Stochastic Passivity of Discrete-Time Markovian Jump Nonlinear Systems

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***Abstract*— In this paper, we analyze the stochastic passivity properties of discrete-time Markovian jump nonlinear systems. We define the notions of general stochastic dissipativity, stochastic QSR dissipativity and then stochastic passivity for these systems. Based on these definitions, the discrete-time stochastic KYP lemma is derived, which gives a necessary and sufficient condition for such a Markovian jump nonlinear system to be stochastic QSR dissipative. Based on the stochastic KYP lemma, we prove that a Markovian jump nonlinear system is locally stochastic feedback passive if and only if its zero dynamics are locally stochastic passive. These results can be directly extended to the case when we have interconnected Markovian jump nonlinear systems. Furthermore, given such interconnected Markovian jump nonlinear subsystems that are stochastic feedback passive, we analyze the stochastic stability of the entire system. We design a feedback control law and obtain the conditions on the weighted Laplacian matrix between all the subsystems to stabilize the entire system in the stochastic sense.**

***Index Terms*— Stochastic (Feedback) Passivity; Markovian Jump Systems; Discrete-Time Nonlinear Systems**

I. Introduction

Markovian jump systems are stochastic systems whose dynamics are subject to random changes due to, for example, changing subsystem interconnections, component failures or repairs, sudden environmental disturbances, and abrupt variations of the operating point [1], [2]. Such a system consists of multiple operating modes and the switching between these modes is governed by a time varying parameter taking values on a finite state Markovian chain. There is rich literature on the stability, observability, controllability, $\mathcal{H}_2$ and $\mathcal{H}_\infty$ norm of Markovian jump systems and, especially Markovian jump linear systems [3]–[5]. However, in this paper, we are interested in the passivity properties of the more general class of Markovian jump nonlinear systems in the discrete-time setting.

A dynamical system is said to be dissipative if it satisfies the dissipativity inequality, i.e., its increase in storage function is bounded by the energy supplied to the system [6], [7]. Passivity is one of the most useful forms of dissipativity and it is a desirable system property in addition to stability. This is because a passive system can achieve asymptotic stability using feedback given that it is zero state detectable [8] and the parallel and negative feedback interconnections of two passive systems remain passive.

The notion of stochastic dissipativity/passivity has appeared in a large amount of work that consider continuous-time systems. These include the problems of stochastic stabilization [9]–[11], passivity-based control [12], stochastic $\mathcal{H}_2$ control [13], stochastic $\mathcal{H}_\infty$ like control [14], [15], stochastic ergodic control [16], and robust simultaneous stabilization of a set of deterministic systems with uncertain parameters [17]. Much fewer work ([5], [18], [19]) can be found in the literature on the stochastic dissipativity/passivity of discrete-time stochastic nonlinear systems and their definitions do not seem to be unified.

In this paper, we propose the notions of stochastic dissipativity, stochastic QSR dissipativity and stochastic passivity as a special case in the discrete-time setting. The counterparts in the deterministic continuous-time setting can be found in [7], [20], [21]. The closest definitions to our presentation in the stochastic setting are in [19] where a nonlinear discrete-time system described by a stochastic difference equations with Markovian switching is considered. However, the paper studies the robust simultaneous stabilization problem and proposes the notion of exponential dissipativity instead. Based on our proposed definitions, we obtain the stochastic KYP lemma which provides a necessary and sufficient condition for a Markovian jump nonlinear system to be stochastic QSR dissipative. However, since the stochastic KYP lemma relies on the existence of some real functions that satisfy the stochastic KYP property, the lemma itself does not provide a direct method to determine if a given Markovian jump nonlinear system is stochastic QSR dissipative/passive or not. Inspired by the authors' work in [22] which discusses the generalized passivity of discrete-time switched nonlinear systems, we introduce the definition of stochastic feedback passivity and investigate the zero dynamics [6] of the system. By extending the stochastic KYP lemma, we prove that a Markovian jump nonlinear system is locally stochastic feedback passive if and only if its zero dynamics are locally stochastic passive. These results can be extended to N interconnected Markovian jump nonlinear systems. If these systems are stochastic feedback passive, we can design a feedback control law for each subsystem and derive the conditions on the interconnection matrix of these subsystems, i.e., the weighted Laplacian matrix, such that the entire system achieves stochastic stability.

The rest of the paper is organized as follows. Section

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II introduces the system model and definitions. Section III provides the main results of the paper. Section III-A presents the stochastic KYP property and lemma. Section III-B analyzes the zero dynamics of a given Markovian jump nonlinear system and studies the stochastic feedback passivity based on zero dynamics. Section III-C investigates the problem of stochastic stabilization for N interconnected Markovian jump nonlinear systems that are stochastic feedback passive. A simulation example of a Markovian-type wireless sensor network with six sensor nodes that switch between three different topologies according to a Markovian chain is provided in Section IV. Section V concludes the paper.

II. Problem Formulation

A. System Model

Consider a discrete-time Markovian jump nonlinear system that is affine in control

$$\begin{aligned} \mathbf{x}(k+1) &= f(\mathbf{x}(k), r(k)) + g(\mathbf{x}(k), r(k))\mathbf{u}(k) \\ \mathbf{y}(k) &= h(\mathbf{x}(k), r(k)) + J(\mathbf{x}(k), r(k))\mathbf{u}(k) \end{aligned} \tag{1}$$

where $\mathbf{x} \in \mathbf{X} \subset \mathbb{R}^n$ is the state vector with an initial condition $\mathbf{x}_0$, $\mathbf{u} \in \mathbf{U} \subset \mathbb{R}^m$ is the control input, $\mathbf{y} \in \mathbb{R}^p$ is the output, $f : \mathbf{X} \times \mathbb{Z}^+ \to \mathbb{R}^n, g : \mathbf{X} \times \mathbb{Z}^+ \to \mathbb{R}^{n\times m}, h : \mathbf{X} \times \mathbb{Z}^+ \to \mathbb{R}^m, J : \mathbf{X} \times \mathbb{Z}^+ \to \mathbb{R}^{m\times m}$ are time-varying smooth nonlinear mappings. The finite state Markovian chain $r(k)$ takes value in the set $\{1, 2, \cdots, M\}$. The transition probability of the Markovian chain $r(k)$ is given as

$$\begin{aligned} &P(r(k+1) = l | r(k) = m) \\ &= q_{ml},\ m, l = 1, \cdots, M. \end{aligned} \tag{2}$$

According to the law of total probability, we have $\sum_{q=1}^{M} q_{ml} = 1$. All considerations are restricted to an open set $\mathbf{X} \times \mathbf{U}$ which is a neighbourhood of the origin $\mathbf{x}^* = \mathbf{0}, \mathbf{u}^* = \mathbf{0}$. We assume that the origin $(\mathbf{x}^*, \mathbf{u}^*) = (\mathbf{0}, \mathbf{0})$ is an isolated equilibrium and that $f(\mathbf{0}, r(k)) = \mathbf{0}, g(\mathbf{0}, r(k)) = \mathbf{0}, h(\mathbf{0}, r(k)) = \mathbf{0}$, and $J(\mathbf{0}, r(k)) = \mathbf{0}$. The system is assumed to have local relative degree zero [23] and J is invertible in a neighborhood of the origin[1].

