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IMPLEMENTATIONS OF TWO DEGREES OF FREEDOM CONTROLLERS

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Abstract

Some issues in the implementation of two degrees of freedom controllers are considered. In particular, a systematic way to implement a two degrees of freedom controller via the {R;G,H} controller configuration is given and it is shown that three "two degrees of freedom" controllers may introduce undesirable response limitations.

I. Introduction

In control problems with multiple objectives, the design must be made independent of the controller configuration in order to avoid unnecessary restrictions on the attainable control properties. The design of a linear multivariable control system is made independent of the controller configuration by using a two degrees of freedom control law, which represents the most general linear controller that can be used to independently specify the desired responses to the command and to the disturbance inputs. Once the design specifications are met, the controller must be implemented using a suitable configuration. Then, it is possible to conjecture that any two degrees of freedom controller implementation should give satisfactory results. A remark to this effect appears in the classical textbook by Horowitz [1, p. 249]. However, this is not the case; some two degrees of freedom controller configurations can introduce undesirable response limitations [2, p.655].

In this paper, we show that three specific "two degrees of freedom" controllers are not suitable in some problems because the response to command and disturbance and/or noise inputs cannot be attained independently. To this effect, we first review some basic facts of configuration independent two degrees of freedom control systems. Then we consider the {R;G,H} controller because it includes the three unsuitable configurations as special cases. It is shown that the {R;G,H} controller can implement all admissible command/output, command/control, and output-disturbance/output maps with internal stability. A systematic approach to synthesize R, G, and H is outlined.

Another important problem that needs to be considered when implementing a controller is the hidden modes. The hidden modes correspond to the compensated system's eigenvalues which are uncontrollable and/or observable from a given input or output, respectively. If the plant and controller are completely described by their transfer matrices then the only hidden modes are those introduced by the interconnection. These hidden modes are usually a consequence of trying to meet several control specifications.

Another source of hidden modes are the plant and controller if they are not controllable and observable. We have learned to cope with the hidden modes of the plant as long as the plant is stabilizable and detectable. We believe that the the study of the effect of the hidden modes of the controller also deserves special attention. One control design method that leads to hidden modes in the controller is the *synthetic design approach* where additional compensators are introduced to handle a new problem. For example, one compensator could handle stability, a second one regulation, and a third one steady–state performance. To understand the significance of the problem notice that the design of the controller specifies all the input–output maps and, hence, the output (y) and control (u) signals. The signals that are not specified are those internal to the plant and to the controller, In the design of fault-tolerant/reliable controllers the signals "inside" of the controller must also meet some specifications. In addition, the solution of robust control problems such as robust tracking imposes constraints that must be satisfied by the controller implementation. These additional specifications on the controller usually result in hidden modes. Thus conditions to minimize the number of hidden modes and to understand their effect when they are needed must be developed. The hidden modes of two degrees of freedom control systems have been studied from the control design point of view in [3,16–18]

A Brief Historical Background

Control systems design using the general two degrees of freedom control law has received considerable attention in the research literature. The implementation of the controller has not been as well investigated. The published results either do not address this issue or assume a specific controller configuration.

The implementation of two degrees of freedom controllers has been considered when the plant has only one input and one output. For example, Truxal in [4], gives a sequential procedure for the design of disturbance-to-output and input-to-output transfer functions, using a particular controller configuration. Horowitz in [1] gives a comprehensive study of two degrees of freedom controllers in the design of feedback systems. Recently, Åström in [5] uses a particular controller configuration in the design of robust controllers.

The following papers consider the implementation of two degrees of freedom controllers when the plant has multiple inputs and outputs.

Wolovich during the treatment of the model matching problem in [6] gives a procedure to realize a transfer matrix M, where T = PM, with P representing the plant and T the desired input–output model, in terms of linear state feedback plus observer and a precontroller. A similar controller configuration is considered by Chen and Zhang in [7] where they give general guidelines for the implementation of a transfer matrix via some controller configuration. Also, optimal methods like LQG incorporate observer type realizations in their controller structure.

