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LIMITATIONS OF VIBRATION SUPPRESSION IN FLEXIBLE SPACE STRUCTURES

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Abstract

The uncertainties inherent in the dynamics of flexible spacecraft make robustness questions very important when designing vibration suppression systems for these vehicles. This often leads to the use of sensors and actuators which are collocated on the structure, so as to avoid the potentially destabilizing problem of unknown modal phase differences between non-collocated sensors and actuators. The closed-loop performance attainable is, of course, reduced if we restrict ourselves to collocated pairs: the object of this paper is to investigate whether the special properties of the transmission zeros of such structures can be used to quantify these performance limitations.

Introduction

The dynamics of Flexible Space Structures (FSS) are generally rather poorly known before launch. This is partly a consequence of the fact that producing finite-element models for complex structures is still something of a "black art", with no guarantees given for the resulting accuracy. This would not be so serious, though, if it were possible to correct the model based on the results of accurate vibration tests before launch. Unfortunately, as FSS are quite weak, any such tests must be conducted with the structure extensively supported against gravity, so the dynamics that are actually measured are those of the structure plus support system. Furthermore, the presence of air resistance makes it very difficult to determine the very low levels of damping present in the structure itself. It is therefore difficult to gain more from ground tests than an approximate idea of how the structure will actually behave in orbit.

A consequence of these considerations is that it is very important that any control system used to suppress vibrations on an FSS is not too adversely affected by differences between the true and nominal dynamics. This in turn often leads to the use of sensors and actuators which are collocated on the structure, as the use of non-collocated sensors and actuators on a poorly-defined structure can easily lead to instability. To see this, suppose that the vibration rate measured by a sensor on a structure is fed back to generate the control input at an actuator some distance away. Suppose further that, in the nominal structural model, the i^{th} mode shape is such that the sensor and actuator stations vibrate in phase. It is therefore clear that negative feedback will serve to increase the damping of this mode, giving better closed-loop response. However, it is particularly difficult to obtain accurate mode shapes for FSS, so it is entirely possible that the sensor and actuator points on the true structure actually vibrate out of phase, and negative feedback will destabilize mode i. It is to avoid this problem of unknown phase differences between sensors and actuators that collocated pairs are often used.

Of course, restricting ourselves to the special case of collocated sensors and actuators is bound to have an adverse affect on the attainable closed-loop performance. The purpose of this paper is to begin to look at describing this performance in some manner. The approach taken here is based on the results recently proved [21, 22]concerning the transmission zeros of flexible structures. It was shown there that, as long as only collocated pairs are used, these zeros have very detailed generic properties. It therefore seems likely that some generic performance information can be derived for those vibration suppression techniques in which the zeros play a central role. The approaches looked at in this paper are the optimal linear regulator together with its derivative, high-authority/low-authority control [2], and the pole/zero cancellation method of [22].

Properties of FSS Transmission Zeros

Consider an n-mode model for the structural dynamics of a non-gyroscopic, non-circulatory Flexible Space Structure (FSS) with m *compatible* sensor/actuator pairs. (By compatible we mean that the direction of the linear/angular motion measured by each sensor is the same as that of the force/torque applied by the actuator which is collocated with it.) This model can be written in modal form [3] as

$$\begin{aligned}\ddot{\boldsymbol{\eta}} &+ C\dot{\boldsymbol{\eta}} + diag(\omega_i^2)\boldsymbol{\eta} = \Phi^T V\mathbf{u},\\ \mathbf{y} &= W_r\Phi\dot{\boldsymbol{\eta}} + W_d\Phi\boldsymbol{\eta},\end{aligned} \tag{1}$$

where $\boldsymbol{\eta}$ is the vector of modal coordinates, $\mathbf{u}$ that of applied actuator inputs, and $\mathbf{y}$ that of sensor outputs. $\Phi = (\phi_{ij})$ is the $(m \times n)$ modal influence matrix (ϕ_{ij} is the value of mass-normalized mode j, corresponding to natural frequency ω_j, at sensor/actuator station i) $C = C^T \geq 0$ is the damping matrix in modal form, and V is an $(m \times m)$ non-singular matrix describing how actuator inputs translate to the physical forces applied to the structure. Typically, V is diagonal if the forces at each station are independent, while it has a column of the form $(\mathbf{0}, a, \mathbf{0}, -a, \mathbf{0})^{\mathbf{T}}$ if equal and opposite reaction forces are applied between two stations; W_r and W_d are defined similarly for rate and displacement sensors, respectively.

