STABILITY PARAMETERIZATIONS AND HIDDEN MODES * IN TWO DEGREES OF FREEDOM CONTROL DESIGN

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ABSTRACT

The general two degrees of freedom controller provides a unifying framework in approaching complicated control problems involving multiple objectives, in a manner independent of specific, and possibly restrictive, control configurations. A stability theorem is used which clearly shows the role of the feedback part of the controller and it allows the introduction of alternative stabilizing controller parameterizations in a natural way. The stable uncontrollable and/or unobservable hidden modes of the control system are then characterized and expressed in terms of the design parameters; this leads to strategies to avoid the hidden modes if so desired and in general it leads to better, more efficient control design.

INTRODUCTION

The objective of any multivariable control design method is to synthesize an appropriate control law, represented here by the multivariable controller C. It was noted earlier [1,2] that, for single-input, single-output plants, a single degree of freedom control configuration, such as error (unity) feedback, imposes restrictions on the attainable control properties. Not surprisingly, similar restrictions are present in the multi-input, multi-output case (e.g. [3]). This is of course to be expected, since an error feedback controller generates the plant input u by processing only the error r-y, where r is the reference (set-point, command) external input and y the plant output (the sensor output); that is, the two natural degrees of freedom, corresponding to the availability of the signals r and y, are not fully utilized by a single degree of freedom control configuration [4]. These two signals should be processed independently if unnecessary constraints on the attainable control objectives are to be avoided.

Our interest here is in linear plants P and linear controllers C. The general linear controller can be written as $u = -C_y y + C_r r$ where $[-C_y, C_r] = C$, with C_y, C_r transfer matrices to be determined. This controller, called two degrees of freedom controller, has been utilized by several researchers [4-11] and recently, parametric characterizations of all internally stabilizing controllers C have been derived and used in the design of control systems [4,7,8,12-14]. The two degrees of freedom controller provides a unifying framework in approaching complicated control problems involving multiple objectives, in a manner which is independent of specific, and possibly restrictive, control configurations. There is much renewed interest in the two degrees of freedom controller mainly due to more demanding control problems but also due to recent advances in understanding and effectively utilizing such control laws.

In this paper, a stability theorem is introduced first. It separates the effect of C_y, the feedback portion of the controller C, and it clearly shows that stability in two degrees of freedom control design is based on the stability of the well studied single degree of freedom configuration

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and it is an extension of previous results (due to $C_r \neq C_y$) as expected. Expressing the stability conditions in terms of (i) the single degree of freedom (C_y) stability condition, and (ii) a condition on C_r given C_y to guarantee internal stability, naturally allows the use of any single degree of freedom stabilizing feedback parameterization used in the literature. In other words, we build upon the single degree of freedom stability results to generate the two degrees of freedom results. Furthermore, this theorem directly leads to parametric characterizations of all input-output maps attainable with internal stability from r, the vector of request inputs; also to controller configurations that attain these maps.

The hidden modes of a compensated system correspond to the compensated system's eigenvalues which are uncontrollable and/or unobservable. The hidden modes for single degree of freedom controllers have been studied in the literature [3,15-17]. On the other hand, the hidden modes for two degrees of freedom controllers have not been studied in the literature except for in special cases [15,16,18]. In this paper we consider the hidden modes of the two degrees of freedom control system in Figure 1. It is assumed that the plant and controller are controllable and observable. In this way the hidden modes of the compensated system are due exclusively to the system interconnection. Later this assumption is relaxed to consider the implications of non-irreducible realizations of the controller.

Observe that the controller C is designed to guarantee at least internal stability of the system. Therefore the hidden modes of the compensated system, if any, will be stable. Stable hidden modes need to be studied because they can increase the necessary order of the controller and they can degrade the performance. Consequently, a complete understanding of stable hidden modes will lead to better control design algorithms.

