Model-Based Event-Triggered Control of Nonlinear Dissipative Systems

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***Abstract*—The Model-Based Networked Control System (MB-NCS) framework can be used to improve performance in applications where communication rates are limited. This framework has been studied for linear systems with state feedback, but little has been done for the nonlinear case or for the output feedback case. This paper approaches the problem of networking discrete-time nonlinear systems in the MB-NCS framework by using dissipativity theory and output feedback measurements. By combining the MB-NCS approach and aperiodic event-triggered updates, the NCS may operate open loop for long time intervals. Additionally, this paper considers model uncertainties and non-vanishing input disturbances as these factors can be destabilizing in the absence of continuous feedback. While these issues can be mitigated, traditional notions of stability are simply not achievable. The main contribution of this paper is a boundedness result on the system output with a constructive bound. The bound is guaranteed despite the presence of aperiodic updates, model uncertainties, and input disturbances.**

I. Introduction

The presence of networks in control systems brings many benefits including lower cost and the ability to easily reconfigure. However, the use of networks introduces many new issues [1]. Primarily, the network is multi-purpose and may not be available to use continuously. It is possible to free the network for critical tasks by reducing communication between nodes. This problem was studied in [2] where a protocol was used to allocate network resources in a control system. Alternative approaches were used to reduce data rates by more efficiently using the payload in a standard packet structure [3], [4].

The present paper uses the Model-Based Networked Control System (MB-NCS) framework and the event-triggered control framework to reduce the load on the network. In MB-NCS [5]–[9], communication is reduced by implementing a model of the plant on the controller side of the network. In the time intervals between updates, the model is used to predict the state of the system. This predicted state is used to produce a control input that can improve performance over a zero order hold. When the model is sufficiently accurate, communication rates may be significantly reduced while performance is maintained.

Previous work in the model-based approach [5]–[9] mainly focused on the linear case with nonlinear approaches only considering the state feedback case [7], [10], [11]. In contrast, the present paper considers output stabilization of nonlinear discrete time systems. When feedback measurements are intermittent, sensitivity to model uncertainties and disturbances increases greatly. To address this issue, this paper considers both plant-model mismatch and non-vanishing input disturbances. Initial work in this direction was presented in the report [12].

While the MB-NCS framework reduces communication, data rates are further reduced by using aperiodic event-triggered communication [9], [13]–[15]. Output feedback stabilization has been recently addressed using event-triggered control strategies [16]–[19]. Output feedback stabilization of linear systems subject to external disturbances was considered in [20]. The present paper addresses a similar problem with the focus being on nonlinear discrete time systems. The use of the model-based approach also provides significant reduction of network communication, as it was discussed in [21] and [22], compared to the zero-order-hold model used in [20].

When considering model uncertainty and disturbances, the system output will not, in general, converge to zero. While asymptotic stability or ℓ_2 stability are desirable, they are not achievable. When considering input-output stability, the notion of ℓ_2 stability must be relaxed to a bound on the output. The form of boundedness considered in this paper, an average bound on the squared output, is shown in two parts. First the problem is recast as a negative feedback design problem. Then, dissipativity theory is applied to show boundedness for the original MB-NCS. The notion of dissipativity was formalized in [23], with the specific form of QSR dissipativity used in this paper was given in [24].

The main contribution of this paper is in proposing an alternative approach for analyzing MB-NCS with nonlinear systems. This method is based on dissipativity theory and is robust to model uncertainties, aperiodic updates, and non-vanishing disturbances. Additionally, this approach is based on an input-output model of the system and does not depend on an internal model of the system dynamics. A brief background on dissipativity and QSR dissipativity is provided in Section II. Section III outlines the network structure including the MB-NCS framework with event-triggered updates. Section IV presents the main boundedness result of this approach. An example is given in Section V and the paper is concluded in Section VI.