Taking Laplace transforms, we obtain the polynomial matrix respresentation [8,13,23]

$$\begin{aligned}P(s)\boldsymbol{\eta}(s) &= \Phi^T V\mathbf{u}(s),\\ \mathbf{y}(s) &= W(s)\Phi\boldsymbol{\eta}(s)\end{aligned} \tag{2}$$

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for the given FSS, where $P(s) = s^2\mathbf{I} + sC + diag(\omega_i^2)$ and $W(s) = sW_r + W_d$. Note that $P(s)$ is symmetric, i.e., Eq. (2) respects the special structure of the FSS equations of motion. This is in contrast to the state space representation $\{A, B, C\}$ obtained by setting $\mathbf{x} = (\dot{\boldsymbol{\eta}}^T, \boldsymbol{\eta}^T)^T$, where A no longer preserves this valuable symmetric structure.

As long as its actuators and sensors have been positioned in such a way as to make it completely controllable and observable, i.e., so that each mode can be both excited and sensed, the transmission zeros of this (invertible) system are those s_i which reduce the rank of the system matrix

$$S(s) = \begin{pmatrix} P(s) & \Phi^T V \\ -W(s)\Phi & 0 \end{pmatrix}. \qquad (3)$$

Associated with each transmission zero s_i is a *zero mode shape* $\boldsymbol{\eta}_i$ which satisfies

$$S(s_i)\begin{pmatrix} \boldsymbol{\eta}_i \\ -\mathbf{u} \end{pmatrix} = 0. \qquad (4)$$

$\boldsymbol{\eta}_i$ can be regarded as the solution of a *constrained modes problem* [8], with the constraint being that the mode have zero deflection/slope at each linear/angular sensor location.

It can be shown from Eq. (3) that $\det(W(s)) = 0$ specifies q_s finite *sensor zeros* ($0 \le q_s \le m$); there are $2m - q_s$ zeros at infinity, and the remaining $2(n - m)$ *structural zeros* are defined by the physical properties of the structure and the positions chosen for sensor/actuator pairs. The structural zeros always lie in the left-half plane (LHP): furthermore, if, as is often the case, the structure is *modally damped* [3] with damping ratios $\{\zeta_i\}$, i.e., $C = diag(2\zeta_i\omega_i)$, then [17] the poles $-\zeta_i\omega_i \pm j\omega_i\sqrt{[1 - \zeta_i^2]}$ of the system define a portion of the LHP in which all these zeros must lie, regardless of the specific locations chosen for sensor/actuator pairs. This generic result, consisting of upper and/or lower bounds on the real and imaginary parts, moduli, and damping ratios of all zeros, is a consequence of the special form of the equations of motion of structural dynamics: it can be regarded as a generalization of the classical observation [10] that the zeros of a single input/single output undamped structure alternate with its poles along the imaginary axis. It admits a very simple graphical interpretation, as shown in Fig. 1 for an arbitrary distribution of poles 'x'.

