

# Adaptive Stabilization of Model-Based Networked Control Systems

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**Abstract**—In this paper Model Based Networked Control Systems (MB-NCS) are considered and on-line identification of system parameters in state space representation is used to upgrade the model and the controller of the system. The updated model is used to control the real system when feedback information is unavailable. The Extended Kalman Filter (EKF) is analyzed in the context of parameter identification and implemented in the MB-NCS framework. Emphasis is placed on global asymptotic estimators for the case when sensors provide noiseless measurements of the state of a linear system; it can be shown that the identification of parameters in this case is a linear problem, in contrast to the nonlinear combined state-parameter estimation problem. We propose new estimation models that offer better convergence properties than the EKF in this case. This estimation strategy is also applied to the MB-NCS framework resulting in a better usage of the network by allowing longer intervals without need for a measurement update.

## I. INTRODUCTION

IN Networked Control Systems (NCS), dynamical systems are controlled by using feedback over a shared communication network. NCS offer a large number of advantages compared to traditional configurations where control systems are interconnected using dedicated wires; NCS reduce wiring, increase reliability, and reduce time and cost of maintenance [1]. At the same time, undesirable situations may be encountered due to bandwidth limitations of the communication channel, which induces network delays and packet dropouts especially when many nodes attempt to broadcast information very frequently [2], [3]. A type of NCS called Model Based Networked Control Systems (MB-NCS) aims to reduce communication over the network by incorporating an explicit model of the system to be controlled. It has been shown that the MB-NCS framework reduces the bandwidth needed to safely operate a control system, consequently, it reduces the size of network induced delays and probability of packet dropouts and releases the network so it can be used for other tasks [4].

Montestruque and Antsaklis provided stability conditions of MB-NCS using periodic updates [4] and when the update intervals are time-varying and follow different probability distributions [5]. The reduction in network communication that we are able to achieve, i.e. the longest we can wait for a new update without compromising stability is directly related to the accuracy of the model.

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Even when an accurate model is initially available, in many applications the parameters of a system may change over time due to the use and working conditions of the plant or of its components.

Estrada and Antsaklis introduced the notion of intermittent MB-NCS [6], [7]. In this case the measurement updates are not only given for a time instant but they last for a period of time making the system to operate in closed loop mode for a finite interval  $\tau < h$ , where  $h$  is the update period and  $\tau$  is the time the system is working in closed loop.

In the present paper we focus on applying identification algorithms in the MB-NCS context. Better knowledge of the plant dynamics will provide an improvement in the control action over the network, i.e. we can operate over longer open loop time intervals without need for feedback. We use Kalman filters to estimate the unknown parameters of the system, since they provide global convergence properties for deterministic systems. We estimate parameters in state space non-canonical form and we do not require a persistently exciting input signal. The last two represent important advantages compared to traditional closed loop system identification algorithms. With respect to MB-NCS, we overcome the usual assumption that the controller is designed to stabilize the real system; this may be unrealistic since our knowledge of the plant dynamics is limited. As we will see, the identification process allows us to update not only the model but the controller itself so it can better respond to the dynamics of the real plant being controlled. In contrast to common adaptive techniques we do not redefine the controller at each time instant but only when the estimated parameters are received in the controller node. When the controller node receives updated model parameters, an LQR controller is calculated solving a discrete-time algebraic Riccati equation using the same weights and the new parameters that were just obtained. The rest of the paper is organized as follows: section II states the problem. Stability and identification of parameters of deterministic MB-NCS are discussed in sections III and IV, respectively. Section V presents similar results for stochastic MB-NCS. Section VI contains illustrative examples and section VII provides relevant conclusions.

## II. PROBLEM STATEMENT

MB-NCS make use of an explicit and in most times inexact model of the plant which is added to the controller node to compute the control input based on the state of the model rather than on the plant state for the intervals of time when feedback is unavailable. Sensor measurements are used

by the controller to update the state of the model resetting any possible mismatch between the state of the model and the state of the controlled system. The sensor and the controller are connected using a communication network whereas the controller is connected directly to the plant as shown in fig. 1.a. where  $p$  is a vector containing the estimated parameters. The plant and model dynamics are given, respectively, by:

$$x(k+1) = Ax(k) + Bu(k) \quad (1)$$

$$\hat{x}(k+1) = \hat{A}\hat{x}(k) + \hat{B}u(k) \quad (2)$$

where  $x, \hat{x} \in R^n$  are the states of the plant and the model, respectively, and  $u = K\hat{x}$ .