II. Background Material

A. Mathematical Preliminaries

The signal space ℓ_2 is the set of finite energy discrete time signals. A function $w(k)$ is in ℓ_2 if it has finite ℓ_2 norm:

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$$||w(k)||_2^2 = \lim_{K\to+\infty} \sum_{k=0}^{K-1} w^T(k)w(k) < \infty. \quad (1)$$

The extended ℓ_2 space, or ℓ_{2e}, is the set of signals that have finite energy on any finite time interval, i.e.

$$||w_K(k)||_2^2 = \sum_{k=0}^{K-1} x^T(k)x(k) < \infty \quad (2)$$

$\forall K < \infty$. The systems of interest in this paper map input signals $u(k) \in \ell_{2e}$ to signals $y(k) \in \ell_{2e}$. A system is ℓ_2 stable if $u(k) \in \ell_2$ implies that $y(k) \in \ell_2$ for all $u(k) \in U$. An important special case of this stability is finite-gain ℓ_2 stability where there exists a γ and β such that,

$$||y_K(k)||_2 \leq \gamma\, ||u_K(k)||_2 + \beta \quad (3)$$

$\forall K > 0$ and $\forall u_K(k) \in U$. The ℓ_2 gain of the system is the smallest γ such that there exists a β to satisfy the inequality.

The systems considered in this paper are nonlinear discrete-time systems. While the systems are controlled using only output feedback, internal models are used to study the dissipative properties of a system. These state space models are given by,

$$\begin{aligned} x(k+1) &= f(x(k), u(k)) \\ y(k) &= h(x(k), u(k)), \end{aligned} \quad (4)$$

where $x(k) \in \mathbb{R}^n$, $u(k) \in \mathbb{R}^m$, and $y(k) \in \mathbb{R}^p$.

B. Dissipativity Theory

Dissipativity is an energy-based property of dynamical systems. This property relates energy stored in a system to the energy supplied to the system. The energy stored in the system is defined by an energy storage function $V(x)$. As a notion of energy, this function must satisfy $V(x) > 0$ for $x \neq 0$ and $V(0) = 0$. The supplied energy is captured by an energy supply rate $\omega(u, y)$. A system is dissipative if it only stores and dissipates energy, with respect to the specific energy supply rate, and does not generate energy on its own.

Definition 1. *A nonlinear discrete-time system (4) is dissipative with energy supply rate $\omega(u, y)$ if there exists a positive energy storage function $V(x)$ such that the following inequality holds,*

$$\sum_{k=k_1}^{k_2} \omega(u(k), y(k)) \geq V(x(k_2+1)) - V(x(k_1)), \quad (5)$$

for all times k_1 and k_2 such that $k_1 \leq k_2$,

A particularly useful form of dissipativity with additional structure is the quadratic form, QSR dissipativity.

Definition 2. *A discrete-time system (4) is QSR dissipative if it is dissipative with respect to the supply rate*

$$\omega(u, y) = \begin{bmatrix} y \\ u \end{bmatrix}^T \begin{bmatrix} Q & S \\ S^T & R \end{bmatrix} \begin{bmatrix} y \\ u \end{bmatrix}. \quad (6)$$

where $Q = Q^T$ and $R = R^T$.

The QSR dissipative framework generalizes many areas of nonlinear system analysis. The property of passivity can be captured when $Q = R = 0$ and $S = \frac{1}{2}I$, where I is the identity matrix. Systems that are finite-gain ℓ_2 stable can be represented by $S = 0$, $Q = -\frac{1}{\gamma}I$, and $R = \gamma I$ where γ is the gain of the system. The following theorems give stability results for QSR dissipative systems and dissipative systems in feedback.

Theorem 1. *[24] A discrete-time system is finite-gain ℓ_2 stable if it is QSR dissipative with $Q < 0$.*

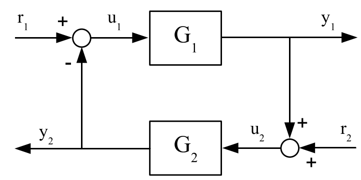

Fig. 1. The feedback interconnection of systems G_1 and G_2.

Theorem 2. *[24] Consider the feedback interconnection of two QSR dissipative systems (Fig. 1). System G_1 is dissipative with respect to Q_1, S_1, R_1 and system G_2 with respect to Q_2, S_2, R_2. The feedback interconnection is ℓ_2 stable if there exists a positive constant a such that the following matrix is negative definite,*

$$\tilde{Q} = \begin{bmatrix} Q_1 + aR_2 & -S_1 + aS_2^T \\ -S_1^T + aS_2 & R_1 + aQ_2 \end{bmatrix} < 0. \quad (7)$$

QSR dissipativity can be used to assess stability of a single system as well as systems in feedback. From a control design perspective, the QSR parameters of a given plant can be determined and used to find bounds on stabilizing QSR parameters of a potential controller.

