Event-Triggered Output Feedback Control For Networked Control Systems Using Passivity: Achieving $\mathcal{L}_2$ Stability in the Presence of Communication Delays and Signal Quantization ☆

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Abstract

When network induced delays are considered in the event-triggered control literature, they are typically delays from the plant to the network controller and a tight bound on the admissible delays is usually imposed based on the analysis of inter-event time to guarantee stability of the event-triggered control systems. In this paper, we introduce a framework for output feedback based event-triggered networked control systems(NCSs). The triggering condition is derived based on passivity theorem which allows us to characterize a large class of output feedback stabilizing controllers. The proposed set-up enables us to consider network induced delays both from the plant to the network controller and from the network controller to the plant. We also take quantization of the transmitted signals in the communication network into consideration and we show that finite-gain $\mathcal{L}_2$ stability can be achieved in the presence of time-varying (or constant) network induced delays with bounded jitters, without requiring that the network induced delays are upper bounded by the inter-event time.

Keywords: Event-triggered Control; Output Feedback Stabilization; Networked Control Systems; Passivity; Communication Delays; Quantization; $\mathcal{L}_2$ Stability.

1. Introduction

Feedback control laws nowadays are typically implemented on digital platforms since microprocessors offer many advantages. In such an implementation, the control task consists of sampling the output of the plant, computing and implementing new actuation signals. Traditionally, the control task is executed periodically; this allows the closed-loop system to be analyzed and the controller to be designed using the well-developed theory on sampled-data systems (cf. Jury, 1958; Ragazzini & Franklin, 1958; Aström et al., 1990). However, the control strategy obtained based on this approach is conservative in the sense that resource usage (i.e., sampling rate, CPU time) is more frequent than necessary to ensure a specified performance level, since stability is guaranteed in the worst case scenarios under sufficiently fast periodic execution of the control action. To overcome this drawback, several researchers suggested the idea of *event-triggered control*. In the literature, the triggering mechanism is referred to as Lebesgue sampling (Aström & Bernhardsson, 2002), dead-band control (Otanez et al., 2002), level-crossing sampling (Kofman & Braslavsky, 2006), event-based-sampling (Aström, 2008), event-driven sampling (Heemels et al., 2008), state-triggered sampling (Tabuada, 2007) and self-triggered sampling (Wang & Lemmon, 2009) with slightly different meanings. In all cases, the control signal is kept constant until violation of a *triggering condition* on certain signals of the plant triggers re-computation of the control signals. The possibility of reducing the number of re-computations, and thus of transmissions, while guaranteeing desired level of performance makes event-triggered control very appealing in networked control systems(NCSs).

Although the advantages of event-triggered control are significant, there are still problems that need to be addressed before event-triggered control can be fruitfully applied to NCSs. Most of the results on event-triggered control are obtained under the assumption that the feedback control law provides input-to-state stability(ISS) in the sense of (Sontag, 1989) with respect to some signal novelty errors of the plant (cf. Tabuada, 2007; Wang & Lemmon, 2009; Anta & Tabuada, 2010). The ISS framework provides insight into the triggering condition by exploring the relation between stabilization and the current full-state information. However, in many control applications, the full state information is not available for measurement, so extensions to event-triggered output feedback based control are important. Early work on event-triggered control using dynamic output feedback based controllers in Kofman & Braslavsky (2006) does not include a thorough analysis of the minimum time between two subsequent events, the so-

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called *inter-event time*. A recent work on output feedback based event-triggered control scheme with guaranteed $\mathcal{L}_\infty$-gain for linear time-invariant control system is reported in Donkers et al. (2010), where the event-triggered control system is modeled as an impulsive system and linear matrix inequalities are applied to study the stability and performance of the event-triggered control systems. This framework cannot be easily extended to nonlinear control systems, and the triggering mechanism requires synchronization between the event-detector and the network controller. In Yu & Antsaklis (2011a), a static output feedback based event-triggered control scheme is introduced for stabilization of passive and output feedback passive(OFP) NCSs, where the triggering condition and the static output feedback gain are derived based on the output feedback passivity indices of the plant. The results in Yu & Antsaklis (2011a) only apply to passive and output feedback passive systems.

Although the above work on output feedback based event-triggered control have recently appeared in the literature in addition to the ISS framework, robustness issues with respect to the imperfect communication networks still have not been addressed. In particular, most of the work on event-triggered control for NCSs assume that the network induced delay is upper bounded by the inter-event time so that stability of the NCS can be guaranteed. However, in real time NCSs, the network induced delay is usually unknown, and it is very likely having network induced delay larger than the inter-event time. Moreover, in the presence of external disturbances, the inter-event time implicitly determined by the triggering condition, could be extremely small. Thus, it is not practical to schedule the data transmissions at the plant side based on the analysis of the inter-event time. Another limitation of many existing work on event-triggered control for NCSs is that only network induced delays from the plant to the network controller have been considered. However, non-trivial delays from the network controller to the actuator (which is collocated with the plant) could also jeopardize the stability of the event-triggered control system.

In this paper, we propose a dynamic output feedback based event-triggered control framework for NCSs that allows us to take both signal quantization and network uncertainties into consideration. The present work applies to Input Feed-forward Output Feedback Passive(IF-OFP) systems which are more general than the results reported in Yu & Antsaklis (2011a), since the plant is not necessarily passive but dissipative. A triggering condition based on the *main passivity theorem* is derived which enables us to characterize a large class of output feedback stabilizing controllers. A rectified scattering transformation has been employed in our framework to deal with time-varying (or constant) network induced delays with bounded jitters both from the plant to the network controller and from the network controller to the plant; finite-gain $\mathcal{L}_2$ stability of the event-triggered NCSs is achieved under the proposed framework. Note that scattering transformation was first applied in the literature of telecommunication to achieve stability independently of time delays provided that the plant and the controller are both passive (cf. Anderson & Spong, 1989; Lozano et al., 2002). It was then applied to networked control systems for output strictly passive systems (cf. Chopra & Spong, 2007; Chopra, 2008). The more general scattering transformation was examined in the work of Hirchea et al. (2009), for IF-OFP systems. The basic idea of applying scattering transformation in all of these work is to preserve the passive or dissipative properties of the original systems through the communication network in the presence of network induced delays. However, in all of those previous work, continuous or periodic communication between the plant and the controller is assumed, and quantization of the transmitted data is not considered. Thus, how to use scattering transformation to deal with network induced delays when the data transmission between the plant and the network controller is event-based remains as an interesting problem. Part of our results have appeared in Yu & Antsaklis (2011b) and Yu & Antsaklis (2011c), without considering quantization effects of the transmitted signals in the communication network. The work presented in this paper are important extensions on applying event-triggered control to NCSs, especially when signal quantization has to be taken into account and when the delays in the communication network could be larger than the inter-event time implicitly determined by the triggering condition.

The rest of this paper is organized as follows: we first introduce some background on passive and dissipative systems in Section 2; the problem is stated in Section 3; an event-triggering condition to achieve L_2 stability of the NCSs derived based on the passivity theorem without considering network induced delays is presented in Section 4; analysis on the corresponding inter-event time is provided in Section 5; we discuss our proposed framework in Section 6; finally, Section 7 summarizes the main results.

2. Background Material

We first introduce some basic concepts on passive and dissipative systems. Consider the following control system, which could be linear or nonlinear:

$$H_p : \begin{cases} \dot{x}_p = f_p(x_p, u_p) \\ y_p = h_p(x_p, u_p) \end{cases} \tag{1}$$

where $x_p \in X_p \subset \mathbb{R}^n$, $u_p \in U_p \subset \mathbb{R}^m$ and $y_p \in Y_p \subset \mathbb{R}^m$ are the state, input and output variables, respectively, and X_p, U_p and Y_p are the state, input and output spaces, respectively. The representation $\phi_p(t, t_0, x_{p0}, u_p)$ is used to denote the state at time t reached from the initial state x_{p0} at the time t_0 under the control u_p.

Definition 1. (Supply Rate (Willems, 1972)) The *supply rate* $\omega_p(t) = \omega_p(u_p(t), y_p(t))$ is a real valued function defined on $U_p \times Y_p$, such that for any $u_p(t) \in U_p$ and $x_{p0} \in X_p$

and $y_p(t) = h_p(\phi_p(t, t_0, x_{p0}, u_p), u_p)$, $\omega_p(t)$ satisfies

$$\int_{t_0}^{t_1} |\omega_p(\tau)| d\tau < \infty. \quad (2)$$

Definition 2. (Dissipative System(Willems, 1972)) System H_p with supply rate $\omega_p(t)$ is said to be *dissipative* if there exists a nonnegative real function $V_p : X_p \to \mathbb{R}^+$, called the storage function, such that, for all $t_1 \geq t_0 \geq 0$, $x_{p0} \in X_p$ and $u_p \in U_p$,

$$V_p(x_{p1}) - V_p(x_{p0}) \leq \int_{t_0}^{t_1} \omega_p(\tau) d\tau, \quad (3)$$

where $x_{p1} = \phi_p(t_1, t_0, x_{p0}, u_p)$ and $\mathbb{R}^+$ is a set of nonnegative real numbers. If V_p is $\mathcal{C}^1$, then we have $\dot{V}_p \leq \omega_p(t)$, $\forall t \geq 0$.

