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PSEUDO-INVERSE METHODS FOR RECONFIGURABLE CONTROL WITH GUARANTEED STABILITY

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Abstract. In this paper, the reconfigurable control problem is first formalized mathematically and the common failures in flight control systems are classified; mathematical models for each failure are provided. One of the key approaches to the problem, the Pseudo-Inverse Method(PIM), is then analyzed and new insight is obtained for both the state feedback and output feedback cases. It provides, for the first time, justification for the successes and failures of this practical approach. The main shortcoming of this method, which is the lack of stability guarantees, is discussed and a new approach is proposed. In this new approach, recent results on stability robustness of linear systems are used to provide stability constraints for the solutions of the PIM. Finally, a novel scheme for failure accommodation of a stuck actuator is also proposed.

Keywords: Automatic reconfiguration; self-adjusting system; Stability; flight control; control system design.

INTRODUCTION

Reconfigurable/Restructurable Control Systems (RCS) are control systems that posses the ability to accommodate system failures automatically based upon a-priori assumed conditions. The research in this area is largely motivated by the control problems encountered in the aircraft control system design. In that case, the ideal goal is to achieve the so called "fault-tolerant", or, "self-repairing" capability in the flight control systems, so that the unanticipated failures in the system can be accommodated and the airplane can be, at least, landed safely whenever possible. This problem has drawn the attention of many researchers and many results have been published (Caglayan, 1988; Chandler, 1984; Gao, 1989a, 1989b; Gavito, 1987; Eslinger, 1988; Howell, 1983; Huber, 1984; Looze, 1985; Ostroff, 1983, 1984; Potts, 1981; Rattan, 1985; Raza, 1985).

The Pseudo-Inverse Method(PIM), has been accepted as a key approach to reconfigurable control and it has been used quite successfully in flight simulations as reported by (Caglayan, 1988a; Huber, 1984; Rattan, 1985; Raza, 1985). In this approach, the feedback gain is to be modified so that the reconfigured system approximates the nominal system in some sense. This method is attractive because of its simplicity in computation and implementation. The main drawback of the PIM is that the stability of the reconfigured system is not guaranteed. As a result, the PIM, if applied with no appropriate safeguard, can lead to instability. It is perhaps as worrisome the fact that the results in literature do not provide adequate insight as to why the PIM actually works when it does, or under what circumstances it will fail. These are very important questions that need to be answered because, in the control system redesign, the first thing to recover has to be stability; once stability is guaranteed, the performance, such as command following, can then be taken into consideration. So it is necessary in any approach to reconfigurable control, no matter what method is used, to guarantee the stability. Another drawback in the current literature on reconfigurable control is that the overall problem has not been clearly defined mathematically. Here the mathematical model of the impaired system under different failures will also be developed. The reconfigurable control theory can only be established after the above issues have been addressed.

For convenience, the pseudo-inverse of a constant matrix is defined here (Stewart, 1973; Golub,1983): Let $A \in R^{nxm}$ has the singular value decomposition of the form:

$$A = V \begin{pmatrix} \Sigma & 0 \\ 0 & 0 \end{pmatrix} U^H \tag{1.1}$$

where V and U are unitary matrices. For any unitary matrix W, we have $W^H W = I$ and $\|W\|_2 = 1$; W^H is the complex conjugate of W.

Definition (Stewart, 1973, pp325): The Pseudo-Inverse of A is defined as

$$A^+ = U \begin{pmatrix} \Sigma^{-1} & 0 \\ 0 & 0 \end{pmatrix} V^H \tag{1.2}$$

Note that $x = A^+b$ is the solution of smallest norm for the linear least squares problem of minimizing $\| Ax - b \|_2^2$.