III. Network Structure

One of the main problems in NCS is the design of control schemes that account for the absence of feedback measurements for possibly long intervals of time. While classical closed loop control with continuous feedback reduces sensitivity to model uncertainties, this benefit is absent when feedback measurements are missing. The MB-NCS approach considers model uncertainties in the absence of continuous feedback. This framework uses a model of the plant to compute the control input based on the predicted state of the plant rather than the actual state. In contrast to previous work in MB-NCS, the work in this paper does not assume that the entire state vector is available for measurement but only the output of the system. It is assumed that the dynamics of each system in the network (Fig. 2) are decoupled so the analysis can focus on a particular system/model pair without loss of generality. In MB-NCS the actuator/controller node with output feedback can be represented as in Fig. 3.

This work considers Single-Input Single-Output (SISO) uncertain and unstable nonlinear discrete-time systems that can be described by:

$$y(k) = f_{io}(y(k-1), ..., y(k-n), u(k), ..., u(k-m)) \quad (8)$$

and the dynamics of the model are given by:

$$\hat{y}(k) = \hat{f}_{io}(\hat{y}(k-1), ..., \hat{y}(k-n), u(k), ..., u(k-m)) \quad (9)$$

where the nonlinear function $\hat{f}_{io}(\cdot)$ represents the available model of the system function $f_{io}(\cdot)$. This input-output representation can be transformed into the state space form, shown in [12], in order to evaluate the QSR parameters.

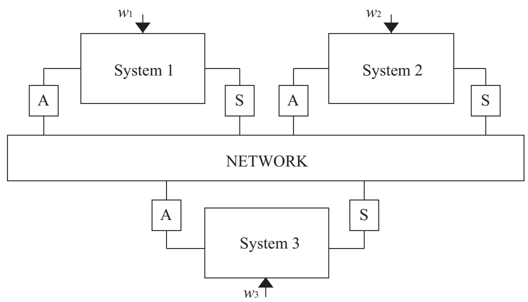

Fig. 2. Representation of Networked Control Systems with actuator nodes (A), sensor nodes (S), and external disturbances (ω_i).

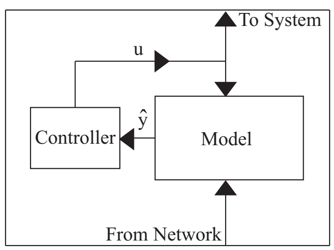

Fig. 3. Model-Based Networked Control System actuator node containing the model and controller.

The main reason for using this configuration is to operate in open-loop for as long as possible while maintaining desirable boundedness properties. This is done by using the estimated output $\hat{y}$ provided by the model to generate the control input u. When needed for the event-triggered control scheme, the actual output y is used to update the model without need of implementing a state observer. The model requires the current value and previous n values of the output based on the dimension of the system (8). The sensor contains a copy of the model and controller so it has access to the model output. It continuously measures the actual output and computes the model-plant output error:

$$e(k) = \hat{y}(k) - y(k). \tag{10}$$

In the MB-NCS literature the update measurements are implemented periodically. This paper discards the periodicity assumption for updating the model, and instead uses an aperiodic event-triggered rule as in [9]. The sensor node monitors the output error and communicates the plant output when an event is triggered. In this case, the event is the error $e(k) > \alpha$ where $\alpha > 0$ is a fixed positive threshold. At this time instant, the model output $\hat{y}(k)$ is updated to equal $y(k)$ and the output error (10) is zero. Assuming no delay in updating the output, the error is always bounded:

$$|e(k)| \leq \alpha. \tag{11}$$

It should be noted that the reduction in network traffic is significant compared to the case in which a measurement of $y(k)$ is sent at every sampling instant. This is true even when the order of the system n is large compared to the inter-update intervals. In this case, nearly every sample of the output is transmitted eventually. The average data rates are still significantly reduced when considering that bandwidth can be lost due to packet overhead and the minimum size of payload for each packet. As the minimum payload in a packet is typically much larger than a single measurement, by saving measurements and sending them all at the same time, a larger portion of the payload can be utilized, similar to the approaches in [3], [4].

One of the original contributions of this paper is in reformulating the MB framework into a traditional feedback problem as in Fig. 1. This is done by representing the output of the model $\hat{y}(k)$ as the sum of the plant output $y(k)$ and the error $e(k)$ which is now treated as an external input. In this equivalent, model abstracted, representation the plant mapping from input $u_p(k)$ to output $y(k)$ and controller mapping from input $u_c(k)$ to output $y_c(k)$ are directly interconnected as in Fig. 4.