Passive systems are special cases of dissipative systems as defined below.

Definition 3. (Passive System (Willems, 1972)) System H_p is said to be *passive* if there exists a storage function V_p such that

$$V_p(x_{p1}) - V(x_{p0}) \leq \int_{t_0}^{t_1} u_p^T(\tau) y_p(\tau) d\tau. \quad (4)$$

If V_p is $\mathcal{C}^1$, then

$$\dot{V}_p \leq u_p^T(t) y_p(t), \ \forall t \geq 0. \quad (5)$$

Definition 4. (IF-OFP systems (Sepulchre et al., 1997)) System H_p is said to be *Input Feed-forward Output Feedback Passive*(IF-OFP) if it is dissipative with respect to the supply rate

$$\omega_p(u_p, y_p) = u_p^T y_p - \rho_p y_p^T y_p - \nu_p u_p^T u_p, \ \forall t \geq 0, \quad (6)$$

for some $\rho_p, \nu_p \in \mathbb{R}$.

For the rest of this paper, we will denote an $m-$inputs $m-$outputs dissipative system with supply rate (6) by *IF-OFP*$(\nu_p, \rho_p)^m$ and we will call (ν_p, ρ_p) the *passivity indices* of the system.

Theorem 1. *(Passivity Theorem(Khalil, 2002)) Consider a well-posed feedback interconnection as shown in Figure 1, and suppose each feedback component satisfies the inequality*

$$\dot{V}_i \leq u_i^T y_i - \rho_i y_i^T y_i - \nu_i u_i^T u_i, \ \textit{for } i = 1, 2, \quad (7)$$

for some storage function V_i. Then, the closed-loop map from $\omega = [\omega_1^T, \omega_2^T]^T$ to $y = [y_1^T, y_2^T]^T$ is finite-gain $\mathcal{L}_2$ stable if

$$\rho_1 + \nu_2 > 0, \quad \rho_2 + \nu_1 > 0. \quad (8)$$

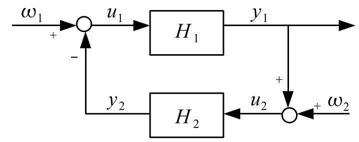

Figure 1: Feedback Interconnection of Two IF-OFP Systems

Lemma 1. *(Matiakis et al., 2006) Without loss of generality the domain of ρ_p, ν_p in IF-OFP system (6) is $\Omega = \Omega_1 \cup \Omega_2$ with $\Omega_1 = \{\rho_p, \nu_p \in \mathbb{R} | \rho_p \nu_p < \frac{1}{4}\}$, $\Omega_2 = \{\rho_p, \nu_p \in \mathbb{R} | \rho_p \nu_p = \frac{1}{4}; \rho_p > 0\}$.*

3. Problem Statement

We consider the control system given in (1). We assume H_p is IF-OFP$(\nu_p, \rho_p)^m$ with a $\mathcal{C}^1$ storage function V_p. Based on Theorem 1, we know that if we design an IF-OFP$(\nu_c, \rho_c)^m$ controller with a $\mathcal{C}^1$ storage function V_c such that $\rho_c + \nu_p > 0$, $\rho_p + \nu_c > 0$, then the closed-loop system is finite-gain $\mathcal{L}_2$ stable.

In real time NCSs, the implementation of the feedback control law on an embedded processor is typically done by sending the value of the plant's output $y_p(t)$ at time instant t_k (for $k = 0, 1, 2, \ldots$) to the network controller through the communication network; the transmitted output information of the plant arrives at the controller at time instant $t_k + \Delta_k$, where $\Delta_k \geq 0$ represents the network induced delay from the plant to the network controller; the controller computes the control action based on the received information of the plant and sends the control laws back to the actuator (located at the plant side) through the communication network. In event-triggered NCSs, new output information is sent to the network controller only when the *output novelty error* $\widetilde{e}_p(t) = y_p(t) - y_p(t_k)$ (where $y_p(t_k)$ denotes the last output information sent to the network controller at the event time t_k) in the event-detector (which is usually embedded hardware in the sampler) satisfies a triggering condition. So transmission of plant's information is essentially scheduled by "demand". A triggering condition based on the stabilizing control action is derived to guarantee stability of the NCS.

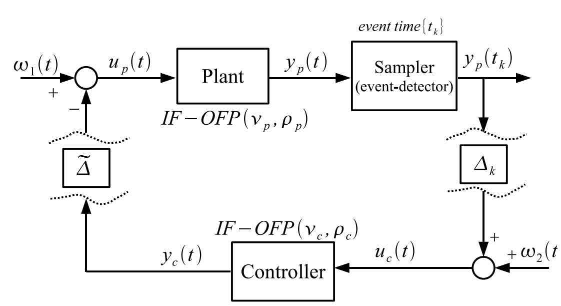

Figure 2: Event-Triggered Control NCSs (actuator is assumed to be collocated with the plant)

In most of the event-triggered NCS's results presented in the literature (cf. Tabuada, 2007, Mazo & Tabuada, 2008, Wang & Lemmon, 2009, Yu & Antsaklis, 2011a), only network induced delays (Δ_k as shown in Figure 2) from the plant to the network controller have been considered and a bound on the admissible network induced delays is usually imposed based on the analysis of the inter-event time, while the network induced delays from the network controller to the actuator ($\widetilde{\Delta}$ as shown in Figure 2) are neglected. As we have discussed in Section 1, it is not very practical to schedule the data transmissions at the plant side based on the inter-event time because of the uncertainties of the network induced delays. Moreover, network induced delay from the controller to the actuator should not be neglected for stability analysis.

In this paper, we propose a framework for output feedback based event-triggered NCSs to address the problems just mentioned above. We summarize the problems investigated in the present paper as follows:

1. If the plant is IF-OFP$(\nu_p, \rho_p)^m$, what should be the output feedback stabilizing controller and accordingly, what is the event-triggering condition? Is the condition shown in Theorem 1 still sufficient to guarantee finite-gain $\mathcal{L}_2$ stability of event-triggered control system?
2. Can we estimate the lower bound on the inter-event time $[t_{k+1} - t_k]$ implicitly determined by the triggering condition?
3. If we consider delay and quantization effects of the transmitted signals in the communication network, can we still achieve finite-gain $\mathcal{L}_2$ stability of the event-triggered control system when the network induced delays could be larger than the inter-event times?

4. Triggering Condition

In this section, we derive a triggering condition to achieve finite-gain $\mathcal{L}_2$ stability of the event-triggered NCSs with an ideal network model being assumed.

Theorem 2. *Consider the event-triggered control system shown in Figure 2, where the plant is IF-OFP$(\nu_p, \rho_p)^m$ with a $\mathcal{C}^1$ storage function V_p, while the controller is IF-OFP$(\nu_c, \rho_c)^m$ with a $\mathcal{C}^1$ storage function V_c; $\nu_c + \rho_p > 0$ and $\nu_p + \rho_c > 0$. Assume that the network induced delays $\Delta_k \equiv 0$ and $\widetilde{\Delta} \equiv 0$. If the event time t_k is explicitly determined by the time whenever*

$$\|\widetilde{e}_p(t)\|_2 > \frac{\delta}{\zeta}\left[\sqrt{\beta(\rho_p+\nu_c)+\frac{\nu_c^2}{\zeta^2}}-\frac{|\nu_c|}{\zeta}\right]\|y_p(t)\|_2,\ \forall t \geq 0, \tag{9}$$

where $\widetilde{e}_p(t) = y_p(t) - y_p(t_k)$, for $t \in [t_k, t_{k+1}]$,

$$\zeta = \left[\frac{1}{4\alpha(\nu_p+\rho_c)} + |\nu_c| - \nu_c\right]^{\frac{1}{2}}, \tag{10}$$

$\delta \in (0, 1]$ and $0 < \alpha,\ \beta < 1$, then the event-triggered control system shown in Figure 2 is finite gain $\mathcal{L}_2$ stable from $\omega(t) = [\omega_1^T(t),\ \omega_2^T(t)]^T$ to $y(t) = [y_p^T(t),\ y_c^T(t)]^T$.

