MODEL BASED NETWORKED CONTROL SYSTEMS: A DISCRETE TIME LIFTING APPROACH.

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Abstract.

In this paper, discrete-time Model-Based Networked Control Systems (MB-NCS) are studied. A lifting process is applied to the original MB-NCS configuration resulting in a Linear Time-Invariant (LTI) system. Necessary and sufficient conditions are given for the asymptotic stability of the system with instantaneous feedback in terms of h, the periodic update constant, and in terms of h and τ for the intermittent feedback case, where τ is the time interval in which the loop remains closed. The lifting process is also applied to a more general MB-NCS configuration in which the controller is connected to the actuator and plant by means of the digital communication network resulting in a multirate system; now, the controller is updated every nT time units while the plant is updated every mT time units, similarly, necessary and sufficient conditions for asymptotic stability are given in terms of the parameters n and m.

I. Introduction.

In recent years, control networks have been replacing traditional point-to-point wired systems. In networked control systems, the different elements, plants, controllers, sensors, and actuators are connected through a digital communication network with limited bandwidth. The new challenges that this implementation has brought are well documented [2, 11, 18, 21]. Perhaps the most relevant is the limitation on bandwidth; many researchers have studied different problems related to bandwidth restrictions such the state estimation problem under limited network capacity [24] or the minimum bit rate required to stabilize a Network Control System (NCS), [5, 19]. Other authors have focused on reducing network communication maintaining the system stable or keeping some level of performance. Georgiev and Tilbury [10] use the packet structure more efficiently, that is, reduction on communication is obtained by sending packets of information using all data bits available (excluding overhead). For the sequence of sensor data received, the controller needs to find a control sequence instead of a single control value. Otanez *et. al*. [20] use deadbands at each node to record the last value sent to the network and compares that value to the current one, taking a decision on sending the current information or not; the node does not send the new data if it is within the deadband. Stability and performance are studied for this implementation and an optimal deadband is found through simulations according to the network usage and the Integral of Absolute Value Error (IAE). Walsh, *et. al.* [23] introduced a network control protocol Try-Once-Discard (TOD) to allocate network resources to the different nodes in a Networked Control System; all of them may access the network at any time assuming each access occurs before the Maximum Allowable Transfer Time (MATI).

A type of NCS called Model Based Networked Control Systems (MB-NCS) aims to reduce communication over the network by incorporating an explicit model of the system to be controlled. The state of this model is used for control when no feedback is available (open loop). When the loop is closed, the state of the model is updated with new information, namely, the state of the real system. The MB-NCS framework is able to reduce network communication; consequently, the network is available for other uses, reducing time delays and bandwidth limitations.

Pioneering work in MB-NCS by Montestruque and Antsaklis [16, 17] provided necessary and sufficient conditions for stability for the case when the update intervals are constant; the output feedback and network delay case were also studied. In an extension, the same authors [18] also presented results when the update intervals are time-varying and follow different probabilistic distributions. In a related work Estrada, *et. al.* [6] introduced MB-NCS to intermittent feedback control resulting in improved performance and longer permissible update intervals. Recently, the intermittent control concept has been successfully applied to control systems [8, 9, 13, 23]. Ronco, *et. al.* introduced intermittent control in the context of Model Predictive Control to take account of the open-loop inter-sample behavior of an underlying continuous time system. Gawthrop and Wang [8, 9] extended the previous work by using basis function generator for the input signal for the unconstrained optimization problem and by substituting a clock trigger by an event trigger respectively. Li *et. al.* [13] discussed the application of intermittent control for the stabilization of a class of nonlinear systems.

In all the mentioned work on MB-NCS it is assumed that the network exists only between the sensor and the controller node while the controller is connected directly to the actuator and plant, that is the input generated by the controller is available to the plant at all times without delays or losses.

In this paper we will revisit the traditional configuration for MB-NCS and then we will study the case when a communication network exists on both sides of the control loop; lifting techniques are going to be applied to the analysis of the resulting multi-rate system.

II. Discrete-time and continuous-time lifting.

The lifting process has the purpose of extending the input and output spaces properly in order to obtain a Linear Time Invariant (LTI) system description for sampled-data, multi-rate, or linear time-varying periodic systems. This section provides a brief discussion on lifting based on [3]. These mathematical tools will be later used in the analysis of discrete-time MB-NCS.

A. Lifting discrete-time signals and systems.

Let us first consider a discrete-time signal *v(k)* that is referred to the sub-period *h/n* of the underlying clock with base period *h*, and *n* is some positive integer. This means that *v(0)* occurs at time *t=0*, *v(1)* at *t=h/n*, *v(2)* at *t=2h/n*, etc.

The lifted signal $\underline{v}$ is defined as follows:

If $v = \{v(0), v(1), v(2), \ldots\}$ then

$$\underline{v} = \left\{ \begin{bmatrix} v(0) \\ v(1) \\ \vdots \\ v(n-1) \end{bmatrix}, \begin{bmatrix} v(n) \\ v(n+1) \\ \vdots \\ v(2n-1) \end{bmatrix}, \ldots \right\} \tag{II.1}$$

The dimension of the lifted signal $\underline{v}(k)$ is *n* times the dimension of the original signal *v(k)* and is regarded to the base period i.e. $\underline{v}(k)$ occurs at time *t=kh*.

The lifting operator *L* is defined to be the map $v \rightarrow \underline{v}$. The inverse operator L^{-1} exists and is defined as follows:

If

$$\psi = \left\{ \begin{bmatrix} \psi_1(0) \\ \psi_2(0) \\ : \\ \psi_n(0) \end{bmatrix}, \begin{bmatrix} \psi_1(1) \\ \psi_2(1) \\ : \\ \psi_n(1) \end{bmatrix}, \dots \right\} \tag{II.2}$$

and $v = L^{-1}\psi$

then $v = \{\psi_1(0), \psi_2(0), \dots, \psi_n(0), \psi_1(1), \psi_2(1), \dots, \psi_n(1), \dots \}$.

A relevant feature about lifting is that it preserves inner products and norms. Let us see this for the case of l_2- norms:

$$v \in l_2(\mathbb{Z}_+, \mathbb{R}^m)$$
$$\underline{v} \in l_2(\mathbb{Z}_+, \mathbb{R}^{nm})$$

$$\|v\|_2^2 = v(0)^T v(0) + v(1)^T v(1) + \dots \tag{II.3.a}$$

$$\begin{aligned} \|\underline{v}\|_2^2 &= \underline{v}(0)^T \underline{v}(0) + \underline{v}(1)^T \underline{v}(1) + \dots \\ &= \begin{bmatrix} v(0) \\ : \\ v(n-1) \end{bmatrix}^T \begin{bmatrix} v(0) \\ : \\ v(n-1) \end{bmatrix} + \begin{bmatrix} v(n) \\ : \\ v(2n-1) \end{bmatrix}^T \begin{bmatrix} v(n) \\ : \\ v(2n-1) \end{bmatrix} + \dots \\ &= v(0)^T v(0) + v(1)^T v(1) + \dots \\ &= \|v\|_2^2 \end{aligned} \tag{II.3.b}$$

Now, let us consider a discrete-time Finite Dimensional Linear Time Invariant (FDLTI) system G_d with underlying period *h/n*. Lifting the input and output signals so that the lifted signals correspond to the base period *h* results in the lifted system: $\underline{G}_d = L G_d L^{-1}$.

Assuming the state space representation of the original system G_d is known and given by:

$$x(k+1) = Ax(k) + Bu(k)$$
$$y(k) = Cx(k) + Du(k) \tag{II.4}$$

Then the state space representation for the lifted system $\underline{G}_d$ is given by:

$$x((k+1)h) = A^n x(kh) + \begin{bmatrix} A^{n-1}B & A^{n-2}B & \ldots & B \end{bmatrix} \underline{u}(kh)$$
$$\underline{y}(kh) = \begin{bmatrix} C \\ CA \\ : \\ CA^{n-1} \end{bmatrix} x(kh) + \begin{bmatrix} D & 0 & \ldots & 0 \\ CB & D & \ldots & 0 \\ : & & & \\ CA^{n-2}B & CA^{n-3}B & \ldots & D \end{bmatrix} \underline{u}(kh) \tag{II.5}$$

B. Lifting continuous-time signals and systems.

Although continuous-time lifting is not going to be used in this paper, it is interesting to see the specific characteristics of the resulting lifted system.

Let us start with a signal *u(t)* in the extended space $l_{2e}(\mathbb{R})$, i.e.

$$\int_0^T u(\tau)^T u(\tau)\, d\tau \quad < \infty \qquad \forall T > 0 \tag{II.6}$$

Define the sampling intervals:

$\ldots,[-h,0),[0,h),[h,2h),\ldots$

And denote u_k the piece of u in the k^{th} sampling interval $[kh,(k+1)h)$, $0 \le t < h$.