PROBLEM DEFINITION

Let the open-loop nominal plant be given by

$$\begin{aligned} \dot{x} &= Ax + Bu \\ y &= Cx \end{aligned} \tag{2.1}$$

where $A \in R^{nxn}$, $B \in R^{nxm}$ and $C \in R^{pxn}$. If the nominal closed-loop system is designed by using the state feedback $u = Kx$, $K \in R^{mxn}$, then the closed-loop system is

$$\begin{aligned} \dot{x} &= (A+BK)x \\ y &= Cx \end{aligned} \tag{2.2a}$$

where K is the state feedback gain. Similarly, for the output feedback $u = Gy$, $G \in R^{mxp}$, the closed-loop system is

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$$\dot{x} = (A+BGC)x \qquad y = Cx \tag{2.2b}$$

Suppose that the model of the system, in which failures have occurred, is given by

$$\dot{x}_f = A_f x_f + B_f u_f + Dw \qquad y_f = C_f x_f \tag{2.3}$$

where $A_f \in R^{l\times l}$, $B_f \in R^{l\times r}$, $C_f \in R^{q\times l}$, and $D \in R^{l\times(m-r)}$; w contains the disturbances due to actuator failures. Let the new control law be

$$u_f = K_f \bar{x}_f \tag{2.4a}$$

or

$$u_f = G_f C_f \bar{x}_f \tag{2.4b}$$

where $K_f \in R^{r\times s}$ or $G_f \in R^{r\times q}$ is the new feedback gain to be determined, $\bar{x}_f$ is the new measured signal given by

$$\bar{x}_f = L x_f \tag{2.5}$$

$\bar{x}_f$ is different from x_f only when there is a sensor failure.

Reconfigurable Control: Problem Formulation

Redesign the feedback gain, K_f, automatically so that it will
i)stabilize the impaired system (2.3) if it is stabilizable, and
ii)recover the performance, such as command following and disturbance rejection, as much as possible.

There are a number of important characteristics which make the Reconfigurable Control Problem unique. The control redesign must be accomplished automatically and rapidly. The following remarks shed some light into the problem requirements.

Remarks:
1. The importance of part i) in the formulation is obvious. In the flight control systems of an aircraft, the stability takes the highest priority in failure accommodation. Once the stability is recovered, the immediate danger of disaster will be removed and this gives time for the reconfiguration system to recover the performance of the impaired system. The performance recovery in part ii) is almost as important as part i) because merely regaining the stability is usually not enough in failure accommodation. For example, to save an impaired aircraft one must regain the control of the system as well as the stability; in this way, safe landing of the aircraft will be possible. This problem is usually difficult to formulate since how much of the performance can be recovered is very much dependent on the severeness of the impairment.

2. Here we deal with potentially large variations in the parameters of the plant. These variations may happen very rapidly during operation corresponding to step like changes in the values. It appears that these problems can not be resolved just by using conventional fixed control or adaptive control techniques. First, the parameter space may be so large that no fixed control law will be able to cover the entire space. Second, the parameter changes may be too fast for any conventional adaptive control system to follow.

3. The control redesign procedure should be automated to some degree because in many cases of failure, such as the failures in an unmanned spacecraft, the failure accommodation has to be done with little or without human intervention. Furthermore, by eliminating the human factor, the speed of the process can be greatly improved.

4. Since the failures could be catastrophic and the time to respond is often very limited, the control redesign procedure should be efficient. A good example of the desired characteristics of such procedure was reported in (NTSB, 1979) where the pilot had only 15 seconds to save the airplane but he failed.

Failure Classifications

It is important, for the understanding of the Reconfigurable Control problem, to classify and model the types of failures. The classification below is in terms of state space description (2.3) and it is novel. The failures of the physical plant are classified into three categories:

Plant Structural Failures:
For this kind of failure, we assume that the dynamics of the plant are changed, but all the sensors and actuators are fully capable. In this case the dimensions of the system, that is the number of states and the number of inputs and outputs are not changed. Here the impaired system is

$$\dot{x}_f = A_f x_f + B_f u_f \qquad y_f = C_f x_f \tag{2.6}$$

This is derived from (2.3) with $l = n$, $r = m$, $q = p$, $s = l$, $\bar{x}_f = x_f$, and $w = 0$.