Proof. Consider a storage function for the event-triggered control system given by $V = V_c + V_p$, with $u_p(t) = \omega_1(t) - y_c(t)$, and $u_c(t) = \omega_2(t) + y_p(t_k)$ for $t \in [t_k, t_{k+1})$, we have

$$\begin{aligned}\dot{V} \leq\ & \omega_1^T(t)y_p(t) - \nu_p\omega_1^T(t)\omega_1(t) + 2\nu_p\omega_1^T(t)y_c(t)\\ &+ \omega_2^T(t)y_c(t) - \nu_c\omega_2^T(t)\omega_2(t) - 2\nu_c\omega_2^T(t)\big[y_p(t) - \widetilde{e}_p(t)\big]\\ &- y_c^T(t)y_p(t) - (\nu_p+\rho_c)y_c^T(t)y_c(t) - \rho_p y_p^T(t)y_p(t)\\ &+ [y_p(t) - \widetilde{e}_p(t)]^T y_c(t) - \nu_c y_p^T(t_k)y_p(t_k),\end{aligned} \tag{11}$$

since $2\nu_c\omega_2^T(t)\widetilde{e}_p(t) \leq |\nu_c|\omega_2^T(t)\omega_2(t) + |\nu_c|\widetilde{e}_p^T(t)\widetilde{e}_p(t)$, we can further get

$$\begin{aligned}\dot{V} \leq\ & \omega^T(t)\begin{bmatrix}1 & 2\nu_p\\ -2\nu_c & 1\end{bmatrix}y(t) - \omega^T(t)\begin{bmatrix}\nu_p & 0\\ 0 & \nu_c - |\nu_c|\end{bmatrix}\omega(t)\\ &- (\nu_p+\rho_c)y_c^T(t)y_c(t) - \widetilde{e}_p(t)y_c(t) + |\nu_c|\widetilde{e}_p^T(t)\widetilde{e}_p(t)\\ &- \rho_p y_p^T(t)y_p(t) - \nu_c y_p^T(t_k)y_p(t_k).\end{aligned} \tag{12}$$

Let

$$A = \begin{bmatrix}1 & 2\nu_p\\ -2\nu_c & 1\end{bmatrix},\quad B = \begin{bmatrix}\nu_p & 0\\ 0 & \nu_c - |\nu_c|\end{bmatrix}, \tag{13}$$

and since $y_p(t_k) = y_p(t) - \widetilde{e}_p(t)$, we can get

$$\begin{aligned}\dot{V} \leq\ & \omega^T(t)Ay(t) - \omega^T(t)B\omega(t) - (\nu_p+\rho_c)y_c^T(t)y_c(t)\\ &- \widetilde{e}_p^T(t)y_c(t) + |\nu_c|\widetilde{e}_p^T(t)\widetilde{e}_p(t) - \rho_p y_p^T(t)y_p(t)\\ &- \nu_c y_p^T(t)y_p(t) + 2\nu_c\widetilde{e}_p^T(t)y_p(t) - \nu_c\widetilde{e}_p^T(t)\widetilde{e}_p(t),\end{aligned} \tag{14}$$

if we choose $0 < \alpha, \beta < 1$ and let

$$C = \begin{bmatrix}(1-\beta)(\rho_p+\nu_c) & 0\\ 0 & (1-\alpha)(\nu_p+\rho_c)\end{bmatrix}, \tag{15}$$

then we can get

$$\begin{aligned}\dot{V} \leq\ & \omega^T(t)Ay(t) - \omega^T(t)B\omega(t) - y^T(t)Cy(t)\\ &- \Big\|\sqrt{\alpha(\nu_p+\rho_c)}y_c(t) + \frac{1}{2\sqrt{\alpha(\nu_p+\rho_c)}}\widetilde{e}_p(t)\Big\|_2^2\\ &- \beta(\rho_p+\nu_c)\|y_p(t)\|_2^2 + 2\nu_c\widetilde{e}_p^T(t)y_p(t)\\ &+ \left(\frac{1}{4\alpha(\nu_p+\rho_c)} + |\nu_c| - \nu_c\right)\|\widetilde{e}_p(t)\|_2^2,\end{aligned} \tag{16}$$

thus

$$\begin{aligned}\dot{V} \leq\ & \omega^T(t)Ay(t) - \omega^T(t)B\omega(t) - y^T(t)Cy(t)\\ &+ \left(\frac{1}{4\alpha(\nu_p+\rho_c)} + |\nu_c| - \nu_c\right)\|\widetilde{e}_p(t)\|_2^2\\ &- \beta(\rho_p+\nu_c)\|y_p(t)\|_2^2 + 2\nu_c\widetilde{e}_p^T(t)y_p(t).\end{aligned} \tag{17}$$

We can further obtain

$$\begin{aligned}\dot{V} \leq\ & \omega^T(t)Ay(t) - \omega^T(t)B\omega(t) - y^T(t)Cy(t)\\ &+ \left(\frac{1}{4\alpha(\nu_p+\rho_c)} + |\nu_c| - \nu_c\right)\|\widetilde{e}_p(t)\|_2^2\\ &+ 2\nu_c\widetilde{e}_p^T(t)y_p(t) + \frac{\nu_c^2}{\frac{1}{4\alpha\left(\nu_p+\rho_c\right)} + |\nu_c| - \nu_c}\|y_p(t)\|_2^2 \\ &- \left[\beta(\rho_p+\nu_c) + \frac{\nu_c^2}{\frac{1}{4\alpha(\nu_p+\rho_c)} + |\nu_c| - \nu_c}\right]\|y_p(t)\|_2^2,\end{aligned} \tag{18}$$

and one can verify that if

$$\|\widetilde{e}_p(t)\|_2 \leq \frac{1}{\zeta}\left[\sqrt{\beta(\rho_p+\nu_c) + \frac{\nu_c^2}{\zeta^2}} - \frac{|\nu_c|}{\zeta}\right]\|y_p(t)\|_2,\ \forall t \geq 0, \tag{19}$$

we have

$$\dot{V} \leq \omega^T(t)Ay(t) - \omega^T(t)B\omega(t) - y^T(t)Cy(t),\ \forall t \geq 0. \tag{20}$$

Let $c = \min\{(1-\alpha)(\nu_p+\rho_c),\ (1-\beta)(\rho_p+\nu_c)\}$, $a = \|A\|_2$, and $b = \|B\|_2$, we can get

$$\begin{aligned}\dot{V} &\leq -c\|y(t)\|_2^2 + a\|\omega(t)\|_2\|y(t)\|_2 + b\|\omega(t)\|_2^2\\ &= -\frac{1}{2c}\left(a\|\omega(t)\|_2 - c\|y(t)\|_2\right)^2 + \frac{a^2}{2c}\|\omega(t)\|_2^2\\ &- \frac{c}{2}\|y(t)\|_2^2 + b\|\omega(t)\|_2^2 \leq \frac{k^2}{2c}\|\omega(t)\|_2^2 - \frac{c}{2}\|y(t)\|_2^2,\end{aligned} \tag{21}$$

where $k^2 = a^2 + 2bc$. Integrating (21) over $[0, \tau]$ and using $V(x) \geq 0$, then taking the square root, we arrive at

$$\|y_\tau\|_{\mathcal{L}_2} \leq \frac{k}{c}\|\omega_\tau\|_{\mathcal{L}_2} + \sqrt{\frac{2V(0)}{c}}, \tag{22}$$

where y_τ and ω_τ denote the truncated signals of $y(t)$ and $\omega(t)$. Note that the triggering condition (9) ensures that (19) is satisfied, which completes the proof. □

Remark 1. Since $\|\widetilde{e}_p(t)\|_2 = \|y_p(t) - y_p(t_k)\|_2$, we have $\|\widetilde{e}_p(t)\|_2 \geq \|y_p(t_k)\|_2 - \|y_p(t)\|_2$, thus $\|y_p(t)\|_2 \geq \|y_p(t_k)\|_2 - \|\widetilde{e}_p(t)\|_2$, for $t \in [t_k, t_{k+1})$. Based on this, if we define

$$\sigma_o = \frac{1}{\zeta}\left[\sqrt{\beta(\rho_p+\nu_c) + \frac{\nu_c^2}{\zeta^2}} - \frac{|\nu_c|}{\zeta}\right], \tag{23}$$

then one can verify that a sufficient condition for (19) to be satisfied is given by

$$\|\widetilde{e}_p(t)\|_2 \leq \frac{\sigma_o}{1+\sigma_o}\|y_p(t_k)\|_2, \quad \text{for } t \in [t_k, t_{k+1}), \forall k. \tag{24}$$

So an alternative triggering condition to (9) is given by

$$\|\widetilde{e}_p(t)\|_2 > \frac{\delta\sigma_o}{1+\sigma_o}\|y_p(t_k)\|_2, \quad \text{for } t \in [t_k, t_{k+1}),\ \forall k, \tag{25}$$

with some $\delta \in (0, 1]$. This is a more conservative triggering condition compared with (9) which will be used later for the analysis of the inter-event time.