Then each u_k exists in the space: $\mathbb{K} := l_2([0,h),\mathbb{R}^n)$, which is an infinite-dimensional Hilbert space with inner product and norm:

$$\langle v, w \rangle = \int_0^h v(\tau)^T w(\tau) d\tau$$
$$\|v\| = \left(\int_0^h v(\tau)^T v(\tau)\, d\tau \right)^{1/2} \tag{II.7}$$

Let the discrete-time signal

$$\underline{u} = \begin{bmatrix} \vdots \\ u_{-1} \\ u_0 \\ u_1 \\ u_2 \\ \vdots \end{bmatrix} \tag{II.8}$$

be the lifted signal. Note that it exists in the space $l(\mathbb{Z},\mathbb{K})$, where each u_k exists in $\mathbb{K}$.

In a similar way to the discrete-time case, it can be shown that the continuous lifting operator L that maps $u \to \underline{u}$ preserves inner products and norms. Assume that continuous lifting is applied to the input and output of a continuous-time FDLTI (input, output, and state evolve in finite-dimensional Euclidean spaces with dimension ε in general) system G with state space representation:

$$\begin{aligned} \dot{x}(t) &= Ax(t) + Bu(t) \\ y(t) &= Cx(t) + Du(t) \end{aligned} \tag{II.9}$$

The resulting lifted system $\underline{G} = LG\,L^{-1}$ has a state space representation given by

$$\begin{aligned} \xi(k+1) &= \underline{A}\xi(k) + \underline{B}u(k) \\ y(k) &= \underline{C}\xi(k) + \underline{D}u(k) \end{aligned} \qquad \text{(II.10)}$$

Where the operators $\underline{A}, \underline{B}, \underline{C}, \underline{D}$ are determined by:

$$\begin{aligned} &\underline{A} : \varepsilon \to \varepsilon, \qquad \underline{A}x = e^{hA}x \\ &\underline{B} : \mathbb{K} \to \varepsilon, \qquad \underline{B}u = \int_0^h e^{(h-\tau)A}Bu(\tau)d\tau \\ &\underline{C} : \varepsilon \to \mathbb{K}, \qquad (\underline{C}x)(t) = Ce^{tA}x \\ &\underline{D} : \mathbb{K} \to \mathbb{K} \qquad (\underline{D}u)(t) = Du(t) + \int_0^t Ce^{(t-\tau)A}Bu(\tau)d\tau \end{aligned} \qquad \text{(II.11)}$$

As it can be seen, the continuous-time lifted system is a system whose input and output are infinite-dimensional while the dimension of the state remains the same; the lifted system is represented not by matrices but by operators. In general, lifting results in a LTI system, and the available tools and results for LTI systems are applicable to the lifted system as well.

III. Lifting discrete-time Model Based Networked Control Systems.

A. Instantaneous feedback.

As it was mentioned earlier, MB-NCS make use of an explicit model of the plant which is added to the controller node to compute the control input based on the state of the model rather than on the plant state. Fig. 1 shows a basic MB-NCS configuration, where the network exists only on the sensor-controller side while the controller is connected directly to the actuator and the plant.

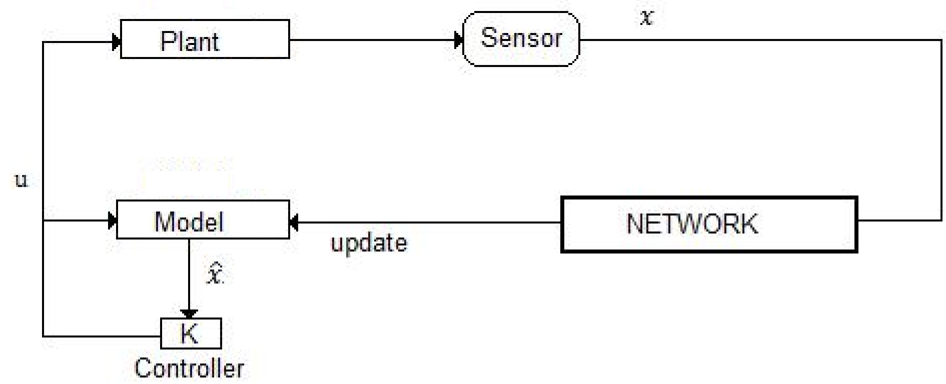


Fig. 1. Representation of a Model-Based Networked Control System.

The dynamics of the plant and the model are given respectively by:

$$\begin{aligned} x(k+1) &= Ax(k) + Bu(k) \\ \hat{x}(k+1) &= \hat{A}\hat{x}(k) + \hat{B}u(k) \end{aligned} \tag{III.1}$$

where x is the state of the plant, $\hat{x}$ is the state of the model, and the matrices $\hat{A}, \hat{B}$ represent the available model of the system matrices A,B.

The input u is defined for the case of instantaneous feedback as:

$$u(k) = \begin{cases} Kx(k) & k = ih \\ K\hat{x}(k) & k = ih + j \end{cases} \tag{III.2}$$

For $i = 0,1,2....$ and $0 < j < h$ (j is also an integer).

The MB-NCS of Fig. 1 can be seen as linear time-varying system as shown in Fig. 2.a.

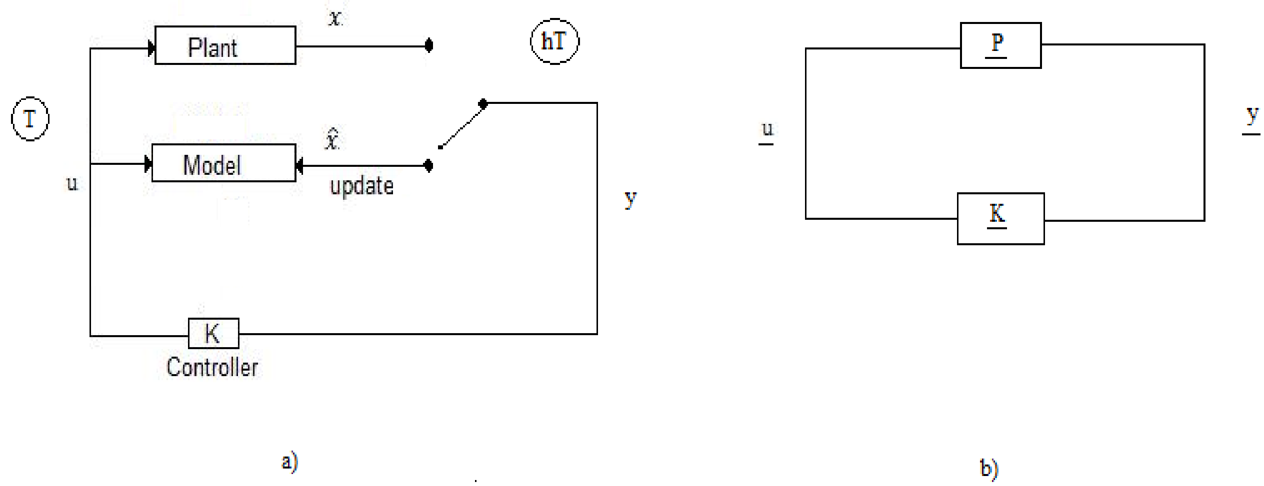


Fig. 2. Equivalent systems to a MB-NCS. a) Linear time-varying system. b) Lifted system.

The system after applying lifting is represented in Fig. 2.b. The input $\underline{u} \in \mathbb{R}^{hm}$ and output $\underline{y} \in \mathbb{R}^{hn}$ of the lifted system are given by the equations:

$$\underline{u}(kh) = \begin{bmatrix} u(kh) \\ u(kh+1) \\ : \\ u((k+1)h-1) \end{bmatrix} = \begin{bmatrix} Kx(kh) \\ K\hat{x}(kh+1) \\ : \\ K\hat{x}((k+1)h-1) \end{bmatrix} \quad \text{(III.3.a)}$$

$$\underline{y}(kh) = \begin{bmatrix} I \\ \hat{A} \\ \hat{A}^2 \\ : \\ \hat{A}^{h-1} \end{bmatrix} x(kh) + \begin{bmatrix} 0 & 0 & \dots & 0 \\ B & 0 & \dots & 0 \\ AB & B & \dots & 0 \\ : & & & : \\ A^{h-2}B & A^{h-3}B & \dots & 0 \end{bmatrix} \quad \text{(III.3.b)}$$

The dimension of the state is preserved, and the state equation expressed in terms of the lifted input is given by:

$$x((k+1)h) = A^h x(kh) + \begin{bmatrix} A^{h-1}B & A^{h-2}B & \dots & B \end{bmatrix} \underline{u}(kh) \quad \text{(III.4)}$$

The new controller is of the form: $\underline{K} = \begin{bmatrix} K & 0 & 0 & .. \\ 0 & K & 0 & \\ 0 & 0 & K & \\ : & & & \end{bmatrix}$

where each zero block has the same dimensions as *K*.

Theorem 1. The lifted system is asymptotically stable if only if the eigenvalues of:

$$A^h + \sum_{j=0}^{h-1} A^{h-1-j} BK(\hat{A} + \hat{B}K)^j \qquad \text{(III.5)}$$

lie inside the unit circle.