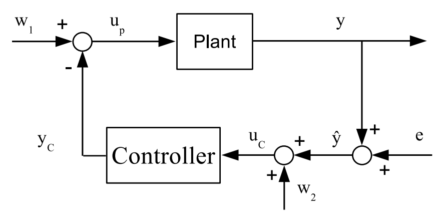

Fig. 4. This figure shows the feedback of the plant and controller/model. The two disturbance signals are w_1 and w_2 while the error in the model output is e.

In the absence of disturbances, the input to the plant is the output of the controller and the input to the controller is the predicted plant output $\hat{y}$. This is consistent with the MB-NCS framework in Fig. 3 and consistent with the definition of the output error, $\hat{y}(k) = e(k) + y(k)$. The error $e(k)$ is still present in the absence of external disturbances due to model uncertainties. It is not possible to show ℓ_2 stability since the error signal $e(k)$ is not an ℓ_2 signal.

Feedback systems with periodic feedback have a low sensitivity to disturbances and unmodeled dynamics. This property is not guaranteed when considering aperiodic communication. This paper explicitly considers a plant input disturbance $w_1(k)$ and a controller input disturbance $w_2(k)$. Both signals are assumed to have bounded magnitude for all time k but may be non-vanishing. These signals can capture unmodeled dynamics as well as error introduced by discretization. These disturbances are unknown but have magnitude bounded by

$$|w_1(k)| \leq W_1(k) + c_1 \tag{12}$$

where the signal $W_1(k) > 0$ is an ℓ_2 signal and $c_1 \geq 0$ is a constant. Likewise, for ℓ_2 signal $W_2(k) > 0$ and positive constant c_2,

$$|w_2(k)| \leq W_2(k) + c_2. \tag{13}$$

An example of such a disturbance and the appropriate bounds is given in Fig. 5.

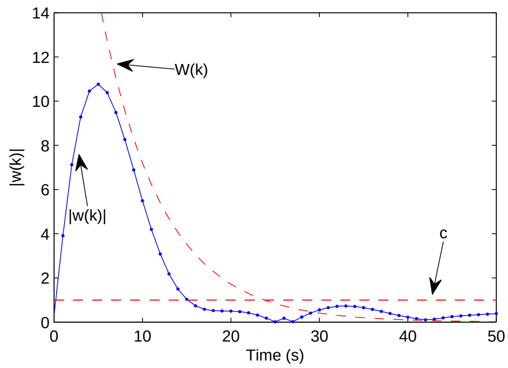

Fig. 5. This figure provides an example of an allowable disturbance $w(k)$ that is bounded by the sum of an ℓ_2 signal $W(k) > 0$ and a constant $c > 0$.

IV. Boundedness Results

This section considers the problem of bounding the size of the system output when operating nonlinear discrete-time systems in the network configuration described in the previous section. As discussed previously, there are two issues with traditional stability for this network setup. The first is due to the aperiodic control updates. Between update events the feedback system is temporarily operating open-loop. With even small model mismatch between the actual plant and the model, the outputs between the two can drift significantly over time. Typically, the system output does not go to zero thus cannot be bounded as in finite-gain ℓ_2 stability. The second issue with traditional notions of stability is that this work allows non-vanishing input disturbances. Traditional dissipativity theory shows stability for disturbances that are in ℓ_2, i.e. the disturbance must converge to zero asymptotically. We generalize existing results to disturbances that may not go to zero but do have an ultimate bound.

While notions of asymptotic stability or finite-gain ℓ_2 stability are appealing, they are simply not achievable in this framework. Instead we relax this to a boundedness result. As this paper considers systems described by an input-output relationship, the notion of ℓ_2 stability is relaxed to a bound on the output as time goes to infinity. With output error and disturbances that are non-vanishing, the output may fluctuate over a large range. It may be difficult to find an ultimate bound the size of the output for all time. Instead, this paper considers an average bound on the squared system output.

Definition 3. *A nonlinear system is average output squared bounded if after time $\bar{k}$, there exists a constant b such that the following bound on the output holds for all times k_1 and k_2 larger than $(\bar{k} \leq k_1 < k_2)$,*

$$\frac{1}{(k_2 - k_1)} \sum_{k=k_1}^{k_2-1} y^T(k)y(k) \leq b. \tag{14}$$

This form of boundedness is a practical form of stability on the system output. While the output does not necessarily converge to zero, it is bounded on average with a known bound as time goes to infinity. It is important to note that this concept is not useful for an arbitrarily large bound b. However, the concept is informative for a small, known bound. The notion should be restricted to being used in the case when the bound is constructive and preferably when the bound can be made arbitrarily small by adjusting system parameters.