Remark 2. In view of (9) and (22), one can see that both the triggering condition and the achievable $\mathcal{L}_2$ gain are related to the passivity indices of the plant and the controller. In general, with larger values of $\nu_c + \rho_p$ and $\nu_p + \rho_c$, we can obtain a larger triggering threshold σ_o and a smaller $\mathcal{L}_2$ gain, which implies a better performance of the event-triggered control system with respect to attenuation of external disturbances.

5. Analysis of The Inter-Event Time

The triggering condition (9) in Theorem 2 explicitly determines when a new output information of the plant should be sent to the network controller for control action update to ensure finite-gain $\mathcal{L}_2$ stability of the event-triggered control system when an ideal network model is assumed. Another problem that needs to be addressed is how often is the data transmitted in the communication network under the triggering condition? This problem is not easy in general, especially when the dynamics of the plant are highly nonlinear and only output information can be measured to generate the control action. Moreover, in the presence of external disturbances, the "Zeno" inter-event time may be unavoidable. The following proposition provides a way to estimate the lower bound on the inter-event time under the triggering condition derived in Section 4, where we assume that the output of the plant being a memoryless function belonging to a bounded sector of the state. One should be aware that while our analysis is similar to Tabuada (2007), there are other ways in the literature to estimate the inter-event time based on different assumptions, see Wang & Lemmon (2009), Anta & Tabuada (2010). Hence, it is possible to derive a less conservative result by taking different approaches under different assumptions. But here, based on the assumptions adopted in the following proposition, the impact of the disturbances on the inter-event time can be shown explicitly.

We assume that the plant is IF-OFP$(\nu_p, \rho_p)^m$ with dynamics given by

$$H_p: \begin{cases} \dot{x}_p = f_p(x_p, u_p)\\ y_p = h_p(x_p), \end{cases} \tag{26}$$

and the controller is IF-OFP$(\nu_c, \rho_c)^m$ with dynamics given by

$$H_c: \begin{cases} \dot{x}_c = f_c(x_c, u_c)\\ y_c = h_c(x_c, u_c). \end{cases} \tag{27}$$

Note we assume that there is no feed-through at the output of the plant. This usually corresponds to the case when the relative degree of the plant is greater than 0 and $\nu_p \leq 0$, see Sepulchre et al. (1997).

Proposition 1. *Consider the event-triggered control system shown in Figure 2, where the plant is IF-OFP$(\nu_p, \rho_p)^m$ and the controller is IF-OFP$(\nu_c, \rho_c)^m$. Assume that the* *network induced delays are negligible ($\Delta_k \equiv 0$ and $\widetilde{\Delta} \equiv 0$). Let the following assumptions be satisfied:*

1) *$f_p(x_p, u_p) : \mathbb{R}^{n_p} \times \mathbb{R}^{n_p} \rightarrow \mathbb{R}^{n_p}$ is locally Lipschitz continuous in x_p on a compact set $S_x \subset \mathbb{R}^{n_p}$ with Lipschitz constant L_x;*
2) *$\|f_p(x_p, u_p) - f_p(x_p, 0)\|_2 \leq L_u \|u_p\|_2$ for all $x_p \in S_x$ with some nonnegative constant L_u;*
3) *$h_p(x_p) : \mathbb{R}^{n_p} \rightarrow \mathbb{R}^{n_p}$ belongs to a sector (K_1, K_2), with $K_1 x_p^T x_p \leq x_p^T h_p(x_p) \leq K_2 x_p^T x_p$, where $K_1 \in \mathbb{R}$, $K_2 \in \mathbb{R}$ and $K_1 K_2 > 0$;*
4) *$\left\|\frac{\partial h_p}{\partial x_p}\right\|_2 \leq \gamma_p$, where $\gamma_p > 0$;*
5) *$\nu_p + \rho_c > 0$, $\rho_p + \nu_c > 0$, $\rho_c > 0$, $x_c(t_0) = 0$;*
6) *$\sup_{t \geq 0} \|\omega_1(t)\|_2 \leq d_1$ and $\sup_{t \geq 0} \|\omega_2(t)\|_2 \leq d_2$, where $d_1, d_2 > 0$.*

Then for any initial condition $x_p(0)$ in a compact set $S_0 \subset S_x$, the inter-event time $\{t_{k+1} - t_k\}$ implicitly determined by the triggering condition (25) is lower bounded by

$$\tau_k = \frac{1}{C_2} \ln\left(1 + \frac{C_3}{C_1}\right), \tag{28}$$

where $C_1 = \frac{(L_x \zeta_p + L_u \Gamma_c)\|y_p(t_k)\|_2 + L_u(d_1 + \Gamma_c d_2)}{L_x \zeta_p}$, $C_2 = \gamma_p L_x \zeta_p$ and $C_3 = \frac{\delta \sigma_o}{1+\sigma_o}\|y_p(t_k)\|_2$, with $\zeta_p = \max\{\frac{1}{|K_1|}, \frac{1}{|K_2|}\}$ and $\Gamma_c = \sqrt{\frac{1+2\rho_c|\nu_c|}{\rho_c^2}}$.

Proof. Since $\widetilde{e}_p(t) = y_p(t) - y_p(t_k)$ for $t \in [t_k, t_{k+1})$, we can get for $t \in [t_k, t_{k+1})$

$$\begin{aligned}
&\frac{d}{dt}\|\widetilde{e}_p(t)\|_2 \leq \|\dot{\widetilde{e}}_p(t)\|_2 = \|\dot{y}_p(t)\|_2 = \|\dot{h}_p(x_p)\|_2 \\
&= \Big\|\frac{\partial h_p}{\partial x_p} f_p(x_p, 0) + \frac{\partial h_p}{\partial x_p}\left[f_p(x_p, u_p) - f_p(x_p, 0)\right]\Big\|_2 \\
&\leq \gamma_p L_x \|x_p(t)\|_2 + \gamma_p L_u \|u_p(t)\|_2 \\
&= \gamma_p L_x \|x_p(t)\|_2 + \gamma_p L_u \|\omega_1(t) - y_c(t)\|_2 \\
&\leq \gamma_p L_x \|x_p(t)\|_2 + \gamma_p L_u d_1 + \gamma_p L_u \|y_c(t)\|_2.
\end{aligned} \tag{29}$$

Since $x_c(t_0) = 0$, with $\rho_c > 0$, one can prove that

$$\|y_{c\tau}\|_{\mathcal{L}_2} \leq \sqrt{\frac{1 + 2\rho_c|\nu_c|}{\rho_c^2}}\|u_{c\tau}\|_{\mathcal{L}_2}, \forall \tau \geq t_0. \tag{30}$$

Thus, we can further obtain

$$\begin{aligned}
\frac{d}{dt}\|\widetilde{e}_p(t)\|_2 &\leq \gamma_p L_x \|x_p(t)\|_2 + \gamma_p L_u d_1 + \gamma_p L_u \Gamma_c \|u_c(t)\|_2 \\
&= \gamma_p L_x \|x_p(t)\|_2 + \gamma_p L_u d_1 + \gamma_p L_u \Gamma_c \|y_p(t_k) + \omega_2(t)\|_2.
\end{aligned} \tag{31}$$

Since $h_p(x_p)$ belongs to the sector (K_1, K_2), one can verify that $\|x_p(t)\|_2 \leq \zeta_p \|y_p(t)\|_2$, and we have

$$\begin{aligned}
\frac{d}{dt}\|\widetilde{e}_p(t)\|_2 &\leq \gamma_p L_x \zeta_p \|y_p(t)\|_2 + \gamma_p L_u d_1 \\
&\quad + \gamma_p L_u \Gamma_c \|y_p(t_k)\|_2 + \gamma_p L_u \Gamma_c \|\omega_2(t)\|_2 \\
&= \gamma_p L_x \zeta_p \|\widetilde{e}_p(t) + y_p(t_k)\|_2 + \gamma_p L_u d_1 \\
&\quad + \gamma_p L_u \Gamma_c \|y_p(t_k)\|_2 + \gamma_p L_u \Gamma_c \|\omega_2(t)\|_2 \\
&\leq \gamma_p L_x \zeta_p \|\widetilde{e}_p(t)\|_2 + \gamma_p (L_x \zeta_p + L_u \Gamma_c)\|y_p(t_k)\|_2 \\
&\quad + \gamma_p L_u (d_1 + \Gamma_c d_2),
\end{aligned} \tag{32}$$

so the evolution of $\|\widetilde{e}_p(t)\|_2$ during the time interval $[t_k, t_{k+1})$ is bounded by the solution to

$$\begin{aligned}
\frac{d}{dt}\phi(t) &= \gamma_p L_x \zeta_p \phi(t) + \gamma_p (L_x \zeta_p + L_u \Gamma_c)\|y_p(t_k)\|_2 \\
&\quad + \gamma_p L_u (d_1 + \Gamma_c d_2),
\end{aligned} \tag{33}$$

with initial condition $\phi(t_k) = 0$. Hence the time for $\|\widetilde{e}_p(t)\|_2$ to evolve from 0 to $\frac{\delta \sigma_o}{1+\sigma_o}\|y_p(t_k)\|_2$ is lower bounded by the solution to $\phi(t_k + \tau_k) = \frac{\delta \sigma_o}{1+\sigma_o}\|y_p(t_k)\|_2$. Then we can get the τ_k given in (28). ☐