Now let us consider N interconnected Markovian jump nonlinear systems of the form (1). Let $G = (\mathcal{V}, \mathcal{E}, \mathcal{A})$ be a weighted/directed graph with the set of nodes $\mathcal{V} = \{v_1, v_2, \cdots, v_N\}$ corresponding to each Markovian jump nonlinear subsystem, set of edges $\mathcal{E} \subset \mathcal{V} \times \mathcal{V}$ connecting each subsystem, and a weighted adjacency matrix $\mathcal{A} = [a_{ij}]$ [24]. If there is a link between node i and j, we assume that the off-diagonal term is $a_{ij} > 0$. The diagonal term is assumed to be $a_{ii} = 0$. We denote L as the weighted graph Laplacian where

$$l_{ij} = \begin{cases} \sum_{j=1,\ j\neq i}^{N} a_{ij} & j = i \\ -a_{ij} & j \neq i \end{cases}.$$

B. Stochastic Dissipativity

In this section, we formally introduce the notion of dissipativity/passivity in the stochastic settings with the intent to be consistent with their deterministic counterparts ([6], [7], [21]).

Definition II.1. *([17]–[19]) Consider a function $W : \mathbb{R}^m \times \mathbb{R}^p \times \mathbb{Z}^+ \to \mathbb{R}$ associated with a system of the form (1). This function is called the* supply rate *if for any $\mathbf{u} \in \mathbf{U} \subset \mathbb{R}^m$ the system with arbitrary initial condition $\mathbf{x}_0$ has the following property*

$$\mathbf{E}\left[\sum_{k=0}^{T} |W(\mathbf{u}(k), \mathbf{y}(k), r(k))|\right] < \infty,\ T = 0, 1, \cdots .$$

Definition II.2. *A system of the form (1) with supply rate $W(\mathbf{u}, \mathbf{y}, r)$ is said to be* locally dissipative in the stochastic sense *if there exists a nonnegative continuous function $V(\mathbf{x}, r)$: $\mathbf{X} \times \mathbb{Z}^+ \to \mathbb{R}^+$, called the* storage function*, such that for all $k \geq 0$,*

$$\begin{aligned} &\mathbf{E}\left[V(\mathbf{x}(k+1), r(k+1))\Big|\mathbf{x}(k), r(k)\right] - V(\mathbf{x}(k), r(k)) \\ &\leq \mathbf{E}\left[W(\mathbf{u}(k), \mathbf{y}(k), r(k))\Big|\mathbf{x}(k), r(k)\right], \\ &\forall \mathbf{x} \in \mathbf{X}, \forall \mathbf{u} \in \mathbf{U}. \end{aligned} \tag{3}$$

Definition II.3. *Suppose that a system of the form (1) is locally stochastic dissipative with storage function $V(\mathbf{x}(k), r(k))$. Let the supply rate be*

$$\begin{aligned} W(\mathbf{u}(k), \mathbf{y}(k), r(k)) &= \mathbf{y}^T(k) Q(r(k)) \mathbf{y}(k) \\ &+ 2\mathbf{y}^T(k) S(r(k)) \mathbf{u}(k) + \mathbf{u}^T(k) R(r(k)) \mathbf{u}(k) \end{aligned} \tag{4}$$

where $Q \in \mathbb{R}^{p\times p}, S \in \mathbb{R}^{p\times m}, R \in \mathbb{R}^{m\times m}$ are constant matrices for each given $r(k)$ with $Q = Q^T, R = R^T$ symmetric. Then the system is said to be locally QSR dissipative in the stochastic sense*, i.e., the following inequality holds*

$$\begin{aligned} &\mathbf{E}\left[V(\mathbf{x}(k+1), r(k+1))\Big|\mathbf{x}(k), r(k)\right] - V(\mathbf{x}(k), r(k)) \\ &\leq \mathbf{E}\left[\mathbf{y}^T(k) Q(r(k)) \mathbf{y}(k) + 2\mathbf{y}^T(k) S(r(k)) \mathbf{u}(k) + \right. \\ &\left. \mathbf{u}^T(k) R(r(k)) \mathbf{u}(k)\Big|\mathbf{x}(k), r(k)\right], \forall \mathbf{x} \in \mathbf{X}, \forall \mathbf{u} \in \mathbf{U}. \end{aligned} \tag{5}$$

We now consider a special class of dissipative systems with supply rate $W(\mathbf{u}, \mathbf{y}, r) = \mathbf{u}^{\mathrm{T}}\mathbf{y}$.

Definition II.4. *A system of the form (1) is said to be* locally passive in the stochastic sense *if it is locally stochastic dissipative with supply rate $W(\mathbf{u}, \mathbf{y}) = \mathbf{u}^T\mathbf{y}$, i.e, the following inequality holds*

$$\begin{aligned} &\mathbf{E}\left[V(\mathbf{x}(k+1), r(k+1))\Big|\mathbf{x}(k), r(k)\right] - V(\mathbf{x}(k), r(k)) \\ &\leq \mathbf{E}\left[\mathbf{u}^T(k)\mathbf{y}(k)\Big|\mathbf{x}(k), r(k)\right],\ \forall \mathbf{x} \in \mathbf{X}, \forall \mathbf{u} \in \mathbf{U}. \end{aligned} \tag{6}$$

[1]This assumption is reasonable because it is shown in [23] that a discrete-time deterministic nonlinear system can be rendered passive if and only if it has relative degree zero and passive zero dynamics.

Remark. It can be shown from Definition II.3 that the locally stochastic QSR dissipativity includes the locally stochastic passivity as special cases. When $Q = 0, S = \frac{I}{2}$ and $R = 0$, the locally stochastic QSR dissipativity corresponds to locally stochastic passivity. •

III. Main Results

A. Discrete-Time Stochastic KYP Lemma

We now investigate the stochastic KYP lemma which gives a necessary and sufficient condition for a Makovian jump nonlinear system of the form (1) to be stochastically QSR dissipative. The deterministic KYP lemma can be found in the literature [7]. Before this, let us consider the stochastic KYP property.