In the recent literature treating two degrees of freedom control systems design, most authors consider a specific configuration during the analysis. For example, in [8,9] C is expressed as

$$C = [-C_y \;\; C_r] = \tilde{D}'^{-1}_c[-\tilde{N}'_y \;\; \tilde{N}'_r] \tag{1.1}$$

a coprime factorization of C, where $\tilde{D}'_c$, $\tilde{N}'_y$ and $\tilde{N}'_r$ are proper, stable rational matrices. A realization of C can be obtained via the {R;G,H} controller in Figure 3.1

with $R=\tilde{N}'_r$, $G=\tilde{D}'^{-1}_c$ and $H=\tilde{N}'_y$.

Pernebo [10] shows that the regulation and servo problems can be separated and solved sequentially. Then, he considers the following control law

$$u = C\begin{bmatrix} y \\ r \end{bmatrix} = R_{fb}\begin{bmatrix} y \\ v \end{bmatrix} = R_{fb}\begin{bmatrix} y \\ R_{ff}r \end{bmatrix} \qquad (1.2)$$

where R_{ff} is a precompensator and R_{fb} corresponds to a compensator in the feedback loop.

Youla and Bongiorno [2] consider the {R;G,H} controller to realize C which satisfies some optimality conditions for the maps Q and M. These authors stress that the selection of the controller configuration is an important issue for future research. They suggest to use the sensitivity to controller parameter variations as a criterion in the selection of a specific control configuration to implement the control law.

A different approach is taken by Desoer and Lin [11] who make a comparative study of seven controller structures —a unity feedback configuration (this is the {R;G,H} controller with R=I, H=I, we denote this as {I;G,I} controller) and six two degrees of freedom configurations. The comparison is based on the stability conditions in terms of the transfer function matrices, the characterizations of all the attainable maps of interest, and the sensitivity of the controller configuration to plant parameter variations. Based on these considerations they selected a particular two degrees of freedom configuration as preferable in some sense. The configuration they chose as preferable has been used in [15]. Extensions to the nonlinear case are also considered.

Of course other two degrees of freedom controller configurations have been used in the literature as in [12–14].

II. Preliminaries

The two degrees of freedom linear controller S_c implements the control law $u = C[y^t, r^t]^t = -C_y y + C_r r$, where $C = [-C_y, C_r]$ as seen in Figure 2.1;

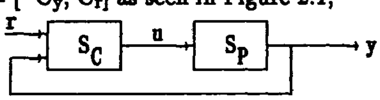

Figure 2.1. The controlled system.

S_p is the linear plant described by $y = Pu$ with P its proper transfer matrix. It is assumed that $|I+PC_y| = |I+C_yP| \neq 0$ and that every input-output map is proper. Under these assumptions, the controlled system is said to be internally stable if the inverse of the denominator matrix in a polynomial matrix description is stable. If the controlled system is internally stable, we say that S_c is an internally stabilizing controller for S_p.

A convenient way to study internal stability of the system in Figure 2.1 is given in Theorem 2.1 [18].

<u>THEOREM 2.1.</u> The compensated system is internally stable if and only if

(i) $u = -C_y y$ internally stabilizes the system $y = Pu$, and

(ii) C_r is such that $M := (I + C_yP)^{-1}C_r$ satisfies $D^{-1}M = X$, a stable rational, where C_y satisfies (i) and $P = ND^{-1}$ a right coprime polynomial factorization.

A valuable tool in control design is the characterization of all internally stabilizing two degrees of freedom controllers C. Using Theorem 2.1 the following characterizations follow in a straightforward manner from the well known results on parametric characterization of all feedback controllers C_y. All internally stabilizing controllers C can be parametrically characterized using two independent stable parameters K and X, or Q and X, or L and X as [2,3,8,9]

$$C = (x_1 - K\tilde{N})^{-1}[-(x_2 + K\tilde{D}),\ X], \qquad (2.1)$$
$$= (I - QP)^{-1}[-Q,\ DX] \qquad (2.2)$$
$$= ((I - LN)D^{-1})^{-1}[-L,\ X], \qquad (2.3)$$