Remark 3. One can see that when $d_1 = d_2 = 0$ (no external disturbance inputs), then we have

$$\tau_k = \frac{1}{\gamma_p L_x \zeta_p} \ln\left(1 + \frac{\frac{\delta \sigma_o}{1+\sigma_o} L_x \zeta_p}{L_x \zeta_p + L_u \Gamma_c}\right) > 0, \tag{34}$$

and in this case we can obtain a common lower bound of the inter-event time. Moreover, a larger triggering threshold σ_o results in a larger τ_k. Since σ_o is related to the passivity indices of the plant and the controller, the interactions between the triggering condition, the passivity indices and the inter-event time are implicitly revealed here. However, when the external disturbances ω_1, ω_2 cannot be neglected, τ_k could be extremely small when $y_p(t)$ approaches the origin, and we may get "Zeno" inter-event time.

Remark 4. Although the triggering condition derived in Theorem 1 and the analysis of the inter-event time shown in Proposition 1 are all obtained with an ideal network model being assumed, stability of the event-triggered NCSs can still be guaranteed as long as the network induced delays are upper bounded by the inter-event time implicitly determined by the triggering condition. This is one of the reasons that most of the work in the literature are trying to get a larger common lower bound on the inter-event time and impose such bound on the *admissible* network induced delays. However, as discussed above, in the presence of external disturbances, the inter-event time could be very small, thus it is usually difficult to obtain a desirable common lower bound on the inter-event time and it is also impractical to impose such bound on the network induced delays.

6. Signal Quantization and Time-varying Network Induced Delays

The analysis on the inter-event time shown in the previous section reveals the main problems that are concerned when applying event-triggered control to networked control systems: stability of the event-triggered control system may not be guaranteed if the network induced delays are larger than the inter-event time, and it is conservative to impose an upper bound on the admissible network induced delays based on the analysis of the inter-event time.

In this section, we consider quantization of the transmitted signals in the network and delays both from the plant to the network controller and from the network controller to the plant. We introduce a set-up which guarantees finite-gain $\mathcal{L}_2$ stability of the event-triggered control system in the presence of time-varying (or constant) network induced delays with bounded jitters.

Consider the set-up for event-triggered networked control system as shown in Figure 3. The plant H_p is IF-OFP$(\nu_p, \rho_p)^m$ with a $\mathcal{C}^1$ storage function V_p; the network controller H_c is IF-OFP$(\nu_c, \rho_c)^m$ with a $\mathcal{C}^1$ storage function V_c; $T_1(t)$ represents the network induced delay from the network controller to the plant, and $T_2(t)$ represents the network induced delay from the plant to the network controller; the "ZOH" block denotes the zero-order holder; the "ED" block represents the "event-detector", which samples the output of the plant with adequately fast sampling rate; whenever ED detects that a specific triggering condition of the plant is satisfied, it will send the output information of the plant at that event time to the ZOH; the "DB" block represents the dead-band control so that the signal $\upsilon_r(t)$ can only be transmitted when

$$\|\upsilon_r(t) - \upsilon_r(t_k)\|_2 = \delta\sigma_o\|\upsilon_r(t)\|_2, \quad \text{with } \|\upsilon_r(t)\|_2 \geq \Delta_{min}, \tag{35}$$

where Δ_{min} is some lower bound on the dead-band designed for practical application; Qc and Qp are passive memoryless quantizers such that

$$\begin{aligned} &\text{Qc:} \;\; a_c u_{Qc}^2(t) \leq u_{Qc}(t) y_{Qc}(t) \leq b_c u_{Qc}^2(t), \\ &\text{Qp:} \;\; a_p u_{Qp}^2(t) \leq u_{Qp}(t) y_{Qp}(t) \leq b_p u_{Qp}^2(t), \end{aligned} \tag{36}$$

for some $b_c > a_c \geq 0$ and $b_p > a_p \geq 0$, where $u_{Qc}(t)$ and $y_{Qc}(t) = q_c\big(u_{Qc}(t)\big)$ denote the input and the quantized output of Qc, $u_{Qp}(t)$ and $y_{Qp}(t) = q_p\big(u_{Qp}(t)\big)$ denote the input and the quantized output of Qp; if $u_{Qc}(t)$ and $u_{Qp}(t)$ are vectors, then $q_c(\cdot)$ and $q_p(\cdot)$ function component wise on the input vectors. M is a local controller implemented at the plant side such that

$$\begin{bmatrix} \upsilon_r(t) \\ u_r(t) \end{bmatrix} = M \begin{bmatrix} \widetilde{u}_c(t) \\ \widetilde{y}_c(t) \end{bmatrix} = \begin{bmatrix} M_{11} I_m & 0 \\ M_{21} I_m & M_{22} I_m \end{bmatrix} \begin{bmatrix} \widetilde{u}_c(t) \\ \widetilde{y}_c(t) \end{bmatrix}, \tag{37}$$

where $I_m \in \mathbb{R}^{m\times m}$ is the identity matrix and M_{11}, M_{21}, M_{22} are chosen such that

$$\begin{aligned} &M_{11}^2 = \frac{\frac{1}{4\rho_c} - \nu_c}{\frac{1}{2\rho_c} + |\nu_c|}, \quad M_{21}^2 = \frac{b_c^2}{2(1 - D_1)\rho_c^2} \\ &M_{22}^2 = \frac{2b_c^2}{1 - D_1}, \quad M_{21}M_{22} < 0. \end{aligned} \tag{38}$$

The implementation of M is also illustrated in Figure 3.

Proposition 2. *Consider the event-triggered networked control system as shown in Figure 3. Let the following conditions be satisfied:*

1) *the controller is designed such that $\nu_c + \rho_p > 0$ and $\rho_c + \nu_p > 0$, with $\rho_c > 0$ and $\rho_c\nu_c < \frac{1}{4}$;*

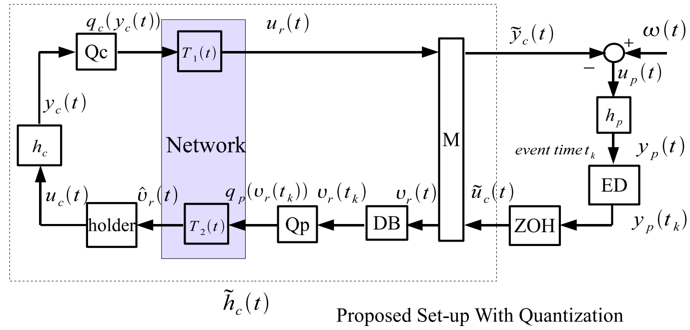

Figure 3: Proposed Set-up for Event-Triggered NCSs With Quantization and Time-varying Delays

2) *$0 \leq \left|\frac{dT_1(t)}{dt}\right| \leq D_1 < 1$, $0 \leq \left|\frac{dT_2(t)}{dt}\right| \leq D_2 < 1$;*
3) *the holder at the controller side yields*

$$u_c(t) = \frac{1}{(1 + \delta\sigma_o) b_p \sqrt{1 + D_2}} q_p\big(\upsilon_r(t_k)\big), \tag{39}$$

$$\textit{for } t \in \Big[t_k + T_2(t_k), t_{k+1} + T_2(t_{k+1})\Big), \forall k.$$

If the event time t_k is explicitly determined by the time whenever

$$\|\widetilde{e}_p(t)\|_2 > \delta\sigma_o\|y_p(t)\|_2, \;\textit{when } \|y_p(t)\|_2 \geq \frac{\Delta_{min}}{M_{11}}, \tag{40}$$

with $\delta \in (0, 1]$ and σ_o as defined in (23), then the event-triggered control system is finite-gain $\mathcal{L}_2$ stable from $\omega(t)$ to $y_p(t)$.

