Digital Control Networks for Continuous Passive Plants Which Maintain Stability Using Cooperative Schedulers

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***Abstract*— This paper provides a sufficient framework to synthesize l^2-*stable* networks in which the controller and plant can be subject to delays and data dropouts. This framework can be applied to control systems which use "soft-real-time" cooperative schedulers as well as those which use wired and wireless network feedback. The framework applies to plants and controllers which are *passive*, therefore these *passive* systems can be either linear, nonlinear, and (or) time-varying. This framework arises from fundamental results related to *passive* control, and scattering theory which are used to design *passive* force-feedback telemanipulation systems, in which we provide a short review. Theorem 3 states how a (non)linear (*strictly input* or *strictly output*) passive plant can be transformed to a discrete (*strictly input*) passive plant using a particular digital sampling and hold scheme. Furthermore, Theorem 4(5) provide new sufficient conditions for l^2 (and L^2)-*stability* in which a *strictly-output passive* controller and plant are interconnected with only *wave-variables*. Lemma 2 shows it is sufficient to use discrete *wave-variables* when data is subject to fixed time delays and dropouts in order to maintain *passivity*. Lemma 3 shows how to safely handle time varying discrete *wave-variable* data in order to maintain *passivity*. Based on these new theories, we provide an extensive set of new results as they relate to *LTI* systems. For example, Proposition 2 shows how a *LTI strictly-output passive* observer can be implemented. We then present a new cooperative scheduler algorithm to implement an l^2-*stable* control network. We also provide an illustrative simulated example which uses a *passive* observer followed with a discussion for future research.**

I. Introduction

The primary goal of this research is to develop reliable wireless control networks. These networks typically consist of distributed-wireless sensors, actuators and controllers which communicate with low cost devices such as the MICA2 and MICAz motes [1]. The operating systems for these devices, typically consist of a very simple scheduler, known as a co-operative scheduler [2]. The cooperative scheduler provides a common time-base to schedule tasks to be executed, however, it does not provide a context-switch mechanism to interrupt tasks. Thus, tasks have to cooperate in order not to delay pending tasks, but this cooperative condition is rarely satisfied. As a result, a controller needs to be designed to tolerate time-varying delays which can be incurred from disruptive tasks which share the cooperative scheduler. Although, other operating systems can be designed to provide a more hard real-time scheduling performance, the time varying delays which will ultimately be encountered with wireless sensing and actuation will be comparable if not more significant. Hence, the primary aim of this paper is to provide the theoretical framework to build l^2-*stable* controllers which can be subject to time-varying scheduling delays. Such results are also of importance as they will eventually allow the plant-controller network depicted in Fig. 5 to run entirely isolated from the plant as is done with telemanipulation systems. Telemanipulation systems have had to address wireless control problems [3] years before the MICA2 mote existed and the corresponding literature provides results to address how to design stable control systems subject to transmission delays in such systems. Much of the theory presented in this paper is inspired and related to work related to telemanipulation systems. Thus, our introduction will conclude (Section I-A) with a brief review of telemanipulation, and how it relates to *passive* control and scattering theory in order too provide the reader some physical insight related to the framework presented in Section II.

Telemanipulation systems are distributed control systems in which a human operator controls a local manipulator which commands a remotely located robot in order to modify a remote environment. The position tracking between the human operator and the robot is typically maintained by a passive proportional-derivative controller. In fact, a telemanipulation system typically consists of a series network of interconnected two-port passive systems in which the human operator and environment terminate each end of the network [4]. These passive networks can remain stable in-spite of system uncertainty; however, delays as small as a few milliseconds would cause force feed back telemanipulation systems to become unstable. The instabilities occur because delayed power variables, force (effort) and velocity (flow), make the communication channel non *passive*. In [3] it was shown that by using a scattering transformation of the power variables into power *wave variables* [5] the communication channel would remain passive in spite of arbitrary fixed delays. For continuous systems, if additional information is transmitted along with the continuous *wave variables*, the communication channel will also remain passive in the presence of time varying delays [6]. However, only recently has it been shown how discrete *wave variables* can remain passive in spite of time varying delays and dropouts [7], [8]. We verified this to be true for fixed time delays and data dropouts (Lemma 2). However, we provide a simple counter example that shows this is not the case for all time-varying delays and provide a lemma which states how to properly handle time varying discrete wave variable data and maintain *passivity* (Lemma 3). The initial results from [7] build upon a novel digital sample and hold scheme which allows the discrete inner-product space and continuous inner-product space to be equivalent [9], [10].

We will build on the results in [9] to show in general how to transform a (non)linear (*strictly input* or *strictly output*) *passive* system into a discrete (*strictly input*) *passive* system (Theorem 3). We then formally show some potentially new l^2*-stability* results related to *strictly-output passive* networks. In particular Theorem 2 shows how to make a discrete *passive* plant *strictly-output passive* and l^2*-stable*. Theorem 2 also makes it possible to synthesize discrete *strictly-output passive* systems from discrete *passive LTI* systems such as those consisting of passive wave digital filters [11]. We will then use the scattering transform to interconnect the controller to the plant with *wave variables*. We use Lemma 3 to show that the cooperative scheduler can allow time varying data transmission delays and maintain passivity between the plant and controller. As a result our digital control system implemented with a cooperative scheduler will remain l^2*-stable*. We conclude this introduction with a brief discussion of telemanipulation systems, *passivity* and scattering theory from continuous time and classic control framework. Section II provides the necessary definitions and theorems necessary to present our main results. Section III shows our main results and outlines how to design a driver which allow the digital controller to be implemented as a cooperative task managed by a cooperative scheduler, such as the one provided by *SOS*. Section IV concludes with a simulation implementing the cooperative scheduler to control a passive system. Section V summarizes our key findings and discusses future research directions.

A. PASSIVE SYSTEMS AND TELEMANIPULATION.

Passive systems are an important class of systems for which Lyapunov like functions exist [12]–[15]. The Lyapunov like function arises from the definition of passivity (1). In passive systems (1), the rate of change in stored energy E_{store} is equal to the amount of power put in to the system P_{in} minus the amount of power dissipated P_{diss} which is greater than or equal to zero.

$$\dot{E}_{store} = P_{in} - P_{diss} \tag{1}$$

As long as all internal states x of the system are associated with stored energy in the system, we can show that a passive system is stable when no input power is present simply by setting $P_{in} = 0$. $P_{diss} \geq 0$ implies that $\dot{E}_{store} \leq 0$ which shows the system is Lyapunov stable. By using either the invariant set theorem or Barbalat's Lemma [13] we can prove asymptotic stability [4]. These passive systems can be interconnected in parallel and feed-back configurations and are fundamental components in telemanipulation systems [4]. Instabilities can occur when a telemanipulation system incurs communication

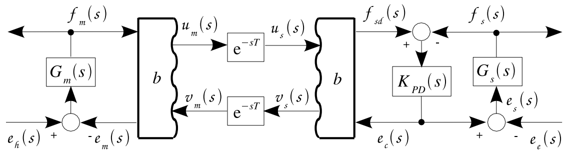

Fig. 1. Telemanipulation system depicted in the s-Domain, subject to communication delays.

delays between the master controller and slave manipulator in which the delays can be as small as a few milliseconds. Instabilities may occur when the communication channel becomes a non-passive element in the telemanipulation system [5]. *Wave variables* are used here to communicate commands and provide feed-back in telemanipulation systems, because they allow the communication channel to remain passive for arbitrarily fixed delays. The variables which traditionally in the past were communicated over a telemanipulation channel were *power variables* such as force and velocity (F,$\dot{x}$). *Power variables*, generally denoted with an *effort* and *flow* pair (e_*,f_*) whose product is power, are typically used to show the exchange of energy between two systems using *bond graphs* [16], [17]. Some other examples of *effort* and *flow* pairs of *power variables* are voltage and current (V,$\dot{q}$), and magneto-motive force and flux rate ($\mathcal{F}$,$\dot{\varphi}$). *Wave variables* are denoted by the following pair of variables (u_*,v_*), the transmission wave impedance $b > 0$ and the channel communication time delay T [5]. The transmission between the master and slave controller (as depicted in Fig. 1 in the s-Domain) are governed by the following delayed equations:

$$u_s(t) = u_m(t - T) \tag{2}$$

$$v_m(t) = v_s(t - T) \tag{3}$$

in which the input waves are computed using

$$u_m(t) = \frac{bf_m(t) + e_m(t)}{\sqrt{2b}} \tag{4}$$

$$v_s(t) = \frac{bf_{sd}(t) - e_c(t)}{\sqrt{2b}} \tag{5}$$

These simple wave variable transformations, which can be applied to vectors, allow us to show that the wave communication channel is both passive and lossless assuming zero initial conditions.

$$E_{store}(t) = \int_0^t P_{in} d\tau = \int_{t-T}^{t} \frac{1}{2}u_m' u_m + \frac{1}{2}v_s' v_s \geq 0 \tag{6}$$

In Fig. 1, the transfer function associated with the master manipulator is denoted $G_m(s)$ and is typically a *passive* mass. Furthermore, the slave manipulator is denoted by the transfer function, $G_s(s)$ and is typically a *passive* mass. The *passive* "proportional-derivative" plant controller $K_{PD}(s)$ has the following form:

$$K_{PD}(s) = \frac{Bs + K}{s} \tag{7}$$

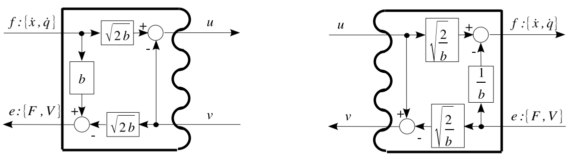

Fig. 2. Block diagrams depicting the wave variable transformation (simplified version of Fig. 3 in [20]).

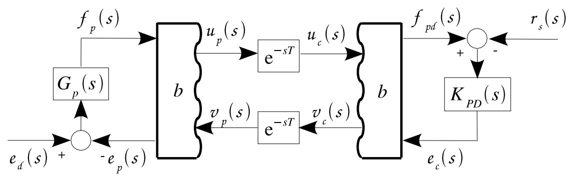

Fig. 3. A delay-insensitive system in which a passive controller commands a passive plant.