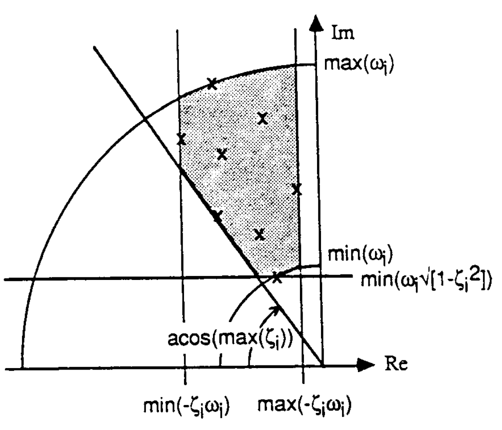

Fig. 1 Zero Region for Modally Damped Structure

Any choice for sensor/actuator positions thus leads to transmission zeros somewhere in the shaded region of Fig. 1. A related question now is: given some specified points in this region can sensor/actuator locations be found which lead to zeros at these points? This is clearly not possible in general, as there are $(n - m)$ zero pairs and m inputs, so if $n - m > m$ (i.e., $m < n/2$; typical of FSS) there is no chance of assigning all zeros arbitrarily. Even if we wish, say, to use a single sensor/actuator pair to only assign the *fundamental* (lowest-frequency) zero, i.e., the one of most practical importance, the following example shows that this is not always possible:
Consider a uniform undamped cantilever beam of length L, cross-sectional area A, second moment of area I, Young's modulus E and density ρ, with a single angular (angle/torque) sensor/actuator pair mounted at distance $l = aL$ from its built-in end. It is well-known [3] that the poles of this structure are $\{j\omega_i\}$, where $\omega_i = \sqrt{[EI/\rho A]}\mu_i^2/L^2$ and the $\{\mu_i\}$ are dimensionless parameters given from the cantilever characteristic equation $\cos\mu_i \cosh\mu_i + 1 = 0$; the first two solutions of this are 1.875 and 4.694. It can be shown that the zeros jz_i are similarly defined by dimensionless parameters $\{\nu_i\}$; unlike the $\{\mu_i\}$, these of course depend on the chosen sensor/actuator position. Fig. 2 plots the first five $\{\nu_i\}$ as functions of a : it can be seen that the zeros interlace the poles as expected, and that it is not possible to find any sensor/actuator position which gives ν_1 above about 3.0. Thus, the entire shaded region of Fig. 1 is not in general attainable.

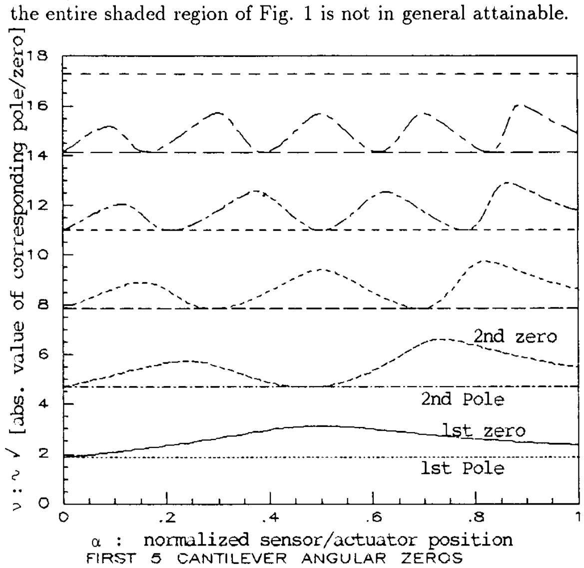

Figure 2.

A final point concerning the properties of FSS zeros is their sensitivity to perturbations in the system parameters. As the pole/zero interlacing property of undamped SISO structures must hold even if the structure is perturbed, it seems likely that the zeros of such systems have about the same sensitivity properties as do their poles. Indeed, it has recently been proved [21], first in terms of partial derivatives and then condition numbers, that this conclusion is actually true for general MIMO damped structures.

Pole/Zero Cancellation in FSS

The results of the last section are central to an analysis of a vibration suppression technique recently derived for FSS. This *pole/zero cancellation* approach is based on using state feedback to make as many closed-loop modes as possible unobservable, so that they do not appear in the sensed outputs [1,6,20]. A practical reason for wishing to do this arises [11] if the open-loop system has a slowly decaying mode which prevents fast output regulation: it is likely to require less control effort to make this undesirable mode unobservable than it would to speed up its response substantially. In the frequency domain, each such mode corresponds to a closed-loop pole and associated mode shape which are made equal to some transmission zero and its associated zero mode shape. Note that the zero bounds illustrated by Fig. 1 guarantee that no structural zero will lie far from the open-loop poles, so the feedback gain required to shift poles to all the zero locations is never likely to be prohibitively large.