where $\tilde{N}$, $\tilde{D}$, x_1, x_2 are polynomial matrices, and they are derived from coprime fractional representations of the plant $P = ND^{-1} = \tilde{D}^{-1}\tilde{N}$ and the associated Bezout–Diophantine equation $x_1D + x_2N = I$ (similar results can be directly derived when proper and stable fractional representations of the plant are used [3]). The parameters K, Q, L, X must be stable and must be such that $D^{-1}(I-QP) = (I-LN)D^{-1}$ stable and $|x_1-K\tilde{N}| \neq 0$, $|I-QP| \neq 0$ or $|I-LN| \neq 0$. The above parameterizations characterize all internally stabilizing controllers C, proper and nonproper. For C proper, M and Q are chosen proper and such that $(I-QP)$ is biproper ($(I-QP)$ and its inverse proper); note that if P is strictly proper, Q proper always implies that $(I-QP)^{-1}$ is proper. In terms of K, for C proper need $D(x_2+K\tilde{D})$ proper and $D(x_1-K\tilde{N})$ biproper.

The relations between the parameters are

$$L = x_2 + K\tilde{D} = D^{-1}Q$$
$$Q = DL = C_y(I + PC_y)^{-1} = (I + C_yP)^{-1}C_y$$
$$X = (x_1 - K\tilde{N})C_r = D^{-1}M$$
$$M = DX = (I + C_yP)^{-1}C_r. \qquad (2.4)$$

It is evident that if exogenous signals (such as disturbances and/or noise) are assumed to be injected at various points in Figure 2.1, all possible transfer matrices from all inputs can be derived in terms of the design parameters —K (or Q or L) and X (or M). This characterizes all "admissible" responses, under internal stability. It follows that each transfer matrix depends on only one parameter and that all the transfer matrices can be characterized using only two parameters. In particular, all the response maps from the command signal r can be characterized in terms of X or M=DX. Similarly all the response maps from disturbance and noise inputs can be characterized in terms of K or Q or L. This shows the fundamental property of two degrees of freedom control systems: it is possible to independently attain the command/output and disturbance/output maps. We call X, M the response parameters [20], and K, Q, and L the feedback parameters.

For the purposes of this paper we consider the command/output (T) and command/control (M) response maps, and the output sensitivity matrix (S). These maps are described by $y = Tr + Sd$, $u = Mr - Qd$, where d is a vector of disturbances at the output of the plant.

<u>THEOREM 2.2.</u> A triple (T,M,S) of proper and stable matrices is realized with internal stability via a two degrees of freedom configuration if and only if there exists proper and stable X, L such that

$$\begin{bmatrix} T \\ M \\ S \end{bmatrix} = \begin{bmatrix} N & 0 \\ D & 0 \\ 0 & -N \end{bmatrix}\begin{bmatrix} X \\ L \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ I \end{bmatrix}. \qquad (2.5)$$

Remarks

1) **Because of the internal stability requirement, it is known that the right half plane zeros of P must be zeros of PC_y and of T; thus, introducing limitations on the transient response and sensitivity minimization [19–23]. In addition, in order to guarantee properness of M, the zeros at infinity of P must be zeros at infinity of T [19].**
2) A fundamental limitation of one degree of freedom systems (when $C_y = C_r$) is still present in two degrees of freedom systems: the specifications on noise attenuation cannot be achieved independently of other feedback properties such as disturbance rejection. Let n be the sensor noise, then $y = Tr + Sd - PQn$. The trade off is given by
$$S + NL = I, \qquad (2.6)$$
where $NL = PQ$. (2.6) clearly states that sensitivity minimization and noise attenuation cannot occur over the same frequency range [24]. For multivariable systems the input sensitivity matrix $S_i = (I + C_yP)^{-1}$ should also be considered [21,24], which leads to a second trade off equation
$$S_i + QP = I. \qquad (2.7)$$
In one degree of freedom systems where $C_y = C_r$, $L = X$, and $Q = M$, it is harder to meet all the control specifications because the command/output response must also be traded–off with disturbance rejection and noise attenuation.