The following boundedness theorem can be applied to the analysis of a plant and controller in the model-based framework. The plant and the model of the plant must be QSR dissipative with respect to parameters Q_P, S_P, and R_P. Although the plant dynamics are not known exactly, sufficient testing can be done to verify that the dissipative rate bounds the actual dissipative behavior of the system. The model-stabilizing QSR dissipative controller has been designed with parameters Q_C, S_C, and R_C.

Theorem 3. *Consider a plant and controller in the MB-NCS framework (Fig. 2-3) where model mismatch may exist between the plant and model. The network structure contains event-triggered, aperiodic updates and non-vanishing disturbances. This feedback system is average output squared bounded if there exists a positive constant a such that the following matrix is negative definite,*

$$\tilde{Q} = \begin{bmatrix} Q_P + aR_C & aS_C^T - S_P \\ aS_C - S_P^T & R_P + aQ_C \end{bmatrix} < 0. \tag{15}$$

Proof. The plant and controller being QSR dissipative implies the existence of positive storage functions V_P and V_C, such that

$$\Delta V_P(x_P) \leq \begin{bmatrix} y \\ u_P \end{bmatrix}^T \begin{bmatrix} Q_P & S_P \\ S_P^T & R_P \end{bmatrix} \begin{bmatrix} y \\ u_P \end{bmatrix}$$

and a similar bound on ΔV_C. A total energy storage function can be defined, $V(x) = V_P(x_P) + aV_C(x_C)$, where $x = [x_P^T \; x_C^T]^T$. The total energy storage function has the dissipative property,

$$\Delta V(x) \leq \begin{bmatrix} y \\ y_C \\ w_1 \\ (w_2+e) \end{bmatrix}^T \begin{bmatrix} \tilde{Q} & \tilde{S} \\ \tilde{S}^T & \tilde{R} \end{bmatrix} \begin{bmatrix} y \\ y_C \\ w_1 \\ (w_2+e) \end{bmatrix}.$$

where

$$\tilde{Q} = \begin{bmatrix} Q_P + aR_C & aS_C^T - S_P \\ aS_C - S_P^T & R_P + aQ_C \end{bmatrix},$$

$$\tilde{S} = \begin{bmatrix} S_P & aR_C \\ -R_P & aS_C \end{bmatrix}, \text{ and } \tilde{R} = \begin{bmatrix} R_P & 0 \\ 0 & aR_C \end{bmatrix}.$$

Due to $\tilde{Q} < 0$ (15), there exists a constant q such that $\tilde{Q} \leq -qI$. As $\tilde{S}$ and $\tilde{R}$ are constant matrices, the largest singular values can be found, $s = \overline{\sigma}(\tilde{S})$ and $f = \overline{\sigma}(\tilde{R})$. This yields the following bound on ΔV

$$\Delta V(x) \leq -q[y^T y + y_c^T y_c] + 2s[y^T w_1 + y_c^T (w_2+e)] + r[w_1^T w_1 + (w_2+e)^T (w_2+e)].$$

Completing the square can be used to remove the cross terms,

$$\Delta V(x) \leq -\frac{q}{2}[y^T y + y_c^T y_c] + \frac{(4s^2+2qr)}{2q}[w_1^T w_1 + w_2^T w_2 + e^T e].$$

Summing this inequality from k_1 to k_2 yields the following,

$$V(x(k_2)) \leq V(x(k_1)) - \frac{q}{2}\sum_{k=k_1}^{k_2-1}[y^T y + y_c^T y_c] + \frac{(4s^2+2qr)}{2q}\sum_{k=k_1}^{k_2-1}[w_1^T w_1 + w_2^T w_2 + e^T e].$$