It is convenient in many applications to drop the usual periodic update implementation in favor of one based on events [20], [21], for example, the event that the plant-model state error is equal to or greater than some predetermined threshold. A sensor node within the network broadcasts its local state only when it is necessary, i.e. when a measure of the local subsystem state error is above some predetermined threshold, the error, in this case, is defined as the difference between the state of the model and the state of the plant:

$$e(k) = \hat{x}(k) - x(k) \quad (3)$$

This form of updating the model has the implicit advantage of extending the interval in which the plant works in open loop, when the model is improved, and provides better estimates of the plant states; a copy of the model is needed in the sensor node to generate the model state and compute (3).

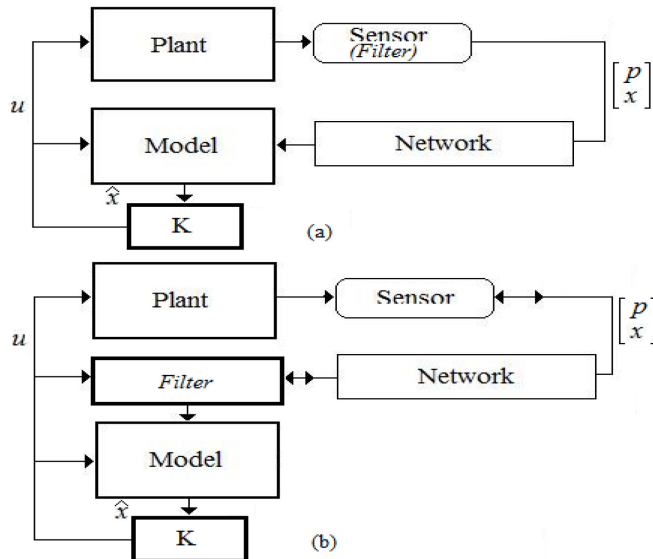


Fig. 1. MB-NCS with filter implemented in a) the sensor node, and b) the controller node.

In view of the fact that we need to be able to identify a system in general state space representation, not necessarily in canonical form as discussed in the literature [8]-[10], we will use the Kalman filter for identification of systems of the form (1) and the extended Kalman filter (EKF) for the case we receive noisy measurements of the state.

### III. STABILITY OF MB-NCS

We are particularly interested in updating the state of the model in one of two ways: either in a periodic fashion, or using an event based strategy. The event based strategy has the advantage that we can immediately obtain longer update intervals after we have upgraded the model.

The next theorem provides stability bounds of the MB-NCS when we update the model based on error events. In the sensor node we measure the state of the plant and compare the magnitude of the error (3) to a fixed threshold  $\alpha < \infty$ ; the plant state is used to update the model if the error is greater than the threshold, i.e. when  $\|e(k)\| > \alpha$ .

**Theorem 1.** For  $\|x(0)\| \leq \beta_1$ ,  $0 < \beta_1 < \infty$ , the networked system described by (1) with state feedback updates triggered when  $\|e(k)\| > \alpha$ , has a bounded state if the eigenvalues of  $A+BK$  lie strictly inside the unit circle.

*Proof:* System (1) can be described by:

$$x(k+1) = (A+BK)x(k) + BKe(k) \quad (4)$$

after (3) and the control input  $u = K\hat{x}$  have been used.

The response of the plant with initial time  $k_0 = 0$  and stable matrix  $A+BK$  at any given time  $k \geq 0$  is given by:

$$x(k) = (A+BK)^k x(0) + \sum_{j=0}^{k-1} (A+BK)^{k-(j+1)} BKe(j) \quad (5)$$

where  $e(k)$  is bounded by:  $\|e(k)\| \leq \alpha$ . We can show that the state of the plant is bounded by evaluating its norm which is done next:

$$\begin{aligned} \|x(k)\| &= \left\| (A+BK)^k x(0) + \sum_{j=0}^{k-1} (A+BK)^{k-(j+1)} BKe(j) \right\| \\ &\leq \|(A+BK)^k\| \|x(0)\| + \sum_{j=0}^{k-1} \|(A+BK)^{k-(j+1)}\| \|BK\| \|e(j)\| \end{aligned}$$