Definition III.1. *A system of the form (1) is said to have* stochastic KYP property *if there exist a nonnegative $\mathcal{C}^2$ storage function $V(\mathbf{x}(k), r(k)) : \mathbf{X} \times \mathbf{Z}^+ \to \mathbb{R}^+$ with $V(\mathbf{0}, r(k)) = 0$, the supply rate $W(\mathbf{u}(k), \mathbf{y}(k), r(k))$ given by Equation (4), and real functions $\rho(\mathbf{x}(k), r(k)) : \mathbf{X} \times \mathbb{Z}^+ \to \mathbb{R}^+, e(\mathbf{x}(k), r(k)) : \mathbf{X} \times \mathbb{Z}^+ \to \mathbb{R}^+$ such that given $r(k) = m$, it follows that*

$$
\begin{aligned}
&\sum_{l=1}^{M} q_{ml} V(f(\mathbf{x}(k), m), l) - V(\mathbf{x}(k), m) = \\
&h^T(\mathbf{x}(k), m) Q(m) h(\mathbf{x}(k), m) - \rho^T(\mathbf{x}(k), m)\rho(\mathbf{x}(k), m), \\
&\frac{1}{2}\sum_{l=1}^{M} q_{ml} \frac{\partial V(z,l)}{\partial z}\Big|_{z=f(\mathbf{x}(k),m)} g(\mathbf{x}(k), m) = \\
&h^T(\mathbf{x}(k), m) Q(m) J(\mathbf{x}(k), m) + h^T(\mathbf{x}(k), m) S(m) \\
&-\rho^T(\mathbf{x}(k), m) e(\mathbf{x}(k), m), \\
&\frac{1}{2}\sum_{l=1}^{M} q_{ml} g^T(\mathbf{x}(k), m) \frac{\partial^2 V(z,l)}{\partial z^2}\Big|_{z=f(\mathbf{x}(k),m)} g(\mathbf{x}(k), m) \\
&= J^T(\mathbf{x}(k), m) Q(m) J(\mathbf{x}(k), m) + 2J^T(\mathbf{x}(k), m) S(m) \\
&+R(m) - e^T(\mathbf{x}(k), m) e(\mathbf{x}(k), m). \qquad (7)
\end{aligned}
$$

Theorem III.1. *A necessary and sufficient condition for a system of the form (1) to be locally stochastic QSR dissipative is that the system has local stochastic KYP property for all $\mathbf{x} \in \mathbf{X}, \mathbf{u} \in \mathbf{U}$.*

Proof: (sufficiency) If a system of the form (1) has local stochastic KYP property, there exists a $\mathcal{C}^2$ storage function such that Equations (7) are satisfied locally. Therefore, according to the second order Taylor series expansion at the origin, it follows that

$$
\begin{aligned}
&\mathbf{E}\left[V(\mathbf{x}(k+1), r(k+1))\Big|\mathbf{x}(k), r(k) = m\right] \\
&-V(\mathbf{x}(k), r(k) = m) \\
=& \sum_{l=1}^{M} q_{ml} \{V(f(\mathbf{x}(k), m), l) \\
&+\frac{\partial V(z,l)}{\partial z}\Big|_{z=f(\mathbf{x}(k),m)} g(\mathbf{x}(k), m)\mathbf{u}(k) \\
&+\frac{1}{2}\mathbf{u}^{\mathrm{T}}(k) g^{\mathrm{T}}(\mathbf{x}(k), m) \frac{\partial^2 V(z,l)}{\partial z^2}\Big|_{z=f(\mathbf{x}(k),m)} \\
&\times g(\mathbf{x}(k), m)\mathbf{u}(k)\} - V(\mathbf{x}(k), m). \qquad (8)
\end{aligned}
$$

In the following proof, we suppress the arguments in the above functions for the sake of simplicity. According to Equations (7), the right hand side of Equation (8) equals to

$$
\begin{aligned}
&h^{\mathrm{T}}Qh - \rho^{\mathrm{T}}\rho + 2h^{\mathrm{T}}QJ\mathbf{u} + 2h^{\mathrm{T}}S\mathbf{u} - 2\rho^{\mathrm{T}}e\mathbf{u} \\
&+\mathbf{u}^{\mathrm{T}}J^{\mathrm{T}}QJ\mathbf{u} + 2\mathbf{u}^{\mathrm{T}}J^{\mathrm{T}}S\mathbf{u} + \mathbf{u}^{\mathrm{T}}R\mathbf{u} - \mathbf{u}^{\mathrm{T}}e^{\mathrm{T}}e\mathbf{u} \\
=& (h + J\mathbf{u})^{\mathrm{T}} Q (h + J\mathbf{u}) + 2(h + J\mathbf{u})^{\mathrm{T}} S\mathbf{u} \\
&+\mathbf{u}^{\mathrm{T}}(k)R\mathbf{u} - (\rho + e\mathbf{u})^{\mathrm{T}}(\rho + e\mathbf{u}) \\
=& \mathbf{y}^{\mathrm{T}}Q\mathbf{y} + 2\mathbf{y}^{\mathrm{T}}S\mathbf{u} + \mathbf{u}^{\mathrm{T}}R\mathbf{u} - (\rho + e\mathbf{u})^{\mathrm{T}}(\rho + e\mathbf{u}) \\
\le& \mathbf{E}\left[W(\mathbf{u}, \mathbf{y}, r)\Big|\mathbf{x}, m\right].
\end{aligned}
$$

Hence, the system of the form (1) is locally QSR dissipative in the stochastic sense according to Definition II.3.

(necessity) If a system of the form (1) is locally stochastic QSR dissipative with a $\mathcal{C}^2$ storage function, then the inequality (3) holds for any $\mathbf{u}(k) \in \mathbf{U}$. Define

$$
\begin{aligned}
&H(\mathbf{x}(k), \mathbf{u}(k)) \\
=& \mathbf{E}\left[V(\mathbf{x}(k+1), r(k+1))\Big|\mathbf{x}(k), r(k)\right] - V(\mathbf{x}(k), r(k)) \\
&-\mathbf{E}\left[W(\mathbf{u}(k), \mathbf{y}(k))\Big|\mathbf{x}(k), r(k)\right] \le 0.
\end{aligned}
$$

Because V is $\mathcal{C}^2$, we have

$$
\begin{aligned}
H(\mathbf{x}, \mathbf{u}) =& \sum_{m=1}^{M} q_{ml}\Big\{V(f, l) + \frac{\partial V(z,l)}{\partial z}\Big|_{z=f} g\mathbf{u} \\
&+\frac{1}{2}\mathbf{u}^{\mathrm{T}}g^{\mathrm{T}}\frac{\partial^2 V(z,l)}{\partial z^2}\Big|_{z=f} g\mathbf{u}\Big\} - V(\mathbf{x}, m) \\
&-\left(\mathbf{y}^{\mathrm{T}}Q\mathbf{y} + 2\mathbf{y}^{\mathrm{T}}S\mathbf{u} + \mathbf{u}^{\mathrm{T}}R\mathbf{u}\right) \le 0. \qquad (9)
\end{aligned}
$$

Therefore, $H(\mathbf{x}(k), \mathbf{u}(k))$ is negative and quadratic in $\mathbf{u}$. We set

$$
\begin{aligned}
H(\mathbf{x}, \mathbf{u}) =& -(\rho(\mathbf{x}, m) + e(\mathbf{x}, m)\mathbf{u})^{\mathrm{T}} \\
&\times(\rho(\mathbf{x}, m) + e(\mathbf{x}, m)\mathbf{u}). \qquad (10)
\end{aligned}
$$

Compare the coefficients of Equations (9) and (10), we obtain the stochastic KYP property (7).