The plant controller is "proportional-derivative" in the sense that the integral of the flow variable f_* yields a displacement variable q_* which is then multiplied by a proportional gain K and derivative term B. Since both the plant and controller are zero-state observable, then: when $r_s(s) = e_d(s) = 0$ the system is stable in regards to the plants velocity and the velocity equilibrium point $= 0$ (note that the final position of the plant is dependent on the systems initial condition) [14, Proposition 3.4.1, Remark 3.4.3]. This velocity equilibrium holds in spite of arbitrary fixed delays in the system, if $e_d(s) = 0$ then it can be shown that $K(s)$ is positive real for $\forall b > 0$ (see [18] for explanation of $K(s)$). We may be able to show that the system is L^2 stable when $e_d(s) = 0$ using Theorem 2 in [19]. However, we will show that it is sufficient for $K_{PD}(s)$ and $G_P(s)$ to be modified to be *strictly-output passive* in order to satisfy L^2 stability for $\forall b > 0$ and both $e_d(s)$ and $r_s(s)$ can be signals in L^2. The sufficient proof for both L^2 and l^2 stability is given in Section III. Although the wave variables (u_*,v_*) do not need to be associated with a particular direction as do the power variables, when interconnected with a pair of effort and flow variables an effective direction is implied. Fig. 3 shows how to implement the wave transform for both cases. Fig. 1 can be modified to yield the following system in which a passive controller $K_{PD}(s)$ is able to command a passive plant $G_p(s)$. The plant will follow the negative flow set-point $\mathbf{r}_s(s)$. If we precede the flow set-point with a causal derivative filter $G_d(s) = \frac{s}{\tau s+1}$ then the plant will track a desired displacement set-point $\mathbf{q}_s(s)$.

The following observations, have been made by simulating this system: If the plant is a *passive* mass, then the plant displacement will equal the negative displacement set-point at steady state. If the plant is *passive* and stable such as a mass-spring-damper, then steady state error will occur. So far the discussion has taken place with respect to the continuous time domain we have shown that delayed data to and from the controller $K_{PD}(s)$ can occur in an isolated manner such that a passive control system can be designed.

II. PASSIVE CONTROL THEORY

Passive control theory is extremely general and broad in that it applies to a large class of controllers for linear, non-linear, continuous and discrete control systems. In [12] control theory for continuous and discrete passive systems is presented. In particular, passive control theory has been used in digital *adaptive control* theory to show stability of various *parameter adaptation algorithms* [21]. Additional texts which discuss non-linear continuous passive control theory are [13]–[15]. In [22] a comprehensive treatment is dedicated to the passive control of a class of non-linear systems, known as *Euler-Lagrange Systems*. *Euler-Lagrange Systems* can be represented by a *Hamiltonian* which possess a Dirac structure that allows dissipative and energy storage elements to be interconnected to ports without causal specification [23, p. 124]. Thus, in [23] an extensive treatment on intrinsically passive control using Generalized Port-Controlled Hamiltonian Systems is presented, in particular as it relates to telemanipulation and scattering theory. Our presentation of passive control theory focuses on laying the groundwork for discrete passive control theorems, mirrors the continuous counterpart results presented in [14], and is based off of the continuous and discrete theorems in [12].

A. l^2 STABILITY THEORY FOR PASSIVE NETWORKS

Definition 1: The l^2 space, is the real space of all bounded, infinitely summable functions $f(i) \in \mathbb{R}^n$. We note that $\mathbb{R}^n$ could be replaced with $\mathbb{C}^n$ in (8) without loss of generality. Denoting $\langle \cdot, \cdot \rangle$ as an inner product space [24], the l^2 space is the set of all functions $f(i)$ which meet the following inequality (8).

$$\sum_{i=0}^{\infty} \langle f^*(i), f(i) \rangle < \infty \tag{8}$$

A truncation operator will be defined as follows:

$$f_N(i) = \begin{cases} f(i), & \text{if } 0 \leq i < N \\ 0, & \text{otherwise} \end{cases} \tag{9}$$

Likewise the extended l^2 space, l^2_e, is the set of all functions $f(i)$ which meet the following inequality (10).

$$\sum_{i=0}^{N-1} \langle f^*(i), f(i) \rangle < \infty, N \geq 1 \tag{10}$$

Note that $l^2 \subset l^2_e$. Typically l^2_e is defined with the summation to N and the truncation includes N [21, p. 75] and [12, p. 172], however, these definitions are equivalent and is convenient for future analysis. Finally we can define our l^2 norms (11) and truncation of the l^2 norm (12) as follows:

$$\|f(i)\|_2 \triangleq \left(\sum_{i=0}^{\infty} \langle f(i), f(i) \rangle\right)^{\frac{1}{2}} \tag{11}$$

$$\|f(i)_N\|_2^2 \triangleq \langle f(i), f(i) \rangle_N \triangleq \sum_{i=0}^{N-1} \langle f(i), f(i) \rangle \tag{12}$$

The following definition for l^2-stability is similar to the one given in [25] which refers to [14] in regards to stating that *finite l^2-gain* is sufficient for l^2-stability, however, in [14] this is only stated for the continuous time case. We provide a short proof for the discrete time case and note where it parallels [14] for completeness.

Definition 2: Let the set of all functions $u(i) \in \mathbb{R}^n$, $y(i) \in \mathbb{R}^p$ which are either in the l^2 space, or l_e^2 space be denoted as $l^2(U)/l_e^2(U)$ and $l^2(Y)/l_e^2(Y)$ respectively. Then define G as an input-output mapping $G : l_e^2(U) \to l_e^2(Y)$, such that it is *$l^2$-stable if*

$$u \in l^2(U) \Rightarrow G(u) \in l^2(Y) \tag{13}$$

The map G has *finite l^2-gain* if there exist finite constants γ and b such that for all $N \geq 1$

$$\|(G(u))_N\|_2 \leq \gamma \|u_N\|_2 + b, \forall u \in l_e^2(U) \tag{14}$$

holds. Equivalently G has *finite l^2-gain* if there exist finite constants $\hat{\gamma} > \gamma$ and $\hat{b}$ such that for all $N \geq 1$ [14, (2.21)]

$$\|(G(u))_N\|_2^2 \leq \hat{\gamma}^2 \|u_N\|_2^2 + \hat{b}, \forall u \in l_e^2(U) \tag{15}$$

holds. If G has *finite l^2-gain* then it is sufficient for l^2-stability. The proof is as simple as letting $u \in l^2(U)$ and $N \to \infty$ which leads (17) to

$$\|(G(u))\|_2 \leq \gamma \|u\|_2 + b, \forall u \in l^2(U) \tag{16}$$

which implies (13) and completes the proof.

Lemma 1: [14, Lemma 2.2.13] The *l^2-gain* $\gamma(G)$ is given as

$$\gamma(G) = \inf\{\hat{\gamma} | \exists \hat{b} s.t. (15) holds\} \tag{17}$$

Next we will present definitions for various types of passivity for discrete time systems.

Definition 3: [12], [14] Let $G : l_e^2(U) \to l_e^2(U)$ then for all $u \in l_e^2(U)$ and all $N \geq 1$:

I. G is *passive* if their exists some constant β such that (18) holds.

$$\langle G(u), u \rangle_N \geq -\beta \tag{18}$$

II. G is *strictly-output passive* if their exists some constants β and $\epsilon > 0$ such that (19) holds.

$$\langle G(u), u \rangle_N \geq \epsilon \|(G(u))_N\|_2^2 - \beta \tag{19}$$

III. G is *strictly-input passive* if their exists some constants β and $\delta > 0$ such that (20) holds.

$$\langle G(u), u \rangle_N \geq \delta \|u_N\|_2^2 - \beta \tag{20}$$

Theorem 1: Let $G : l_e^2(U) \to l_e^2(U)$ be *strictly-output passive*. Then G has *finite l^2-gain*.

Proof: The proof for the discrete case is practically the same as for the continuous case given in [14, Theorem 2.2.14], for completeness we denote $y = G(u)$, and rewrite (19)

$$\begin{aligned} \epsilon \|y_N\|_2^2 &\leq \langle y, u \rangle_N + \beta \\ &\leq \langle y, u \rangle_N + \beta + \frac{1}{2} \|\frac{1}{\sqrt{\epsilon}} u_N - \sqrt{\epsilon} y_N\|_2^2 \\ &\leq \beta + \frac{1}{2\epsilon} \|u_N\|_2^2 + \frac{\epsilon}{2} \|y_N\|_2^2 \end{aligned} \tag{21}$$

thus moving all terms of y to the left, (21), has the final form of (15) with *l^2-gain* $\hat{\gamma} = \frac{1}{\epsilon}$ and $\hat{b} = \frac{2\beta}{\epsilon}$. ■

The requirement for *strictly-output passive* is a relatively easy requirement to obtain for a *passive* plant with map G and input u and output y. This is accomplished by closing the loop relative to a reference vector r with a positive definite feedback gain matrix $K > 0$ such that $u = r - Ky$.

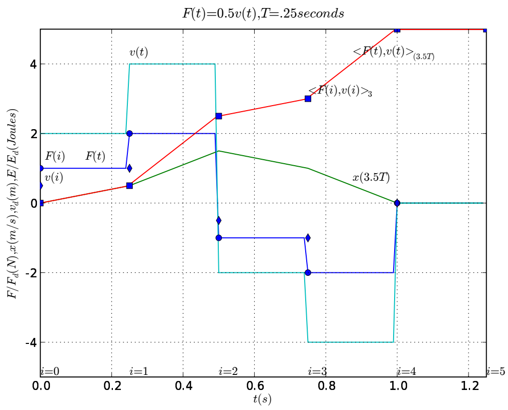

Fig. 4. Illustration showing $\langle v(i), F(i) \rangle_N = \langle v(t), F(t) \rangle_{NT}$

Theorem 2: Given a *passive* system with input u, output $G(u) = y$, a positive definite matrix $K > 0$, and new reference vector r. If the input $u = r - Ky$, then the new mapping $G_{cl} : r \to y$ is *strictly-output passive* which implies *l^2-stability*.