In view of the assumption on the initial condition of the plant and the triggering condition, and using the bound  $\|(A+BK)^k\| \leq \beta_2 \lambda^k$ ,  $\lambda \in (0,1)$ ,  $\beta_2 > 0$ , [17], we can write:

$$\begin{aligned} \|x(k)\| &\leq \beta_1 \beta_2 \lambda^k + \alpha \beta_2 \|BK\| \sum_{j=0}^{k-1} \lambda^{k-(j+1)} \\ &\leq \beta_1 \beta_2 \lambda^k + \frac{\alpha \beta_2 \|BK\| (1 - \lambda^{k-1})}{1 - \lambda} \end{aligned} \quad (6)$$

$$\text{Note that: } \lim_{k \rightarrow \infty} \|x(k)\| = \frac{\alpha \beta_2 \|BK\|}{1 - \lambda}.$$

The fact that the eigenvalues of  $A+BK$  lie strictly inside the unit circle ensures that the first term in the right hand side of (6) decreases exponentially with time and the second term is bounded for all time  $k > 0$ . ■

*Remark 1.* The condition for the state of the plant to be bounded is obtained in terms of the real parameters, but this is not restrictive in an adaptive scheme since we can obtain accurate estimates of those parameters and feedback laws based on the upgraded model that stabilize the plant as well.

**Theorem 2.** For  $\|x(0)\| \leq \beta_1$ ,  $0 < \beta_1 < \infty$ , the networked system (1) with *intermittent* state feedback updates triggered

when  $\|e(k)\| > \alpha$  has a bounded state if the eigenvalues of  $A+BK$  lie strictly inside the unit circle.

*Proof:* The proof is similar to the one in theorem 1 by noting that  $e(k) = \hat{x}(k) - x(k) = 0, \forall k \in [t_k, t_k + \tau)$  i.e. in the closed loop interval, then  $\|e(k)\| \leq \alpha, \forall k$ . ■

The next two theorems describe the stability properties of MB-NCS with periodic updates  $h$ ; for the case of intermittent feedback we assume the closed loop time  $\tau$  is constant.

**Theorem 3.** The networked system (1) with *periodic* state feedback is asymptotically stable if only if the eigenvalues of

$$A^h + \sum_{j=0}^{h-1} A^{h-1-j} BK(\hat{A} + \hat{B}K)^j \quad (7)$$

lie strictly inside the unit circle.

**Theorem 4.** The networked system (1) with *periodic intermittent* feedback is asymptotically stable if only if the eigenvalues of:

$$(A^{h-\tau} + \sum_{j=0}^{h-\tau-1} A^{h-\tau-1-j} BK(\hat{A} + \hat{B}K)^j)(A+BK)^\tau \quad (8)$$

lie strictly inside the unit circle.

*Proof:* For the proofs of theorems 3 and 4 refer to [11].

With respect to the above results, we can see that the event-triggered approach only offers a bounded output compared to the asymptotic properties when using periodic updates. This drawback can be addressed by applying a time-varying threshold as shown in [21].

#### IV. PARAMETER ESTIMATION OF DETERMINISTIC MB-NCS

In this section we focus on deterministic linear systems of the form (1) with no particular form of the matrices  $A$  and  $B$ . This identification problem can be solved using a linear Kalman filter; this implementation provides much better convergence properties than the EKF (see section VI.A).

In the special case when the sensors provide noiseless measurements of the state, it is possible to modify the model that will be used for the Kalman filter equations in order to estimate the parameters  $A$  and  $B$  assuming the order of the system is known but the structure of the system is not, i.e. no canonical form is assumed. In order to show this simple idea let us focus on second order autonomous systems, (the idea can be easily extended to higher order systems with deterministic inputs) with unknown time-invariant parameters,

$$\begin{bmatrix} x_1(k+1) \\ x_2(k+1) \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} x_1(k) \\ x_2(k) \end{bmatrix} \quad (9)$$

We do not know the values of the parameters and we only receive measurements of the states  $x(0) \dots x(k)$ . At any given step due to the iterative nature of the Kalman filter we only need  $x(k)$  and  $x(k-1)$ . Now we rewrite (9) as:

$$\begin{bmatrix} \hat{x}_1(k) \\ \hat{x}_2(k) \end{bmatrix} = \begin{pmatrix} x_1(k-1) & x_2(k-1) & 0 & 0 \\ 0 & 0 & x_1(k-1) & x_2(k-1) \end{pmatrix} \begin{bmatrix} \hat{a}_{11}(k) \\ \hat{a}_{12}(k) \\ \hat{a}_{21}(k) \\ \hat{a}_{22}(k) \end{bmatrix} = \hat{C}(k)\hat{a}(k) \quad (10)$$