B. Stochastic Feedback Passivity & Zero Dynamics

The stochastic KYP lemma states that if there exist real functions $\rho(\mathbf{x}, m)$ and $e(\mathbf{x}, m)$ such that the stochastic KYP property holds then a system of the form (1) is QSR dissipative in the stochastic sense. However, it does not provide a straightforward tool to determine if such real functions exist or not, i.e., if the system is stochastic QSR dissipative/passive or not. In addition, stochastic QSR dissipativity or stochastic passivity is a relatively constrained definition because it requires that the stochastic QSR dissipativity inequality (5) or the passivity inequality (6) to be hold for any $\mathbf{x} \in \mathbf{X}$ and $\mathbf{u} \in \mathbf{U}$. This might not hold true in most cases. Therefore, in this section, we introduce the notion of stochastic feedback passivity. That is to say, a given system of the form (1) may not be stochastic passive, but it can be made passive in the stochastic sense by a suitably designed state feedback control law. Furthermore, based on the stochastic KYP lemma, we prove that the necessary and sufficient condition for a system of the form (1) to be locally stochastic feedback passive is that its zero dynamics are locally stochastic passive.

To this end, let us first obtain the zero dynamics of System (1). Because the system has relative degree zero and J is locally invertible, choose the following feedback control law

$$\begin{aligned}\mathbf{u}(k) = &-J^{-1}(\mathbf{x}(k), m)h(\mathbf{x}(k), m)\\ &+J^{-1}(\mathbf{x}(k), m)\mathbf{v}(k).\end{aligned} \tag{11}$$

The transformed dynamics of System (1) are

$$\begin{aligned}&\mathbf{x}(k+1) = f^*(\mathbf{x}(k), m) + g^*(\mathbf{x}(k), m)\mathbf{v}(k)\\ &\mathbf{y}(k) = \mathbf{v}(k)\end{aligned} \tag{12}$$

where we have $f^* = f - gJ^{-1}h$ and $g^* = gJ^{-1}$. The zero dynamics are the internal dynamics of the system that are consistent with constraining the system output to zero and given by the following equation

$$\begin{aligned}&\mathbf{x}(k+1) = f^*(\mathbf{x}(k), m)\\ &\mathbf{y}(k) = \mathbf{0}.\end{aligned} \tag{13}$$

According to Definition II.4, the zero dynamics (13) are passive, or equivalently, stable in the stochastic sense if the following equality holds

$$\begin{aligned}&\mathbf{E}\left[V(\mathbf{x}(k+1), r(k+1))\Big|\mathbf{x}(k), r(k)\right]\\ &-V(\mathbf{x}(k), r(k)) \le 0.\end{aligned} \tag{14}$$

Now consider a new control input $\mathbf{w}$ for the transformed dynamics (12),

$$\mathbf{y}(k) = \mathbf{v}(k) = \bar{h}(\mathbf{x}(k), m) + \bar{J}(\mathbf{x}(k), m)\mathbf{w}(k),$$

where $\bar{J}$ is assumed to be symmetric and

$$\begin{aligned}\bar{J}(\mathbf{x}(k), m) &= \left(\frac{1}{2}g^{*\mathrm{T}}\frac{\partial^2 V(\bar{z}, l)}{\partial \bar{z}^2}\Big|_{\bar{z}=f^*} g^*\right)^{-1}\\ \bar{h}(\mathbf{x}(k), m) &= -\bar{J}\left(\frac{\partial V(\bar{z}, l)}{\partial \bar{z}}\Big|_{\bar{z}=f^*} g^*\right)^{\mathrm{T}}\end{aligned} \tag{15}$$

The new system dynamics are given by

$$\begin{aligned}\mathbf{x}(k+1) = &f^*(\mathbf{x}(k), m) + g^*(\mathbf{x}(k), m)\bar{h}(\mathbf{x}(k), m)\\ &+g^*(\mathbf{x}(k), m)\bar{J}(\mathbf{x}(k), m)\mathbf{w}(k)\\ \mathbf{y}(k) = &\bar{h}(\mathbf{x}(k), m) + \bar{J}(\mathbf{x}(k), m)\mathbf{w}(k).\end{aligned} \tag{16}$$

We now give the definition of locally feedback passive in the stochastic sense.

Definition III.2. *A system of the form (1) is said to be* locally feedback passive in the stochastic sense *if there exists a nonnegative continuous storage function* $V: \mathbf{X} \times \mathbb{Z}^+ \to \mathbb{R}^+$*, such that for all* $k \ge 0$,

$$\begin{aligned}&\mathbf{E}\left[V(\mathbf{x}(k+1), r(k+1))\Big|\mathbf{x}(k), r(k)\right] - V(\mathbf{x}(k), r(k))\\ &\le \mathbf{E}\left[\mathbf{w}^T(k)\mathbf{y}(k)\Big|\mathbf{x}(k), r(k)\right], \forall \mathbf{x} \in \mathbf{X}, \forall \mathbf{w} \in \mathbf{U}.\end{aligned} \tag{17}$$

Theorem III.2. *Suppose there exists a nonnegative* $\mathcal{C}^2$ *storage function* V *with* $V(\mathbf{x}, r) = 0$ *if and only if* $\mathbf{x} = \mathbf{0}$ *and* $V(f + g\mathbf{u}(k), r)$ *quadratic in* $\mathbf{u}$*. Then a system of the form (1) has a locally feedback passive dynamics in the stochastic sense if and only if its zero dynamics (13) are locally stochastic passive.*

Proof: (necessity) Because System (1) is locally stochastic feedback passive, the inequality (17) holds. The zero dynamics enforces $\mathbf{y}(k) = \mathbf{0}$. Hence, the inequality (17) is converted to the inequality (14). That is, the zero dynamics (13) are locally stochastic passive.