Proof: To prove this theorem we first show that the state equation (III.4) is equivalent to the state equation that characterizes the autonomous linear time invariant system:

$$x((k+1)h) = (A^h + \sum_{j=0}^{h-1} A^{h-1-j} BK(\hat{A} + \hat{B}K)^j)x(kh) \qquad \text{(III.6)}$$

and, since the lifted system is a LTI system we simply apply basic results for LTI systems, i.e. a system given by *x(k+1)=Ax(k)* is asymptotically stable if only if the eigenvalues of *A* lie inside the unit circle.

Equation (III.6) can be obtained by directly substituting (III.3a) in (III.4), and then substituting the value of each individual output by its equivalent in terms of the state *x(kh),* i.e.

$$\begin{aligned} &\hat{x}(kh+1) = \hat{A}x(kh) + \hat{B}u(kh) = (\hat{A} + \hat{B}K)x(kh) \\ &\hat{x}(kh+2) = (\hat{A} + \hat{B}K)^2 x(kh) \\ &: \end{aligned} \qquad \text{(III.7)}$$

The resulting equation can simply be expressed as in (III.6). ■

Example 1. Consider the following plant and model:

$$A = \begin{pmatrix} 1.15 & 0 \\ 0 & 0.9 \end{pmatrix} \qquad B = \begin{pmatrix} 1.01 \\ 1.1 \end{pmatrix}$$

$$\hat{A} = \begin{pmatrix} 1.1497 & 0.0196 \\ 0.0086 & 0.9272 \end{pmatrix} \qquad \hat{B} = \begin{pmatrix} 1.0109 \\ 1.1018 \end{pmatrix}$$

$$K = \begin{pmatrix} -1.2225 & 0.0633 \end{pmatrix}$$

Figure 3 shows the absolute value of the eigenvalues of the test matrix in theorem 1. We can implement our controller and model over a network and receive measurements every h tics and remain stable as long as that h produces eigenvalues of equation (III.5) with absolute value less than one.

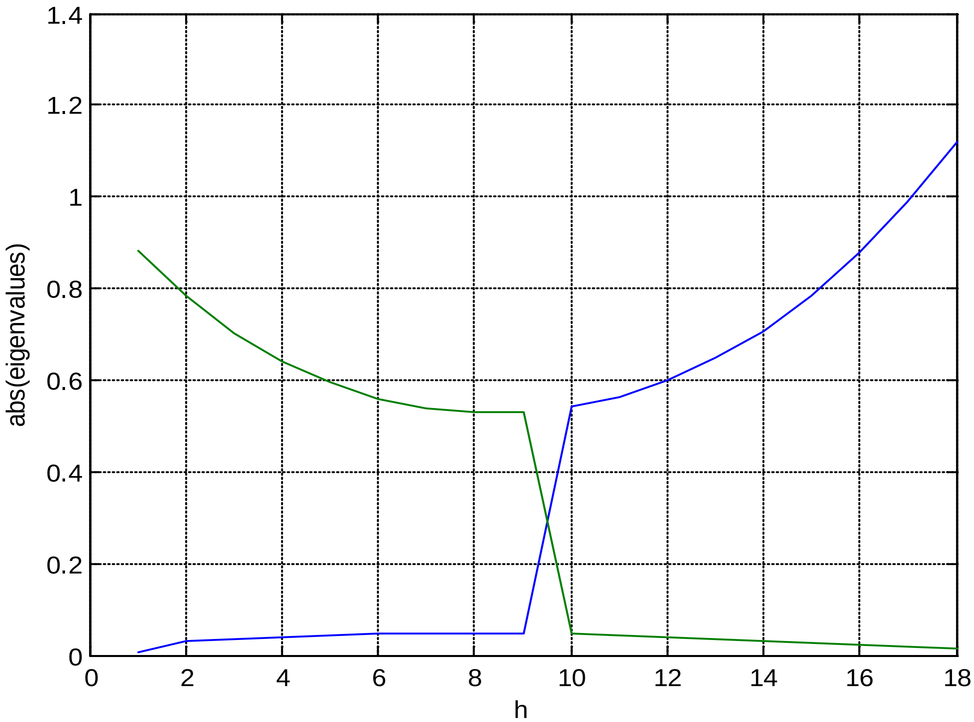
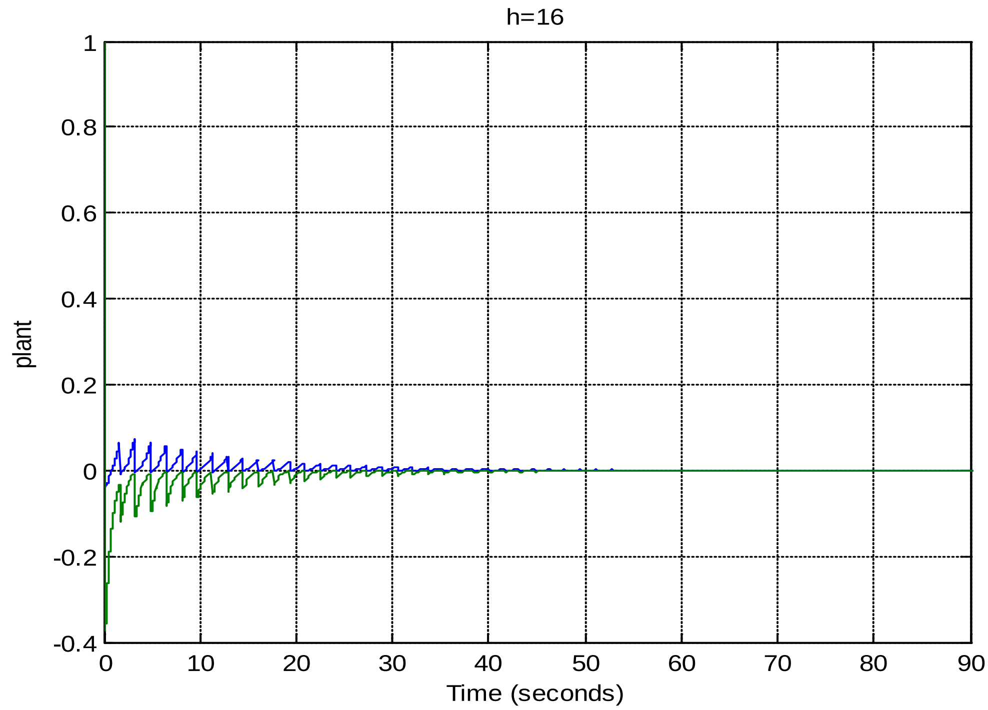
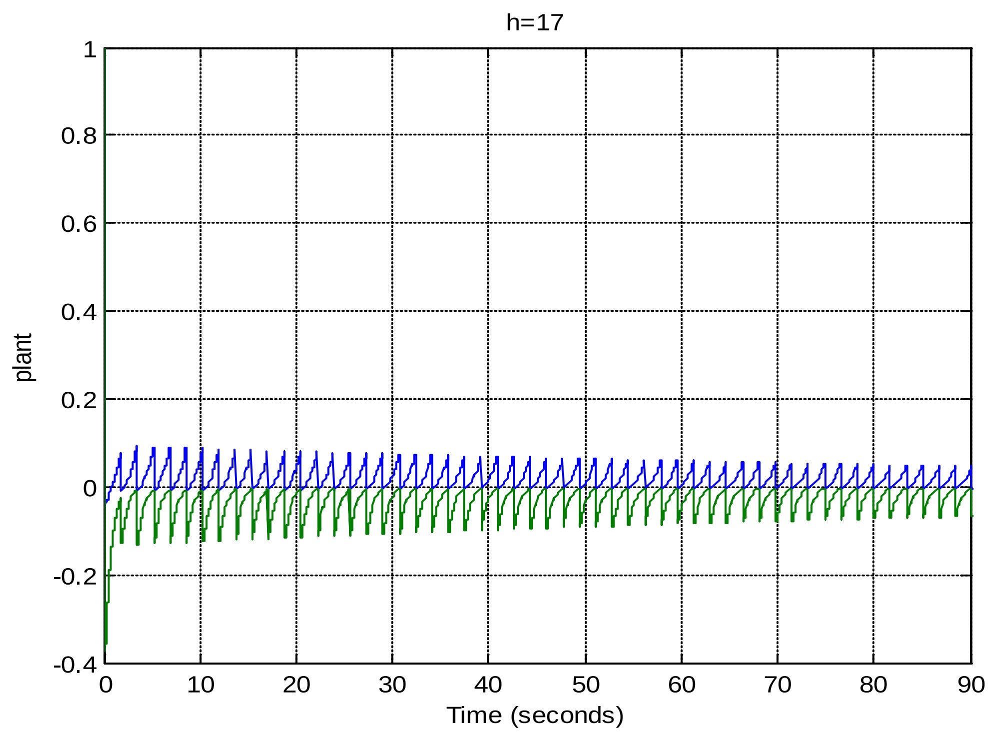