Actuator Failures:
This is the type of failure where at least one of the actuators is either stuck at a constant value, which is called an "actuator stuck" failure, or it is oscillating between two values, which is called an "actuator floating" failure. The actuator failure will change, in general, the dimensions of the system. For example, when the ith actuator is stuck an angle a, the impaired system takes the form of

$$\dot{x}_f = A_f x_f + B_f u_f + b_i a \qquad y_f = C_f x_f \tag{2.7}$$

where, in terms of (2.3), we have $l = n$, $r = m-1$, $q = p$, $s = l$, $\bar{x}_f = x_f$, $w = a$ and D is the ith column of B.

Sensor Failures:
Sensor failures result in at least one of the states not being available for feedback gain calculation in (2.4). For example, when one sensor fails the impaired system takes the form:

$$\dot{x}_f = A_f x_f + B_f u_f \qquad y_f = C_f \bar{x}_f \tag{2.8}$$

where $\bar{x}_f = L x_f$, $\bar{x}_f \in R^{l-1}$ and $s = l-1$

The Pseudo-Inverse Method

The PIM has become a key approach to reconfigurable control. Its main objective is to maintain as much similarity as possible to the original design and thus to provide graceful degradation in performance. This is achieved by reassigning the feedback gains. Here it is assumed that the impaired system is of the type (2.6). In the PIM (Ostroff, 1985), the objective is to find a new feedback gain so that the impaired system approximates, in some sense, the nominal one in (2.2a) or (2.2b). For state feedback, A+BK is equated to $A_f+B_fK_f$ and an approximate solution for K_f is given by

$$K_f = B_f^+(A - A_f + BK) \tag{2.9a}$$

where B_f^+ denotes the pseudo-inverse of B_f. Similarly, for output feedback,

$$G_f = B_f^+(A - A_f + BGC)C_f^+ \tag{2.9b}$$

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Note that K_f or G_f can be calculated from (2.9) for many anticipated failures and stored in the flight control computer. Once the failure has occurred and is identified, that is the model of the system with failure (2.6) is obtained, the feedback gain can then be modified. This is considered as a relatively fast solution to stabilize the impaired airplane(Ostroff, 1985). This PIM method has also been used for on-line accommodation for unanticipated failures (Caglayan, 1988a; Huber, 1984; Rattan, 1985; Raza, 1985) although it appeared in different forms. However, there is one problem which might render the method useless, namely, that the solution from (2.9) does not necessarily make the closed-loop system in (2.6) and (2.4) stable; see example 1 and example 3.

The Properties of the PIM

Although the PIM has been used widely in the literature of the RCS, the theoretical aspects of this methods have not been fully investigated. For example, it is not clear in what sense the closed-loop system in (2.6) and (2.4) approximates the one in (2.2) when the PIM is used. In the following analysis, the state feedback $u = K_f x_f$ is used for convenience. Note that the results applies to the output feedback system as well.

In the PIM, it is desirable to have

$$A + BK = A_f + B_f K_f \tag{2.10}$$

This equation may or may not have an exact solution depending on the row rank of the matrix B_f. If B_f has full row rank, then (2.9a) always satisfies (2.10), otherwise, there is no exact solution to (2.10) and K_f from (2.9a) is only an approximate solution. It is interesting to see what the PIM implies in terms of the eigenvalues, which are often used in the specifications of the performance criteria. First let's look at the following lemma:

Lemma1 Let

$$J = \| (A+BK) - (A_f+B_fK_f) \|_F \tag{2.11}$$

where $\|\cdot\|_F$ stands for the Frobenius norm. Then the K_f obtained from (2.9a) minimizes J.
Proof: see Gao(1989b).