Proof. Since the controller H_c is IF-OFP$(\nu_c, \rho_c)^m$ such that

$$\dot{V}_c \leq u_c^T(t) y_c(t) - \rho_c y_c^T(t) y_c(t) - \nu_c u_c^T(t) u_c(t), \tag{41}$$

with $\rho_c > 0$, we can get

$$\dot{V}_c(t) \leq \big(\frac{1}{2\rho_c} + |\nu_c|\big)\|u_c(t)\|_2^2 - \frac{\rho_c}{2}\|y_c(t)\|_2^2, \tag{42}$$

integrating both sides of (42) from t_0 to t ($\forall t \geq t_0 \geq 0$), we can get

$$\begin{aligned} \Delta V_c &= V_c(t) - V_c(t_0) \\ &\leq \big(\frac{1}{2\rho_c} + |\nu_c|\big)\int_{t_0}^{t}\|u_c(\tau)\|_2^2 d\tau - \frac{\rho_c}{2}\int_{t_0}^{t}\|y_c(\tau)\|_2^2 d\tau. \end{aligned} \tag{43}$$

For Qp, with $q_p(\cdot)$ functioning component wise on the input vector $\upsilon_r(t_k)$, one can verify that $\|q_p(\upsilon_r(t_k))\|_2 \leq b_p\|\upsilon_r(t_k)\|_2$. Since

$$\widehat{\upsilon}_r(t) = \begin{cases} 0, & \text{for } t \in (t_k + T_2(t_k), t_{k+1} + T_2(t_{k+1})), \quad \forall k \\ q_p(\upsilon_r(t_k)), & \text{for } t = t_k + T_2(t_k), \quad \forall k \end{cases} \tag{44}$$

under condition 3), choose $t = \sum_{k=0}^{N}\big[t_{k+1} - t_k + T_2(t_{k+1}) - T_2(t_k)\big]$, we can further obtain

$$\int_{t_0}^{t} \|u_c(\tau)\|_2^2 d\tau = \sum_{k=0}^{N} \frac{t_{k+1} - t_k + T_2(t_{k+1}) - T_2(t_k)}{(1+\delta\sigma_o)^2 b_p^2 (1+D_2)} \|q_p(\upsilon_r(t_k))\|_2^2, \tag{45}$$

under condition 2), we have

$$\begin{aligned}\int_{t_0}^{t} \|u_c(\tau)\|_2^2 d\tau &\leq \sum_{k=0}^{N} \frac{(1+D_2)(t_{k+1}-t_k)}{(1+\delta\sigma_o)^2 b_p^2 (1+D_2)} \|q_p(\upsilon_r(t_k))\|_2^2 \\ &= \sum_{k=0}^{N} \frac{t_{k+1}-t_k}{(1+\delta\sigma_o)^2 b_p^2} \|q_p(\upsilon_r(t_k))\|_2^2 \\ &\leq \sum_{k=0}^{N} \frac{t_{k+1}-t_k}{(1+\delta\sigma_o)^2} \|\upsilon_r(t_k)\|_2^2.\end{aligned} \tag{46}$$

Note that the dead-band control actually guarantees that

$$\|\upsilon_r(t) - \upsilon_r(t_k)\|_2 \leq \delta\sigma_o \|\upsilon_r(t)\|_2, \quad \text{for } t \in [t_k, t_{k+1}], \forall k, \tag{47}$$

since $\|\upsilon_r(t) - \upsilon_r(t_k)\|_2 \geq \|\upsilon_r(t_k)\|_2 - \|\upsilon_r(t)\|_2$, we can conclude that

$$\|\upsilon_r(t_k)\|_2 \leq (1+\delta\sigma_o)\|\upsilon_r(t)\|_2, \text{ for } t \in [t_k, t_{k+1}], \ \forall k, \tag{48}$$

thus

$$\begin{aligned}\int_{t_0}^{t} \|u_c(\tau)\|_2 d\tau &\leq \sum_{k=0}^{N} \big(t_{k+1} - t_k\big) \frac{\|\upsilon_r(t_k)\|_2^2}{(1+\delta\sigma_o)^2} \\ &= \sum_{k=0}^{N} \int_{t_k}^{t_{k+1}} \frac{\|\upsilon_r(t_k)\|_2^2}{(1+\delta\sigma_o)^2} d\tau \\ &\leq \sum_{k=0}^{N} \int_{t_k}^{t_{k+1}} \|\upsilon_r(\tau)\|_2^2 d\tau = \int_{t_0}^{t} \|\upsilon_r(t)\|_2^2 d\tau.\end{aligned} \tag{49}$$

For Qc, with $q_c(\cdot)$ functioning component wise on the input vector $y_c(t)$, one can verify that $\|q_c(y_c(t))\|_2 \leq b_c\|y_c(t)\|_2$. Since $0 \leq \big|\frac{dT_1(t)}{dt}\big| \leq D_1 < 1$, one could get

$$\begin{aligned}\int_{t_0}^{t} \|u_r(\tau)\|_2^2 d\tau &\leq \frac{1}{1-D_1} \int_{t_0}^{t} \|q_c(y_c(\tau))\|_2^2 d\tau \\ &\leq \frac{b_c^2}{1-D_1} \int_{t_0}^{t} \|y_c(\tau)\|_2^2 d\tau,\end{aligned} \tag{50}$$

so

$$-\int_{t_0}^{t} \|y_c(\tau)\|_2^2 d\tau \leq -\frac{1-D_1}{b_c^2} \int_{t_0}^{t} \|u_r(\tau)\|_2^2 d\tau. \tag{51}$$

Replace (51) into (43) and in view of (49), we can further get

$$\Delta V_c \leq \int_{t_0}^{t} \Big[\big(\frac{1}{2\rho_c} + |\nu_c|\big)\|\upsilon_r(\tau)\|_2^2 - \frac{\rho_c(1-D_1)}{2b_c^2}\|u_r(\tau)\|_2^2\Big] d\tau, \tag{52}$$

with $\upsilon_r(t) = M_{11}\widetilde{u}_c(t)$, $u_r(t) = M_{21}\widetilde{u}_c(t) + M_{22}\widetilde{y}_c(t)$, we can rewrite (52) as

$$\begin{aligned}\Delta V_c \leq \int_{t_0}^{t} \Big\{ &\frac{-\rho_c(1-D_1)M_{21}M_{22}}{b_c^2} \widetilde{u}_c^T(\tau)\widetilde{y}_c(\tau) \\ &+ \Big[\big(\frac{1}{2\rho_c} + |\nu_c|\big)M_{11}^2 - \frac{\rho_c(1-D_1)M_{21}^2}{2b_c^2}\Big]\widetilde{u}_c^T(\tau)\widetilde{u}_c(\tau) \\ &- \frac{\rho_c(1-D_1)M_{22}^2}{2b_c^2}\widetilde{y}_c^T(\tau)\widetilde{y}_c(\tau)\Big\} d\tau,\end{aligned} \tag{53}$$

with M_{11}, M_{21}, M_{22} chosen as given in (38), we get

$$\Delta V_c \leq \int_{t_0}^{t} \big[\widetilde{u}_c^T(\tau)\widetilde{y}_c(\tau) - \nu_c\widetilde{u}_c^T(\tau)\widetilde{u}_c(\tau) - \rho_c\widetilde{y}_c^T(\tau)\widetilde{y}_c(\tau)\big] d\tau, \tag{54}$$

which implies that the subsystem $\widetilde{H}_c : \widetilde{u}_c(t) \to \widetilde{y}_c(t)$ shown in Figure3 is IF-OFP$(\nu_c, \rho_c)^m$.

According to Theorem 2, for the feedback interconnection of H_p and $\widetilde{H}_c$, if we schedule the transmission of the output measurement $y_p(t)$ according to the triggering condition (9), then the event-triggered control system will be finite-gain $\mathcal{L}_2$ stable. Furthermore, because the growing rate on the threshold of the dead-band control is the same as the growing rate on the threshold of the triggering condition, one can conclude that the data transmission of $\upsilon_r(t)$ and the triggering process are actually synchronized. Thus, whenever a new output information of the plant is obtained, an updated quantized signal $q_p\big(\upsilon_r(t_k)\big)$ will be sent to the network controller. When $\|\upsilon_r(t)\|_2 < \Delta_{min}$, which could be considered as the case when the output of the plant reaches some safe region for practical application, then no more data transmission is needed. The proof is completed. □

Remark 5. Based on the set-up shown in Proposition 2, one can see that system $\widetilde{H}_c : \widetilde{u}_c(t) \to \widetilde{y}_c(t)$ is IF-OFP$(\nu_c, \rho_c)^m$ as the controller $H_c : u_c(t) \to y_c(t)$, so the networked control system can still be analyzed as a feedback interconnection of an IF-OFP$(\nu_p, \rho_p)^m$ plant with an IF-OFP$(\nu_c, \rho_c)^m$ subsystem $(\widetilde{H}_c)$, and the triggering condition derived in Theorem 2 can be directly applied for event-triggered data transmission in the current set-up. One can further use the analysis shown in Proposition 1 to estimate the inter-event time.