Proof: First we use the definition of passivity for G and substitute the feedback formula for u.

$$\langle y, u \rangle_N = \langle y, r - Ky \rangle_N \geq -\beta \tag{22}$$

Then we can obtain the following inequality

$$\langle y, r \rangle_N \geq \lambda_m(K) \|y\|_2^2 - \beta \tag{23}$$

in which $\lambda_m(K) > 0$ is the mimimum eigenvalue for K. Hence, (23) has the form of (19) which shows *strictly-output passive* and implies *l^2-stability*. ■

It is important to note that for very small maximum eigenvalues, the system is essentially the nominal passive system we started with. This is important, for we can design more general passive digital controllers and modify them with this simple transform to make them *strictly-output passive*.

B. INNER-PRODUCT EQUIVALENT SAMPLE AND HOLD

In this section we prove Theorem 3 which shows how a (non)linear (*strictly input* or *strictly output*) passive plant can be transformed to a discrete (*strictly input*) passive plant using a particular digital sampling and hold scheme. This novel zero-order digital to analog hold, and sampling scheme proposed by [9] was to yield a combined system such that the energy exchange between the analog and digital port is equivalent. This equivalence allows one to interconnect an analog to a digital Port-Controlled Hamiltonian (*PCH*) system which yields an overall passive system. In [10], a correction was made to the original scheme proposed in [9]. In order to prove Theorem 3, we will restate the sample and hold algorithm with a slightly modified nomenclature. Fig. 5 shows a simple example of a continuous force, $F(t)$ (solid blue line), being applied to a damper with damping ratio 0.5 (kg/s-m). The force is updated at a rate of T seconds, such that at $t = iT$ the corresponding discrete force, $F(i)$ (circles), updates $F(t)$ and is held for an additional T seconds. The discrete "velocity", $v(i)$ (diamonds), is defined as $v(i) = (x(i+1) - x(i))$. The discrete "position", $x(i)$, is the sampled integral of the continuous velocity, $v(t)$ (solid magenta line), up to time $t = iT$. Likewise $x(i+1)$ is the sampled integral of the *predicted* continuous velocity up to time $t+T$. Note that the solid green line, $x(t)$ denotes the integral of the continuous velocity. Finally, the continuous inner-product integral, $\langle F(t), v(t) \rangle_{NT} \triangleq \int_0^{NT} \langle F(t), v(t) \rangle$, is denoted by the solid red line. The discrete inner-product summation, $\langle v(i), F(i) \rangle_N$, is indicated at each index i with a blue square, thus showing equivalence to $\langle F(t), v(t) \rangle_{NT}$.

Definition 4: [9], [10] Let a continuous one-port plant be denoted by the input-output mapping $G_{ct} : L_e^2(U) \rightarrow L_e^2(U)$. Denote continuous time as t, the discrete time index as i, the continuous input as $u(t) \in L_e^2(U)$, the continuous output as $y(t) \in L_e^2(U)$, the transformed discrete input as $u(i) \in l_e^2(U)$, and the transformed discrete output as $y(i) \in l_e^2(U)$. The *inner-product equivalent sample and hold* (*IPESH*) is implemented as follows:

I. $x(t) = \int_0^t y(\tau) d\tau$
II. $y(i) = x((i+1)T) - x(iT)$
III. $u(t) = u(i), \forall t \in [iT, i(T+1))$

As a result

$$\langle y(i), u(i) \rangle_N = \langle y(t), u(t) \rangle_{NT}, \forall N \geq 1 \tag{24}$$

holds.

Theorem 3: Using the *IPESH* given in Definition 4, the following relationships can be stated between the continuous one-port plant, G_{ct}, and the discrete transformed one-port plant, $G_d : l_e^2(U) \rightarrow l_e^2(U)$:

I. If G_{ct} is *passive* then G_d is *passive*.
II. If G_{ct} is *strictly-input passive* then G_d is *strictly-input passive*.
III. If G_{ct} is *strictly-output passive* then G_d is *strictly-input passive*.

This is a general result, in which Theorem 3-I was defined for the special case in which the input was a force and the output was a velocity [10, Definition 2] and for the special case when interconnecting *PCH* systems [9], [26, Theorem 1].

Proof:

I. Since the continuous *passive* system G_{ct} satisfies

$$\langle y(t), u(t) \rangle_\tau \geq -\beta, \forall \tau \geq 0 \tag{25}$$

then by substituting (24) into (25) results in

$$\langle y(i), u(i) \rangle_N \geq -\beta, \forall N \geq 1 \tag{26}$$

which satisfies (18) and completes the proof of Theorem 3-I.

II. Let $\tau = NT$, then since the continuous *strictly-input passive* system G_{ct} satisfies

$$\langle y(t), u(t) \rangle_\tau \geq \delta \|u(t)_\tau\|_2^2 - \beta, \forall \tau \geq 0 \tag{27}$$

and Definition 4-III implies

$$\|u(t)_\tau\|_2^2 = T \|u(i)_N\|_2^2 \tag{28}$$

substituting (28) and (24) into (27) results in

$$\langle y(i), u(i) \rangle_N \geq T\delta \|u(i)_N\|_2^2 - \beta, \forall N \geq 1 \tag{29}$$

therefore, the transformed discrete system G_d satisfies (20) and completes the proof of Theorem 3-II.

III. Let $\tau = NT$, then since the continuous *strictly-output passive* system G_{ct} satisfies

$$\langle y(t), u(t) \rangle_\tau \geq \epsilon \|y(t)_\tau\|_2^2 - \beta, \forall \tau \geq 0 \tag{30}$$

however, no direct relationship can be made between $\|y(t)_\tau\|_2^2$ and $\|y(i)_N\|_2^2$. But Definition 4-III still implies (28), and since G_{ct} is *strictly-output passive*, which implies *finite* l^2*-gain* such that

$$\begin{aligned} \|y(t)_\tau\|_2^2 &\leq \frac{1}{\epsilon^2} \|u(t)_\tau\|_2^2 + \frac{2\beta}{\epsilon} \\ &\leq \frac{T}{\epsilon^2} \|u(i)_N\|_2^2 + \frac{2\beta}{\epsilon} \end{aligned} \tag{31}$$

holds. Substituting (31) into (30) results in

$$\langle y(i), u(i) \rangle_N \geq \frac{T}{\epsilon} \|u(i)_N\|_2^2 - \beta(1 - \frac{2}{\epsilon}), \forall N \geq 1 \tag{32}$$

therefore, the transformed discrete system G_d satisfies (20) and completes the proof of Theorem 3-III.

■

Continuous and discrete linear time invariant systems have an important property in that if they are *strictly-input passive* they have *finite* L^2/l^2*-gain* and are *strictly-output passive* (Corollary 8).

Corollary 1: Using the *IPESH* defined by Definition 4, the following relationships can be stated between the continuous *LTI* one-port plant, G_{ct}, and the discrete transformed *LTI* one-port plant, $G_d : l_e^2(U) \rightarrow l_e^2(U)$: If G_{ct} is either *strictly-input passive* or *strictly-output passive* then G_d is both *strictly-input passive* with *finite* l^2*-gain* and *strictly-output passive*.

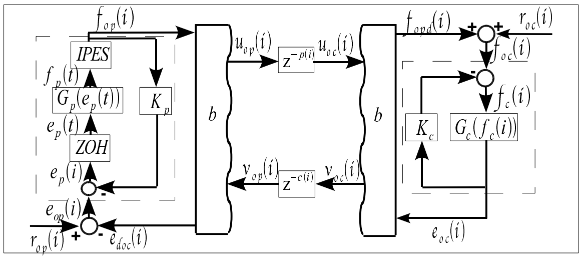

Fig. 5. l^2*-stable* digital control network for cooperative scheduler

III. Main Results

Fig. 5 depicts our proposed control scheme in order to guarantee l^2 stability in which the feedback and control data can be subject to variable delays between the controller and the plant. Depicted is a continuous passive plant $G_p(e_p(t)) = f_p(t)$ which is actuated by a zero-order hold and sampled by an *IPES*. Thus G_p is transformed into a discrete passive plant $G_{dp}(e_p(i)) = f_{op}(i)$. Next, a positive definite matrix K_p is used to create a discrete *strictly-output passive* plant $G_{op}(e_{op}(i)) = f_{op}(i)$ outlined by the dashed line. Next G_{op}

is interconnected in the following feed-back configuration such that

$$\langle f_{op}, e_{doc}\rangle_N = \frac{1}{2}(\|(u_{op})_N\|_2^2 - \|(v_{op})_N\|_2^2) \tag{33}$$

holds due to the wave transform. Moving left to right towards the *strictly-output passive* digital controller $G_{oc}(f_{oc}) = e_{oc}$ we first note that

$$\langle f_{opd}, e_{oc}\rangle_N = \frac{1}{2}(\|(u_{oc})_N\|_2^2 - \|(v_{oc})_N\|_2^2) \tag{34}$$

holds due to the wave transform. The wave variables $u_{oc}(i), v_{op}(i)$ are related to the corresponding wave variables $u_{op}(i), v_{oc}(i)$ and by the discrete time varying delays $p(i), c(i)$ such that

$$u_{oc}(i) = u_{op}(i - p(i)) \tag{35}$$

$$v_{op}(i) = v_{oc}(i - c(i)) \tag{36}$$

(35) and (36) hold. Finally the positive definite matrix K_c is used to make the *passive* digital controller $G_c(f_c(i)) = e_{oc}(i)$ *strictly-output passive*. Typically, r_{oc} can be considered the set-point in which $f_{opd}(i) \approx -r_{oc}(i)$ at steady state, while $r_{op}(i)$ can be thought as a discrete disturbance. Which leads us to the following theorem.

Theorem 4: The system depicted in Fig. 5 is l^2-*stable* if

$$\langle f_{op}, e_{doc}\rangle_N \geq \langle e_{oc}, f_{opd}\rangle_N \tag{37}$$

holds for all $N \geq 1$.