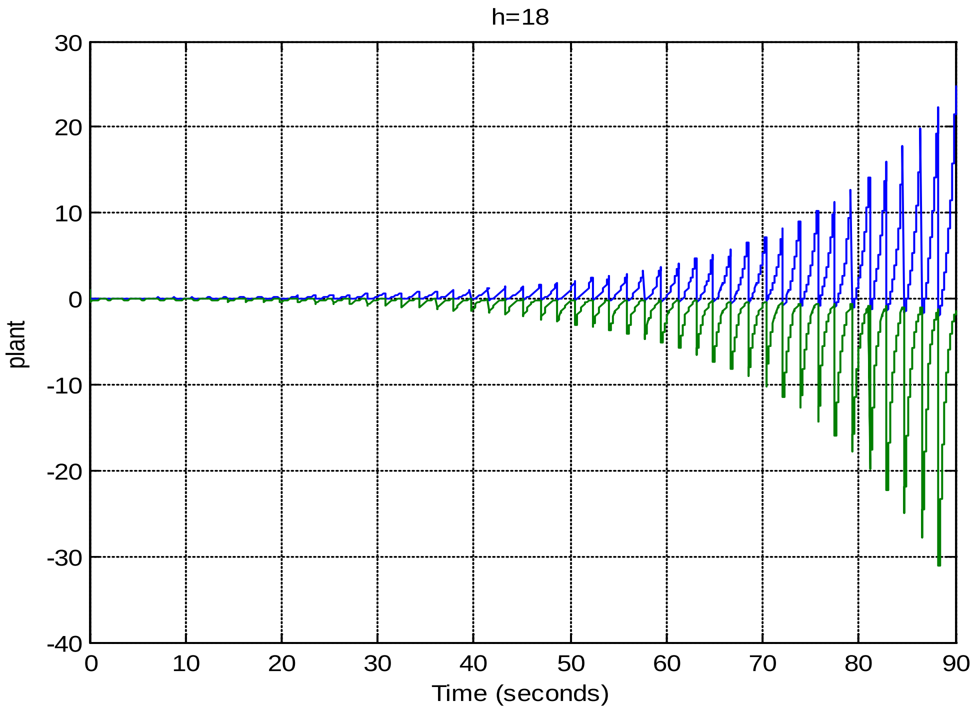
Fig. 3. Absolute value of the eigenvalues of (III.5), showing the values of h for which the system remains stable.


h=16
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
plant
0
10
20
30
40
50
60
70
80
90
Time (seconds)

a)


h=17
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
plant
0
10
20
30
40
50
60
70
80
90
Time (seconds)

b)



c)

Fig. 4. Response of the plant for different values of *h* around the critical value for stability.

B. Intermittent feedback.

The input *u* of the MB-NCS of Fig. 1 with intermittent feedback is defined as

$$u(k) = \begin{cases} Kx(k) & h_l \le k \le h_l + \tau_l \\ K\hat{x}(k) & h_l + \tau_l < k < h_{l+1} \end{cases} \tag{III.8}$$

In this work we consider intermittent feedback with constant updates and constant closed loop times, that means, $h_{l+1} - h_l = h$, which represents how often we close the loop between the sensor and the controller, and $\tau_l = \tau < h$, which represents the constant number of clock tics that the loop remains closed, h and τ are positive integer numbers.

Theorem 2. The lifted system with intermittent feedback (III.8) is asymptotically stable if only if the eigenvalues of:

$$(A^{h-\tau} + \sum_{j=0}^{h-\tau-1} A^{h-\tau-1-j} BK(\hat{A} + \hat{B}K)^j)(A + BK)^\tau \tag{III.9}$$

lie inside the unit circle.

Proof: Similarly to theorem 1, we rewrite the state equation of the lifted LTI system in terms of only the state. To do this, we consider closed loop and open loop behaviors. For the interval $h \le k \le h + \tau$ the response of the system at time $kh + \tau$ is given by (closed loop response):

$$x(kh + \tau) = (A + BK)^\tau x(kh) \tag{III.10}$$

And the response of the system at time $kh + h$ in terms of the state at time $kh + \tau$ can be shown by following the procedure in theorem 1,

$$x(kh + h) = (A^{h-\tau} + \sum_{j=0}^{h-\tau-1} A^{h-\tau-1-j} BK(\hat{A} + \hat{B}K)^j) x(kh + \tau) \tag{III.11}$$

Then, the response of the system at time *(k+1)h* in terms of only the state at time *kh* is given by:

$$x((k+1)h) = (A^{h-\tau} + \sum_{j=0}^{h-\tau-1} A^{h-\tau-1-j} BK(\hat{A} + \hat{B}K)^j)(A + BK)^\tau x(kh) \tag{III.12}$$

Again, the resulting system is a LTI system, and stability is present when the eigenvalues of (III.9) lie inside the unit circle. ■

Example 2: We take the same example as before but with intermittent feedback. For simplicity we fix one of the variables (h or τ) and plot the eigenvalues of the test matrix over a range of interest of the remaining variable, see Fig 5.

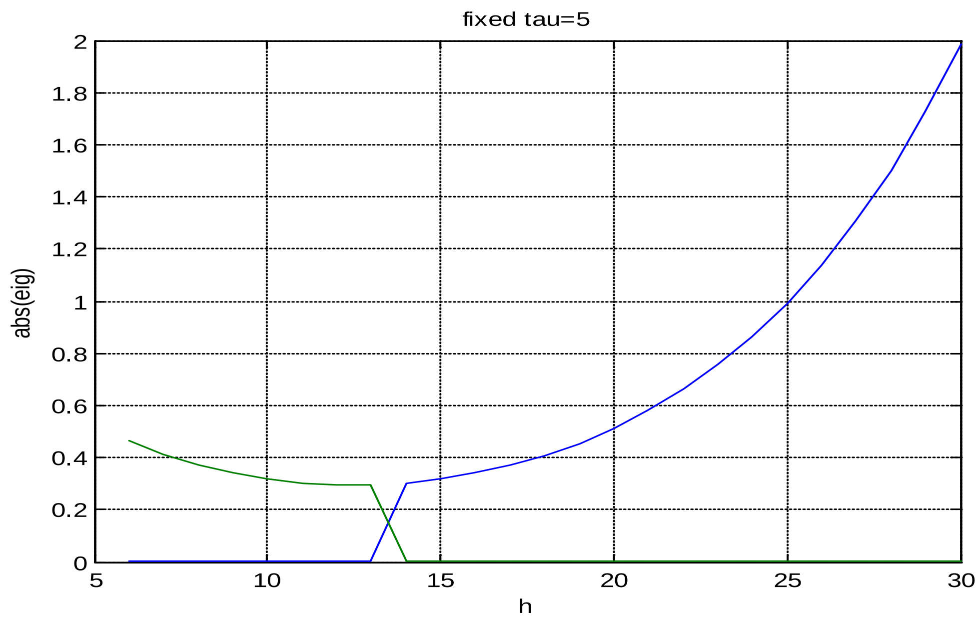
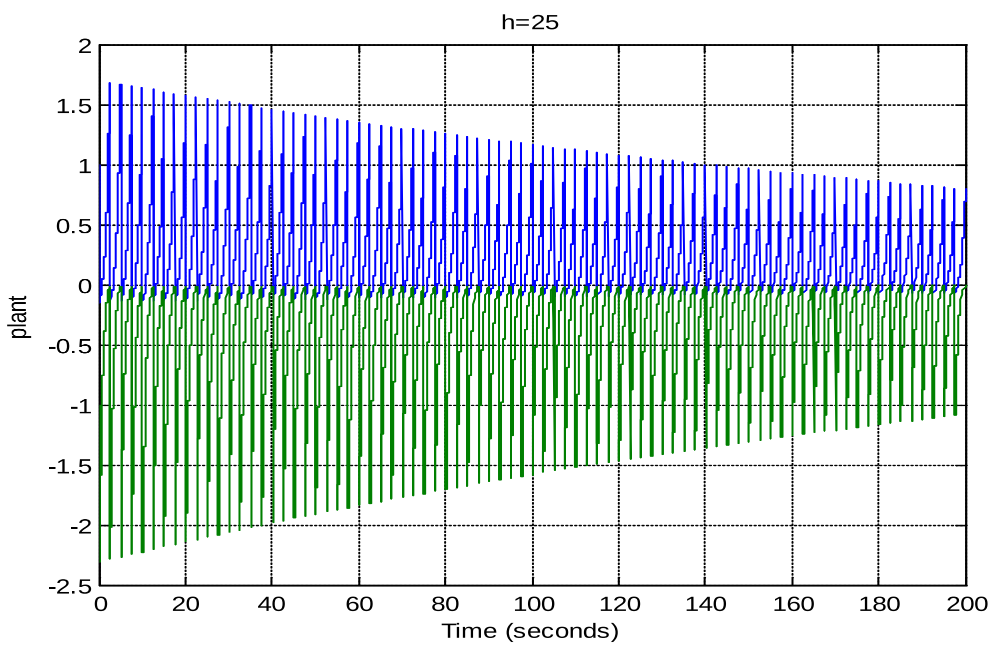
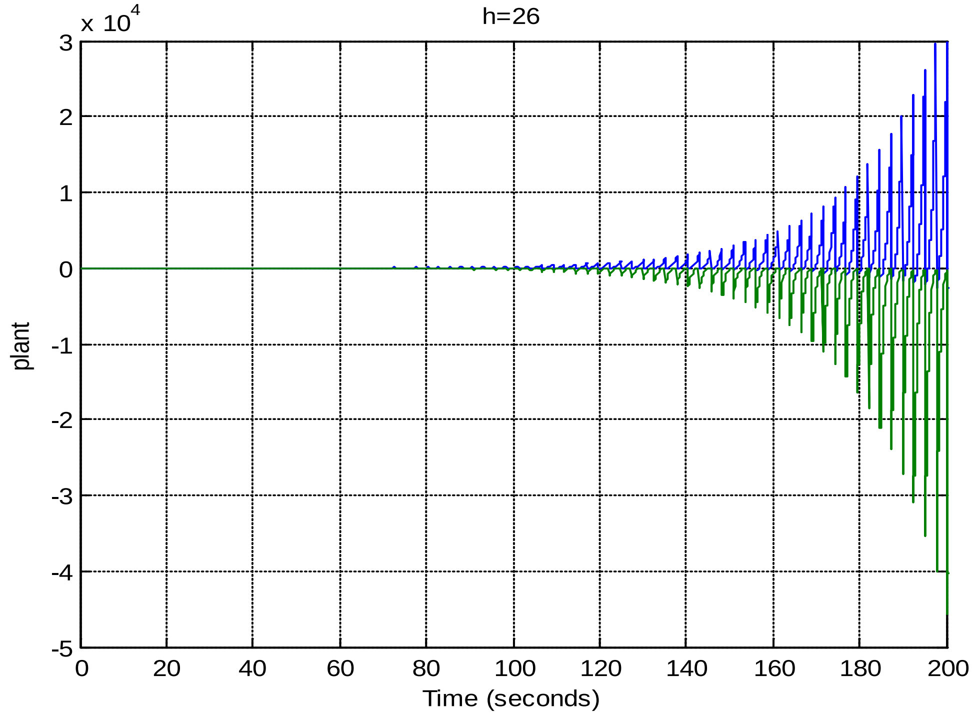