Remark 6. Let us consider a special case when the network controller is a static output feedback gain matrix

$$y_c(t) = Ku_c(t), \quad K \in \mathbb{R}^{m\times m}, \tag{55}$$

then we have

$$\|y_c(t)\|_2^2 = u_c^T(t)K^TKu_c(t) \leq \lambda_{max}\{K^TK\}\|u_c(t)\|_2^2,$$

where $\lambda_{max}\{\cdot\}$ denotes the largest eigenvalue of a square matrix. Choose scalar $\kappa > 0$ such that $\kappa^2 \geq \lambda_{max}\{K^TK\}$, we can obtain

$$\begin{aligned}&\kappa\|u_c(t)\|_2^2-\frac{1}{\kappa}\|y_c(t)\|_2^2\\&\geq\kappa\|u_c(t)\|_2^2-\frac{\lambda_{max}\{K^TK\}}{\kappa}\|u_c(t)\|_2^2\geq 0.\end{aligned}\tag{56}$$

In view of the derivation from (43)-(52), we can obtain

$$0\leq\int_{t_0}^{t}\kappa\|\upsilon_r(\tau)\|_2^2d\tau-\int_{t_0}^{t}\frac{1-D_1}{b_c^2\kappa}\|u_r(\tau)\|_2^2d\tau.\tag{57}$$

Since $\upsilon_r=M_{11}\widetilde{u}_c$ and $u_r=M_{21}\widetilde{u}_c+M_{22}\widetilde{y}_c$, we can obtain

$$\begin{aligned}0\leq&\int_{t_0}^{t}\frac{-2(1-D_1)M_{21}M_{22}}{\kappa b_c^2}\widetilde{u}_c^T(\tau)\widetilde{y}_c(\tau)d\tau\\&+\int_{t_0}^{t}\Big[\kappa M_{11}^2-\frac{(1-D_1)M_{21}^2}{\kappa b_c^2}\Big]\widetilde{u}_c^T(\tau)\widetilde{u}_c(\tau)d\tau\\&-\int_{t_0}^{t}\frac{(1-D_1)M_{22}^2}{\kappa b_c^2}\widetilde{y}_c^T(\tau)\widetilde{y}_c(\tau)d\tau.\end{aligned}\tag{58}$$

By choosing M_{11},M_{21},M_{22} according to

$$\begin{aligned}&M_{11}^2=\frac{\frac{1}{4\rho_c}-\nu_c}{\kappa},\quad M_{21}^2=\frac{\kappa b_c^2}{4(1-D_1)\rho_c}\\&M_{22}^2=\frac{\kappa\rho_c b_c^2}{1-D_1},\quad M_{21}M_{22}<0,\end{aligned}\tag{59}$$

where ρ_c,ν_c are chosen such that condition 1) in Proposition 2 is satisfied, we obtain

$$0\leq\int_{t_0}^{t}\big[\widetilde{u}_c^T(\tau)\widetilde{y}_c(\tau)-\nu_c\widetilde{u}_c^T(\tau)\widetilde{u}_c(\tau)-\rho_c\widetilde{y}_c^T(\tau)\widetilde{y}_c(\tau)\big]d\tau.\tag{60}$$

Thus, subsystem $\widetilde{H}_c:\widetilde{u}_c(t)\rightarrow\widetilde{y}_c(t)$ is IF-OFP$(\nu_c,\rho_c)^m$ and the event-triggered control approach shown in Proposition 2 still applies.

Remark 7. Traditionally, NCSs are referred to direct type remote control loops as shown in Figure 2. One may argue that in our set-up, we need a local controller at the plant side. But as illustrated in Figure 3, the local controller M only requires a direct output feedback loop from $\widetilde{u}_c(t)$ to $\widetilde{y}_c(t)$ with gain $\frac{M_{21}}{M_{22}}$. The tedious and complex control action computation can still be done at the network controller(i.e., the network controller could be an adaptive or optimal controller with its inputs and outputs satisfying the dissipative inequalities).

Remark 8. In our proposed set-up, instead of obtaining an upper bound on the admissible network induced delays based on the triggering condition or based on the past information of the plant, we consider delays both from the plant to the controller and from the controller to the plant, and we have shown that finite-gain $\mathcal{L}_2$ stability can be achieved in the presence of time-varying (or constant) network induced delays with bounded jitters.

Remark 9. One should notice that the implementation of the local controller M at the plant side requires the knowledge of the network controller's passivity indices (ρ_c,ν_c), the parameter b_c of quantizer Qc, and the knowledge of the "jitters" on the network induced delays from the controller to the plant (D_1). The implementation of the "holder" at the controller side requires the knowledge of the "jitters" on the network induced delays from the plant to the controller (D_2), the information on the triggering threshold $(\delta\sigma_o)$, and the parameter b_p of quantizer Qp.

The following example is provided to illustrate the results presented in this paper.
Example. Consider the IF-OFP system given by

$$\begin{aligned}\dot{x}_{p1}(t)&=-3x_{p1}^3(t)+x_{p1}(t)x_{p2}(t)\\\dot{x}_{p2}(t)&=0.2x_{p2}(t)+2u_p(t)\\y_p(t)&=x_{p2}(t),\end{aligned}\tag{61}$$

we can see that the system is ZSD but unstable, and we can only measure x_{p2}. If we choose the storage function $V_p(x_p)=\frac{1}{4}x_{p2}^2(t)$, we can get

$$\dot{V}_p(x_p)=u_p(t)y_p(t)+0.1y_p^2(t),\tag{62}$$

so in this case $\rho_p=-0.1$, $\nu_p=0$, and the plant is IF-OFP(0,-0.1) with respect to x_{p2}.

If we consider an IF-OFP controller, which is given by

$$\begin{aligned}\dot{x}_c(t)&=-3x_c(t)+u_c(t)\\y_c(t)&=7x_c(t)+u_c(t),\end{aligned}\tag{63}$$

with storage function $V_c(x_c)=\frac{49}{26}x_c^2(t)$, we can get

$$\dot{V}_c(x_c)=u_c(t)y_c(t)-\frac{3}{13}y_c^2(t)-\frac{10}{13}u_c^2(t),\tag{64}$$

and in this case $\rho_c=\frac{3}{13}$, $\nu_c=\frac{10}{13}$. So we have $\rho_c+\nu_p>0$ and $\nu_c+\rho_p>0$. If we choose $\alpha=\beta=0.9$, then the triggering condition shown in Theorem 2 with $\delta=1$ is given by

$$\|\widetilde{e}_p(t)\|_2>0.3142\|y_p(t)\|_2,\forall t\geq 0.\tag{65}$$

The external disturbance $\omega(t)$ applied to the plant is an uniformly distributed random signal on the interval $[0,1]$. The quantizers used at the plant side and at the controller side are both uniform mid-tread quantizer with quantization level 1, so in this case one can verify that $a_c=a_p=0$, and $b_c=b_p=2$ (other types of quantizers can also be used as long as the input-output mapping of the quantizers satisfies (36)). Assume that the network induced delay from the plant to the controller is increasing with rate $0.6(T_2(0)=0.5s)$, and the delay from the controller to the plant is increasing with rate $0.2(T_1(0)=0.2s)$, so $D_1=0.2,D_2=0.6$. Based on (38), choose $M_{11}=0.3271$, $M_{21}=6.8516$, $M_{22}=-3.1623$, and note that the output of the holder in this case should be

$u_c(t) = \frac{1}{(1+\delta\sigma_o)b_p\sqrt{1+D_2}} q_p\big(\upsilon_r(t_k)\big) = 0.3008 q_p\big(\upsilon_r(t_k)\big)$, for $t \in \big[t_k + T_2(t_k), t_{k+1} + T_2(t_{k+1})\big)$.