STABILITY THEOREM. PARAMETERIZATIONS

The two degrees of freedom linear controller

$$u = C \begin{bmatrix} y \\ r \end{bmatrix} = [-C_y \quad C_r] \begin{bmatrix} y \\ r \end{bmatrix}, \tag{1}$$

where $C = [-C_y, C_r]$ proper, transfer matrices, generates the plant input u by independently processing the plant output y and the external reference input r (Figure 1),

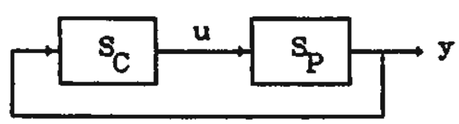

Figure 1. The compensated system.

where S_P is the linear plant described by $y = Pu$ with P its proper transfer matrix and S_C is the controller described in (1). Several researchers have utilized C in a time domain state-space formulation (e.g. Bengtsson [5]). Using a transform domain formulation, C has been incorporated by Pernebo [6] and more recently, by Youla and Bongiorno [4], Desoer and Gustafson [7], Vidyasagar [8], Desoer and Gündes [10] and Sugie and Yoshikawa[11]. The pole-placement algorithm of Åström in [9] also uses C but for SISO plants only.

A significant step towards better understanding the role of C in plant compensation was recently accomplished by parametrically characterizing all stabilizing two degrees of freedom controllers C; thus extending the results on parametric characterization of all feedback controllers C_y [19-23,3,15,12-14] which have greatly contributed to control design methods

[19-20,24-28]. All internally stabilizing controllers C were parametrically characterized in [4,7-8] using two independent proper and stable parameters K and X as

$$C = (x_1 - K\tilde{N})^{-1}[-(x_2 + K\tilde{D}), \ \ X], \tag{2}$$

where $\tilde{N}$, $\tilde{D}$, x_1, x_2 are proper and stable transfer matrices, and they are derived from coprime fractional representations of the plant

$$P = ND^{-1} = \tilde{D}^{-1}\tilde{N}, \tag{3}$$

and the associated Bezout-Diophantine equation

$$x_1D + x_2N = I. \tag{4}$$

In [4], (2) involves polynomial matrices $\tilde{N}$, $\tilde{D}$, x_1, and x_2 with K and X stable transfer matrices; x_1 and x_2 satisfy the Diophantine equation $x_1D+x_2N = D_0$, where D, N and D_0 are polynomial matrices. Note that the development in [4] follows [19], while in [7-8] the development is based on the seminal paper of Desoer, et. al. [22], which among other results, it extends the results of [19,21] to more general (beyond polynomial) rings. The parameter K in (2) is the well known parameter used in the characterization of all stabilizing feedback controllers C_y in [19,22]. Note that the parameter X is actually the response parameter used by Antsaklis and Sain [3] (and Liu and Sung [29]) to parametrically characterize feedback controllers in an error feedback setting. If Dz = u, y = Nz is an internal polynomial matrix representation of the plant P, then it can be shown that

$$z = Xr, \tag{5}$$

that is, X is the transfer matrix between the external input r and the partial state z of the plant.

It is evident that if exogenous signals (such as disturbances, noise) are assumed to be injected at various points in Figure 1, all possible transfer functions and responses to all inputs can be derived in terms of K and X by direct substitutions of (2); in this way all "admissible", under internal stability responses can be characterized [4,7-8,14].

It is advantageous to study internal stability of the system in Figure 1 in a novel alternative way [30].

<u>Theorem 1</u>. Consider a proper controller C as in (1) and the system in Figure 1, and assume that $|I + C_yP| \neq 0$. The compensated system is internally stable if and only if

(i) $u = -C_yy$ internally stabilizes the system $y = Pu$, and

(ii) C_r is such that

$$M := (I + C_yP)^{-1}C_r \tag{6}$$

satisfies $D^{-1}M = X$, a stable rational, where C_y satisfies (i) and $P = ND^{-1}$ is a coprime polynomial factorization.

Theorem 1 provides some advantages over other stability theorems presented so far for the system in Figure 1. These advantages are discussed below and are followed by a proof of Theorem 1.