The effect of the non-vanishing disturbances w_1 and w_2 can be bounded by constants ϵ_1 and ϵ_2 after some time $\bar{k}$, $|w_i(k)| \leq \epsilon_i$, for $k \geq \bar{k}$. Additionally, $|e(k)| < \alpha$ for all k. A single bound can be defined $\epsilon^2 = \epsilon_1^2 + \epsilon_2^2 + \alpha^2$. At this point, either the average output squared is bounded by the following,

$$\frac{1}{(k_2-k_1)}\sum_{k=k_1}^{k_2-1} y^T y \leq \frac{(4s^2+2qr)\epsilon^2}{q^2(1-\delta)}, \tag{16}$$

where $0 < \delta < 1$, or not bounded by it,

$$\frac{1}{(k_2-k_1)}\sum_{k=k_1}^{k_2-1} y^T y > \frac{(4s^2+2qr)\epsilon^2}{q^2(1-\delta)}. \tag{17}$$

When it is larger than this quantity, it is possible to show,

$$V(x(k_2)) \leq V(x(k_1)) - \frac{q\delta}{2}\sum_{k=k_1}^{k_2-1} y^T y - \frac{(4s^2+2qr)}{2q}\sum_{k=k_1}^{k_2-1}[\epsilon^2 - w_1^T w_1 - w_2^T w_2 - e^T e].$$

This can be used to show a bound on y,

$$\sum_{k=k_1}^{k_2-1} y^T y \leq \frac{2}{q\delta} V(x(k_1)). \tag{18}$$

As the sum of $y^T y$ is bounded, the average is also bounded. Either (16) or (18) holds which shows that the average squared system output is bounded, satisfying Definition 3. Furthermore, the parameter δ can be adjusted to vary (and potentially lower) the relative size of the two bounds.

□

One important takeaway is that the bound on the system output is constructive. The bounds can be made smaller by adjusting the values of controller which changes q, s, and r. The bounds also depend on the value of the output error threshold α which can be made smaller, to an extent. The effect of the non-vanishing disturbances may be significant depending on ϵ_1 and ϵ_2. When these disturbances are vanishing, the bound on the output depends mainly on α which may be made arbitrarily small. The value of α may be chosen to tradeoff decreased communication with reduced output error.

V. EXAMPLE

The following example was chosen to be LTI for ease of following, but the results apply to nonlinear systems. The QSR parameters were found using state space models, but the NCS is simulated using the equivalent input-output models. The plant of interest is unstable with model given by:

$$\hat{A} = \begin{bmatrix} -0.7 & 0.52 \\ 0.88 & 0.4 \end{bmatrix}, \hat{B} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \hat{C} = \begin{bmatrix} 0.73 & 2.2 \end{bmatrix}, \hat{D} = 1.2. \tag{19}$$

The model can be shown QSR dissipative ($Q_P = 0.04$, $S_P = 0.15$, and $R_P = 0.1$) by using the storage function:

$$\hat{V}(\hat{x}) = \hat{x}^T \begin{bmatrix} 0.38 & 0.42 \\ 0.42 & 0.73 \end{bmatrix} \hat{x}. \tag{20}$$

An example of a stabilizing controller is given by: $A_C = 0.2$, $B_C = 0.6$, $C_C = 0.8$, and $D_C = 1$. This controller is QSR dissipative ($Q_C = -0.35$, $S_C = 0.15$, and $R_C = -0.3$) which can be shown using storage function, $V_c(x_c) = 0.31 {x_u}^2$. The controller can be shown to stabilize the model by evaluating (15) with $a = 1$,

$$\tilde{Q} = \begin{bmatrix} -0.26 & 0 \\ 0 & -0.25 \end{bmatrix} < 0. \tag{21}$$

As mentioned earlier, the actual plant is dissipative with respect to the same QSR parameters (Q_P, S_P, and R_P), and is given by:

$$A = \begin{bmatrix} -0.71 & 0.55 \\ 0.95 & 0.35 \end{bmatrix}, B = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, C = \begin{bmatrix} 0.75 & 2.3 \end{bmatrix}, D = 1.1. \tag{22}$$

The QSR parameters can be verified using storage function

$$V(x) = x^T \begin{bmatrix} 0.37 & 0.36 \\ 0.36 & 0.55 \end{bmatrix} x. \tag{23}$$

By assumption, the controller also stabilizes the plant and satisfies the inequality for boundedness. This MB-NCS was simulated with input-output models for the plant, model, and controller. The external disturbances $w_1(k)$ and $w_2(k)$ for this example are shown in Fig. 6.