Proof: First, by theorem 3-I, G_p is transformed to a discrete *passive* plant. Next, by theorem 2 both the discrete plant and controller are transformed into a *strictly-output passive* systems. The *strictly-output passive* plant satisfies

$$\langle f_{op}, e_{op}\rangle_N \geq \epsilon_{op}\|(f_{op})_N\|_2^2 - \beta_{op} \tag{38}$$

while the *strictly-output passive* controller satisfies (39).

$$\langle e_{oc}, f_{oc}\rangle_N \geq \epsilon_{oc}\|(e_{oc})_N\|_2^2 - \beta_{oc} \tag{39}$$

Substituting, $e_{doc} = r_{op} - e_{op}$ and $f_{opd} = f_{oc} - r_{oc}$ into (37) yields

$$\langle f_{op}, r_{op} - e_{op}\rangle_N \geq \langle e_{oc}, f_{oc} - r_{oc}\rangle_N$$

which can be rewritten as

$$\langle f_{op}, r_{op}\rangle_N + \langle e_{oc}, r_{oc}\rangle_N \geq \langle f_{op}, e_{op}\rangle_N + \langle e_{oc}, f_{oc}\rangle_N \tag{40}$$

so that we can then substitute (38) and (39) to yield

$$\langle f_{op}, r_{op}\rangle_N + \langle e_{oc}, r_{oc}\rangle_N \geq \epsilon(\|(f_{op})_N\|_2^2 + \|(e_{oc})_N\|_2^2) - \beta \tag{41}$$

in which $\epsilon = \min(\epsilon_{op}, \epsilon_{oc})$ and $\beta = \beta_{op} + \beta_{oc}$. Thus (41) satisfies (19) in which the input is the row vector of $[r_{op}, r_{oc}]$, and the output is the row vector $[f_{op}, e_{oc}]$ and completes the proof. ■

Theorem 5: The system depicted in Fig. 5 without the *IPESH* in which i and t denote continuous time is L^2-*stable* if

$$\langle f_{op}, e_{doc}\rangle_\tau \geq \langle e_{oc}, f_{opd}\rangle_\tau \tag{42}$$

holds for all $\tau \geq 0$.

Proof: With the exception that the *IPESH* is no longer involved and the discrete time delays are replaced with continuous time delays. The proof is completely analogous to the proof given for Theorem 4. ■

In order for (37) to hold, the communication channel/ data-buffer needs to remain *passive*. It has been proved in [26] that the discrete communication channel is passive for both fixed delays [26, Proposition 1] and variable time delays including loss of packets [26, Proposition 2], as we will show with a different and straight forward proof.

Lemma 2: If the discrete time varying delays are fixed $p(i) = p, c(i) = c$ and/or data packets are dropped then (37) holds.

Before we begin the proof, we denote the partial sum from M to N of an extended norm as follows

$$\|x_{(M,N)}\|_2^2 \triangleq \langle x^*, x\rangle_{(M,N)} = \sum_{i=M}^{N-1}\langle x^*, x\rangle \tag{43}$$

Proof: In order to satisfy (37), (33) minus (34) must be greater than zero, or

$$\begin{aligned}
&(\|(u_{op})_N\|_2^2 - \|(v_{op})_N\|_2^2) - (\|(u_{oc})_N\|_2^2 - \|(v_{oc})_N\|_2^2) \geq 0\\
&(\|(u_{op})_N\|_2^2 - \|(u_{oc})_N\|_2^2) + (\|(v_{oc})_N\|_2^2 - \|(v_{op})_N\|_2^2) \geq 0\\
&\qquad(\|(u_{op})_N\|_2^2 - \|(u_{op}(i-p(i))_N\|_2^2)+\\
&\qquad(\|(v_{oc})_N\|_2^2 - \|(v_{oc}(i-c(i))_N\|_2^2) \geq 0
\end{aligned} \tag{44}$$

holds. Clearly (44) holds when the delays are fixed, as (44) can be written to show

$$(\|(u_{op})_{((N-p),N)}\|_2^2 + \|(v_{oc})_{((N-p),N)}\|_2^2) \geq 0 \tag{45}$$

the inequality always holds for all $0 \leq p, c < N$. Note if p and c equal zero, then inequality in (45) becomes an equality. If all the data packets were dropped then, $\|(u_{oc})_N\|_2^2) = 0$ and $\|(v_{op})_N\|_2^2) = 0$, such that (37) holds and all the energy is dissipated. If only part of the data packets are dropped, the effective inequality described by (44) serves as a lower bound ≥ 0; hence dropped data packets do not violate (37). ■

[26, Proposition 2] is to broad in stating that the communication channel is passive in spite of variable time delays when only the transmission of one data packet per sample period occurs. For instance, a simple counter example is to assume $p(i) = i$, then (44) will not hold if $N\|(u_{op})_1\|_2^2 > (\|(u_{op})_N\|_2^2 + \|(v_{oc})_N\|_2^2)$. Clearly other variations can be given such that $p(i)$ eventually becomes fixed and never changes after sending old *duplicate samples*, and still (37) will not hold. Therefore, we state the following lemma:

Lemma 3: The discrete time varying delays $p(i), c(i)$ can vary arbitrarily as long as (44) holds. Thus, the main assumption (37) will hold if:

1) we change $p(i) = (i+1)$, which sets $u_{oc}(i) = u_{op}(-1) = 0$, when ever a duplicate $u_{op}(i-p(i))$ would be received (ie. we eliminate duplicate transmissions). We also need to change $c(i) = (i+1)$, which sets $v_{op}(i) = v_{oc}(-1) = 0$, when ever a duplicate $v_{oc}(i-c(i))$ would be received.
2) we change $p(i) = (i+1)$ and/or $c(i) = (i+1)$ in order that (44) holds. This requires us to track the current energy

storage in the communication channel. A similar energy-storage audit is discussed in [27, Section IV] without using wave-variables. In [6] a similar audit is described for the continuous time case.

A. PASSIVE DISCRETE LTI SYSTEM SYNTHESIS

In [28], using dissipative theory and a longer proof than we will provide, it was shown how to synthesize a discrete passive plant from a linear time invariant (*LTI*) plant. The advantage of such a result is that one does not need to measure an integrated output from the passive plant. However, if one is concerned with controlling the integrated output such as position, one will probably have this measurement as well as the corresponding passive output such as velocity. We will also show how an observer, based on the integrated output measurement can still be used. Such an observer maintains passivity and eliminates the need to directly measure the actual passive output such as the velocity. The proof for the observer will follow a similar proof by [29].

A passive continuous time *LTI* system, $H(s)$, which is described by the following state space representation $\{\mathbf{A} \in \mathbb{R}^{\mathbf{n}\times\mathbf{n}}, \mathbf{B} \in \mathbb{R}^{\mathbf{n}\times\mathbf{p}}, \mathbf{C} \in \mathbb{R}^{\mathbf{p}\times\mathbf{n}}, \mathbf{D} \in \mathbb{R}^{\mathbf{p}\times\mathbf{p}}\}$ is cascaded in series with a diagonal matrix of integrators, $H_I(s)$, described by $\{\mathbf{A_I} = \mathbf{0}, \mathbf{B_I} = \mathbf{I}, \mathbf{C_I} = \mathbf{I}, \mathbf{D_I} = \mathbf{0}\}$. The combined system, $H_o(s) = H(s)H_I(s)$, is described by $\{\mathbf{A_o}, \mathbf{B_o}, \mathbf{C_o}\}$. Where

$$\mathbf{A_o} = \begin{bmatrix} \mathbf{A} & \mathbf{0} \\ \mathbf{C} & \mathbf{0} \end{bmatrix} \in \mathbb{R}^{(n+p)\times(n+p)} \tag{46}$$

$$\mathbf{B_o} = \begin{bmatrix} \mathbf{B} \\ \mathbf{D} \end{bmatrix} \in \mathbb{R}^{(n+p)\times p} \tag{47}$$

$$\mathbf{C_o} = \begin{bmatrix} \mathbf{0} & \mathbf{I} \end{bmatrix} \in \mathbb{R}^{p\times(n+p)} \tag{48}$$

Applying a zero-order-hold, the system is described by [30]

$$\begin{aligned} x(k+1) &= \mathbf{\Phi_o} x(k) + \mathbf{\Gamma_o} u(k) \\ p(k) &= \mathbf{C_o} x(k) \end{aligned} \tag{49}$$

in which

$$\begin{aligned} \mathbf{\Phi_o} &= e^{\mathbf{A_o}T} \\ \mathbf{\Gamma_o} &= \int_0^T e^{\mathbf{A_o}\eta} d\eta \mathbf{B_o} \end{aligned} \tag{50}$$

.

Proposition 1: A *passive* continuous time *LTI* system, $H(s)$, can be converted to a discrete passive *LTI* system, $G_p(z)$ at a sample rate T in which the discrete state equations are

$$\begin{aligned} x(k+1) &= \mathbf{\Phi_o} x(k) + \mathbf{\Gamma_o} u(k) \\ y(k) &= \mathbf{C_p} x(k) + \mathbf{D_p} u(k) \end{aligned} \tag{51}$$

in which $\mathbf{C_p} = \mathbf{C_o}(\mathbf{\Phi_o} - \mathbf{I})$, and $\mathbf{D_p} = \mathbf{C_o}\mathbf{\Gamma_o}$.