This theorem separates the role of C_y, the feedback part of C, from C_r in achieving internal stability. Clearly if only feedback action is considered, only (i) is of interest; and if open loop control is desired, $C_y = 0$, (i) implies that P must be stable, and $C_r = M$ must satisfy (ii). In (ii) the parameter M (=DX) appears rather naturally and in (i) the way is open to use any desired feedback parameterization, not necessarily K of [4,7-8].

From Theorem 1 we can directly characterize the input-output maps attainable from r with internal stability. In particular, consider the two maps described by y = Tr and u = Mr. The characterization is done in Theorem 2.

Theorem 2. A pair (T,M) is realizable with internal stability via a two degrees of freedom configuration if and only if $(T,M) = (NX,DX)$ with X stable.

Proof of Theorem 2. Necessity. Assume (T,M) are realizable with internal stability. Consider $u = Mr$, (5), and (7). Then $Dz = Mr$ or $z = Xr$. Internal stability implies that X is stable.
Sufficiency. If X is stable, then T and M are stable. The only thing left to show is that a controller configuration exists to implement these maps. Consider the two degrees of freedom controller in Figure 2 ($C_r = \hat{M} + C_y\hat{T}$).

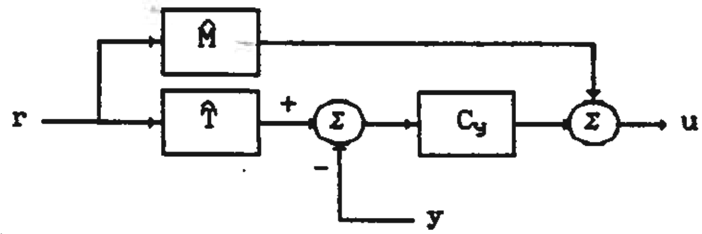

Figure 2. A two degrees of freedom controller.

This configuration implements the desired maps when $\hat{M} = M$ and $\hat{T} = T$. Other configurations to attain these maps are, of course, possible [14].

Q.E.D.

Proof of Theorem 1. Consider an irreducible polynomial internal description of the plant

$$Dz = u, \quad y = Nz, \tag{7}$$

and of the controller

$$\tilde{D}_c z_c = [-\tilde{N}_y \quad \tilde{N}_r] \begin{bmatrix} y \\ r \end{bmatrix}, \quad u = z_c, \tag{8}$$

where (N,D) and $(\tilde{D}_c, [-\tilde{N}_y, \tilde{N}_r])$ are coprime polynomial factorizations. With these definitions an internal description of the compensated system is

$$D_0 z = (\tilde{D}_c D + \tilde{N}_y N)z = \tilde{N}_r r, \quad y = Nz. \tag{9}$$

We say that if $|I + C_y P| \neq 0$ then the compensated sytem is internally stable if D_0^{-1} is stable.
Necessity: Assume that the compensated system is internally stable, that is, D_0^{-1} is stable. Since $C_y = \tilde{D}_c^{-1}\tilde{N}_y$ is not necessarily a coprime polynomial factorization, there exists a polynomial matrix G_ℓ so that

$$[\tilde{D}_c \quad \tilde{N}_y] = G_\ell\, [\tilde{D}_{C_y} \quad \tilde{N}_{C_y}], \tag{10}$$

where G_ℓ is a greatest common left divisor (g.c.l.d.) of $(\tilde{D}_c, \tilde{N}_y)$. Comparing (9) and (10) we notice that

$$\tilde{D}_{C_y} D + \tilde{N}_{C_y} N = G_\ell^{-1} D_0 := \tilde{D}_k, \text{ with } \tilde{D}_k^{-1} \text{ stable}, \tag{11}$$

where $\tilde{D}_k$ is a polynomial matrix. Observe that (11) also implies that G_ℓ^{-1} is stable. Hence (11) shows that $u = -C_y y$ stabilizes $y = Pu$, that is, part (i) of Theorem 1 is satisfied. To show (ii) write

$$\begin{aligned} M &= (I + C_y P)^{-1} C_r = D\tilde{D}_k^{-1}\tilde{D}_{C_y}(\tilde{D}_c^{-1}\tilde{N}_r) \\ &= D\tilde{D}_k^{-1} G_\ell^{-1} \tilde{N}_r \\ &= DX, \end{aligned} \tag{12}$$

where $X := D_o^{-1}\tilde{N}_r$ is a stable tranfer matrix. This shows that (ii) is necessary too.