(sufficiency) We now prove that if the zero dynamics (13) are locally stochastic passive, System (1) is feedback passive in the stochastic sense according to Definition III.2 and the stochastic feedback passivity inequality (17) holds. According to Theorem III.1, this is equivalent to prove that the transformed system (16) satisfies the stochastic KYP property (7) with $Q = 0, S = \frac{I}{2}$ and $R = 0$ given that its zero dynamics are locally stochastic passive. More specifically, we need to prove that

$$\begin{aligned}&\sum_{l=1}^{M} q_{ml}V(f^*(\mathbf{x}, m) + g^*(\mathbf{x}, m)\bar{h}(\mathbf{x}, m), l)\\ &-V(\mathbf{x}, m)\\ &= -\left(\rho(\mathbf{x}, m) + e(\mathbf{x}, m)\bar{h}(\mathbf{x}, m)\right)^{\mathrm{T}}\\ &\times\left(\rho(\mathbf{x}, m) + e(\mathbf{x}, m)\bar{h}(\mathbf{x}, m)\right),\end{aligned} \tag{18}$$

$$\begin{aligned}&\frac{1}{2}\sum_{l=1}^{M} q_{ml}\frac{\partial V(\tilde{z}, l)}{\partial \tilde{z}}\Big|_{\tilde{z}=f^*(\mathbf{x},m)+g^*(\mathbf{x},m)\bar{h}(\mathbf{x},m)}\\ &\times g^*(\mathbf{x}, m)\bar{J}(\mathbf{x}, m)\\ &= \frac{1}{2}\bar{h}^{\mathrm{T}}(\mathbf{x}, m) - \left(\rho(\mathbf{x}, m) + e(\mathbf{x}, m)\bar{h}(\mathbf{x}, m)\right)^{\mathrm{T}}\\ &\times e(\mathbf{x}, m)\bar{J}(\mathbf{x}, m),\end{aligned} \tag{19}$$

$$\begin{aligned}&\sum_{l=1}^{M} q_{ml}\left[g^*(\mathbf{x}, m)\bar{J}(\mathbf{x}, m)\right]^{\mathrm{T}}\\ &\times\frac{\partial^2 V(\tilde{z}, l)}{\partial \tilde{z}^2}\Big|_{\tilde{z}=f^*(\mathbf{x},m)+g^*(\mathbf{x},m)\bar{h}(\mathbf{x},m)}\\ &\times g^*(\mathbf{x}, m)\bar{J}(\mathbf{x}, m)\\ &= \bar{J}(\mathbf{x}, m) + \bar{J}^{\mathrm{T}}(\mathbf{x}, m)\\ &-2\bar{J}^{\mathrm{T}}(\mathbf{x}, m)e^{\mathrm{T}}(\mathbf{x}, m)e(\mathbf{x}, m)\bar{J}(\mathbf{x}, m).\end{aligned} \tag{20}$$

For the sake of simplicity, we suppress the arguments of the functions in the following proof.

Because $V(f + g\mathbf{u}, r)$ is quadratic in $\mathbf{u}$, the Taylor series expansion for $V(f^* + g^*\bar{h}, l)$ can be expressed as follows:

$$\begin{aligned}V(f^* + g^*\bar{h}, l) = &V(f^*, l) + \frac{\partial V}{\partial \bar{z}}\Big|_{\bar{z}=f^*} g^*\bar{h}\\ &+\frac{1}{2}(g^*\bar{h})^{\mathrm{T}}\frac{\partial^2 V}{\partial \bar{z}^2}\Big|_{\bar{z}=f^*} g^*\bar{h}.\end{aligned} \tag{21}$$

Use Equation (15) to Equation (21), since $\bar{J}$ is symmetric, we have

$$V(f^*+g^*\bar{h},l) = V(f^*,l)-\bar{h}^{\mathrm{T}}\left(\bar{J}^{-1}\right)^{\mathrm{T}}\bar{h} + \bar{h}^{\mathrm{T}}\bar{J}^{-1}\bar{h} = V(f^*,l).$$

Since the zero dynamics (13) are locally stochastic passive, it follows that

$$\begin{aligned}&\sum_{l=1}^{M} q_{ml}V(f^*+g^*\bar{h},l)-V(\mathbf{x},m)\\ &=\sum_{l=1}^{M} q_{ml}V(f^*,l)-V(\mathbf{x},m)\\ &=E\left[V(f^*,l)|\mathbf{x},m\right]-V(\mathbf{x},m)\le 0.\end{aligned}$$

Again since $V(f^*+g^*\bar{h},l)$ is quadratic in $\bar{h}$, we set

$$\begin{aligned}&\sum_{l=1}^{M} q_{ml}V(f^*+g^*\bar{h},l)-V(\mathbf{x},m)\\ &=-\left(\rho+e\bar{h}\right)^{\mathrm{T}}\left(\rho+e\bar{h}\right)\le 0.\end{aligned}$$

Hence, Equation (18) holds.

Next we expand $E[V(f^*+g^*\bar{h},l)|\mathbf{x},m]-V(\mathbf{x},m)$ using the Taylor series expansion

$$\begin{aligned}&E[V(f^*+g^*\bar{h},l)|\mathbf{x},m]-V(\mathbf{x},m)\\ &=\sum_{l=1}^{M} q_{ml}V(f^*+g^*\bar{h},l)-V(\mathbf{x},m)\\ &=\sum_{l=1}^{M} q_{ml}\left[V(f^*,l)+\frac{\partial V(\bar{z},l)}{\partial \bar{z}}\Big|_{\bar{z}=f^*}g^*\bar{h}\right.\\ &\quad\left.+\frac{1}{2}\left[g^*\bar{h}\right]^{\mathrm{T}}\frac{\partial^2 V(\bar{z},l)}{\partial \bar{z}^2}\Big|_{\bar{z}=f^*}g^*\bar{h}\right]-V(\mathbf{x},m)\\ &=\sum_{l=1}^{M} q_{ml}V(f^*,l)-V(\mathbf{x},m)+\sum_{l=1}^{M} q_{ml}\times\\ &\left[\frac{\partial V(\bar{z},l)}{\partial \bar{z}}\Big|_{\bar{z}=f^*}g^*\bar{h}+\frac{1}{2}\left[g^*\bar{h}\right]^{\mathrm{T}}\frac{\partial^2 V(\bar{z},l)}{\partial \bar{z}^2}\Big|_{\bar{z}=f^*}g^*\bar{h}\right]\\ &=-\left(\rho+e\bar{h}\right)^{\mathrm{T}}\left(\rho+e\bar{h}\right)+\sum_{l=1}^{M} q_{ml}\times\\ &\left[\frac{\partial V(\bar{z},l)}{\partial \bar{z}}\Big|_{\bar{z}=f^*}g^*\bar{h}+\frac{1}{2}\left[g^*\bar{h}\right]^{\mathrm{T}}\frac{\partial^2 V(\bar{z},l)}{\partial \bar{z}^2}\Big|_{\bar{z}=f^*}g^*\bar{h}\right]\end{aligned}\tag{22}$$