A canonical form that has proved to be very useful for the study of pole/zero cancellation in FSS with compatible sensors and actuators is based on the QR decomposition [5] of the full column rank Φ^T, i.e., $\Phi^T = QR$ with Q orthogonal and $R = (R_1^T, 0)^T$ upper triangular and of full rank. Substituting this factorization into Eq. (3) gives

$$S(s) = Q_s \begin{pmatrix} Q^T P(s) Q & RV \\ -W(s)R^T & 0 \end{pmatrix} Q_s^T \tag{5}$$

where $Q_s = diag(Q, I)$ is orthogonal. Thus, partitioning Q as (Q_1, Q_2), $Q_1 (n \times m)$, we have

$$S(s) = Q_s \left(\begin{array}{cc|c} Q_1^T P(s) Q_1 & Q_1^T P(s) Q_2 & R_1 V \\ Q_2^T P(s) Q_1 & Q_2^T P(s) Q_2 & 0 \\ \hline -W(s) R_1^T & 0 & 0 \end{array} \right) Q_s^T, \tag{6}$$

so the structural zeros are clearly simply those s_i which make singular the $(n-m) \times (n-m)$

$$Q_2^T P(s) Q_2 = s^2 I + s C_2 + K_2, \tag{7}$$

where $C_2 = Q_2^T C Q_2$ and $K_2 = Q_2^T diag(\omega_i^2) Q_2$. Note that Q_2 depends in an explicit way on the positions selected for sensor/actuator pairs: its columns form an orthonormal basis for the orthogonal complement [5] of Φ^T, or equivalently $Ker(\Phi)$. This formulation was used in [18] as the basis for an algorithm to compute the transmission zeros of an FSS which is at least 60 times as fast as the general-purpose zeros method of [4] when applied to an undamped structure, and 15 times as fast for a lightly-damped one.

In the transformed coordinates of Eq. (6), linear state feedback $\mathbf{u} = F_r \dot{\boldsymbol{\eta}} + F_d \boldsymbol{\eta} + G\mathbf{v}$ gives rise to a polynomial matrix representation with denominator matrix

$$P_F(s) = \begin{pmatrix} [Q_1^T P(s) Q_1 - R_1 V F_1(s)] & [Q_1^T P(s) Q_2 - R_1 V F_2(s)] \\ Q_2^T P(s) Q_1 & Q_2^T P(s) Q_2 \end{pmatrix}, \tag{8}$$

where $F_1(s) = F(s)Q_1$, $F_2(s) = F(s)Q_2$ and $F(s) = sF_r + F_d$. Now, it can be seen that choosing $F_2(s)$ such that $Q_1^T P(s) Q_2 - R_1 V F_2(s) = 0$ is sufficient for full pole/zero cancellation; it can also be proved [22] to be necessary. $F_2(s)$ satisfying this condition always exists and is unique, as $R_1 V$ is non-singular. $F_1(s)$ by contrast, is arbitrary: it is equivalent to dynamic output feedback, and can be used to freely place those poles which remain observable in the closed-loop system. Two properties of these residual poles are of interest:

(i) if $F_1(s)$ is chosen to be zero, so minimizing the norm of the feedback gain matrix with the intention of making the control effort small (but not minimized: see [22]), the residual poles are simply the eigenvalues of $Q_1^T P(s) Q_1$, a section [15] of $P(s)$. As this is of precisely the same form as the matrix $Q_2^T P(s) Q_2$ which defined the zeros, the residual poles here have all the properties of transmission zeros: they therefore must also lie in the shaded region of Fig. 1. A physical interpretation of this is that all closed-loop modes obtained by pole/zero cancellation, even for the relatively low-performance case of $F_1(s) = 0$, must have decay rates and frequencies no lower than the lowest open-loop values.