Sufficiency: Let C satisfy (i) and (ii) of Theorem 1. If $C = \tilde{D}_C^{-1}[-\tilde{N}_y, N_r]$ is a coprime polynomial factorization and G_ℓ is a g.c.l.d. of $(\tilde{D}_C, \tilde{N}_y)$, then (10) is true for some left coprime matrices $\tilde{D}_{C_y}$ and $\tilde{N}_{C_y}$ $(C_y = \tilde{D}_{C_y}^{-1}\tilde{N}_{C_y})$. Because (i) is satisfied, $\tilde{D}_{C_y}D + \tilde{N}_{C_y}N = \tilde{D}_k$ is such that $\tilde{D}_k^{-1}$ is stable. The expression for $\tilde{D}_k$ can be premultiplied by G_ℓ to obtain

$$\tilde{D}_C D + \tilde{N}_y N = G_\ell \tilde{D}_k. \tag{13}$$

In view of the internal stability criterion (D_o^{-1} stable) and $\tilde{D}_k^{-1}$ stable it suffices to show that G_ℓ^{-1} is stable. Note that (13) can be written as $G_\ell^{-1}D_o = \tilde{D}_k$ or as $G_\ell^{-1}\tilde{N}_r = \tilde{D}_k X$ with $(G_\ell, \tilde{N}_r)$ left coprime (if they were not coprime, $C = \tilde{D}_C^{-1}[-\tilde{N}_y, \tilde{N}_r]$ would not have been a coprime factorization). Therefore, $D_o = G_\ell \tilde{D}_k$ in (13) satisfies D_o^{-1} stable.

Q.E.D.

There are many choices in parametrically characterizing all feedback stabilizing controllers C_y and these are extensively discussed by Antsaklis and Sain in [15]. Parameterizations of all feedback stabilizing multivariable controllers were first introduced by Youla, Jabr and Bongiorno in [19]; also by Kucera [23]. Antsaklis [21] expressed the parameterization of Youla, et. al., over the polynomial ring, while Desoer, Liu, Murray and Saeks [22] extended the Youla parameterization over more general rings. Zames in [20] (see also [31,32]) introduced the parameter Q for the case of stable plants, and Antsaklis and Sain [3,33] discussed the parameter X ($=D^{-1}Q$, when an error feedback configuration is used) and derived conditions on X for the unstable plant case. Alternative parameterizations are discussed in [15]. The stabilizing controllers C can therefore be expressed, in addition to (2) as (for example):

$$C = (I - QP)^{-1}\,[-Q \quad M] = ((I - LN)D^{-1})^{-1}\,[-L \quad X], \tag{14}$$

where $Q = DL$, $M = DX$ with L, X stable and $D^{-1}(I - QP) = (I - LN)D^{-1}$ stable ($|I - QP| \neq 0$ or $|I - LN| \neq 0$), and $P = ND^{-1}$ a coprime polynomial matrix factorization. (14) gives parametric characterizations of all stabilizing controllers C, proper and nonproper. For C proper, M and Q are chosen proper and such that $(I - QP)^{-1}$ is also proper; note that if P is strictly proper, Q proper always implies that $(I - QP)^{-1}$ is proper. Notice that L or Q in (14) must satisfy certain conditions, in addition to being stable, in contrast to K in (2); however, alternative to K parameterizations, such as in (14), are very useful, since they do have certain additional desirable properties (see [15]).