Take the first order derivative of Equation (22) with respect to $\bar{h}$ and right multiply by $\bar{J}$, we have

$$\begin{aligned}&\sum_{l=1}^{M} q_{ml}\frac{\partial V(\tilde{z},l)}{\partial \tilde{z}}\Big|_{\tilde{z}=f^*+g^*\bar{h}}g^*\bar{J}\\ &=-2\left(\rho+e\bar{h}\right)^{\mathrm{T}}e\bar{J}+\sum_{l=1}^{M} q_{ml}\left[\frac{\partial V(\bar{z},l)}{\partial \bar{z}}\Big|_{\bar{z}=f^*}g^*\bar{J}\right.\\ &\quad\left.+\bar{h}^{\mathrm{T}}(g^*)^{\mathrm{T}}\frac{\partial^2 V(\bar{z},l)}{\partial \bar{z}^2}\Big|_{\bar{z}=f^*}g^*\bar{J}\right].\end{aligned}$$

Use Equation (15), it follows that

$$\begin{aligned}&\sum_{l=1}^{M} q_{ml}\frac{\partial V(\tilde{z},l)}{\partial \tilde{z}}\Big|_{\tilde{z}=f^*+g^*\bar{h}}g^*\bar{J}\\ &=-2\left(\rho+e\bar{h}\right)^{\mathrm{T}}e\bar{J}+\sum_{l=1}^{M} q_{ml}\bar{h}.\end{aligned}$$

Because at time step k, $\bar{h}(\mathbf{x}(k),m)$ is a constant, $\sum_{l=1}^{M} q_{ml}\bar{h}=\bar{h}$ and hence Equation (19) holds.

Similarly, we take the second order derivative of Equation (22) with respect to $\bar{h}$, left multiply by $\bar{J}^{\mathrm{T}}$ and right multiply by $\bar{J}$, we have

$$\begin{aligned}&\sum_{l=1}^{M} q_{ml}\left[g^*\bar{J}\right]^{\mathrm{T}}\frac{\partial V^2(\tilde{z},l)}{\partial \tilde{z}^2}\Big|_{\tilde{z}=f^*+g^*\bar{h}}g^*\bar{J}\\ &=-2\bar{J}^{\mathrm{T}}e^{\mathrm{T}}e\bar{J}+\sum_{l=1}^{M} q_{ml}(g^*\bar{J})^{\mathrm{T}}\frac{\partial^2 V(\bar{z},l)}{\partial \bar{z}^2}\Big|_{\bar{z}=f^*}g^*\bar{J}\\ &=-2\bar{J}^{\mathrm{T}}e^{\mathrm{T}}e\bar{J}+\sum_{l=1}^{M} q_{ml}\left(\bar{J}^{\mathrm{T}}+\bar{J}\right)\\ &=-2\bar{J}^{\mathrm{T}}e^{\mathrm{T}}e\bar{J}+\bar{J}^{\mathrm{T}}+\bar{J}.\end{aligned}$$

The second equation follows by using Equation (15) and the third equation holds because $\bar{J}^{\mathrm{T}}(\mathbf{x}(k),m)+\bar{J}(\mathbf{x}(k),m)$ is a constant at time step k. Therefore, Equation (19) is satisfied. This completes the proof.

C. Stochastic Stabilization for Interconnected Systems

The above passivity results hold true for interconnected Markovian jump nonlinear systems, where each subsystem is of the form (1). In this section, we consider the stochastic stability of N interconnected Markovian jump nonlinear systems given that each subsystem $i,\ i=1,\cdots,N$, is stochastic feedback passive according to Definition III.2. We design feedback control law and obtain the conditions on the interconnections between these subsystems, or equivalently, the weighted Laplacian matrix L, to guarantee the stochastic stability of the entire system.

Theorem III.3. *Consider N interconnected Markovian jump nonlinear systems of the form (1). Assume that each subsystem $i,\ i=1,\cdots,N$, is stochastic feedback passive. Choose the external feedback control as*

$$\begin{aligned}\mathbf{w}(k)=&-k_g\left(1+k_g\mathbf{L}(r(k))\bar{\mathbf{J}}(\mathcal{X}(k),r(k))\right)^{-1}\\ &\times\mathbf{L}(r(k))\bar{\mathbf{h}}(\mathcal{X}(k),r(k))\end{aligned}\tag{23}$$

where $r(k)$ is the Markovian chain, $\mathcal{X}=[\mathbf{x}_1^T,\cdots,\mathbf{x}_N^T]^T$, $\mathbf{w}=[\mathbf{w}_1^T,\cdots,\mathbf{w}_N^T]^T$, $\bar{\mathbf{J}}=diag(\bar{J}_i)$, $\bar{\mathbf{h}}=[\bar{h}_1^T,\cdots,\bar{h}_N^T]^T$, $\mathbf{L}(r(k))=\mathbf{I}_{n\times n}\otimes L(r(k))$ is the augmented weighted Laplacian matrix, and $k_g>0$ is some constant gain. The entire system is locally stochastic stable if $L(r(k))\ge 0$.

Proof: Define the storage function for the system with N interconnected Markovian jump nonlinear systems (1) as $V(\mathcal{X},r)=\sum_{i=1}^{N}V_i(\mathbf{x}_i,r)$. Based on the proof of

Theorem III.2, we have

$$
\begin{aligned}
& E\left[V(\mathcal{X}(k+1), r(k+1))\Big|\mathcal{X}(k), r(k)\right] - V(\mathcal{X}(k), r(k)) \\
&= E[\sum_{i=1}^{N} V_i(\mathbf{x}_i(k+1), r(k+1))\Big|\mathbf{x}_i(k), r(k)] \\
&\quad - \sum_{i=1}^{N} V_i(\mathbf{x}_i(k), r(k)) \\
&= \sum_{i=1}^{N}\Big\{E[V_i(\mathbf{x}_i(k+1), r(k+1))\Big|\mathbf{x}_i(k), r(k)] \\
&\quad -V_i(\mathbf{x}_i(k), r(k))\} \\
&= \sum_{i=1}^{N}\Bigg[\sum_{l=1}^{M} q_{ml}\Big(V_i(f_i^* + g_i^*\bar{h}_i, l) \\
&\quad + \frac{\partial V_i(\tilde{z}, l)}{\partial \tilde{z}}\Big|_{\tilde{z}=f_i^*+g_i^*\bar{h}_i} g_i^*\bar{J}_i\mathbf{w}_i + \frac{1}{2}\left[g_i^*\bar{J}_i\mathbf{w}_i\right]^{\mathrm{T}} \\
&\quad \times \frac{\partial^2 V_i(\tilde{z}, l)}{\partial \tilde{z}^2}\Big|_{\tilde{z}=f_i^*+g_i^*\bar{h}_i} g_i^*\bar{J}_i\mathbf{w}_i\Big) - V_i(\mathbf{x}_i, m)\Bigg] \\
&= \sum_{i=1}^{N}\Big[-\left(\rho_i + e_i\bar{h}_i\right)^{\mathrm{T}}\left(\rho_i + e_i\bar{h}_i\right) + \bar{h}_i^{\mathrm{T}}\mathbf{w}_i \\
&\quad -2(\rho_i + e_i\bar{h}_i)^{\mathrm{T}} e_i\bar{J}_i\mathbf{w}_i + \frac{1}{2}\mathbf{w}_i^{\mathrm{T}}(\bar{J}_i^{\mathrm{T}} + \bar{J}_i)\mathbf{w}_i \\
&\quad -\mathbf{w}_i\bar{J}_i^{\mathrm{T}} e_i^{\mathrm{T}} e_i\bar{J}_i\mathbf{w}_i] \\
&= \mathbf{y}^{\mathrm{T}}\mathbf{w} - \sum_{i=1}^{N}\|(\rho_i + e_i\bar{h}_i + e_i\bar{J}_i\mathbf{w}_i)\|^2
\end{aligned}
$$