Proof: From Definition 4 it is a simple exercise to compute the passive output $y(k) = p(k+1) - p(k)$ as follows

$$\begin{aligned} x(k+1) &= \mathbf{\Phi_o} x(k) + \mathbf{\Gamma_o} u(k) \\ y(k) &= \mathbf{C_o}(\mathbf{\Phi_o} - \mathbf{I}) x(k) + \mathbf{C_o}\mathbf{\Gamma_o} u(k) \end{aligned} \tag{52}$$

hence $\mathbf{C_p} = \mathbf{C_o}(\mathbf{\Phi_o} - \mathbf{I})$, and $\mathbf{D_p} = \mathbf{C_o}\mathbf{\Gamma_o}$ which completes the proof. ■

Using Proposition 1 and Theorem 2 the following corollary can be shown:

Corollary 2: Given a positive definite matrix $\mathbf{K_x} > 0$ and discrete *passive* system described by (51), the system

$$\begin{aligned} x(k+1) &= \mathbf{\Phi_{sp}} x(k) + \mathbf{\Gamma_{sp}} u(k) \\ y(k) &= \mathbf{C_{sp}} x(k) + \mathbf{D_{sp}} u(k) \end{aligned} \tag{53}$$

is *strictly-output passive* or *strictly positive real*. Here

$$\begin{aligned} \mathbf{\Phi_{sp}} &= \mathbf{\Phi_o} - \mathbf{\Gamma_o}\mathbf{K_x}(\mathbf{I} + \mathbf{D_p}\mathbf{K_x})^{-1}\mathbf{C_p} \\ \mathbf{\Gamma_{sp}} &= \mathbf{\Gamma_o}(\mathbf{I} - \mathbf{K_x}(\mathbf{I} + \mathbf{D_p}\mathbf{K_x})^{-1}\mathbf{D_p}) \\ \mathbf{C_{sp}} &= (\mathbf{I} + \mathbf{D_p}\mathbf{K_x})^{-1}\mathbf{C_p} \\ \mathbf{D_{sp}} &= (\mathbf{I} + \mathbf{D_p}\mathbf{K_x})^{-1}\mathbf{D_p} \end{aligned} \tag{54}$$

With our discrete *strictly-output passive* system we can scale the gain so that its steady state gain matches the *strictly-output passive* continuous systems steady state gain.

Corollary 3: Given a diagonal matrix $\mathbf{K_s} > 0$ and discrete *strictly-output passive* system described by (53), the following system is *strictly-output passive* or *strictly positive real*

$$\begin{aligned} x(k+1) &= \mathbf{\Phi_{sp}} x(k) + \mathbf{\Gamma_{sp}} u(k) \\ y(k) &= \mathbf{K_s}\mathbf{C_{sp}} x(k) + \mathbf{K_s}\mathbf{D_{sp}} u(k) \end{aligned} \tag{55}$$

in which each diagonal element

$$k_s(i) = \begin{cases} y_c(i)/y_d(i) \forall i \in \{1, \ldots, p\} & \text{if } y_c(i) \text{ and } y_d(i) \neq 0; \\ \frac{1}{T} & \text{otherwise} \end{cases} \tag{56}$$

The vectors y_c/y_d correspond to the respective steady state continuous/discrete output of a *strictly-output passive* plant given a unit step input. These vectors can be computed as follows:

$$\begin{aligned} &y_c = (-\mathbf{C_c}\mathbf{A_c}^{-1}\mathbf{B_c} + \mathbf{D_c})\mathbf{1} \\ &y_d = H_{sp}(z=1)\mathbf{1},\ H_{sp}(z) = \mathbf{C_{sp}}(z\mathbf{I} - \mathbf{\Phi_{sp}})^{-1}\mathbf{\Gamma_{sp}} + \mathbf{D_{sp}} \end{aligned} \tag{57}$$

where

$$\begin{aligned} \mathbf{G_x} &= \mathbf{I} + \mathbf{D}\mathbf{K_x} \\ \mathbf{C_c} &= \mathbf{G_x}^{-1}\mathbf{C} \\ \mathbf{D_c} &= \mathbf{G_x}^{-1}\mathbf{D} \\ \mathbf{A_c} &= \mathbf{A} - \mathbf{B}\mathbf{K_x}\mathbf{C_c} \\ \mathbf{B_c} &= \mathbf{B}(\mathbf{I} - \mathbf{K_x}\mathbf{D_c}) \end{aligned} \tag{58}$$

Next, the following corollary provides a method to compute $u_{op}(k), f_{op}(k)$ given r_{op}, v_{op}, b. We can also synthesize the digital controller from a continuous model using the *IPES* with *ZOH* as well, so an additional corollary will show how to compute $v_{oc}(k), e_{oc}(k)$ given $u_{oc}(k), r_{oc}(k)$.

Corollary 4: The following state equation describes the relationship between the inputs r_{op}, v_{op} and scattering gain b to the outputs $u_{op}(k), f_{op}(k)$.

$$\begin{aligned} x(k+1) &= \mathbf{\Phi_{ef}} x(k) + \mathbf{\Gamma_{ef}}(\sqrt{2b} v_{op}(k) + r_{op}(k)) \\ f_{op}(k) &= \mathbf{C_{ef}} x(k) + \mathbf{D_{ef}}(\sqrt{2b} v_{op}(k) + r_{op}(k)) \\ u_{op}(k) &= \sqrt{2b} f_{op}(k) - v_{op}(k) \end{aligned} \tag{59}$$

Here

$$
\begin{aligned}
\mathbf{G} &= \mathbf{I} + b\mathbf{K_s}\mathbf{D_{sp}} \\
\mathbf{C_{ef}} &= \mathbf{G}^{-1}\mathbf{K_s}\mathbf{C_{sp}} \\
\mathbf{D_{ef}} &= \mathbf{G}^{-1}\mathbf{K_s}\mathbf{D_{sp}} \\
\mathbf{\Phi_{ef}} &= \mathbf{\Phi_{sp}} - b\mathbf{\Gamma_{sp}}\mathbf{C_{ef}} \\
\mathbf{\Gamma_{ef}} &= \mathbf{\Gamma_{sp}}(\mathbf{I} - b\mathbf{D_{ef}}) \qquad (60)
\end{aligned}
$$

Corollary 5: The following state equation describes the relationship between the inputs r_{oc}, u_{oc} and scattering gain b to the outputs $v_{oc}(k), e_{oc}(k)$.

$$
\begin{aligned}
x(k+1) &= \mathbf{\Phi_{fe}}x(k) + \mathbf{\Gamma_{fe}}(\sqrt{\frac{2}{b}}u_{oc}(k) + r_{oc}(k)) \\
e_{oc}(k) &= \mathbf{C_{fe}}x(k) + \mathbf{D_{fe}}(\sqrt{\frac{2}{b}}u_{oc}(k) + r_{oc}(k)) \\
v_{oc}(k) &= u_{oc}(k) - \sqrt{\frac{2}{b}}e_{oc}(k) \qquad (61)
\end{aligned}
$$

Where

$$
\begin{aligned}
\mathbf{G_1} &= \mathbf{I} + \frac{1}{b}\mathbf{K_s}\mathbf{D_{sp}} \\
\mathbf{C_{fe}} &= \mathbf{G_1}^{-1}\mathbf{K_s}\mathbf{C_{sp}} \\
\mathbf{D_{fe}} &= \mathbf{G_1}^{-1}\mathbf{K_s}\mathbf{D_{sp}} \\
\mathbf{\Phi_{fe}} &= \mathbf{\Phi_{sp}} - \frac{1}{b}\mathbf{\Gamma_{sp}}\mathbf{C_{fe}} \\
\mathbf{\Gamma_{fe}} &= \mathbf{\Gamma_{sp}}(\mathbf{I} - \frac{1}{b}\mathbf{D_{fe}}) \qquad (62)
\end{aligned}
$$

In order to prove that a state observer can be used in a *strictly-input passive* manner, we require the following lemma.

Lemma 4: [31] The discrete *LTI* system (51) is *strictly-input passive* (*strictly-positive real*(*SPR*)) if and only if a symmetric positive definite matrix $\mathbf{P}$ exists and satisfies the following *LMI*:

$$
\begin{bmatrix} \mathbf{\Phi_o}^\top\mathbf{P}\mathbf{\Phi_o} - \mathbf{P} & (\mathbf{\Gamma_o}^\top\mathbf{P}\mathbf{\Phi_o} - \mathbf{K_s}\mathbf{C_p})^\top \\ \mathbf{\Gamma_o}^\top\mathbf{P}\mathbf{\Phi_o} - \mathbf{K_s}\mathbf{C_p} & -(\mathbf{K_s}\mathbf{D_p} + \mathbf{D_p}^\top\mathbf{K_s}^\top - \mathbf{\Gamma_o}^\top\mathbf{P}\mathbf{\Gamma_o}) \end{bmatrix} < 0 \qquad (63)
$$

Therefore by Theorem 3-(II,III) any continuous *strictly-input passive* or *strictly-output passive LTI* system which is sampled and actuated by an *IPESH* will satisfy (63). Note that we added K_s in order to show that any positive diagonal matrix can be used to scale the output $y(k)$ as is done with our observer described by (64).

We now propose the following state observer, based on the sampled integrated output of the *strictly-input passive* or *strictly-output passive* plant and the corresponding output estimate $\hat{y}(k)$:

$$
\begin{aligned}
\hat{x}(k+1) &= \mathbf{\Phi_o}\hat{x}(k) + \mathbf{\Gamma_o}u(k) - \mathbf{K_e}(\hat{p}(k) - p(k)) \\
\hat{p}(k) &= \mathbf{C_o}\hat{x}(k) \\
\hat{y}(k) &= \mathbf{K_s}\mathbf{C_p}\hat{x}(k) + \mathbf{K_s}\mathbf{D_p}u(k) \qquad (64)
\end{aligned}
$$

This observer is along similar lines to the observer proposed in [29] except that our observer is based on the sampled integrated output and we specifically focus on how it applies to *strictly-input passive* and *strictly-output passive* plants. Defining the error in the state estimate as $e(k) \triangleq \hat{x}(k) - x(k)$ and the augmented observer state vector as $x_{ob}(k) \triangleq [x(k), e(k)]$ the system dynamics are

$$
\begin{aligned}
x_{ob}(k+1) &= \mathbf{\Phi_{ob}}x_{ob}(k) + \mathbf{\Gamma_{ob}}u(k) \\
\hat{y}(k) &= \mathbf{K_s}\mathbf{C_{ob}}x_{ob}(k) + \mathbf{K_s}\mathbf{D_p}u(k) \qquad (65)
\end{aligned}
$$

where

$$
\begin{aligned}
\mathbf{\Phi_{ob}} &= \begin{bmatrix} \mathbf{\Phi_o} & \mathbf{0} \\ \mathbf{0} & \mathbf{\Phi_o} - \mathbf{K_e}\mathbf{C_o} \end{bmatrix} \\
\mathbf{\Gamma_{ob}} &= \begin{bmatrix} \mathbf{\Gamma_o} \\ \mathbf{0} \end{bmatrix} \\
\mathbf{C_{ob}} &= \begin{bmatrix} \mathbf{C_p} & \mathbf{C_p} \end{bmatrix} \qquad (66)
\end{aligned}
$$

Proposition 2: If the sampled *LTI* system is either *strictly-input passive* or *strictly-output passive* and K_e is chosen so that the eigenvalues of $\mathbf{\Phi_o} - \mathbf{K_e}\mathbf{C_o}$ are inside the unit circle the observer described by (64) is both *strictly-input passive* with *finite l^2-gain* and *strictly-output passive*.