Fig. 5. Absolute value of the eigenvalues of (III.9), showing the values of h for which the system remains stable.



a)



b)

Fig. 6. Response of the plant for τ=5 and for values of h for which: a) stability is still preserved and b) system becomes unstable.

C. Double network path MB-NCS.

We call double network path MB-NCS a NCS in which not only the path from the sensor to the controller is implemented using a digital network but also the path from the controller to the actuator as well. The configuration is shown in Fig. 7; here the two switches are closed at different constant rates. As a starting point we wish to find the bounding values of *m* and *n* that preserve stability of the MB-NCS using instantaneous feedback. Note that in this case, between input updates, the input to the plant is held constant in the actuator and it is equal to the last received value.

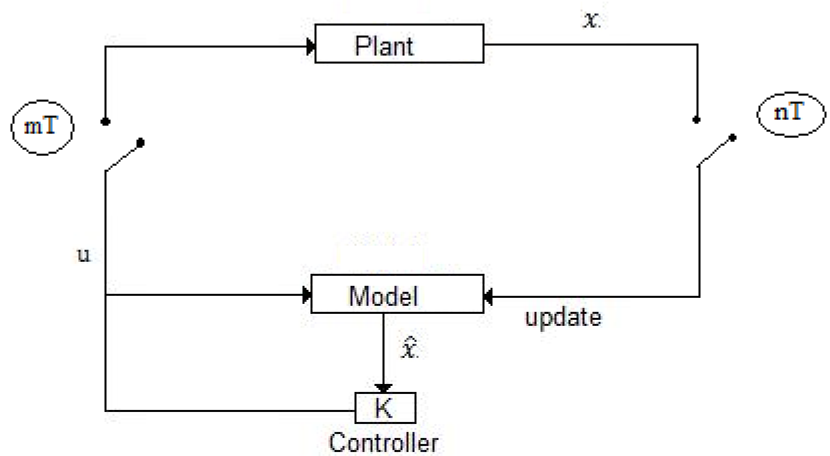


Fig. 7. Model-Based Networked Control System with communication network between sensor-controller and controller-actuator (plant).

Assume that $n \geq m$ (low measurement rate, which is typical in many implementations of physical systems) and $p=n/m$ is assumed to be an integer, for simplicity. Then we have:

Theorem 3: The lifted system corresponding to Fig. 7 is asymptotically stable if only if the eigenvalues of:

$$A^n + \sum_{i=0}^{p-1} H_i \Gamma^i \quad \text{(III. 13)}$$

lie inside the unit circle, where

$$H_i = \sum_{j=1}^{m} A^{n-im-j} BK \quad \text{(III.14a)}$$

$$\Gamma = (\hat{A} + \hat{B}K)^m \quad \text{(III.14b)}$$

Proof: Since $p=n/m$ is an integer the period of the equivalent linear time-varying periodic system is n. Taking a similar approach as in the last two cases we obtain a LTI system. In order to find the state equations of the lifted system let us describe the response of the system as a function of the input updates that take place every *m* clock tics. From equation (III.4) we obtain:

$$x(kn+n) = A^n x(kn) + [A^{n-1} \quad A^{n-2} \quad \dots \quad A^{n-m}]Bu(kn) +$$
$$[A^{n-m-1} \quad A^{n-m-2} \quad \dots \quad A^{n-2m}]Bu(kn+m) + \tag{III.15}$$
$$[A^{n-2m-1} \quad A^{n-2m-2} \quad \dots \quad A^{n-3m}]Bu(kn+2m) + \dots.$$

The input *u* is a function of the state of the plant at times *kn* and a function of the state of the model otherwise. The state of the model between sensor updates can be expressed in terms of the state of the plant at times *kn* as follows:

$$\begin{aligned} &u(kn) = Kx(kn) \\ &u(kn+m) = K\hat{x}(kn+m) = K(\hat{A}+\hat{B}K)^m x(kn) \\ &u(kn+2m) = K\hat{x}(kn+2m) = K(\hat{A}+\hat{B}K)^{2m} x(kn) \\ &\vdots \end{aligned} \tag{III.16}$$

Note that the model has access to the input that it generates at all times as it can be deduced from Fig 7. The network connection is between the controller and the plant, and the model is part of the controller. Equation (III.15) becomes:

$$x((k+1)n) = (A^n + H_0\Gamma^0 + H_1\Gamma^1 + H_2\Gamma^2 + \dots + H_{p-1}\Gamma^{p-1})x(kn)$$
$$= (A^n + \sum_{i=0}^{p-1} H_i\Gamma^i)x(kn) \tag{III.17}$$

where H_i and Γ are given in (III.14). Equation (III.17) represent a discrete-time LTI, therefore stability is achieved when the eigenvalues of (III.13) lie inside the unit circle. ■

Example 3:

$$A = \begin{pmatrix} 0.9 & 0.1 \\ 0 & 1.07 \end{pmatrix} \qquad B = \begin{pmatrix} 0.01 \\ 0.01 \end{pmatrix}$$

$$\hat{A} = \begin{pmatrix} 0.9117 & 0.1054 \\ 0.0360 & 1.0672 \end{pmatrix} \qquad \hat{B} = \begin{pmatrix} 0.0109 \\ 0.0117 \end{pmatrix}$$

$$K = \begin{pmatrix} -2.3294 & -17.6266 \end{pmatrix}$$

As in the intermittent case we have two variables, *n* and *m*, but for this case we expect a decrease in the necessary value of *n* as *m* increases. An appropriate way to proceed here then is as follows: first find the largest value of *n* for $m=1$, that is, find the value of *h* in theorem 1; then we select a value for *n* less than the value of *h* that we just found and we find the divisors of *n*, with this information we can plot the eigenvalues of (III.13) as a function of *m*. For the example in hand, the highest value of *h*, from theorem 1, is 29, so for illustrative purposes we can fix $n=12$.