Note that when output feedback is used G_f is to be obtained by (2.9b), which minimize the Frobenius norm: $J = \|(A+BGC) - (A_f+B_fG_fC_f)\|_F$ (Penrose, 1966). Lemma1 shows that the solution (2.9) makes the closed-loop system (2.6) and (2.4) approximate the nominal one (2.2) in the sense that the Frobenius norm of the difference of the closed-loop "A" matrices is minimized. The underlining idea in the PIM is that if the norm J is minimized, hopefully the behavior of the reconfigured system will be close to that of the nominal system. It goes without saying that we would like to know just how close it is going to be and in what sense exactly. The relation between variations of closed-loop eigenvalues and the Frobenius norm in (2.11) is given by the following theorem:

Theorem1: Let (A+BK) be non-defective, that is, it can be reduced to diagonal form by a similarity transformation, and let X be the eigenvector matrix of A+BK in (2.2) and $X^{-1}(A+BK)X = \mathrm{diag}(\lambda_1, \lambda_2, ..., \lambda_n)$, the eigenvalues of $A_f+B_fK_f$ lie in the union of the disks

$$D_i = \{ \lambda : |\lambda - \lambda_i| \le \|X\|_2 \|X^{-1}\|_2 J \} \tag{2.12}$$

where J is defined in (2.11) and $\|\cdot\|_2$ denotes the matrix 2-norms.
Proof: see Gao (1989b).

The significance of this theorem is that it provides a bound to the variation of the eigenvalues in terms of eigenvectors of the nominal system and the Frobenius norm J. If the nominal system is robust in the sense that all the eigenvalues are relatively far left of the $j\omega$ axis, then the PIM has a good chance to work provided J is small enough; J can be made small when the failure is not "too severe". This fact must be taken into consideration in the design of the nominal system. Theorem1 also shows the limitations of the PIM. It is clear that by just minimizing J, stability is not guaranteed; the minimum J can be large enough to allow the eigenvalues of the closed-loop system shift to the right half plane. That is, the use of the PIM in the automatic reconfiguring control system may result in an unstable closed-loop system unless it is restricted to only certain classes of failure where the value of J is small enough, in which case all the disks in (2.12) lie in the left half plane.

Example 1: Let the nominal plant in (2.1) be

$$A = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix} \quad B = \begin{bmatrix} 1 \\ 5 \end{bmatrix} \quad C = I$$

and the feedback gain be given by $K = [-1, 0]$ which assigns the closed-loop poles at (-1,-2). Let the model of the impaired plant be

$$A_f = A \quad B_f = \begin{bmatrix} -1 \\ 1 \end{bmatrix} \quad C_f = C$$

By using the PIM, (2.9a) gives

$$K_f = B_f^+(A - A_f + BK) = [-1/2, 1/2]\begin{bmatrix} -1 & 0 \\ -5 & 0 \end{bmatrix} = [-2, 0]$$

But the eigenvalues of $(A_f + B_fK_f)$ are {1, -1}. This shows that the PIM can led to an unstable system. In this case, J = 4.24, $D_i = \{ \lambda : |\lambda - \lambda_i| \le 42.9 \}$ for $\lambda_1 = -1$ and $\lambda_2 = -2$.

In the next section, a new method for reconfigurable control with guaranteed stability is proposed. It is based on the PIM and recent results on the stability robustness of systems with structured uncertainties.

THE MODIFIED PSEUDO-INVERSE METHOD

In the implementation of the RCS, the stability is probably the most important property of the system. Although the PIM has many good aspects, the lack of stability guarantees puts severe constraints on its application. In this section, an alternative approach is proposed. This method utilizes the recent research results on the stability robustness of linear systems with structured uncertainty (Barlett, 1988; Barmish, 1984; Yedavalli, 1988; Zhou, 1987; Siljak, 1989). The objective of this approach is to maintain the closed-loop stability while recovering the performance as much as possible. Equivalently, the objective is to keep the closed-loop system in (2.6) and (2.4) stable, while minimizing the norm in (2.11). Clearly this is a constraint minimization problem(CMP). In order to use standard, easy to implement, numerical algorithms on the CMP, the stability constraints will be expressed explicitly in terms of the individual elements of K_f as simple inequalities.