Equation (10) becomes the output equation of our filter, and the state equation is described by:

$$\begin{bmatrix} \hat{a}_{11}(k+1) \\ \hat{a}_{12}(k+1) \\ \hat{a}_{21}(k+1) \\ \hat{a}_{22}(k+1) \end{bmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{bmatrix} \hat{a}_{11}(k) \\ \hat{a}_{12}(k) \\ \hat{a}_{21}(k) \\ \hat{a}_{22}(k) \end{bmatrix} = A_f \hat{a}(k) \quad (11)$$

Equations (10)-(11) represent a linear model; therefore we can use a linear filter to obtain estimates of the parameters  $a_{ij}$  of the state matrix of the original system. Note that we do not need any external input, only nonzero initial conditions on the state. For systems of the form (1) we can estimate the elements of both matrices  $A$  and  $B$  if we receive measurements of the state and the deterministic input  $u(k)$ . Any common inputs such as steps and sinusoidal inputs can be used for identification purposes. Sinusoidal inputs do not need to have in any particular frequency i.e. there is no requirement of a persistently exciting input which makes this approach a suitable tool for adaptive stabilization. In this case we need to include estimates of the parameters of  $B$  in the state vector of the filter model and the input values in  $\hat{C}(k)$ ; this model can be easily applied to higher order systems by following the structures of (10)-(11). A limitation is that the order of the filter is  $n^2$  where  $n$  is the dimension of the state of the original system.

Song and Grizzle [18] have shown that the linear time-varying Kalman filter (LTV-KF) is a global asymptotic observer for the underlying deterministic system. Consider the deterministic system described by (10)-(11), and the associated noisy system:

$$\begin{aligned} \hat{a}(k+1) &= A_f \hat{a}(k) + Nw(k) \\ \hat{y}(k) &= \hat{C}(k)\hat{a}(k) + Mv(k) \end{aligned} \quad (12)$$

where the design parameters  $M, N$  are chosen as positive definite matrices and the artificial noise processes  $w, v$  are white, zero-mean, uncorrelated, and have known covariance matrices  $Q$  and  $R$  respectively. The next theorem states the convergence of the estimation error.

**Theorem 5.** Consider the deterministic system (10)-(11), and the Kalman filter associated with (12). Suppose that the deterministic system is uniformly observable and  $A_f(k)$  is invertible for all  $k$ , and that  $\|A_f\| := \sup\{\|A_f(k)\| : k \geq 0\}$  and  $\|\hat{C}\| := \sup\{\|\hat{C}(k)\| : k \geq 0\}$  are bounded. Then the Kalman filter for the noisy system (12) is a global, uniform asymptotic observer for the deterministic system (10)-(11).

For the proof of this theorem refer to [18]. We will now focus on the details pertaining to our specific model. From (11) we can see that  $A_f = I$  is constant, bounded, and invertible for all  $k$ . The output matrix is built by using the measurements of the deterministic system, for unstable systems it is required that the initial conditions of the system are finite. The matrices  $M, N$  are simply chosen to be identity matrices of appropriate dimensions. The problem with our model is that the pair  $(A_f, \hat{C}(k))$  is not observable; a simple

solution is to increase the number of measurements used in the output equation (10), although this is not a necessary condition. A single previous measurement is used in all examples in section VI.

## V. ADAPTIVE STABILIZATION OF STOCHASTIC MB-NCS

In this section we will study the case in which stochastic systems of the form:

$$\begin{aligned} x(k+1) &= Ax(k) + Bu(k) + w(k) \\ y(k) &= x(k) + v(k) \end{aligned} \quad (13)$$

are implemented using the configurations of Fig.1. The noise processes  $w$  and  $v$  are white, Gaussian, uncorrelated, zero-mean, and have known covariance matrices  $Q$  and  $R$  respectively. The model of the system is still given by (2) and, since we only measure  $y(k)$ , the error is now given by:

$$e(k) = \hat{x}(k) - y(k) \quad (14)$$

**Theorem 6.** Assume  $x(0)$  is a random variable with Gaussian distribution  $(\mu_0, \Sigma_0)$ ; then the state of the stochastic model-based networked system (13) with feedback based on error events has finite mean and covariance for all  $k$  if the eigenvalues of  $A+BK$  lie strictly inside the unit circle.