where $\mathbf{y} = [\mathbf{y}_1^{\mathrm{T}}, \cdots, \mathbf{y}_N^{\mathrm{N}}]^{\mathrm{T}}$. Because the control law (23) gives

$$
\left(1 + k_g\mathbf{L}(r)\bar{\mathbf{J}}(\mathcal{X}, r)\right)\mathbf{w}(k) = -k_g\mathbf{L}(r)\bar{\mathbf{h}}(\mathcal{X}, r),
$$

we have

$$
\begin{aligned}
& \mathbf{w}(k) \\
&= -k_g\mathbf{L}(r(k))\left(\bar{\mathbf{h}}(\mathcal{X}(k), r(k)) + \bar{\mathbf{J}}(\mathcal{X}(k), r(k)))\mathbf{w}(k)\right) \\
&= -k_g\mathbf{L}(r(k))\mathbf{y}(k)
\end{aligned}
$$

Therefore,

$$
\begin{aligned}
& E\left[V(\mathcal{X}(k+1), r(k+1))\Big|\mathcal{X}(k), r(k)\right] - V(\mathcal{X}(k), r(k)) \\
&= -k_g\mathbf{y}^{\mathrm{T}}\mathbf{L}(r)\mathbf{y} - \sum_{i=1}^{N}\|(\rho_i + e_i\bar{h}_i + e_i\bar{J}_i\mathbf{w}_i)\|^2
\end{aligned}
$$

If the weighted Laplacian matrix is such that $L(r) \geq 0$, the above inequality becomes

$$
\begin{aligned}
& E\left[V(\mathcal{X}(k+1), r(k+1))\Big|\mathcal{X}(k), r(k)\right] \\
& -V(\mathcal{X}(k), r(k)) \leq 0,
\end{aligned}
$$

i.e., the entire system is stochastically stable according to the inequality (14). This completes the proof.

Remark. This result is consistent with the classical deterministic conditions derived in [25]. •

Corollary III.1. *Consider N interconnected Markovian jump nonlinear systems of the form (1). If each subsystem has stochastic passive zero dynamics, and an external control input*

$$
\mathbf{w}_i(k) = -\sum_{i=1} l_{ij}(r(k))\mathbf{y}_i(k) \tag{24}
$$

with $L(r(k)) \geq 0,\ L(r(k)) = \{l_{ij}(r(k))\}$, then the entire system is stochastically stable.

Proof: According to Theorem III.2, if each subsystem i has stochastic passive zero dynamics, then it is locally stochastic feedback passive. From the proof in Theorem III.3, the control law (24) for system i is the i_{th} element in the vector control law (23) for $i,\ i = 1, \cdots, N$. According to Theorem III.3, the system is stochastically stable.

IV. Simulation

In this section, we give an example of a wireless sensor network consisting of 6 sensors whose dynamics are of the form (1). The network randomly switches between 3 different topologies according to a Markovian chain $r(k)$. The discrete-time nonlinear dynamics of node $i,\ i = 1, 2, \cdots, 6$ are

$$
\begin{aligned}
x_1^i(k+1) &= 0.6\sin x_1^i(k) + Tx_3^i(k) \\
x_2^i(k+1) &= 0.8\sin x_2^i(k) + Tx_4^i(k) \\
x_3^i(k+1) &= x_3^i(k) - \frac{b(r(k))}{m(r(k))}Tx_1^i(k)x_3^i(k) \\
&\quad -\frac{K_s(r(k))}{m(r(k))}Tx_1^i(k)^{\frac{1}{3}} + \frac{1}{m(r(k))}Tu_1^i(k) \\
x_4^i(k+1) &= x_4^i(k) - \frac{b(r(k))}{m(r(k))}Tx_2^i(k)x_4^i(k) \\
&\quad -\frac{K_s(r(k))}{m(r(k))}Tx_2^i(k)^2 + \frac{1}{m(r(k))}Tu_2^i(k) \\
y_1^i(k) &= x_1^i(k) + u_1^i(k) \\
y_2^i(k) &= x_2^i(k) + u_2^i(k)
\end{aligned} \tag{25}
$$

where $\mathbf{x}_i = [x_1^i, x_2^i, x_3^i, x_4^i]^{\mathrm{T}}$ are the states, $\mathbf{y}_i = [y_1^i, y_2^i]^{\mathrm{T}}$ are the outputs, and $\mathbf{u}_i = [u_1^i, u_2^i]^{\mathrm{T}}$ are the control inputs. T is the sampling rate and chosen as 0.1 in the simulation. The 3 different network topologies are shown in Figure 1.

According to the above graphs, the corresponding weighted Laplacian matrix of wireless sensor network is chosen as

$$
L_1 = \begin{bmatrix}
0.1 & -0.1 & 0 & 0 & 0 & 0 \\
0 & 0.1 & -0.1 & 0 & 0 & 0 \\
0 & 0 & 0.1 & -0.1 & 0 & 0 \\
0 & 0 & 0 & 0.1 & -0.1 & 0 \\
0 & 0 & 0 & 0 & 0.1 & -0.1 \\
-0.1 & 0 & 0 & 0 & 0 & 0.1
\end{bmatrix},
$$

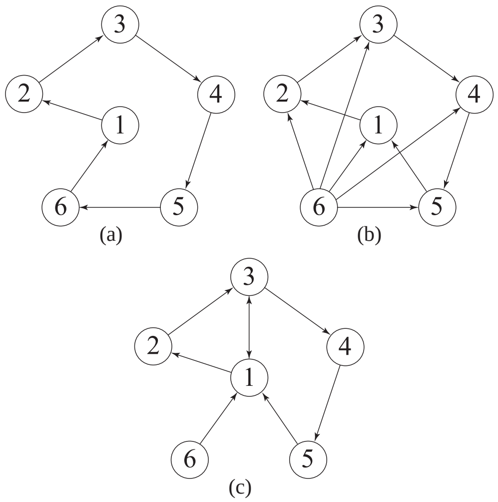

Fig. 1. Three different network topologies: (a), (b) and (c).

$$L_2 = \begin{bmatrix} 0.1 & -0.1 & 0 & 0 & 0 & 0 \\ 0 & 0.1 & -0.1 & 0 & 0 & 0 \\ 0 & 0 & 0.1 & -0.1 & 0 & 0 \\ 0 & 0 & 0 & 0.1 & -0.1 & 0 \\ -0.1 & 0 & 0 & 0 & 0.1 & 0 \\ -0.1 & -0.1 & -0.1 & -0.1 & -0.1 & 0.5 \end{bmatrix},$$

$$L_3 = \begin{bmatrix} 0.2 & -0.1 & -0.1 & 0 & 0 & 0 \\ 0 & 0.1 & -0.1 & 0 & 0 & 0 \\ -0.1 & 0 & 0.2 & -0.1 & 0 & 0 \\ 0 & 0 & 0 & 0.1 & -0.1 & 0 \\ -0.1 & 0 & 0 & 0 & 0.1 & 0 \\ -0.1 & 0 & 0 & 0 & 0 & 0.1 \end{bmatrix}.$$

and we have $L_1 = 0, L_2 > 0, L_3 > 0$.