Let's assume that the pair (A_f, B_f) given in (2.6) is stabilizable. Without loss of generality, we assume that A_f is stable. Suppose that the state feedback is used and a stability bound δ can be found, by (Barlett, 1988; Barmish, 1984; Yedavalli, 1988; Zhou, 1987; Siljak, 1989), such that if

$$|K_f(i,j)| < \delta \quad \text{for } i = 1,2, ..., m \text{ and } j = 1,2, ..., n \tag{3.1}$$

the system in (2.6) and (2.4) will be stable; for convenience,

$$|K_f(i,j)| \le \delta' \quad \text{for } i = 1,2, ..., m \text{ and } j = 1,2, ..., n \tag{3.1a}$$

will be used instead of (3.1), where $\delta' = \delta - \varepsilon$ for some small ε. Now the reconfigurable control problem can be described as follows:

Problem Formulation:
Determine K_f to minimize J in (2.11) subject to (3.1a) (3.2)

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Since J^2 can be decomposed as a sum of the square 2-norm of column vectors(see (3.4)), problem (3.2) can be solved as a set of constrained least square problems(CLSP). Algorithms to solve constrained minimization problems can be found in (Lawson, 1974; Gill, 1984).

When the plant is a single-input system, or it can be decomposed into SIMO subsystems, then the following theorem gives an explicit solution to the reconfigurable control problem (3.2).

Assume that the PIM solution (2.9a) makes the closed-loop system in (2.3) and (2.4) unstable.

Theorem2: If $B_f \in R^{n \times 1}$, then the K_f below solves (3.2):

$$K_f(i,j) = \begin{cases} \tilde{K}_f(i,j) & \text{if } |\tilde{K}_f(i,j)| \le \delta' \\ \mathrm{sgn}(|\tilde{K}_f(i,j)|)\delta' & \text{otherwise} \end{cases} \qquad (3.3)$$

where $\tilde{K}_f$ is obtained from (2.9a).
Proof: see Gao(1989b)

For single input systems we have the following algorithm:

Algorithm 1: (A-1)
- Step1: Calculate K_f from (2.9a);
- Step2: Check the stability of (2.6) with feedback (2.4) for the K_f obtained in step1;
- Step3: If the closed-loop system is stable, stop; otherwise calculate K_f using (3.3).

We call this approach the Modified Pseudo-Inverse Method (MPIM).

Example 2:
For the system in example 1, E can be constructed as:

$$E = B_f K_f = k_{f1}\begin{bmatrix} -1 & 0 \\ 1 & 0 \end{bmatrix} + k_{f2}\begin{bmatrix} 0 & -1 \\ 0 & 1 \end{bmatrix}$$

The bound on K_f is found to be $\delta = 0.5$ using Yedavalli's(1988) result; this is for k_{f1} and k_{f2} nonzero in general. If we consider $k_{f2} = 0$, and $E = k_{f1}\begin{bmatrix} -1 & 0 \\ 1 & 0 \end{bmatrix}$ then $\delta = 0.828$. Let $\delta' = 0.8$, and use Theorem2; then $K_f = [-.8, 0]$ is a solution to the reconfigurable control problem (3.2).

Remarks:
1. The stability constraints obtained in (Barlett, 1988; Barmish, 1984; Yedavalli, 1988; Zhou, 1987; Siljak, 1989) are sufficient conditions only. They tend to be more conservative as the number of uncertain parameters gets larger. Thus for high dimensional systems, the stability bounds could be too restrictive to be useful. In such case, it is suggested that the E matrix be constructed as $E = \beta(B_f \tilde{K}_f)$, where $\tilde{K}_f$ is obtained in (2.9a), and the bound on β is to be found. In this way, there are bounds to be established only on one parameter β. These bounds appear to be less conservative than before. This is illustrated in example 3.