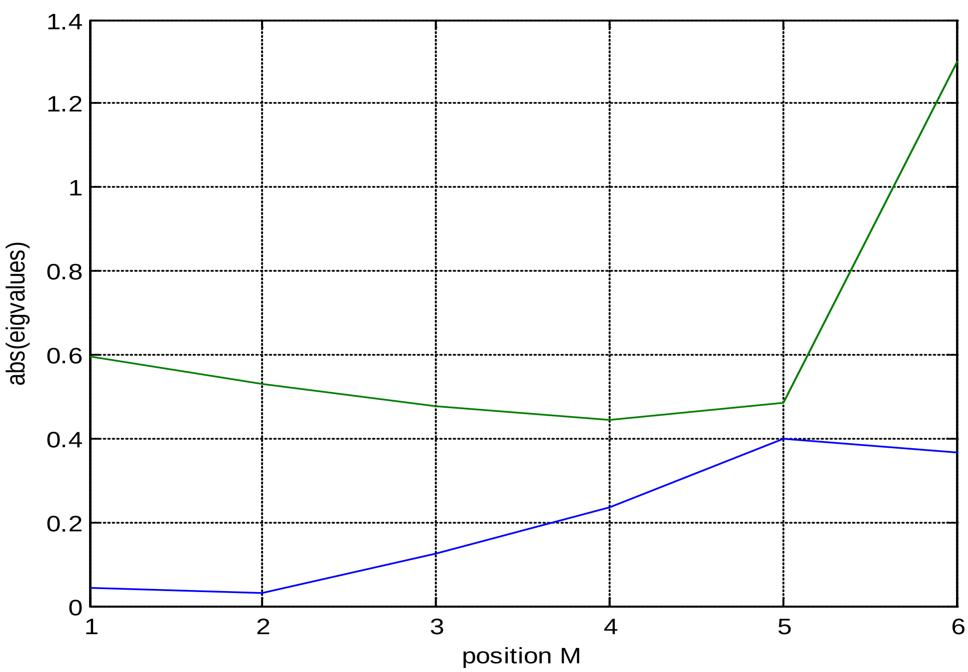
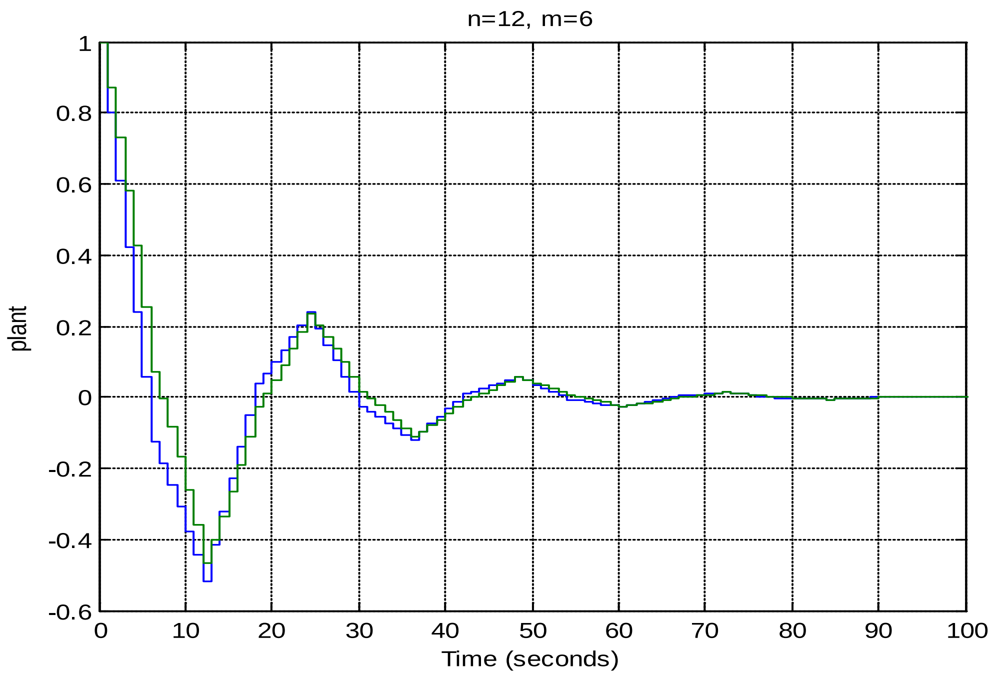
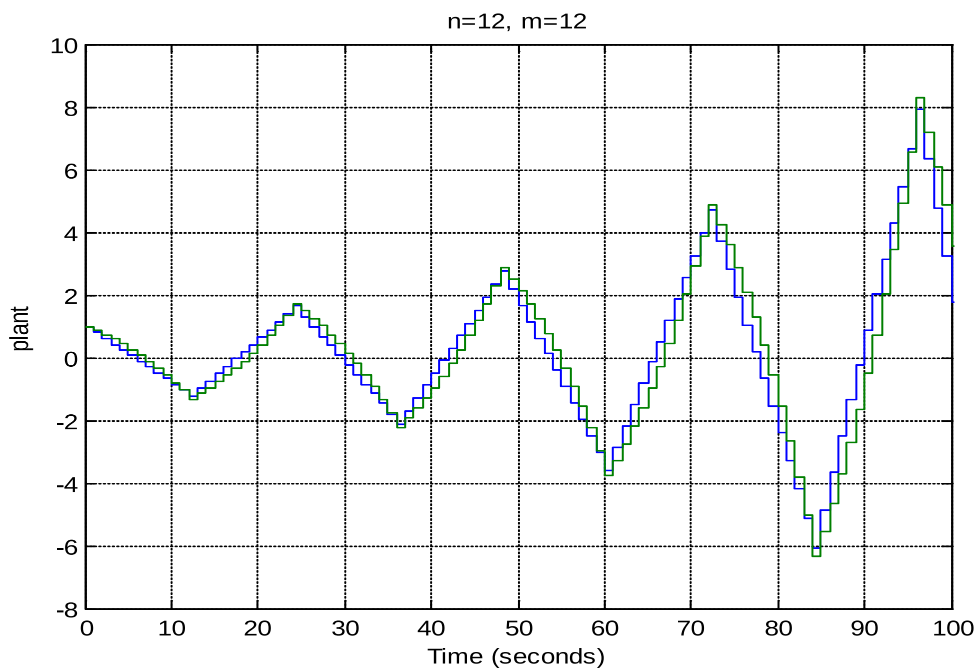


Fig. 8. Absolute value of the eigenvalues of (III.13) with $n=12$, showing the values of *m* for which the system remains stable.



a)



b)

Fig. 9. Response of the plant for n=12 and different values of m for which: a) stability is still preserved and b) system becomes unstable.

Note that in Fig. 8 the horizontal axis refers to the value in the vector $M = [1, 2, 3, 4, 6, 12]$ at that position; this vector contains the actual values that *m* takes, i.e. the last value of *m* is *M* at *6* which is *m=12*.

We can also consider to take *n=24* which is still less than the value of *h* (the choice of *n* is taken considering the existence of a large number of divisors). For this case the eigenvalues of equation (III.13) are shown in Fig. 10 and the response of the plant in Fig. 11. For Fig. 10 the vector that contains the possible values of *m* (horizontal axis) is defined as *M* = [1,2,3,4,6,8,12,24].