The interconnection of a controller C, implemented via a minimal Kalman realization, controlling a minimal plant P as in Figure 1, might cause unobservable, uncontrollable modes from y, r respectively. In addition, when the controller C is put together by interconnecting separately designed subcontrollers, the overall controller's internal description might not be controllable and observable, even though the interconnected subcontrollers are assumed to be realized via minimal Kalman realizations. The uncontrollable and/or unobservable hidden modes, if not appropriately accounted for during the design, might lead to deterioration of system properties and even the loss of internal stability [15]; they could also lead to unnecessarily higher order of the controller. The hidden modes for certain controller configurations have been studied in [3,15,16,18]. Here we study the hidden modes of the system in Figure 1. A complete treatment is presented in [14]. Some of the results are outlined below:

Recall that $Dz = u$, $y = Nz$ and $\tilde{D}_c z_c = -\tilde{N}_y y + \tilde{N}_r r$, $u = z_c$ are irreducible descriptions of the plant and controller, respectively, assuming (N,D) and $(\tilde{D}_c, [-\tilde{N}_y, \tilde{N}_r])$ are coprime polynomial factorizations. Combining these two descriptions gives the following internal description of the feedback system in Figure 1

$$D_0 z = \tilde{N}_r r$$
$$\begin{bmatrix} y \\ u \end{bmatrix} = \begin{bmatrix} N \\ D \end{bmatrix} z, \tag{15}$$

where $D_0 = \tilde{D}_c D + \tilde{N}_y N$. The roots of the determinant of D_0 are the closed-loop eigenvalues. The not necessarily irreducible internal description in (15) leads directly to the following preliminary characterization of hidden modes of the system in Figure 1: *The uncontrollable modes from r correspond to the poles of D_0^{-1} that cancel in $D_0^{-1}\tilde{N}_r$. The unobservable modes from y (u) correspond to the poles of D_0^{-1} that cancel in ND_0^{-1} (DD_0^{-1}).* The concept of cancellations in a product of transfer matrices has been clarified in [34-36,14]; the cancellations should be taken as pole cancellations rather than pole-zero cancellations.

A more useful characterization of hidden modes can be given in terms of the transfer matrices. The characterization is done in this way as in SISO systems because it gives insight into the mechanism of introducing hidden modes. These characterizations are also proven using internal descriptions. *The unobservable modes from y correspond to those poles of C which cancel in the product PC, and the unobservable modes from u correspond to those poles of P that cancel in $\tilde{N}_y P$. The uncontrollable modes from r correspond to the poles of $(I+C_yP)^{-1}$ that cancel in $(I+C_yP)^{-1}C_r$ and those poles of P that cancel in PM and C_yP.*. For comparison note that in the error feedback configuration $C_y = C_r$. In this case, the unobservable modes from y are the poles of C_y which cancel in PC_y whereas the uncontrollable modes from r are the poles of P which cancel in PC_y [3,15].

These results can be more useful in the design of the controller if they are written in terms of some of the design parameters, like L and X; in this way insight will be gained and the designer will be able to deal with the hidden modes using a variety of design tools and methodologies. Note that $L = D^{-1}(I + C_yP)^{-1}C_y$ and $X = D^{-1}(I + C_yP)^{-1}C_r$. *The unobservable modes from y correspond to the poles of [X, L] that cancel in N[X, L] and* *the unobservable modes from u correspond to the poles of* [X, L] *that cancel in* D[X, L] and to the poles of P that cancel in $D_0^{-1}[\tilde{N}_y, \tilde{N}_r]$. *The uncontrollable modes from r correspond to the poles of* P *that do not cancel in* $(I-LN)D^{-1}$ *and to the poles of* L *which are not poles of* X.

When a controller consists of the interconnection of separately designed subcontrollers, typically additional hidden modes are introduced because the resulting controller is not controllable and observable. The subcontrollers are designed to handle a particular problem such as stability and regulation. The hidden modes introduced depend on the particular interconnection. For example, when an observer is implemented with feedforward and feedback controllers, the poles of the observer appear as stable poles of the feedback controller and as stable zeros of the feedforward controller, and correspond to unobservable modes from y. It can be shown that the uncontrollable modes from $[y^t, r^t]^t$ (^{t} denotes transpose) of the controller become uncontrollable modes from r of the compensated system. Similarly, the unobservable modes from u of the controller become unobservable modes from y of the compensated system. It is also possible for the hidden modes of the controller to lead to other kinds of hidden modes of the compensated system. The characterization of these additional hidden modes also reduces to the well known case when $C_y=C_r$.