2. In implementing the MPIM in practice, the cases when the impaired systems are unstable must be considered. In these cases, the system has to be stabilized first by modifying the feedback gain. Many methods can be used here, but the key issue is that this process has to be computationally efficient. Only after the system is stabilized can the MPIM be applied to improve the performance of the system.

3. From 1. and 2. above, it can be seen, in general, that one way to adjust the reconfigured feedback gain is to use $k_{mpim} = k_1 + \bar{\beta}\Delta k$, where k_1 is the stabilizing feedback gain, $\bar{\beta}$ is the stability bound found for β in 1. above and Δk is obtained by the PIM: $\Delta k = B_f^+((A+BK) - (A_f + B_f k_1))$.

In a general MIMO system, a closed-form solution to problem (3.2) does not appear to exist. Note that (3.2) can be viewed as a minimization problem subject to simple bounds, and this is a well studied problem in optimization theory. The choice for the particular optimization algorithm used should be guided by the requirement for rapid feasible solutions. For a quick solution with a reasonable sacrifice in optimality, an algorithm is proposed in the following:

Let $\bar{A} = A+BK - A_f = [\,\bar{a}_1, \bar{a}_2, \cdots \bar{a}_n]$, $K_f = [\,k_{f1}, k_{f2}, \cdots k_{fn}]$, where $\bar{a}_i$ and k_{fi}, $i = 1, 2, \ldots, n$, are column vectors. Then J can be written as

$$J^2 = \|[\,\bar{a}_1 - B_f k_{f1},\ \bar{a}_2 - B_f k_{f2}, \cdots, \bar{a}_n - B_f k_{fn}]\|_F^2$$

$$= \|\,\bar{a}_1 - B_f k_{f1}\|_2^2 + \|\,\bar{a}_2 - B_f k_{f2}\|_2^2 + \cdots + \|\,\bar{a}_n - B_f k_{fn}\|_2^2 \qquad (3.4)$$

For the K_f obtained form (2.9), its ith column has the form: $k_{fi} = B_f^+ \bar{a}_i$, which is the least square solution of the equation, $\bar{a}_i = B_f k_{fi}$. Such k_{fi} minimizes the vector norm $\|\,\bar{a}_i - B_f k_{fi}\|_2^2$ and $\|k_{fi}\|_2^2$ for $i = 1, 2, \ldots n$. Let

$$\tilde{k}_{fi} = k_{fi} + \psi_i \qquad (3.5)$$

where ψ_i is a column vector with its elements chosen such that

i) all elements in $\tilde{k}_{fi}$ satisfy the stability constraints (3.1a)

ii) and $\|\psi_i\|_2$ is the smallest possible.

Algorithm2: (A-2)
- Step 1: Calculate K_f from equation (2.9a);
- Step 2: Check the closed-loop stability, If it is stable, stop. Otherwise
- Step 3: Set $\psi_i(j) = 0$ if $k_{fi}(j)$ satisfies (3.1a), otherwise set $\psi_i(j) = \delta' - \mathrm{sgn}(k_{fi}(j)) \cdot |k_{fi}(j)|$ for $i = 1, 2, \ldots, n$ and $j = 1, 2, \ldots, m$.
- Step4: Calculate the solution

$$\tilde{K}_f = K_f + \Psi \qquad (3.6)$$

where Ψ has ψ_i, $i = 1, 2, \ldots, n$, as its columns.