The transition probability matrix of the Markovian chain is given as follows

$$P(r(k+1) = i | r(k) = j) = \begin{bmatrix} 0.5 & 0.2 & 0.3 \\ 0.2 & 0.45 & 0.35 \\ 0.3 & 0.1 & 0.6 \end{bmatrix},$$

where $i, j = 1, 2, 3$. The set of elements that changing according to the network topologies are chosen as

$$\begin{aligned} m = 1, K_s = 0.1, b = 10, \text{ if } r(k) = 1, \\ m = 1, K_s = 0.2, b = 9, \text{ if } r(k) = 2, \\ m = 2, K_s = 0.1, b = 20, \text{ if } r(k) = 3. \end{aligned}$$

Under this setting, all the sensor nodes have stochastic passive zero dynamics and hence are stochastic feedback passive. The storage function is chosen as $V_i(\mathbf{x}_i(k), r(k)) = \frac{1}{2}K_s(r(k))(x_1^i(k) + x_2^i(k)) + \frac{1}{2}m(r(k))(x_3^i(k) + x_4^i(k))$. For each sensor node i, we have

$$\bar{h}_1^i(k) = \bar{h}_2^i(k) = 2\frac{m(r(k))}{T^2},$$

$$\begin{aligned} \bar{J}_1^i(k) = -2\frac{m(r(k))}{T}\Bigg(\left(1 - \frac{b(r(k))}{m(r(k))}T\right)x_3^i(k) \\ -\frac{K_s(r(k))}{m(r(k))}T(x_1^i(k))^{\frac{1}{3}}\Bigg), \\ \bar{J}_2^i(k) = -2\frac{m(r(k))}{T}\Bigg(\left(1 - \frac{b(r(k))}{m(r(k))}T\right)x_4^i(k) \\ -\frac{K_s(r(k))}{m(r(k))}T(x_1^i(k))^2\Bigg). \end{aligned}$$

Therefore, the control inputs are

$$u_1^i(k) = -x_1^i(k) + \bar{h}_1^i(k) + \bar{J}_1^i(k)w_1^i(k),$$

$$u_2^i(k) = -x_2^i(k) + \bar{h}_2^i(k) + \bar{J}_2^i(k)w_2^i(k),$$

where $w_1^i(k), w_2^i(k)$ is given by Equation (23) to stabilize the system in the stochastic sense. We choose $k_g = 1$. Figure 2(a) checks the stochastic stability inequality $\Delta V = E[V(\mathcal{X}(k+1), r(k+1))|\mathcal{X}(k), r(k)] - V(\mathcal{X}(k), r(k)) \leq 0$, which is satisfied at every time step. Figures 2(b) and 2(c) show the states x_1^i, x_2^i, x_3^i and x_4^i for all the 6 sensors. It is shown that all the states approach to the equilibrium.

V. CONCLUSION

In this paper, we study the passivity and feedback passivity properties in the stochastic setting for discrete-time Markovian jump nonlinear systems. We first introduce the notions of stochastic dissipativiy and stochastic QSR dissipativity. We then derive the stochastic KYP lemma which provides a sufficient and necessary condition to determine if a given Markovian jump nonlinear system is QSR dissipative. The definition of stochastic passivity follows as a special case of stochastic QSR dissipativity. However, the stochastic KYP lemma relies on the existence of some real functions and does not provide a direct tool to check the stochastic passivity of a given system. Therefore, we investigate the stochastic feedback passivity of a Markovian jump nonlinear system and relate it with the system zero dynamics. Based on the stochastic KYP lemma, we prove that such a Markovian jump nonlinear system is locally stochastic feedback passive if and only if its zero dynamics are locally stochastic passive. Furthermore, we consider the stochastic stability of N interconnected Markovian jump nonlinear systems which are stochastic feedback passive. We design the state feedback control law and investigate the conditions on the interconnection matrix of the subsystems to guarantee the stochastic stability of the entire system. An example of a Markovian-type wireless sensor network is provided. Future work will focus on the extension of stochastic passivity systems to stochastic conic systems and its relationship with stochastic QSR dissipative systems. More general stochastic system models than the Markovian jump nonlinear systems will be considered.

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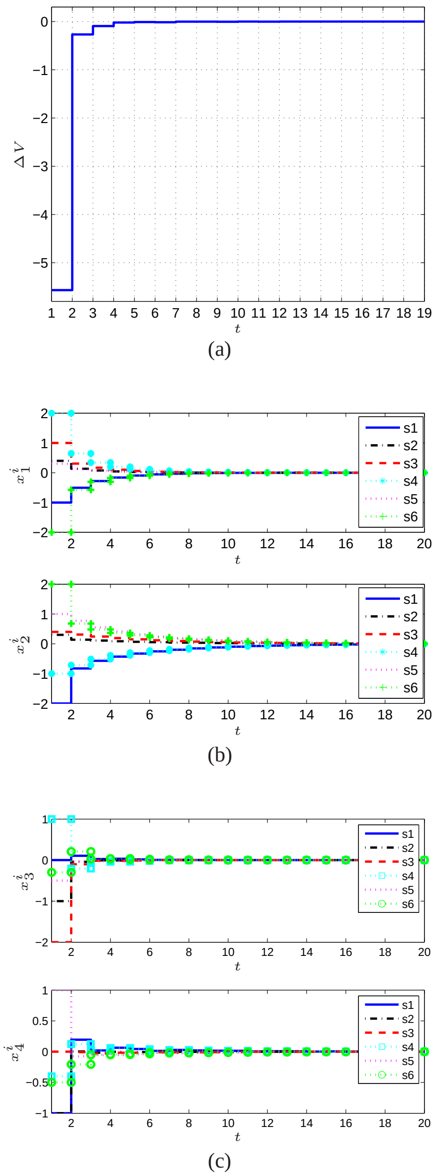

Fig. 2. (a) Check stochastic stability inequality, (b) states x_1^i and x_2^i, and (c) states x_3^i and x_4^i.

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