III. Implementation Via {R;G,H} Compensation

The two degrees of freedom controller is implemented via the {R;G,H} controller as seen in Figure 3.1,

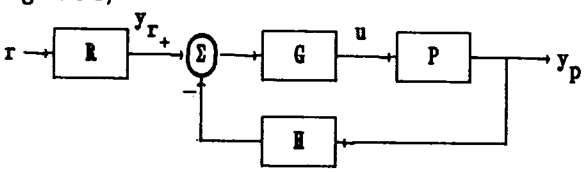

Figure 3.1. $\Sigma(\{R;G,H\},P)$ system.

where the interconnected subsystems are completely described by their transfer matrices P (p×m), R (m×q), H (m×p), and G (m×m). Notice that $C = [-C_y, C_r] = G[-H, R]$. In a rough sense, we have two matrix equations with three unknowns (R,G,H). In most control problems the dimensions are such that there is freedom in the choice of R, G, and H. In this section it will be seen that a systematic implementation requires the specification of another matrix parameter.

First, examine the conditions imposed on G, H, and R due to the internal stability requirement.

<u>Theorem 3.1.</u> The system $\Sigma(\{R;G,H\},P)$ is internally stable if and only if

(i) The control law $u = -GHy$ internally stabilizes P with no right half plane pole cancellations in the product GH.

(ii) The product GR is such that $M = (I + GHP)^{-1}GR$ satisfies $D^{-1}M = X$, a stable transfer matrix, where R is stable, GH satisfies (i), and $P = ND^{-1}$ is coprime.

Theorem 3.1 is a direct application of Theorem 2.1. Note that the poles of R and any poles that cancel in the product GH are closed-loop eigenvalues; hence, they must be in the open left half of the complex plane.

Second. if the {R;G,H} controller satisfies the internal stability conditions, then the following theorem shows that it can implement any realizable triple (T,M,S).

<u>Theorem 3.2.</u> Assume that (T, M, S) = (NX, DX, I–NL) with proper and stable X and L. Any such triple (T, M, S) can be realized with internal stability via {R;G,H} compensation with G proper, and R and H proper and stable.

Note that H stable is a desirable condition but it is not necessary. If H is not stable then its right half plane zeros may result in unnecessary right half plane zeros of T. The following example serves to prove Theorem 3.2.

<u>Example 1.</u> If P is stable, a feasible realization of C is
$$G = (I - QP)^{-1},\quad H = Q,\quad \text{and}\quad R = DX = M, \qquad (3.1)$$
while if P is unstable, a feasible realization of C is
$$G = (I - QP)^{-1}D',\quad H = (D')^{-1}Q,\quad \text{and}\quad R = X, \qquad (3.2)$$
where $P = N'(D')^{-1}$ is a proper and stable coprime factorization. In these implementations, the feedback loop compensators G and H take care only of the feedback properties, and the precompensator R takes care of the desired command/output response. To get a better understanding of these two implementations it helps to consider the {R;G,H} compensated system as the cascade connection of R followed by the {I;G,H} compensated system. The feedback system has command/output, command/control maps described by $y = T_fy_r$ and $u = M_fy_r$, respectively. In the first implementation we have that $T_f = P$ and $M_f = I$, while in the second one $T_f = N'$ and $M_f = D'$. In both cases the control input is given by $u = Mr$. ☐

In Example 1 the two degrees of freedom system was implemented in a way that the command/output response was taken care of outside the feedback loop and, of course, the feedback properties were taken care of inside the loop. Clearly, for some problems, as in those solvable with one degree of freedom systems, the feedback loop can also be used to implement the command/output map. In fact, for the {R;G,H} controller it is usually desirable that the command/output response be taken care of by the feedback loop, since the precompensator R should only be used to "fine tune" the command/output map.

These comments suggest that the additional parameter that needs to be introduced should be one that characterizes the "control action" that needs to be undertaken by the feedback loop. We can choose either M_f or X_f, since there is a one-to-one relation between them as seen by application of Theorem 2.2 to the system $\Sigma(\{I;G,H\},P)$ which yields the following characterization of maps
$$\begin{bmatrix} T_f \\ M_f \\ S \end{bmatrix} = \begin{bmatrix} N & 0 \\ D & 0 \\ 0 & -N \end{bmatrix} \begin{bmatrix} X_f \\ L \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ I \end{bmatrix}, \qquad (3.3)$$
where X_f and L are proper and stable maps. Notice that the set of all command/output, command/control and sensitivity matrices is the same in (2.5) and (3.3). It will be seen, however, that not all admissible (T,M,S) can be implemented with the {I;G,H} controller.