Note that the solution given in (3.6) satisfy the stability constraints (3.1a) by the nature of the algorithm. Although it is no longer the optimal solution, the distance from this solution to the optimal one is bounded by

$$\|\,\tilde{K}_f - K_f\|_F \le \|\Psi\|_F \qquad (3.7)$$

A bound on the increase of the performance index can also be found. Let $\tilde{J} = \|\,(A+BK) - (A_f + B_f \tilde{K}_f)\|_F$, the J is bounded by

$$\tilde{J} - J \le \|B_f\|_F \|\Psi\|_F \qquad (3.8)$$

This can be seen from

$$\begin{aligned} \tilde{J} - J &= \|\,(A+BK) - (A_f + B_f \tilde{K}_f)\|_F - \|\,(A+BK) - (A_f + B_f K_f)\|_F \\ &\le \|B_f \tilde{K}_f - B_f K_f\|_F \\ &\le \|B_f\|_F \|\Psi\|_F \end{aligned}$$

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Example 3: This is a design example by Friedland (1986). Let the nominal system (A, B, C) be given as:

$$A = \begin{bmatrix} -.0507 & -3.861 & 0. & -32.17 \\ -.0012 & -.5164 & 1.0 & 0. \\ -.0001 & 1.4168 & -.4932 & 0. \\ 0. & 0. & 1.0 & 0. \end{bmatrix}$$

$$B = \begin{bmatrix} 0. \\ -.0717 \\ -1.645 \\ 0. \end{bmatrix} \quad C = \begin{bmatrix} 0. \\ 0. \\ 1.0 \\ 0. \end{bmatrix}^T$$

A full state feedback is used with the gain: $k = [-.0043, -3.872, -.7186, -.0988]$, which assigns the closed-loop eigenvalues to the desired locations: $\{-.0095 \pm .0941i, -1.25 \pm 2.1655\}$. Suppose a failure occurred and the model of the impaired system is given as:

$$A_f = A, \quad B_f = \begin{bmatrix} 0. \\ -.0717 \\ -.1645 \\ 0. \end{bmatrix}, \quad C_f = C$$

The PIM method (equation(2.9a)) gives an unacceptable solution, $k_{pim} = [-.0367\ -33.17\ -6.154\ -.8456]$, where the closed-loop 'A' matrix $(A_f + B_f k_{pim})$ is unstable.

Using the MPIM method, we must first stabilize the impaired system since neither A_f nor $(A_f + B_f k)$ is stable. There are many ways to accomplish this. Here, it is done by solving the matrix algebraic Riccati equation:

$$0 = SA_f + A_f^T S - SB_f R^{-1} B_f^T S + Q$$

where R and Q are standard weighting matrices used in LQR. The stabilizing feedback gain has the form

$$k_1 = -R^{-1} B_f^T S$$

Note that this solution will guarantee the stability if the impaired system is stabilizable and detectable. To make k_1 reasonably small, we choose Q as identity and R = 10, and the resulting k_1 is $k_1 = [.2925\ -8.83\ -13.86\ -16.74]$. Although this gain stabilizes the system it does not provide a desirable performance, see Fig.1. To recover the performance, the PIM is used in the form:

$$\Delta k = B_f^+((A+BK) - (A_f + B_f k_1))$$

The idea is to adjust k by the amount of Δk so that $(A_f + B_f(k_1 + \Delta k))$ is close to the nominal closed-loop 'A' matrix (A+BK). Unfortunately the Δk obtained does not preserve stability, therefore, the stability bound should be found and enforced. As mentioned, the gain obtained by MPIM can be used to establish stability bound on each element of Δk. However, less conservative results were obtained using the form $k_{mpim} = k_1 + \bar{\beta}\Delta k$, where $\bar{\beta} = .6$ is the stability bound found by using the method in (Yedavalli, 1988). The performance is much improved by using this new gain, see Fig.1.

It is interesting to mention that in some cases the performance can be further improved by tuning both k_1 and Δk. That is

$$k_{mpim} = (1-\alpha)k_1 + \alpha\Delta k$$

where α is increased from zero to the largest number (smaller than one) before the system becomes unstable. This is true in this example. It turns out the gain obtained in this way (α = .48) gives the best transient response, see Fig. 1. In fact, this gain gives smaller changes in eigenvalues and eigenvectors than the gains obtained above. This approach worked very well for this example. Further investigation is necessary to establish the validity of the approach in general.