Proof: First by choosing the eigenvalues to be inside the unit circle there exists two matrices $\mathbf{Q_2} > 0$ and $\mathbf{P_o} > 0$ such that the following Lyapunov inequality is satisfied

$$
-\mathbf{Q_2} = (\mathbf{\Phi_o} - \mathbf{K_e}\mathbf{C_o})^\top\mathbf{P_o}(\mathbf{\Phi_o} - \mathbf{K_e}\mathbf{C_o}) < 0 \qquad (67)
$$

In order to satisfy the requirements of Lemma 4 we consider the following symmetric positive definite matrix

$$
\mathbf{P_{ob}} = \begin{bmatrix} \mathbf{P} & \mathbf{0} \\ \mathbf{0} & \mu\mathbf{P_o} \end{bmatrix} > 0 \qquad (68)
$$

and show that there exists a $\mu > 0$ that satisfies (72). Note the following inequalities hold from our original *strictly-input passive* system.

$$
\begin{aligned}
-\mathbf{Q_1} &= \mathbf{\Phi_o}^\top\mathbf{P}\mathbf{\Phi_o} - \mathbf{P} < 0 \\
-\mathbf{Q_3} &= -(\mathbf{K_s}\mathbf{D_p} + \mathbf{D_p}^\top\mathbf{K_s}^\top - \mathbf{\Gamma_{ob}}^\top\mathbf{P_{ob}}\mathbf{\Gamma_{ob}}) \\
&= -(\mathbf{K_s}\mathbf{D_p} + \mathbf{D_p}^\top\mathbf{K_s}^\top - \mathbf{\Gamma_o}^\top\mathbf{P}\mathbf{\Gamma_o}) < 0 \qquad (69)
\end{aligned}
$$

To simplify the expression we define

$$
\mathbf{C_1} \triangleq \mathbf{\Gamma_o}^\top\mathbf{P}\mathbf{\Phi_o} - \mathbf{K_s}\mathbf{C_p} \qquad (70)
$$

Therefore the proposed passive system described by (65) has to satisfy

$$
\begin{bmatrix} \mathbf{Q_1} & \mathbf{0} & -\mathbf{C_1^\top} \\ \mathbf{0} & \mu\mathbf{Q_2} & -\mathbf{C_p}^\top\mathbf{K_s}^\top \\ -\mathbf{C_1} & -\mathbf{K_s}\mathbf{C_p} & \mathbf{Q_3} \end{bmatrix} > 0 \qquad (71)
$$

Using a similarity transformation, (71) is equivalent to

$$
\begin{bmatrix} \mathbf{Q_1} & -\mathbf{C_1^\top} & \mathbf{0} \\ -\mathbf{C_1} & \mathbf{Q_3} & -\mathbf{K_s}\mathbf{C_p} \\ \mathbf{0} & -\mathbf{C_p}^\top\mathbf{K_s}^\top & \mu\mathbf{Q_2} \end{bmatrix} > 0 \qquad (72)
$$

The following upper block matrix, $\mathbf{O}$, satisfies (63) due to Proposition 1, Theorem 3-(II,III), and Lemma 4.

$$
\mathbf{O} = \begin{bmatrix} \mathbf{Q_1} & -\mathbf{C_1^\top} \\ -\mathbf{C_1} & \mathbf{Q_3} \end{bmatrix} > 0 \qquad (73)
$$

Since $\mathbf{O} > 0$, and $Q_2 > 0$, then from using Proposition 8.2.3-v in [32] which is based on the Schur Complement Theory we need to show that

$$\mathbf{O} > 0, \text{ and} \tag{74}$$

$$\mu\mathbf{Q_2} - \begin{bmatrix}\mathbf{0} & -\mathbf{C_p}^{\mathsf{T}}\mathbf{K_s}^{\mathsf{T}}\end{bmatrix} \mathbf{O}^{-1} \begin{bmatrix}\mathbf{0} \\ -\mathbf{K_s}\mathbf{C_p}\end{bmatrix} > 0$$
$$\mu\mathbf{Q_2} - \mathbf{C_p}^{\mathsf{T}}\mathbf{K_s}^{\mathsf{T}}\mathbf{O}^{-1}\mathbf{K_s}\mathbf{C_p} > 0 \tag{75}$$

Thus denoting $\lambda_m(\cdot)/\lambda_M(\cdot)$ as the minimum/maximum eigenvalues for a matrix, noting that the similarity transform of $\mathbf{Q_2} = \mathbf{P_2}\mathbf{\Lambda_2}\mathbf{P_2}^{\mathsf{T}}$, and defining $\mathbf{M} \triangleq \mathbf{C_p}^{\mathsf{T}}\mathbf{K_s}^{\mathsf{T}}\mathbf{O}^{-1}\mathbf{K_s}\mathbf{C_p}$, μ needs to satisfy

$$\mu > \frac{\lambda_M(\mathbf{P_2}^{\mathsf{T}}(\mathbf{M} + \mathbf{M}^{\mathsf{T}})\mathbf{P_2})}{2\lambda_m(\mathbf{Q_2})} \tag{76}$$

Therefore μ exists and satisfies (72) which completes the proof. ■

The proof emphasizes the fact that the one given in [29] only shows *sufficiency* for *passive* systems and implicitly assumes that their discrete sampled plant is *strictly-input passive*. Furthermore, their results can not be applied for our desired design of an observer which uses the integrated output of a *strictly-input passive* or *strictly-output passive* plant.

Since we are using the observer on continuous *LTI* systems which are either *strictly-input passive* with *finite L^2-gain*, or *strictly-output passive* and the corresponding discrete observer is both *strictly-input passive* with *finite l^2-gain* and *strictly-output passive* we can simplify our implementation by setting the feedback gain $K_p = 0$ in Fig. 5. We note that K_p may still be helpful in converting a continuous passive signal into a discrete *strictly-output passive* signal with an observer, however we found the analysis to be quite difficult. Similar to Corollary 4 we state for the observer of a *strictly-output passive* plant.

Corollary 6: If using an observer for either a *LTI* system which is *strictly-input passive* with finite gain or is *strictly-output passive*, the following state equation describes the relationship between the inputs r_{op}, v_{op} and scattering gain b to the outputs $\hat{u}_{op}(k), \hat{f}_{op}(k)$.

$$\hat{x}(k+1) = \mathbf{\Phi_{efo}}\hat{x}(k) + \mathbf{\Gamma_{efo}}(\sqrt{2b}v_{op}(k) + r_{op}(k)) + \mathbf{K_e}p(k)$$
$$\hat{f}_{op}(k) = \mathbf{C_{efo}}\hat{x}(k) + \mathbf{D_{efo}}(\sqrt{2b}v_{op}(k) + r_{op}(k))$$
$$\hat{u}_{op}(k) = \sqrt{2b}\hat{f}_{op}(k) - v_{op}(k) \tag{77}$$

In which

$$\mathbf{G} = \mathbf{I} + b\mathbf{K_s}\mathbf{D_p}$$
$$\mathbf{C_{efo}} = \mathbf{G}^{-1}\mathbf{K_s}\mathbf{C_p}$$
$$\mathbf{D_{efo}} = \mathbf{G}^{-1}\mathbf{K_s}\mathbf{D_p}$$
$$\mathbf{\Phi_{efo}} = \mathbf{\Phi_o} - \mathbf{K_e}\mathbf{C_o} - b\mathbf{\Gamma_o}\mathbf{C_{efo}}$$
$$\mathbf{\Gamma_{efo}} = \mathbf{\Gamma_o}(\mathbf{I} - b\mathbf{D_{efo}}) \tag{78}$$

Note that Corollary 6 describes a standard observer not connected to a wave junction when $b = 0$.

Corollary 7: If using an observer for either a *LTI* system which is *strictly-input passive* with finite gain or is *strictly-output passive*, the following state equation describes the relationship between the inputs r_{oc}, u_{oc} and scattering gain b to the outputs $\hat{v}_{oc}(k), \hat{e}_{oc}(k)$.

$$\hat{x}(k+1) = \mathbf{\Phi_{feo}}\hat{x}(k) + \mathbf{\Gamma_{feo}}(\sqrt{\frac{2}{b}}u_{oc}(k) + r_{oc}(k)) + \mathbf{K_e}p(k)$$
$$\hat{e}_{oc}(k) = \mathbf{C_{feo}}\hat{x}(k) + \mathbf{D_{feo}}(\sqrt{\frac{2}{b}}u_{oc}(k) + r_{oc}(k))$$
$$\hat{v}_{oc}(k) = u_{oc}(k) - \sqrt{\frac{2}{b}}\hat{e}_{oc}(k) \tag{79}$$

In which

$$\mathbf{G_1} = \mathbf{I} + \frac{1}{b}\mathbf{K_s}\mathbf{D_p}$$
$$\mathbf{C_{feo}} = \mathbf{G_1}^{-1}\mathbf{K_s}\mathbf{C_p}$$
$$\mathbf{D_{feo}} = \mathbf{G_1}^{-1}\mathbf{K_s}\mathbf{D_p}$$
$$\mathbf{\Phi_{feo}} = \mathbf{\Phi_o} - \mathbf{K_e}\mathbf{C_o} - \frac{1}{b}\mathbf{\Gamma_o}\mathbf{C_{feo}}$$
$$\mathbf{\Gamma_{feo}} = \mathbf{\Gamma_o}(\mathbf{I} - \frac{1}{b}\mathbf{D_{feo}}) \tag{80}$$