*Proof:* The state of equation (13) can be expressed using the linear system:

$$x(k+1) = (A+BK)x(k) + BKe(k) + BKv(k) + w(k)$$

then the state  $x(k)$  is a Gaussian random variable for all  $k$  with mean and covariance given by:

$$\mu_k = (A+BK)^k \mu_0 + \sum_{j=0}^{k-1} (A+BK)^{k-(j+1)} BKe(j) \quad (15.a)$$

$$\begin{aligned} \Sigma_k &= (A+BK)\Sigma_{k-1}(A+BK)^T + (BKR(BK)^T + Q) = (A+BK)^k \Sigma_0 ((A+BK)^k)^T \\ &+ \sum_{j=0}^{k-1} (A+BK)^j (BKR(BK)^T + Q) ((A+BK)^j)^T \end{aligned} \quad (15.b)$$

In theorem 1 we showed that (15.a) is bounded if the eigenvalues of  $A+BK$  lie strictly inside the unit circle. In order for the covariance  $\Sigma_k$  to converge we need the series (15.b) to be summable as  $k \rightarrow \infty$  and this is obtained by making  $(A+BK)^j$  to converge to 0, i.e. if  $A+BK$  is stable then the covariance converges to a finite value. ■

The combined estimation of states and parameters problem has been studied by different researchers, see for example [14], [18]. The use of the Extended Kalman Filter (EKF) to deal with this problem was first proposed by Kopp and Orford [13]; a detailed derivation of the EKF as a parameter estimator may be found in [12].

The extended Kalman filter whether is used as estimator of states of nonlinear systems or combined estimation of states and parameters is prone to divergence as it lacks the robustness and the convergence properties of the linear Kalman filter. Many of the causes for the estimates to be biased or divergent have been illustrated and somewhat successful remedies have been proposed in many papers and books, see for instance [8], [15], [18], and [19]. The most common causes of divergence in the EKF are related to the fact that the EKF is based on linearization about the current

estimate, and so if the a priori state estimates are poor, or if later estimates should take the filter out of the linear region, the estimates often diverge. A more rigorous analysis of the local convergence properties of the EKF used as a parameter estimator for linear systems is offered in [14].

## VI. IMPLEMENTATION CASES AND EXAMPLES

### A. Comparison of EKF and LTV-KF for parameter identification of deterministic systems.

The EKF can also be used as parameter estimator of deterministic systems of the form (1); this is a special case of the approach in the last section. However, for the special case when noiseless measurements of the state are available there is a significant improvement in the quality of the estimated parameters given by the LTV-KF described in section IV compared with those obtained using the EKF particularly for higher order systems.

As explained before, the EKF will diverge or provide biased estimates due to many factors, especially if the initial estimates are not sufficiently close to the real parameters we try to estimate. We present a simulation-based comparison between the LTV-KF model using only one previous measurement that we proposed in section IV and the EKF, both working under similar assumptions.

**Example 1.** A fourth order deterministic system with random initial conditions is given by.

$$x(k+1) = Ax(k)$$

with

$$A = \begin{pmatrix} 1.7209 & -1.1484 & -2.8700 & -1.8609 \\ 0.9510 & 2.9805 & 2.3617 & -0.3365 \\ -2.6334 & -2.4172 & 0.1133 & 2.6316 \\ 2.8223 & 0.5102 & -2.1791 & -2.7730 \end{pmatrix}$$

First, for both filters we assume we receive noiseless measurements of the state and that the elements of matrix  $A$  are unknown but lie somewhere in the range  $[-3,3]$ . The main difference given by the simulations of the two filters with the same deterministic system is that both converge to some constant value but the EKF tends to provide biased estimates, that is, the error defined as the difference between the real parameters and their estimates is of greater order in the EKF than in the LTV-KF. In the case we have some knowledge of the a priori estimates of the parameters then the results of the EKF show some improvement, emphasizing the dependence of the EKF to initial estimates  $a(0)$ . Results of simulations are shown in Table 1.

Filter	Random initial conditions for filters	Error order in 200 simulations
LTV-KF	Uniform: $[-3,3]$	$10^{-7} - 10^{-4}$
	$a_{ij}(0) \pm \alpha$	$10^{-8} - 10^{-4}$
EKF	Uniform: $[-3,3]$	$10^1$
	$a_{ij}(0) \pm \alpha$	$10^{-1} - 10^1$

Table 1. Error order results for the LTV-KF and the EKF. ( $\alpha$  is a random variable with uniform distribution in  $[0, 0.1]$ ).

