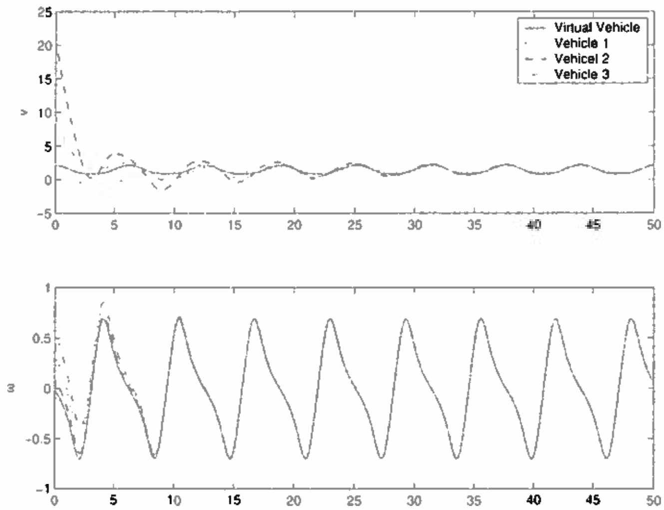

Fig. 15.8. Vehicle control signals v_i's and ω_i's.

15.4.1 Discussions on Formation ISS

In the proof of Theorem 15.4, it was assumed that the reference velocities $v_{r_i}(t)$ and $\omega_{r_i}(t)$ for any vehicle i in the group will eventually approach to $v_r(t)$ and $\omega_r(t)$. In fact, $v_{r_i}(t)$ and $\omega_{r_i}(t)$ cannot always follow time-varying $v_r(t)$ and $\omega_r(t)$ without errors, due to the low-pass nature of all consensus schemes. But the tracking errors between $v_r(t)$ and $v_{r_i}(t)$, $\omega_r(t)$ and $\omega_{r_i}(t)$ are known to be bounded, provided that:

- The formation graph has a spanning tree, and,
- $v_r(t)$ and $\omega_r(t)$ are uniformly bounded rate signals, i.e., $|\dot{v}_r(t)| \leq m_1$ and $|\dot{\omega}_r(t)| \leq m_2$ (see Proposition 2 in [14]).

A question that is raised naturally is the following: Does a variant of Theorem 15.4 hold with $|v_r - v_{r_i}| \leq \varepsilon_1$, $|\omega_r - \omega_{r_i}| \leq \varepsilon_2$, where ε_1 and ε_2 are reference velocities tracking errors? The answer is yes. To state this new result, one must introduce first the concept of leader-to-formation stability (LFS) [23].

Definition 15.6: A formation is called LFS if there exist a class *KL* function β and a class *K* function γ, such that for any initial formation error $\tilde{x}(0)$ and for any bounded inputs of the formation leaders, $\{w_l\}$ the formation error satisfies:

$$\|\tilde{x}(t)\| \leq \beta(\|\tilde{x}(0)\|, t) + \sum_{l \in L_F} \gamma_l (\sup_{0 \leq \tau \leq t} \|w_l(\tau)\|) \tag{15.23}$$

As a variant of Theorem 15.4, the following theorem takes into account the effects of time-varying reference velocities on the formation stability.

Theorem 15.5: Consensus-based FTC (15) results in LFS if the formation graph has a spanning tree and the reference velocities are uniformly bounded rate signals.

Proof: The proof follows from Corollary 15.1 and the invariance property of LFS [23].

15.5 Conclusions and Future Work

This Chapter addressed the formation tracking problem for multiple mobile unmanned vehicles with nonholonomic constraints. A basic formation tracking controller (FTC) was developed as well as a consensus-based one using only neighboring leaders information. The stability properties of the multiple vehicle system in closed-loop with these FTCs were studied using cascaded systems theory and nonlinear synchronization theory. In particular, connections were established between stability of consensus-based FTC and Laplacian matrices for formation graphs. The simple formation tracking strategy holds great potential to be extended to the case of air and marine vehicles.

Collision avoidance and formation error propagation problems were not discussed. The proposed FTC does not guarantee avoidance of collisions and there is a need to consider them in future work. Theorem 15.5 showed that consensus-based FTC leads to LFS. The invariance properties of LFS under cascading could be explored to quantify the formation errors when individual vehicle's tracking errors are bounded. Formation tracking in a higher dimension is another interesting problem for future study.

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