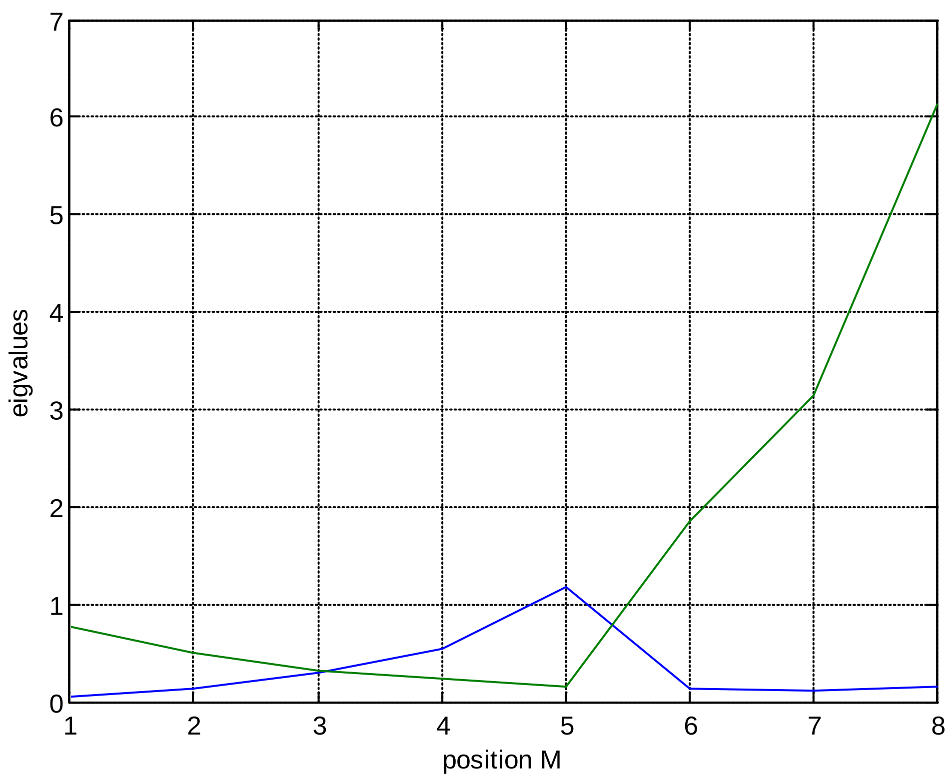
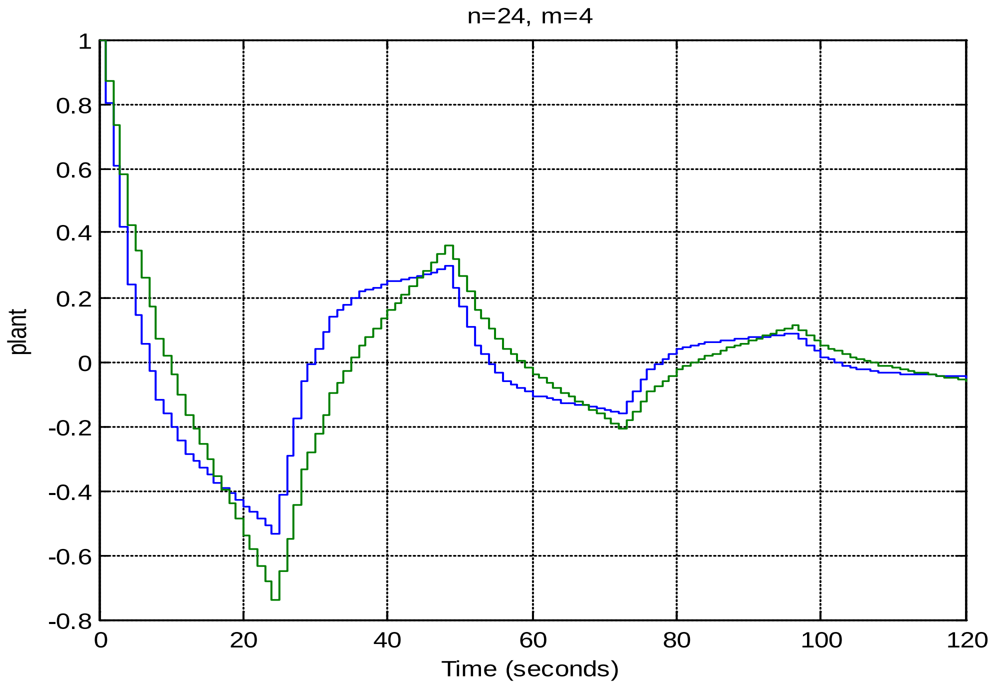
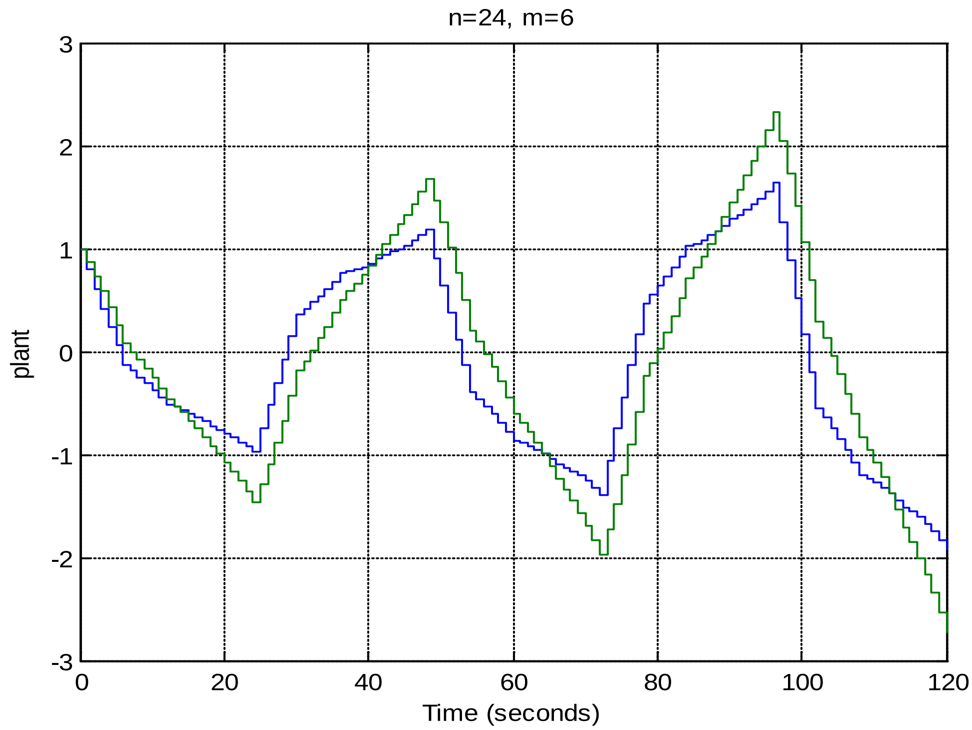


Fig. 10. Absolute value of the eigenvalues of (III.13) with *n=24*.



a)



b)

Fig. 11. Response of the plant for n=24 and different values of m for which: a) stability is still preserved and b) system becomes unstable.

Example 4. We could think that if a system is stable for some *n* and *m*, the same system will be stable for the same *n* and lower values of *m*, i.e. a faster input update; but this is not true in general. We take the same plant as in example 3 but in this case the knowledge of the plant dynamics is very limited so we choose the model to be:

$$\hat{A} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} \qquad \hat{B} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

For $m=1$ this system is stable only for very low values of *n* (1, 2), then we cannot proceed as in example 3; we try higher values of *n* in order to find the eigenvalues of equation (III.13) for different values of *m*, a case of interest is shown in Fig. 12, here the system is stable only for $m=4=M(4)$ and $m=6=M(5)$. The selected model holds the control input only for the first *mT* units of time after an update in the model has occurred, and then the state of the model is zero which makes the input zero until a new state update takes place. This situation makes sense since our knowledge of the dynamics of the plant is very imprecise; our estimate of the state becomes zero as we expect to stabilize the plant using the last measurement.