(ii) The sensitivity of each closed-loop pole produced by pole/zero cancellation to perturbations in the open-loop system parameters can be shown [21] to be bounded from below by that of the corresponding zero. Approximate equality is obtained if $F_1(s)$ is chosen so as to shift all residual poles to locations far from all zeros (as would be desired in any case for fast output response). But the zeros themselves have sensitivities approximately equal to those of the open-loop poles, so the sensitivities of the open-and closed-loop (for appropriate $F_1(s)$) poles are closely related.

It is interesting to note from points (i) and (ii) that pole/zero cancellation using collocated sensors and actuators can lead to somewhat poor performance in the minimum-norm ($F_1(s) = 0$) case. For, (i) shows that the residual poles cannot be appreciably faster than the open-loop poles. Furthermore, (ii) implies that the nominally unobservable closed-loop poles may be quite sensitive to perturbations in the FSS dynamics: the resulting near pole-zero cancellations will then lead to closed-loop modes which are actually observable, albeit only slightly. Thus, it is indeed possible to say something meaningful about the performance of minimum norm pole/zero cancellations directly from the properties of the zeros of flexible structures. It should be noted that it is also possible to conclude that a suitable choice for $F_1(s) \neq 0$ will serve to overcome both these limitations, as demonstrated in the following example:

Consider the uniform vertical steel plate used in [22] for pole/zero cancellation simulation studies. This plate, based on the DFVLR laboratory test article described in [14], has horizontal length 1.50m, vertical length 2.75m, thickness 2mm, and isotropic material properties $E = 2.0 \times 10^{11} N/m^2, \rho = 8.0 \times 10^3 kg/m^3$ and $\nu = 0.3$. For simplicity, it is assumed to be simply-supported along all four edges, leading [3] to a lowest natural frequency of 2.741 Hz and ten modes below 20 Hz. These modes make up the model studied here; a somewhat high damping ratio of 1% is chosen for each mode in order to demonstrate the effects of damping on zeros sensitivity more clearly.

If a single linear sensor/actuator pair is placed at a horizontal distance 0.6m and vertical distance 1.2m from the lower-left tip of the plate, i.e., offset slightly from the central node point, none transmission zeros result, the lowest being at 4.321 Hz. The single residual mode that results from minimum-norm feedback is at 11.607 Hz: it is clearly seen in the closed-loop response to a 0.1N impulse shown in Fig. 3. The performance limitations described above result in the closed-loop response only being superior to the open-loop after about 2 seconds. By contrast, if additional rate feedback is used to shift the real part of the residual pole to the quite large $-50s^{-1}$ the resulting closed-loop response is essentially at rest after about 0.12s. It is interesting to note (Fig. 4) that, although the norm of the gain matrix F is increased somewhat in the "fast residual" case, the actual control effort applied to the system is reduced considerably over the minimum-norm case. This results from the fact that the displacements used to construct the feedback control force are now damped out much faster than before.

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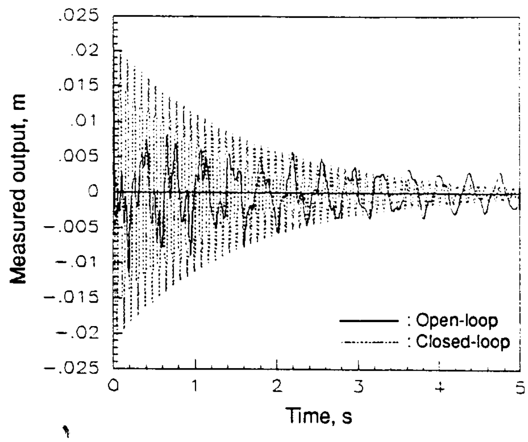

Figure 3.