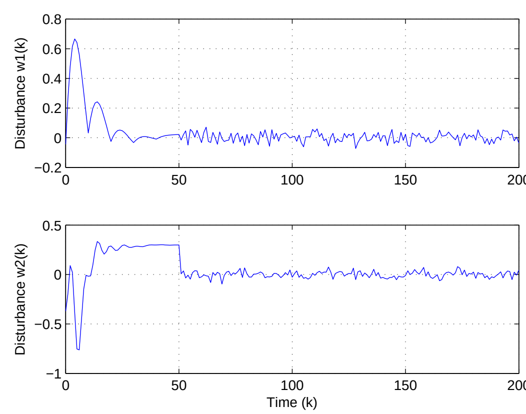

Fig. 6. This figure shows the non-vanishing disturbances $w_1(k)$ and $w_2(k)$.

The disturbances are bounded by 0.1 after time $k = 50$. With this magnitude of disturbance, it is not possible to guarantee that the output error stay less than 0.1. The threshold for the output error was chosen to be 0.2. These systems were simulated and the system outputs are shown in Fig. 7. The evolution of the output error over time is shown in the first subplot of Fig. 8. This plot shows the error after each update takes place, that is, when the error is reset to zero. As a result, the error is always bounded as stated in (11). The second subplot shows the time instants at which output measurements are sent from the sensor node to the controller node. The rest of the time the networked system operates in open-loop.

Fig. 7 shows that the outputs of the MB-NCS are bounded, as expected. Clearly the outputs do not converge to zero,

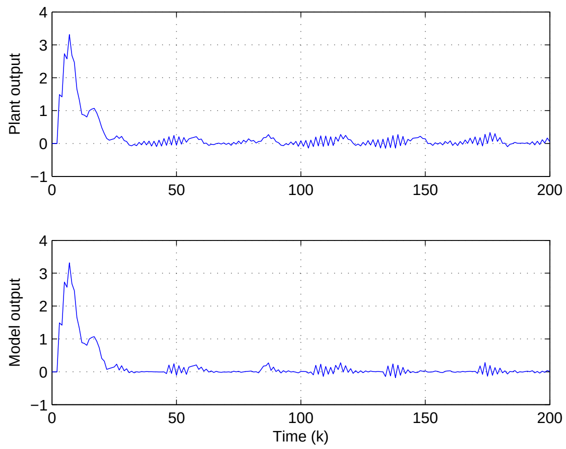

Fig. 7. This figure shows the output of the plant (top) and the model (bottom). The model tracks the plant closely within the update threshold.

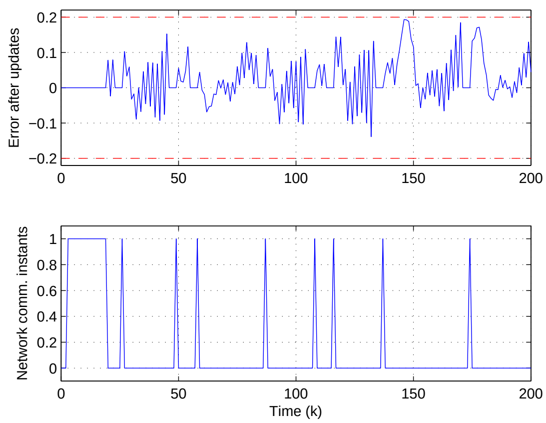

Fig. 8. This figure shows the output error (top) and communication instants. A value of 1 indicates data is transmitted while 0 indicates no data sent. The communication instants align with the error growing to above 0.2.

but the model output tracks the plant output closely. In this example, communication is relatively constant for the first 20 time instants as the system responds to the large disturbances. After this point, the communication rate drops significantly. While the error stays bounded by 0.2, the communication rate is reduced by 88.9%. For comparison, a simulation was run with the output being transmitted at every time instant. For this case, the output error grew to as large as 0.17 after $k = 50$. The continuous output feedback provides a small reduction in output error, with an average data transmission rate that is nearly 10 times higher.

VI. Conclusions

This paper presented a boundedness result for discrete-time nonlinear systems in the MB-NCS framework with only output measurements. It is assumed that the system has uncertain dynamics and is perturbed by non-vanishing disturbances. The magnitude of the disturbances were bounded by the sum of an ℓ_2 signal with a constant offset. The MB-NCS framework and the event-triggered feedback are both utilized to reduce communication rates. While the output does not converge to zero, the squared output is on average bounded with a constructive bound. This bound can be made small by varying the parameters of the controller and by reducing the threshold on the acceptable error of the model output. An example was provided to illustrate how these methods can be used in practice.

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