The above discussion on hidden modes is necessarily short. The results are based on well developed theory which is not presented here due to space limitations. It is hoped that the example at the end of the paper illustrates some of the main points.

The study of hidden modes and their classification as uncontrollable and unobservable leads to better understanding of the phenomena which occur in control design. For example, it can be shown that the hidden modes which can result in undesirable "ringing" of the input u in digital control (an example is given in [9]) are exactly those unobservable modes from y which are observable from u. Understanding the hidden modes leads to strategies on how to avoid them and in general it leads to better, more efficient control designs.

EXAMPLE

Consider the plant

$$P = \frac{s-1}{(s-2)(s+1)} \tag{16}$$

and suppose that a compensator should be designed to stabilize the plant and to attain

$$T = \frac{s-1}{(s+\alpha)(s+\beta)}. \tag{17}$$

One compensator that meets these specifications is

$$C = \left[\frac{-8(s+1)}{(s-13/3)}, \quad \frac{(s+1)^2(s+2/3)}{(s-13/3)(s+\alpha)(s+\beta)}\right]. \tag{18}$$

The closed-loop characteristic polynomial, $(s+\alpha)(s+\beta)(s+1)^2(s+2/3)$, is Hurwitz and of fifth order. Then it is clear that there are three stable hidden modes corresponding to the eigenvalues $\{-1,-1,-2/3\}$. By the discussion above these three hidden modes are uncontrollable from r and the pole of P at -1 corresponds to an unobservable mode from u.

Another controller C can be designed that will satisfy the specifications and will introduce less hidden modes. First, notice that no single degree of freedom controller can meet the specifications [14]. A simplified design approach for a two degrees of freedom controller is explained next. Since T is the same and T=NX=(s-1)X, X is the same. So we need to design C_y. We can do this by designing L. In order to minimize the number of hidden modes, the poles of L should be the same as the poles of X and no poles of L should be zeros of the plant. Then a possible choice for L is

$$L = \frac{\kappa}{(s+\alpha)(s+\beta)}, \tag{19}$$

where κ is a constant. The conditions for the compensator to be internally stabilizing and proper are X, L and $(1-LN)D^{-1}$ must be proper and stable. Only the latter one needs further checking:

$$(1-LN)D^{-1} = \frac{(s+\alpha)(s+\beta) - (s-1)\kappa}{(s-2)(s+1)(s+\alpha)(s+\beta)}. \tag{20}$$

The transfer function in (20) is stable if and only if $(2+\alpha)(2+\beta)=\kappa$, that is, $\alpha=(\kappa-4-2\beta)/(2+\beta)$. Then, for $\alpha>0$ need

$$0 < \beta < \frac{\kappa-4}{2} \tag{21}$$

or choose $\alpha>0$, $\beta>0$ and $\kappa=(2+\alpha)(2+\beta)$. By choosing κ in this way, a compensator $C=[-C_y \quad C_r]$ that attains the desired closed-loop transfer funtion is

$$C = \left[\frac{-\kappa(s-2)(s+1)}{(s+\alpha)(s+\beta)-(s-1)\kappa} \quad \frac{(s-2)(s+1)}{(s+\alpha)(s+\beta)-(s-1)\kappa} \right], \tag{22}$$

where there is a pole-zero cancellation at s=-2 in C_y and C_r. Suppose $\alpha=1$ and $\beta=3$, then $\kappa=15$ and

$$C = \left[-\frac{15(s+1)}{s-9} \quad \frac{s+1}{s-9} \right]. \tag{23}$$

For C in (23) there is only one hidden mode due to the interconnection of the controller and the plant; the pole of P at s=-1 corresponds to an uncontrollable mode from r and to an unobservable mode from u.

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