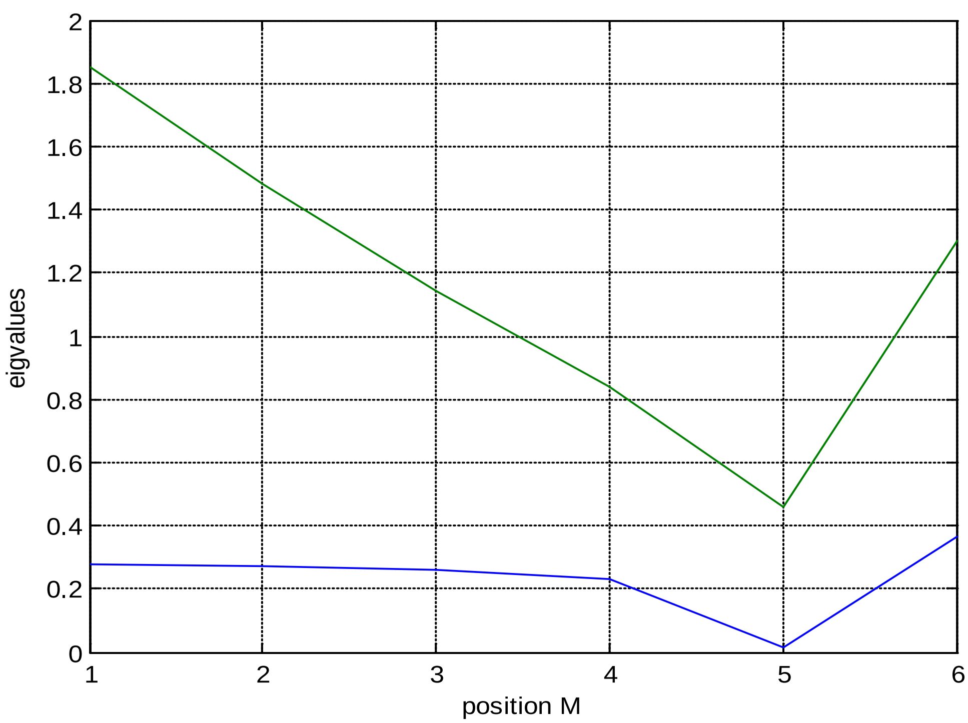
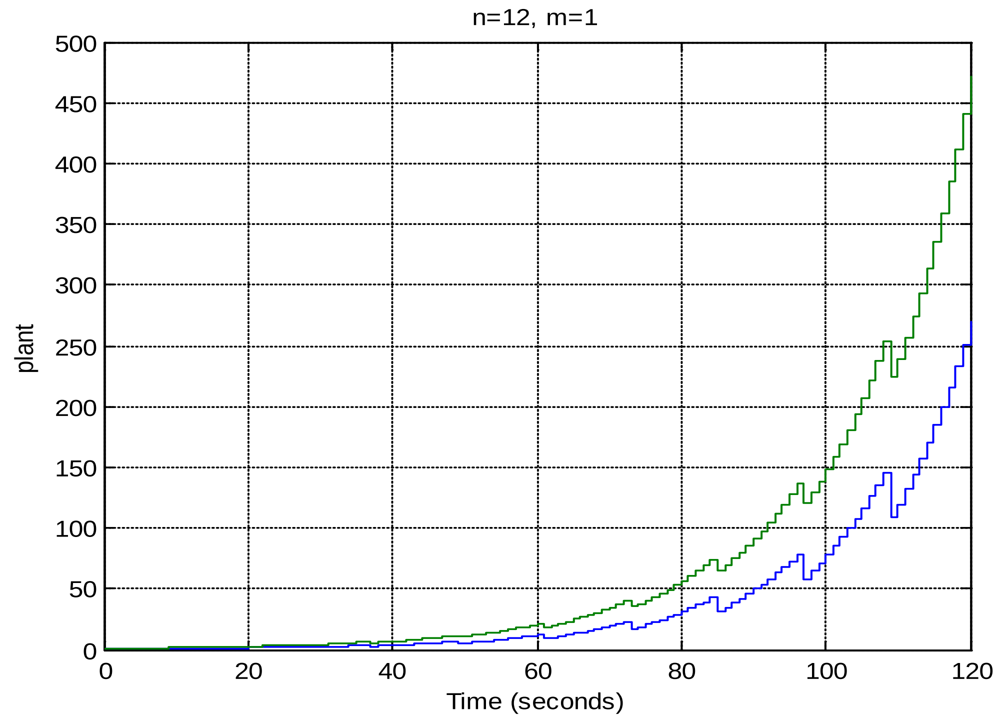
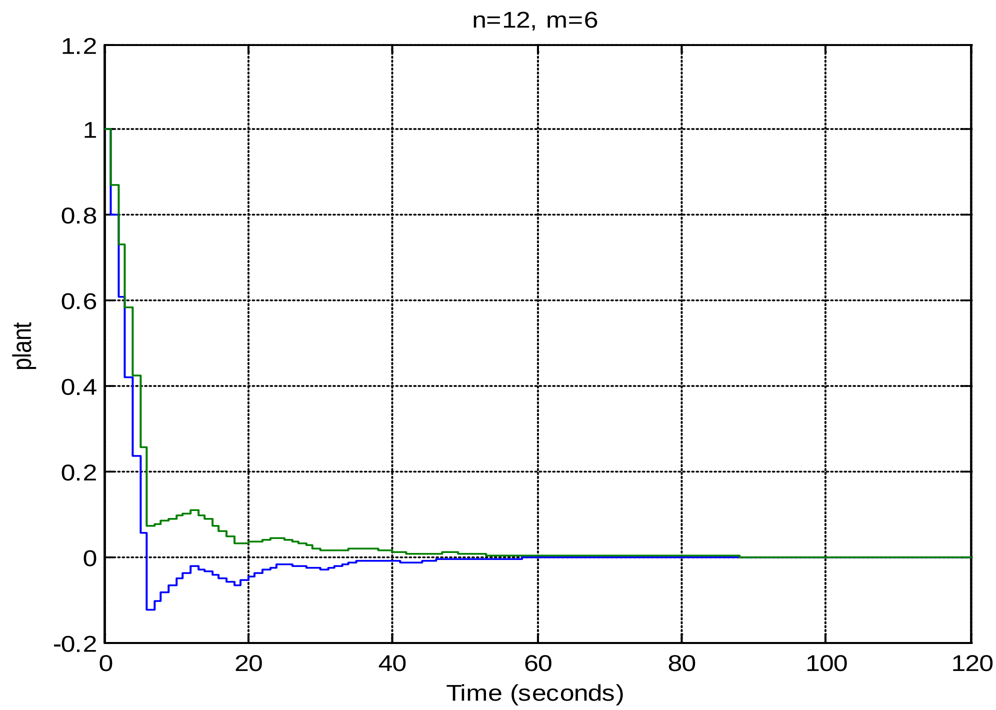


Fig.12. Absolute value of the eigenvalues of (III.13) with $n=12$ and model as in example 4.



a)



b)

Fig. 13. Response of the plant for n=12 and a) m=1 (unstable system) and b) m=6 (stable system).

IV. Conclusions and future work.

The Model-Based control of networked systems has been revisited in this paper making use of lifting procedures. Basic details on lifting were provided and this technique was applied to different configurations of Networked Control Systems that use a model of the plant to generate an estimate of the state between updates. The typical MB-NCS configurations (those in which the network is implemented only between the sensor and the controller) were analyzed and conditions for asymptotic stability were obtained. In addition, the more general configuration where the network is also present between the controller and the plant was also studied and analogous results were derived. These results represent the main contributions of the paper. There exists the interest in continuing working with the latest, more general configuration because it offers more flexibility in implementation.

References.

[1] B. Bamieh, J. B. Pearson, B.A. Francis, A. Tannenbaum, “A lifting technique for linear periodic systems with applications to sampled-data control”, Systems & Control Letters vol. 17, pp 79-88, 1991.

[2] J. Baillieul, P. J. Antsaklis, “Control and communication challenges in Networked Real-Time Systems”, Proceedings of the IEEE vol. 95, no.1, pp. 9-28, January 2007.

[3] T. Chen and B. Francis, “Optimal sampled data control systems”, Springer, 1995

[4] F. Ding and T. Chen, “Hierarchical identification of lifted state space models for general dual-rate systems”, IEEE Transactions on Circuits and Systems, Vol. 52, No. 6, June 2005.

[5] N. Elia, S. K. Mitter, “Stabilization of linear systems with limited information” IEEE Transactions on Automatic Control, Vol. 46, No.9, pp. 1384-1400, 2001.

[6] T. Estrada, H. Lin, and P. J. Antsaklis “Model based control with intermittent feedback” Proceedings of the 14th Mediterranean Conference on Control and Automation, 2006.

[7] E Fridman, A Seuret, J. P. Richard (2004), "Robust sampled-data stabilization of linear systems: an input delay approach", Automatica, Vol. 40, No.8, 2004.

[8] P. J. Gawthrop, L. Wang, “Event driven intermittent control” International Journal of Control, 2009.

[9] P. J. Gawthrop, L. Wang ¨Intermittent model predictive control¨ Proceedings of the Institution of Mechanical Engineers, Pt. B, Journal of Engineering Manufacture vol. 221 no.7, 2007.

[10] D. Georgiev, D. M. Tilbury “Packet-Based control” Proc. of the American Control Conference, pp. 329 – 336, 2004.

[11] J. P. Hespanha, P. Naghshtabrizi, and Y. Xu, “A survey of recent results in Networked Control Systems” Proceedings of the IEEE vol. 95, no.1, pp. 138-162, January 2007.

[12] P. P. Khargoneskar, K. Poolla, A. Tannenbaum, “Robust control of linear time-invariant plants using periodic compensation” IEEE Trans. on Automatic Control vol. AC-30, Novemeber 1985.

[13] C. Li, G. Feng, X. Liao, “Stabilization of nonlinear systems via periodically intermittent control” IEEE Trans. on Circuits and Systems, 2007.

[14] D. Li, S. L. Shah, T. Chen, “Analysis of dual-rate inferential control systems” Automatica, vol. 38, pp. 1053-1059, 2002.

[15] D. Li, S. L. Shah, T. Chen, “Identification of fast-rate models from multirate data” International Journal of Control, Vol. 74 No. 7, 2001.

[16] L. A. Montestruque, and P. J. Antsaklis, “Model-based networked control systems: necessary and sufficient conditions for stability” Proceedings of the 10th Mediterranean Conference on Control and Automation, 2002.

[17] L. A. Montestruque, and P. J. Antsaklis “On the model-based control of networked systems” Automatica, pp. 1837 – 1843, 2003.

[18] L. A. Montestruque, P. J. Antsaklis “Stability of model-based networked control systems with time varying transmission times”. IEEE Transactions on Automatic Control, Vol. 49, No. 9, pp. 1562-1572, Sept. 2004.

[19] J. M. Moyne, D. M. Tilbury, “The emergence of industrial control networks for manufacturing control, diagnostics, and safety data” Proc. of the IEEE, Vol. 95, No. 1, pp. 29 – 47, January 2007.

[20] G. Nair, R. Evans, “Communication-limited stabilization of linear systems” Proc. of the Conference on Decision and Control, pp.1005-1010, 2000.

[21] P. G. Otanez, J.R. Moyne, D.M. Tilbury “Using deadbands to reduce communication in Networked Control Systems” Proc. of the American Control Conference 2002 pp. 3015 - 3020.

[22] L. Schenato, B. Sinopoli, M. Franceschetti, K. Poolla, and S. Sastry, “Foundations of control and estimation over lossy networks”, Proceedings of the IEEE, Vol. 95, No. 1, January 2007.

[23] E. Ronco, T. Arsan,P. J. Gawthrop, “Open-loop intermittent feedback control: practical continuous-time GPC” IEE Proceedings-Control Theory and Applications, 1999

[24] G. C. Walsh, H. Ye, L. Bushnell, “Stability analysis of Networked Control Systems”. IEEE Trans. on Control Systems Technology. May 2002.

[25] W. S. Wong, R. W. Brockett “Systems with finite communication bandwidth constraints - Part I: State estimation problems” Transactions on Automatic Control, vol. 42, No. 9, September 1997, pp. 1294 -1299.