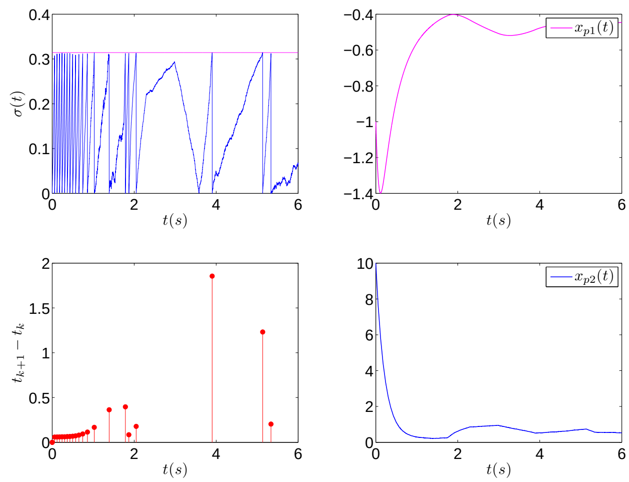

Figure 4: simulation result with time-varying network induced delays and quantizations: event times and state evolution

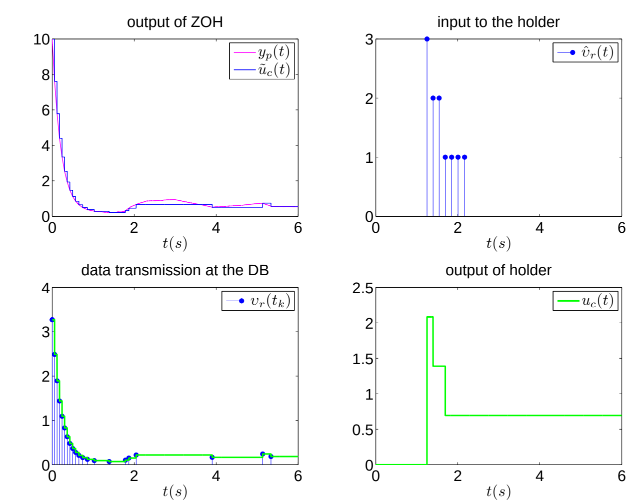

Figure 5: simulation result with time-varying network induced delays and quantizations: process from $y_p(t)$ to $u_c(t)$

The simulation results are shown in Figure 4 and Figure 5. In Figure 4, $\sigma(t)$ shows the evolution of $\frac{\|\widetilde{e}_p(t)\|_2}{\|y_p(t)\|_2}$, $\{t_{k+1} - t_k\}$ shows the evolution of the inter-event time, x_{p1} and x_{p2} show the evolution of the states of the plant. The process from $y_p(t)$ to $u_c(t)$ is shown in Figure 5.

7. Conclusion

In this paper, it is assumed that the plant and the controller are Input Feed-forward Output Feedback Passive(IF-OFP), and an output feedback based event-triggered control framework for network control systems(NCSs) is proposed. The contributions of this work are summarized as follows:

- This framework studies event-triggered control for network control systems from an I/O perspective based on the dissipative properties of the plant and the network controller; the triggering condition is derived based on the passivity theorem, which enables us to characterize a large class of output feedback stabilizing controller.
- Signal quantization of the transmitted signals in the network has been considered and a scattering transformation has been applied to deal with time-varying (or constant) network induced delays with bounded jitters. The key idea is to use the limited computation power at the plant side to implement a local controller so that the plant and the network controller can still be analyzed as a feedback interconnection of two IF-OFP systems through the communication networks, while the scheduling of the data-transmissions at the plant side is event-triggered.
- Network induced delays both from the plant to the network controller and from the network controller to the plant have been considered. Finite-gain $\mathcal{L}_2$ stability from external disturbance to the plant output is achieved under the proposed set-up.
- This framework is an important extension on applying event-triggered control to networked control systems, especially for the cases when the delays in the network could be larger than the inter-event time implicitly determined by the triggering conditions.

We believe these problems have not been addressed in the open literature yet.

References

Anderson, R. and Spong, M. W. (1989). "Bilateral Control of Teleoperators with Time Delay,"IEEE Transaction on Automatic Control, vol. 34, pp. 494-501, 1989.

Aström, K. J., & Wittenmark, B. (1990). *Computer Controlled Systems: theory and design.* Prentice-Hall, Englewood Cliffs, NJ.

Årzén, K. E. (1999). A simple event based PID controller. In IFAC World Congress. China (pp.423-428).

Aström, K. J., & Bernhardsson, B. M. (2002). Comparison of Riemann and Lebesgue sampling for first order stochastic systems (I). In IEEE Conf. on decision and control. Las Vegas, NV(pp.2011-2016).

Aström, K. J. (2008). Event Based Control. *Analysis and Design of Nonlinear Control Systems, Part 3.* pp.127-147. Springer.

Anta, A., & Tabuada, P. (2010). To sample or not to sample: Self-triggered control for nonlinear systems. *IEEE Transactions on Automatic Control*, 55(9), 2030-2042.

Chopra, N. and Spong, M. W. (2007). Delay independent stability for interconnected nonlinear systems with finite L_2 gain. IEEE conference on Decision and Control, page 3847-3852, 12-14 Dec. 2007.

Chopra, N. (2008). Passivity results for interconnected systems with time delay, 47th IEEE Conference on Decision and Control. page 4620-4625, 9-11 Dec. 2008.

Donkers, M. C. F., & Heemels, W. P. M. H. (2010). Output-Based Event-Triggered Control with Guaranteed $\mathcal{L}_\infty$-gain and Improved Event-Triggering. In *IEEE Conf. on Decision and Control*. Atlanta, GA (pp.3246-3251).

Haddad, W. M., & Chellaboina, VS. (2008). *Nonlinear Dynamical Systems and Control: A Lyapunov-Based Approach*. Princeton University Press.

Heemels, W. P. M. H., Sandeeb, J. H., & Van Den Boscha, P. P. J. (2008). Analysis of event-driven controllers for linear systems. *International Journal of Control*, 81(4), 571-590.

Hirchea, S., Matiakis, T., and Bussa, M. (2009). A distributed controller approach for delay-independent stability of networked control systems. *Automatica* , Volume 45, Issue 8, Page 1828-1836, August, 2009.

Jury, E. I. (1958). *Sample-Data Control Systems*. Wiley, New York.

Krichman, M., Sontag, E. D., & Wang, Y. (1999). Input-Output-to-State Stability. *SIAM J. Control Optim*, pp.1874-1928.

Khalil, H. (2002). *Nonlinear systems*. Prentice Hall, 3 edition.

Kofman, K., & Braslavsky, J. H. (2006). Level Crossing Sampling in Feedback Stabilization under Data-Rate Constraints. In *IEEE Conf. on Decision and Control*. San Diego, CA (pp.4423-4428).

LaSalle, J. P. (1960). Some extensions of Liapunov's second method. *IRE Transactions on Circuit Theory*, CT-7, 520-527.

Lozano, R., Chopra, N., and Spong, M. W. (2002). Passivation of Force Reflecting Bilateral Teleoperators with Time Varying Delay, in Proceedings of the 8th Mechatronics Forum, (Enschede, Netherlands). 2002.

Matiakis, T., Hirche, S., & Buss, M.(2006). A novel Input-Output Transformation Method to Stabilize Networked Control Systems of Delay. 17th International Symposium on Mathematical Theory of Networks and Systems. Kyoto, Japan (pp.2890-2897).

Mazo, M., & Tabuada, P.(2008). On event-triggered and self-triggered control over sensor/actuator networks. In *IEEE Conf. on Decision and Control*. Cancun, Mexico (pp.435-440).

Otanez, P. G., Moyne, J. R., & Tilbury, D. M. (2002). Using deadbands to reduce communication in networked control systems. In *American Control Conference*. Anchorage, AK (pp.3015-3020).

Ragazzini, J. R., & Franklin, G. F. (1958). *Sampled-Data Control Systems*. McGraw-Hill, New York.

Sontag, E. D. (1989). Smooth stabilization implies coprime factorization. *IEEE Transactions on Automatic Control*, 34(4), 435-443.

Sepulchre, R., Jankovic, M., & Kokotovic, P. (1997). *Constructive Nonlinear Control*. Springer-Verlag.

Tabuada, P. (2007). Event-triggered real-time scheduling of stabilizing control tasks. *IEEE Transaction on Automatic Control*, 52(9), 1680-1685.

Willems, J. C. (1972). Dissipative dynamical systems part I: General theory. *Archive for Rational Mechanics and Analysis*. 45(5), 321-351. Springer Berlin.

Wang, X., & Lemmon, M. D. (2009). Self-triggered feedback control systems with finite-gain $\mathcal{L}_2$ stability. *IEEE Transactions on Automatic Control*, 54(3), 452-467.

Yu, H., & Antsaklis, P. J. (2011a). Event-Triggered Real-Time Scheduling For Stabilization of Passive/Output Feedback Passive Systems. In *American Control Conference*. San Francisco, CA (pp. 1674-1679).

Yu, H., & Antsaklis, P. J. (2011b). Event-Triggered Output Feedback Control for Networked Control Systems using Passivity: Triggering Condition and Limitations. In IEEE Conf. on Decision and Control (CDC-11) and ECC-11. Orlando, FL (pp. 199-204).

Yu, H., & Antsaklis, P. J. (2011c). Event-Triggered Output Feedback Control for Networked Control Systems using Passivity: Time-varying Network Induced Delays. In IEEE Conf. on Decision and Control (CDC-11) and ECC-11. Orlando, FL (pp. 205-210).

Yu, H., Zhu, F., & Antsaklis, P. J. (2011d). Stabilization of Large-scale Distributed Control Systems using I/O Event-driven Control and Passivity, In IEEE Conf. on Decision and Control (CDC-11) and ECC-11. Orlando, FL (pp. 4245-4250).