B. STABLE CONTROL WITH A COOPERATIVE SCHEDULER

SOS is an operating system which uses a high priority and low priority queue with timers which signal a task through the queue in order to implement the soft real time scheduler (note that most other operating systems such as *TinyOS* which use just a single FIFO message queue could be used to notify the control task as well) [2]. For simplicity we will use *SOS* to discuss one possible implementation for our *l^2-stable* control system illustrated in Fig. 5. As a future project, we will write a device driver which does the following:

1) Provide an interface for the controller to register a function to enable the device driver to send $u_{op}(i)$ to. Also allow the controller to specify a desired sample time T, wave impedance b, and K_p (note K_p does not need to be a matrix, it could be a scalar to modify all parts of $f_{op}(i)$ equally. Note that the driver will buffer $v_{oc}(i)$ while the controller will buffer $u_{op}(i)$.
2) Provide an interface for the controller to send outgoing $v_{oc}(i)$ to.
3) Calculate $f_{op}(i)$ based on the *IPES* given in Definition 4-I,II.
4) Calculate the corresponding $u_{op}(i)$, and $e_{doc}(i)$ based on the buffered $v_{oc}(i)$, the servicing of the buffer is where the $v_{op}(i - c(i))$ delay comes in effect. Since data can be popped directly from the buffer, we do not need to worry about counting duplicate data. For simplicity if the buffer begins to get full we can safely drop data.
5) With the new $e_{doc}(i)$ and $f_{op}(i)$, calculate $e_p(i) = -e_{doc}(i) - K_p f_{op}(i)$ and apply to *ZOH*.

The controller, is notified by the driver through the high-priority queue and implements the right side of Fig. 5. Note, that the lower-priority queue can be used for more time-consuming tasks, such as changing control parameters and loading new modules. This may cause temporary delays, however, *l^2-stability* will be maintained. Note that old data does not have to be simply dropped (which satisfies

Lemma 2) in order for the system to recover from these longer periodic delays. Using Lemma 3-2 we can calculate the two-norm of all M, in which $i = 0, 1, ..., M-1$ of the non-processed inputs $s(u_{op}, M) = \|u_{op}(i)\|_2$ and multiply it by the sign of the sum of the non-processed inputs $sn(u_{op}, M) = \texttt{sgn}(\sum_{i=0}^{M-1} u_{op}(i))$ such that the input for $u_{oc}(i) = sn(u_{op}, M)s(u_{op}, M)$. This will improve tracking and highlights why we split the buffers appropriately. The driver can do a similar calculation in order to calculate $v_{op}(i)$.

IV. SIMULATION

We shall control a motor with an ideal current source, which will allow us to neglect the affects of the motor inductance and resistance for simplicity. The fact that the current source is non-ideal, leads to a non-passive relationship between the desired motor current and motor velocity [20]. There are ways to address this problem using passive control techniques by controlling the motors velocity indirectly with a switched voltage source and a minimum phase current feedback technique [33], and more recently incorporating the motors back voltage measurement which provides an exact tracking error dynamics passive output feedback controller [34].

The motor is characterized by its torque constant, $K_m > 0$, back-emf constant K_e, rotor inertia, $J_m > 0$, and damping coefficient $B_m > 0$. The dynamics are described by

$$\dot{\omega} = -\frac{B_m}{J_m}\omega + \frac{K_m}{J_m}i \tag{81}$$

and are in a (strictly) positive real form which is a necessary and sufficient condition for (strict input) passivity [35, Section V.A.2)] [31, Defintion 1]. We choose to use the passive “proportional-derivative” controller described by (7) and define $\tau = \frac{B}{K}$ in order to factor out K and yield

$$K_{PD}(s) = K\frac{\tau s + 1}{s} \tag{82}$$

Using loop-shaping techniques we choose $\tau = \frac{J_m}{B_m}$ and choose $K = \frac{J_m \pi}{10 K_m T}$. This will provide a reasonable crossover frequency at roughly a tenth the Nyquist frequency and maintain a 90 degree phase margin. We choose to use the same motor parameter values given in [34] in which $K_m = 49.13 mV$rad sec, $J_m = 7.95 \times 10^{-3} kgm^2$, and $B_m = 41 \mu Nm$sec. With $T = .05$ seconds, we use Corollary 5 to synthesize a *strictly-output passive* controller from our continuous model (82), and Corollary 6 to implement the observer. We also use Corollary 3 in order to compute the appropriate gains for both the controller $K_{s_c} = 1$ and the *strictly-output passive* plant $K_{s_p} = 20$. Note that by arbitrarily choosing $K_{s_c} = \frac{1}{T} = 20$ would have led to a incorrectly scaled system in which the crossover frequency would essentially equal the Nyquist frequency (only because a zero exists extremely close to -1 in the z-plane). Fig. 6, Fig. 7, and Fig. 8 indicates that our baseline system performs as expected. We chose $K_e = [16.193271, 1.799768]^\mathsf{T}$ for our observer in which the poles are equal to a tenth of the poles of the discrete *passive* plant synthesized by Proposition 1, this by definition forces all the poles inside the unit circle. Since the plant is *strictly-output passive* we chose $K_p = 0$. For the controller we

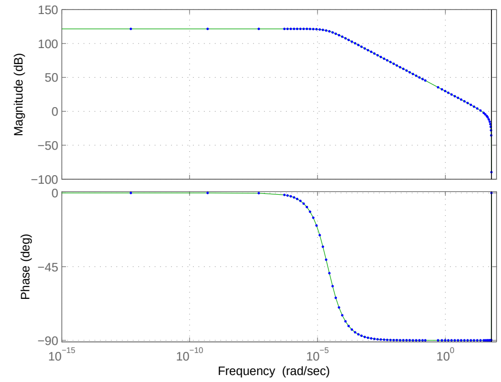

Fig. 6. Bode plot depicting crossover frequency for baseline plant with observer and controller.

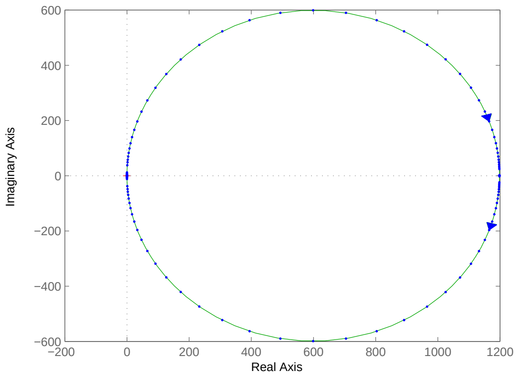

Fig. 7. Nyquist plot for the continuous plant (solid line) and the synthesized discrete counterpart (solid dots) with observer.

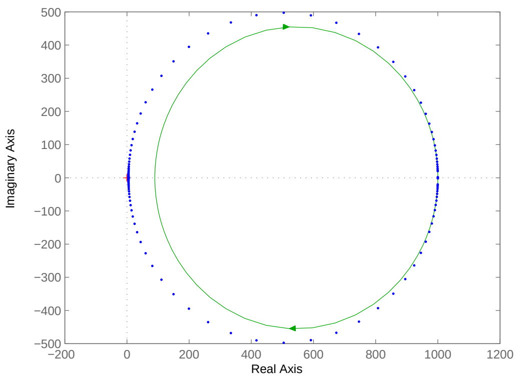

Fig. 8. Nyquist plot for the continuous controller (solid line) and the synthesized discrete counterpart (solid dots).

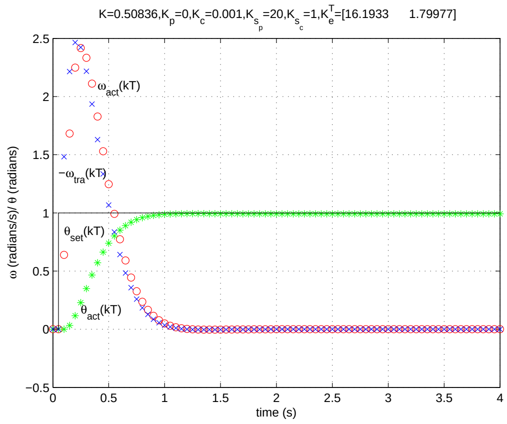

Fig. 9. Baseline step response for motor with *strictly-output passive* digital controller and *strictly-output passive* observer.

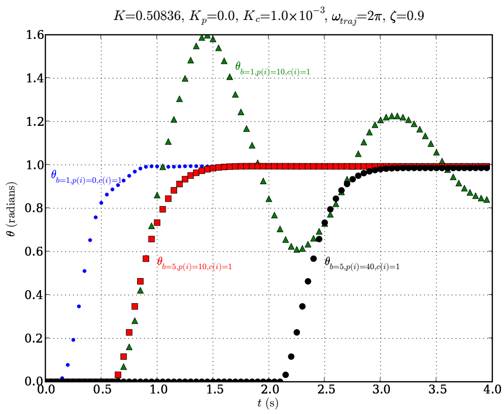

Fig. 10. Step response for motor with *strictly-output passive* digital controller and *strictly-output passive* observer as depicted in Fig. 5 with delays.

chose $K_c = 0.001$ in order to make it *strictly-output passive*. Fig. 9 shows the step response to a desired position set-point $\theta_d(k)$ which generates an approximate velocity reference for $r_{oc}(z) = -H_t(z)\theta_d(z)$. $H_t(z)$ is a zero-order hold equivalent of $H_t(s)$, in which $\omega_{traj} = 2\pi$ and $\zeta = .9$.

$$H_t(s) = \frac{\omega_{traj}^2 s}{s^2 + 2\zeta\omega_{traj} + \omega_{traj}^2} \tag{83}$$

Note, that it is important to use a second order filter in order to achieve near perfect tracking, a first order filter resulted in significant steady state position errors for relatively slow trajectories. Finally in Fig. 10 we see that the proposed control network maintains similar performance as the baseline system. Note that by increasing $b = 5$ significantly reduced the overshoot caused by a half second delay (triangles $b = 1$/squares $b = 5$). Also note that even a two second delay (large circles $b = 5$) results in only a delayed response nearly identical to the baseline system.