Whether a linear Kalman filter or the EKF is used we can implement the filter in the MB-NCS framework using one of two approaches.

*B. Filter collocated with sensor.*

In the configuration shown in Fig. 1.a the filter is implemented in the sensor node. We assume that a copy of the model and controller are contained in the sensor to generate the state that is compared to the measured state, and the input that is needed by the filter. The sensor estimates the parameters (and compares the state error, in the event-triggered case) constantly. It will transmit the measured state and the new value of the estimated parameters, or it can send a smaller packet containing only the state of the model if no significant variation has been detected in the parameter values; intermittent feedback is not necessary in this case.

*C. Filter collocated with controller.*

Due to several factors, especially computational limitations in the sensor node, it may be necessary to implement the identification algorithm in the controller node. In this configuration (shown in Fig. 1.b) the filter in the controller receives a set of measurements (intermittent feedback is needed) that uses for estimation of the parameters of interest. When the estimated variables pass a converge test, the model is updated with the new value of the parameters and the state of the model is updated using the last measurement available. That is, we use intermittent feedback for parameter identification and instantaneous feedback for control. No model of the plant is needed in the sensor node and the filter updates directly the model in the controller immediately after its estimates have converged since no network exists between filter and model. For the case when we send the measurements based on checking the state error, we need a copy of the model in the sensor node in order to generate the model state. For this scenario we require the controller node to send back to the sensor node the new estimated parameters and the new calculated controller to update the model in the sensor as it does with the model in the controller.

*D. Adaptive stabilization examples.*

In the two previous cases, *B* and *C*, when the controller node receives or obtains new estimates of the parameters, a discrete-time algebraic Riccati equation is solved using the same weights and the new parameters in order to obtain a stabilizing control law *K* that reflects the new acquired knowledge of the plant in the control action.

**Example 2.** Consider the second order system described by (13) with time-invariant but partially unknown parameters and time index  $T=0.01$  seconds. Assume that  $B=[1 \ 1]^T$ , the elements  $a_{12}=0.3, a_{21}=-1.05$  are known, and  $a_{11}, a_{22}$  are unknown constants. We implement an EKF in the controller node using intermittent feedback triggered by the state error. Fig. 2 shows the simulation results. Fig. 2.a

shows the measured states that are used to compute the state error (14) and to make the decision about whether sending or not new measurements to the controller node. Fig. 2.b shows the error on the estimated parameters by comparing them to the real values that were used for the unknown parameters, those values in this example are  $a_{11}=-0.985, a_{22}=-0.7$ .

Fig. 2.c shows the time intervals that the sensor establishes communication with the controller to send its measurements.

The initial model was inadequate and made the real plant go unstable in the beginning of the execution until we updated the model and recomputed the controller. Every time the controller receives a set of measurements, the filter estimates again the parameters, the rest of the time the system runs in open loop.

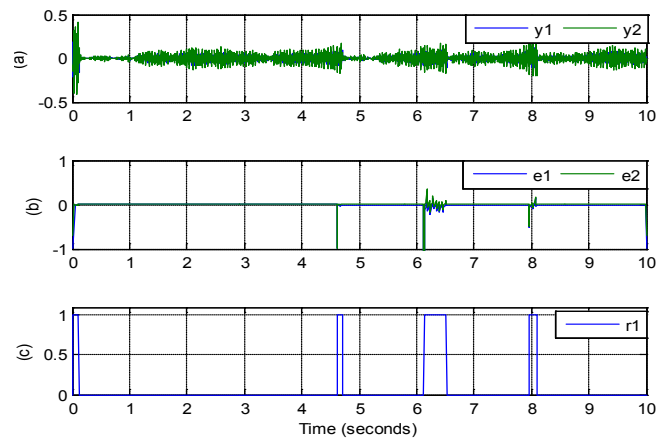


Fig. 2. Estimation of two parameters and stabilization of stochastic MB-NCS with intermittent feedback based on events. (a) Measured states. (b) Error on the estimated parameters. (c) Network communication.