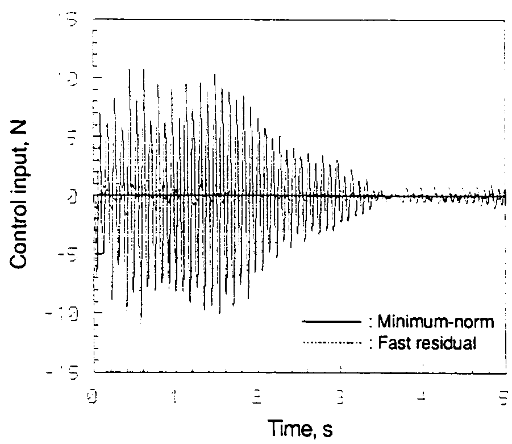

Figure 4.

Optimization-Based Approaches

Another type of vibration suppression technique in which the zeros play a clear and central role is the Linear optimal Quadratic Regulator (LQR). This approach is often taken in FSS applications, as many typical structural problems (e.g., minimizing the RMS surface deflection or line-of-sight error of a flexible antenna) are easily expressed in a quadratic optimization form. Furthermore, the optimal regulator generally forms the basis for another common structural control technique, that of high-authority/low-authority control (HAC/LAC) [2]. The idea here is that control of a very lightly-damped structure by a "high-authority" outer loop could be made in some sense easier if a "low-authority" inner loop were first used to increase structural damping appreciably. The LAC loop is generally thought of as being made up of passive dampers, dissipative layers on the structure, or a simple arrangement of independent local rate sensor/actuator pairs. The HAC loop, on the other hand, can consist of any desired control law, in most applications though, a linear optimal regulator HAC loop is used. (See e.g., [12] for a recent discussion.)

It is very well established that the zeros of a system are closely related to the behavior achievable if an LQR is used to control it. For, the zeros of the system (or the mirror images of any non-minimum phase zeros about the imaginary axis) are precisely the asymptotic values of those branches of the optimal root locus [9] which remain finite as the allowable control gain tends to infinity. The properties of the zeros of flexible structures shown in Fig. 1 therefore have clear implications for the high-gain behavior of FSS. This is not all that can be said about the optimal root loci of flexible structures though. In fact, it has recently been shown [19] that those closed-loop poles which tend to infinity in the high-gain case, i.e., the zeros at infinity, also have simple properties: these depend only on whether the system outputs are made up of rates or displacements.

The low-gain behavior of lightly-damped structures also can be characterized fairly easily and completely. In this case it can be shown [19] that the branches of the optimal root locus set off roughly horizontally (to the left) from each open-loop pole, at rates proportional to the contribution of the corresponding mode to the output to be regulated. It should be noted that all these properties, and so the conclusions that can be drawn concerning attainable closed-loop performance, are simpler and more complete than those that hold for general linear systems. There is, however, some correspondence between the results of [19] and the conclusions in [7] concerning performance limitations in single-input two-state linear systems under an optimal regulator. The latter paper pointed out that, if the weighting matrix Q in the quadratic objective function $J = \int_0^\infty [\mathbf{x}^T Q \mathbf{x} + \mathbf{u}^T R \mathbf{u}]dt$ is restricted to be positive definite, as is usually done, then there are "forbidden" regions of the left half-plane in which closed-loop poles cannot be placed for any choice of Q and R. For the simple $n = 2$ case of a 1-mode flexible structure, this "forbidden" region corresponds very well with the generic properties derived for flexible structures: for instance, both treatments show that no closed-loop poles will lie closer to the imaginary axis than does the open-loop pole. It appears likely that the greater simplicity of the optimal root loci generic results will make this the easier approach to apply to the study of such performance-limitation questions in multi-mode flexible systems.

As a final point, we return to the high-authority/low-authority control scheme, with the HAC loop taken to be an optimal regulator. The key question in the design of the LAC loop is just where these dampers should be placed so as to yield the best possible performance for the final composite closed-loop system. In the past, it has tended to be assumed that what is important is that the poles of the composite system (FSS + LAC) be as highly damped as possible. However, if the HAC optimal regulator is operating at fairly high gains, i.e., speeding up the closed-loop system significantly, it appears that it is actually the zeros of (FSS + LAC) that are important in determining the performance of the overall system, not its poles. The question of whether these zeros can be used to determine good damper locations in a simple way is still under investigation.

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