ACTUATOR STUCK ACCOMMODATION

For the approaches described above, it is required that the number of states, of input and of outputs are unchanged under the failure. This is usually the case for the plant structural failures as it was defined in (2.6). But for actuator and sensor failures in general, this assumption does <u>not</u> hold. For example, when an actuator is stuck at a position, it corresponds to one of the elements in the input vector u being set to a constant value. In this case, we can not simply redesign the feedback gain, K_f, and set $u_f = K_f x_f$, because this control law can not be physically implemented. It should be realized that there is hardly any method that can be used alone to deal with all possible failures; the failures have to be classified and treated accordingly. This fact, however, has neither been clarified nor adequately addressed in the literature of reconfigurable control. This restriction puts significant limitations on the applicability of existing results.

To address these problems, a general model for the impaired system was defined in (2.3). Also given in the same section were the specific models for the plant structural failures, the actuator failures and the sensor failures. As an illustration, consider the model of a single actuator failure:

$$\dot{x}_f = A_f x_f + B_f u_f + b_i a$$
$$y_f = C_f x_f \qquad (2.7)$$

The failure could be actuator "stuck" or "floating". From (2.7), the actuator failure can be viewed as input disturbance. For actuator stuck, assume the value a is known, then a disturbance cancellation technique (Skelton, 1988) can be used. Let

$$u_f = u_{fc} + u_{fw} \qquad (4.1)$$

where u_{fw} satisfies

$$B_f u_{fw} + b_i a = 0 \qquad (4.2)$$

and u_{fc} can be designed using the PIM approach. Note that the accommodation of such failures in (2.7) have not been fully investigated; the previous methods in reconfigurable control can not be used directly in this case. Further investigation of actuator failure accommodation is to be carried out for different types of actuator failures.

CONCLUSIONS

In this paper, the reconfigurable control problem was formulated and mathematic models for three different types of failures are given. One of the key approaches to the problem, the Pseudo-Inverse method(PIM) was analyzed and significant new insight has shed into the method. It was pointed out that the key problem with the PIM is the stability of the reconfigured system. To guarantee stability, the reconfigurable control problem was formulated as a constrained minimization problem and a modified Pseudo-Inverse Method was proposed which guarantees the stability of the reconfigured system. A closed form solution was derived for single-input systems. For general multi-input multi-output systems, the problem was formulated as a minimization problem subject to simple bounds. A simple, near optimum solution was found by sacrificing the optimality of the solution to the stability of the reconfigured system. This new method is illustrated in an airplane control example. Also proposed was a formulation for failure accommodation when an actuator was stuck; an approach to that problem was also introduced.

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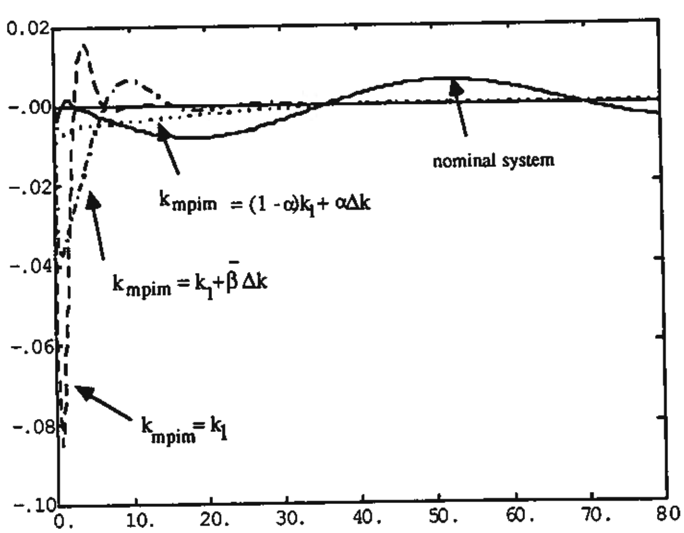

Fig. 1 The comparison of outputs of the reconfigured system with different gains