**Example 3.** A second order system of the form (1) ( $T=0.01$  seconds) is interconnected to a model-based controller as shown in Fig. 1.a. All of the elements of the matrices *A* and *B* are unknown, the Kalman filter implemented in the sensor provides estimates of all parameters; we design the filter by using only one previous measurement. When the state error is greater than a predefined threshold the sensor sends the most recent estimates, if there is a significant variation with respect to the previous update, and the latest measured state to the controller. For illustration purposes we construct the communication signal  $r(k)$  as:

$$r(k) = \begin{cases} 0 & \text{if no packet is sent} \\ 1 & \text{if only the state is sent} \\ 2 & \text{if both, parameters and state are sent} \end{cases}$$

The initial model contained random estimates of the parameters and the control input obtained from that model does not stabilize the real plant as it can be seen in the beginning of the simulation in Fig. 3.a. After a few iterations we are able to obtain estimates of the parameters, redesign the controller based on the upgraded model, and update the state of the model as well. Variations in the main diagonal components of matrix *A* of the plant at  $t=2$  seconds were

introduced and successfully identified as shown in Fig. 4. The real values used in this example were:

$$\begin{aligned} a_{11} &= -1.231 \rightarrow_{t=2} -1.303 & a_{12} &= 0.503 \\ a_{21} &= -0.034 & a_{22} &= 0.820 \rightarrow_{t=2} 1.086 \\ b_1 &= 1.8 & b_2 &= -1.4 \end{aligned}$$

In the absence of measurement noise we are able to identify with great precision all parameters of the plant and if we use a linear filter there is no restriction on the initial estimates compared to for example the EKF.

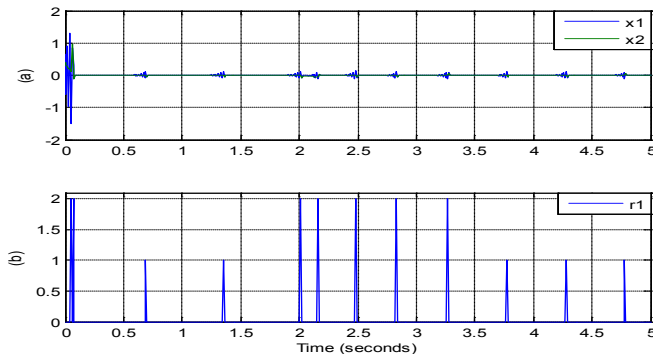


Fig. 3. Stabilization of MB-NCS in example 3. (a) Measured states. (b) Network communication.

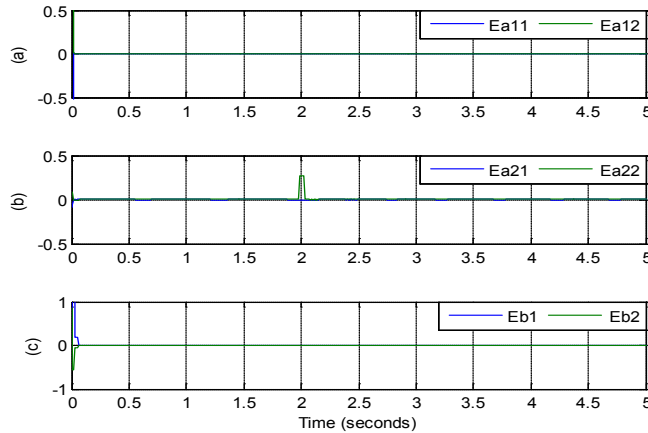


Fig. 4. Identification of parameters  $A$  and  $B$  in example 3. (a) Error on parameters  $a_{11}$  and  $a_{12}$ . (b) Error on parameters  $a_{21}$  and  $a_{22}$ . (c) Error on parameters  $b_1$  and  $b_2$ .

## VII. CONCLUSION

Adaptive stabilization of MB-NCS was studied in this paper. By using Kalman filters we are able to overcome two typical restrictions in the parameter estimation literature: estimating state-space parameters in general form, not necessarily in canonical form, and performing a successful estimation without persistent excitation requirement by the input signal. Both of these issues are of great importance in MB-NCS applications. The typical problem in MB-NCS is the stabilization of a system using a state-space model and using the network as little as possible. The adaptation of the control law based on upgraded versions of the model permits a faster stabilization rate and reduction of bandwidth utilized by the system to communicate to its controller by appropriately designing a broadcasting strategy. The particular case in which we can use noiseless measurements

of the state for identification purposes allows for the implementation of a linear filter in order to obtain the correct parameters independently of the initial values of the parameters of the model.

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