V. CONCLUSIONS

We have presented the necessary theory to design a digital control network which maintains $l^2 - stability$ in spite of time varying delays caused by cooperative schedulers. We presented a fairly complete, and needed l^2 stability analysis, in particular the results in Theorem 1, and Theorem 2 (for the discrete-time case) appeared to be lacking from the open literature and were necessary in order to complete our proof. The other new results (not available in the open literature) which led to a l^2-*stable* controller design are as follows:

1) Theorem 3-I is an improvement which captures all *passive* systems (not just *PCH*) systems.
2) Theorem 3-II, and Theorem 3-III are completely original (the latter forced us to require that the driver had to implement the additional feedback (K_p) calculation to obtain passivity for the non-linear case).
3) Corollary 1 allows us to set $K_p = \mathbf{0}$ if the continuous *LTI* plant is either *strictly-input passive* or *strictly-output passive*.
4) Theorem 4 is a new and general theorem to interconnect continuous non-linear passive plants which we hope will lead to more elaborate networks interconnected in the discrete time domain. Theorem 5 is also new, in which no knowledge of the energy storage function is required to show stability of the network.
5) Proposition 1 shows how to synthesize a discrete *passive LTI* system from a continuous one.
6) Corollary 2 and Corollary 3 show how to respectively make the discrete *passive* plant *strictly-output passive* (*strictly-positive real*) and scale the output so that it will match the steady state output for its continuous counterpart.
7) Corollary 4 and Corollary 5 show how to implement the *strictly-output passive* network depicted in Fig. 5.
8) Proposition 2 shows how to implement a discrete *strictly-output passive LTI* observer for either a *strictly-input passive* or *strictly-output passive* continuous *LTI* system.
9) Corollary 6 and Corollary 7 show how to implement the observer when attached to a scattering junction.

We are excited about Theorem 2 because it allows us to directly design *low-sensitivity strictly-output passive* controllers using the *wave-digital filters* described in [11]. We plan on extending this networking theory as it applies to multiple plants controlled by either a single or possibly multiple controllers.

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APPENDIX I
STRICTLY POSITIVE REAL AND STRICTLY INPUT/OUTPUT PASSIVE *LTI* SYSTEMS

In our research related to *passivity* theory, as it relates to *LTI* systems, it is not clear that anyone has formally stated that if a *LTI* system is *strictly-input passive* it is also *strictly-output passive*. Possibly this was implicitly understood in the earlier literature for *LTI* systems [12], [31], [35] in which the definition for a *strictly-input passive* system (Definition 3) was termed *strictly passive*. Although there is an emphasis in the literature that a *strictly passive* (*strictly-input passive*) systems may or may not have a *finite* l^2*-gain* which is necessary for a *strictly-output passive* system. There is no specific statement that a *strictly-input passive LTI* systems has *finite* l^2*-gain*. This is emphasized by the fact that both [36, Corollary 1] [37, Corollary 2] note that discrete *SPR LTI* systems are indeed stable.

Definition 5: [37], [38] Let $H(s)$ be a square rational transfer matrix in s. $H(s)$ is said to be *SPR* if

a) All elements of $H(s)$ are analytic in $\mathtt{Re}(s) \geq 0$;
b) $H(j\omega) + H^{\mathsf{T}}(-j\omega) > 0$, $\forall \omega \in \mathbb{R}$
c) i. $\lim_{\omega \to \infty} \omega^2 [H(j\omega) + H^{\mathsf{T}}(-j\omega)] > 0$, if $|D + D^{\mathsf{T}}| = 0$
   ii. $\lim_{\omega \to \infty} [H(j\omega) + H^{\mathsf{T}}(-j\omega)] > 0$, if $|D + D^{\mathsf{T}}| \neq 0$

Definition 6: [37], [38] Let $H_d(z)$ be a square rational transfer matrix in z. $H_d(z)$ is said to be *SPR* if

a) All elements of $H_d(z)$ are analytic in $|z| \geq 1$
b) $H_d(e^{j\theta}) + H_d^{\mathsf{T}}(e^{-j\theta}) > 0$, $\forall \theta \in [0, 2\pi]$

Note that both definitions are slightly more restrictive than those given by [38], however they are consistent with previous statements relating *SPR* to *strictly-input passive* systems [12], [35], [37]. Thus by definitions 5,6 continuous and discrete *SPR LTI* systems are stable which implies that *strictly-input passive* or *strictly-output passive* systems are also stable. It has already been shown that *strictly-output passive* systems have *finite* l^2*-gain*, it remains to be shown that *LTI strictly-input passive* systems also have *finite* l^2*-gain*.

Theorem 6: [39] The L^2/l^2*-gain* of a *LTI* system described by a transfer matrix $H(p)$ equals the H_∞ norm of H defined by

$$||H||_\infty = \sup_{p\in\Omega} ||H(p)|| \tag{84}$$

where Ω is the right half plane $\Omega = \mathbb{C}_0$ for the continuous time (*CT*) case, and the exterior of the unit circle $\Omega = \mathbf{D}_1$ in the discrete time (*DT*) case. Moreover, for rational transfer matrices with no poles in Ω (such as *SPR* systems), the supremum can be calculated on the boundary of Ω (the imaginary axis in the *CT* case and the unit circle in the *DT* case).

Therefore, from Theorem 6 a continuous/discrete *LTI strictly-input passive* system which is *SPR* has *finite* L^2/l^2*-gain* which implies the *LTI* system is *strictly-output passive* [14, Remark 2.3.5].

Corollary 8: Every continuous/discrete *LTI* system which is *strictly-input passive* has *finite* L^2/l^2*-gain*, therefore it also *strictly-output passive*.

Appendix II
Observer Simulation Equations

In order to simulate an observer for a continuous *LTI* plant in which the actual state space matrices for the actual passive plant are denoted $\{\mathbf{A_a} \in \mathbb{R}^{\mathbf{n}\times\mathbf{n}}, \mathbf{B_a} \in \mathbb{R}^{\mathbf{n}\times\mathbf{p}}, \mathbf{C_a} \in \mathbb{R}^{\mathbf{p}\times\mathbf{n}}, \mathbf{D_a} \in \mathbb{R}^{\mathbf{p}\times\mathbf{p}}\}$. The actual discrete equivalent matrices for a passive system are computed appropriately as described by (46), (47), (48), (49), and (50), and denoted as $\{\mathbf{\Phi_{oa}}, \mathbf{\Gamma_{oa}}, \mathbf{C_{oa}}\}$. If the observer is implemented on the plant side for a *LTI strictly-input passive* or *strictly-output passive* plant as depicted in Fig. 5 and described by Corollary 6, then the system can be described by

$$\begin{aligned}\begin{bmatrix}\hat{x}(k+1)\\ x(k+1)\end{bmatrix} &= \begin{bmatrix}\mathbf{\Phi_{efo}} & \mathbf{K_e C_{oa}}\\ -b\mathbf{\Gamma_{oa}C_{efo}} & \mathbf{\Phi_{oa}}\end{bmatrix}\begin{bmatrix}\hat{x}(k)\\ x(k)\end{bmatrix}\\ &+ \begin{bmatrix}\mathbf{\Gamma_{efo}}\\ \mathbf{\Gamma_{efoa}}\end{bmatrix}(\sqrt{2b}v_{op}(k) + r_{op}(k))\\ \begin{bmatrix}\hat{f}_{op}(k)\\ p(k)\end{bmatrix} &= \begin{bmatrix}\mathbf{C_{efo}} & \mathbf{0}\\ \mathbf{0} & \mathbf{C_{oa}}\end{bmatrix}\begin{bmatrix}\hat{x}(k)\\ x(k)\end{bmatrix}\\ &+ \begin{bmatrix}\mathbf{D_{efo}}\\ \mathbf{0}\end{bmatrix}(\sqrt{2b}v_{op}(k) + r_{op}(k))\end{aligned} \tag{85}$$

in which

$$\mathbf{\Gamma_{efoa}} = \mathbf{\Gamma_{oa}}(\mathbf{I} - b\mathbf{D_{efo}}) \tag{86}$$

. Similarly, if we implement the observer for a continuous plant on the “controller side” (i.e. when the plant is more accurately depicted as having a flow input and corresponding effort output) as depicted in Fig. 5 and described by Corollary 7 then the system can be described by

$$\begin{aligned}\begin{bmatrix}\hat{x}(k+1)\\ x(k+1)\end{bmatrix} &= \begin{bmatrix}\mathbf{\Phi_{feo}} & \mathbf{K_e C_{oa}}\\ -\frac{1}{b}\mathbf{\Gamma_{oa}C_{feo}} & \mathbf{\Phi_{oa}}\end{bmatrix}\begin{bmatrix}\hat{x}(k)\\ x(k)\end{bmatrix}\\ &+ \begin{bmatrix}\mathbf{\Gamma_{feo}}\\ \mathbf{\Gamma_{feoa}}\end{bmatrix}(\sqrt{\frac{2}{b}}u_{oc}(k) + r_{oc}(k))\\ \begin{bmatrix}\hat{e}_{oc}(k)\\ p(k)\end{bmatrix} &= \begin{bmatrix}\mathbf{C_{feo}} & \mathbf{0}\\ \mathbf{0} & \mathbf{C_{oa}}\end{bmatrix}\begin{bmatrix}\hat{x}(k)\\ x(k)\end{bmatrix}\\ &+ \begin{bmatrix}\mathbf{D_{feo}}\\ \mathbf{0}\end{bmatrix}(\sqrt{\frac{2}{b}}u_{oc}(k) + r_{oc}(k))\end{aligned} \tag{87}$$

in which

$$\mathbf{\Gamma_{feoa}} = \mathbf{\Gamma_{oa}}(\mathbf{I} - \frac{1}{b}\mathbf{D_{feo}}) \tag{88}$$

.