Suppose X_f is chosen, hence $T_f = NX_f$ and $M_f = DX_f$ are specified. Then the following theorem can be used to synthesize G, R, and H.

<u>Theorem 3.2.</u> The set of all the internally stabilizing {R;G,H} controllers that implements the triple (T,M,S) is
$$G = (I - QP)^{-1}M_f,\qquad M_f[H,\ R] = [Q,\ M], \qquad (3.4)$$
where $[M_f, M] = D[X_f, X]$ with X_f, and X stable; $Q = C_y(I + PC_y)^{-1}$, $D^{-1}[Q, (I - QP)]$ stable, $|I - QP| \neq 0$; R stable and no right half plane pole cancellations in GH.

The first equation in (3.4) establishes a one–to–one relation between G and M_f. The second equation in (3.4) is a model matching type relation and can also be written as $X_f[H, R]=[L, X]$.

SPECIAL CASES

It has been demonstrated that any triple (T,M,S) satisfying the admissibility conditions in Theorem 2.2 can be realized via {R;G,H} compensation. In addition, the set of maps attainable with {I;G,H} compensation is the same. This appears to indicate that any realizable triple (T,M,S) can be implemented via {I;G,H} compensation. However, that is not the case even though {I;G,H} is a two degrees of freedom controller. In this section we consider three two degrees of freedom configurations that are a subset of the {R;G,H} controller and show that they may introduce some undesirable limitations.

Case 1. {I;G,H} Controller. In this case R=I, which makes $M=M_f$ and L=XH. Substituting the latter equality in the trade off relation in (2.6) gives

$$S + NXH = I, \tag{3.5}$$

or

$$S + TH = I. \tag{3.6}$$

This shows that under some conditions this "two degrees of freedom" controller cannot achieve independent output disturbance rejection and command/output response. The independent specification of these maps is still possible, for example, when m=q and X is invertible, then let $H=X^{-1}\underline{H}$, which would introduce stable hidden modes. Its effect could be study using [18].

In order to avoid the introduction of unnecessary right half plane zeros in T, G should be designed to have no right half plane zeros and H should be stable. An extreme case of this fact was given in an example in [2, p.655] which it is put in our terms below.

Example 2. Consider {I;G,H} compensation and suppose that the design parameter X is such that it has a finite transmission zero in $\mathbb{C}^+$, the right half plane. For this case C_y=GH and C_r=G, and if K and X have been determined, then using (2.1) leads to $G=(x_1-K\tilde{N}_1)^{-1}X$ and $GH = (x_1-K\tilde{N}_1)^{-1}(x_2+K\tilde{D}_1)$. Suppose that $(x_1-K\tilde{N}_1)$ and $(x_2+K\tilde{D}_1)$ do not have as a zero the transmission zero of X in $\mathbb{C}^+$, then this zero of X must be a zero of G and a pole of H, becoming an unstable hidden mode of the compensated system and the internal stability requirement is not met. Note that if $(x_1-K\tilde{N}_1)$ has the zero of X in $\mathbb{C}^+$, in a way that it cancels when forming G, and that $(x_2+K\tilde{D}_1)$ does not have it, then H would have it as a pole and internal stability would be maintained. □

Case 2. {R;G,I} Controller. In this case H=I which makes X=LR and the trade off equation in (2.6) becomes

$$T = (I - S)R, \tag{3.7}$$

showing that independent command/output and noise attenuation is not possible. Another limitation is that C_y should not have right half plane zeros; otherwise, unnecessary right half plane zeros are introduced in T.

Case 3. {R;I,H} Controller. In this case G=I which makes M_f=(I–QP) and the trade off equation in (2.7) becomes

$$S_i = M_f = DX_f, \tag{3.8}$$

or

$$S_iR = X \tag{3.9}$$

showing that command/output and input disturbance rejection may not be achieved independently. In addition this configuration has two very restrictive conditions: C_r must be stable, and C_y should be stable (to avoid introducing unnecessary right half plane zeros in T).

IV. Conclusions

It has been shown that any realizable triple (T,M,S) can be implemented via {R;G,H} compensation. However, when R or G or H are set to identity, the resulting "two degrees of freedom" controller may not realize the desirable triple because it cannot attain independently the response and feedback properties. Examples demonstrating this limitations will be presented at the conference.

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