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OPTIMAL STABILIZATION OF DISCRETE EVENT SYSTEMS

Kevin M. Passino
Dept. of Electrical Engineering
The Ohio State University
2015 Neil Ave.
Columbus, OH 43210

Panos J. Antsaklis
Dept. of Electrical Engineering
University of Notre Dame
Notre Dame, IN 46556

Abstract

We utilize a class of not necessarily finite state "logical" discrete event system (DES) models which can also model the *costs* for events to occur. Let P and A denote two such models. Suppose that P characterizes the *valid* behavior of a dynamical system and A represents certain design objectives which specify the *allowable DES behavior* which is "contained in" the valid behavior. An optimal control problem for P and A is how to choose the sequence of inputs to P so that the DES behavior lies in A (i.e., it is allowable) and so that a performance index defined in terms of the costs of the events is minimized. Here we provide two solutions to an *optimal stabilization problem*, i.e. how to find a sequence of inputs that results in an optimal state trajectory which cycles in a pre-specified set. The results here are based on those in [11,13].

1. INTRODUCTION

Recently there have been investigations into the foundations of the optimal control of DES. Extensions to the theory of heuristic search in [11,13] showed that the A^* algorithm [4] can be adapted to provide a computationally efficient method to find solutions to several optimal control problems for DES. These results were applied to various manufacturing and planning system problems in [9-11]. In [12] the authors developed a heuristic search approach to find solutions to various *near-optimal* control problems for DES. The results in [11,13] have been extended and applied to flexible manufacturing systems and other examples in [14]. In this paper we show that the approach in [14] can be extended to solve optimal stabilization problems.

2. AN OPTIMAL STABILIZATION PROBLEM FOR DISCRETE EVENT SYSTEMS

We consider DES that can be accurately modelled with

$$P=(X,Q,\delta,\chi,x_0,X_f) \tag{1}$$

where (i) X is the possibly infinite set of plant states, (ii) Q is the possibly infinite set of plant inputs, (iii) $\delta:Q\times X\rightarrow X$ is the plant state transition function, (iv) $\chi:X\times X\rightarrow \mathbb{R}^+$ is the *event cost function*, (v) x_0 is the initial plant state, and (vi) $X_f\subset X$ is the non-empty finite set of *final states*. $\mathbb{R}^+$ denotes the set of positive reals and $\mathbb{R}_+=\mathbb{R}^+\cup\{0\}$. The set

$$E(P)=\{(x,x')\in X\times X:x'=\delta(q,x)\}\cup\{(x_d,x_0)\} \tag{2}$$

denotes the (possibly infinite) set of events for the DES P (x_d is a dummy state, and (x_d,x_0) a dummy event added for convenience). The event cost function $\chi(x,x')$ is defined for all $(x,x')\in E(P)$; it specifies the "cost" for each event (state transition) to occur and it is required that there exist a $\delta'>0$ such that $\chi(x,x')\geq\delta'$ for all $(x,x')\in E(P)$. Finally, we require that for each $x\in X$, $|\{\delta(q,x):q\in Q\}|$ is finite. The mathematical notation in this paper is as follows: Let Z be an arbitrary set. Z^* denotes the set of all finite strings over Z including the empty string $\emptyset$. For any $s,t\in Z^*$ such that $s=zz'\cdots z''$ and $t=yy'\cdots y''$, st denotes the concatenation of the strings s and t, and $t\in s$ is used to indicate that t is a substring of s, i.e., $s=zz'\cdots t\cdots z''$. A (finite directed) *cycle* is a string $s\in Z^*$ such that $s=zz'\cdots z''z$ has the same first and last element $z\in Z$. A string $s\in Z^*$ is *cyclic* if it contains a cycle (for $t_{zz}\in Z^*$, $t_{zz}\in s$), and *acyclic* if it does not.

A string $s\in X^*$ is called a *state trajectory* or *state path* of P if for all successive states $xx'\in s$, $x'=\delta(q,x)$ for some $q\in Q$. Let $E_s(P)\subset E(P)$ denote the set of all events needed to define a particular state path $s\in X^*$ that can be generated by P. For some state path $s=xx'x''x'''\cdots$, $E_s(P)$ is found by simply forming the pairs (x,x'), (x',x''), (x'',x'''),$\cdots$. An *input sequence* $u\in Q^*$ that produces a state trajectory $s\in X^*$ is constructed by concatenating $q\in Q$ such that $x'=\delta(q,x)$ for all $xx'\in s$. A set $X_z\subset X$ is said to be *invariant* if for all $x\in X_z$, $\delta(q,x)\in X_z$ for all $q\in Q$. Let $X_z\subset X$ then

$$\mathcal{X}(P,x,X_z)\subset X^* \tag{3}$$

denotes the set of all state trajectories $s=xx'\cdots x''$ of P beginning with $x\in X$ and ending with $x''\in X_z$. Let $X_z\subset X$ then

$$\mathcal{X}_s(P,x,X_z)\subset X^* \tag{4}$$

will be used to denote the set of all state trajectories $s=s's''$ of P with $s'\in\mathcal{X}(P,x,X_z)$ and s'' a (finite) cycle where for each $x\in s''$, $x\in X_z$. Then, for instance, $\mathcal{X}_s(P,x_0,X_f)$ denotes the set of all state trajectories of P that begin with the initial state x_0 and end in a cycle whose elements are all contained in X_f. A plant P is said to be *(x,X_z)-stabilizable* if there exists a sequence of inputs $u\in Q^*$ that produces an state trajectory $s\in\mathcal{X}_s(P,x,X_z)$.

There are other ways to define (x,X_z)-stabilizability. For instance we could have said that P is (x,X_z)-stabilizable if there exists a sequence of inputs $u\in Q^*$ that produces an state trajectory $s=s's_c$ where s_c is a cycle that contains at least one element of X_z. For other studies of stability and stabilizability of logical DES see, for instance, [1,7,8,15].

Let $P=(X,Q,\delta,\chi,x_0,X_f)$ specify the valid behavior of the plant and

$$A=(X_a,Q_a,\delta_a,\chi_a,x_{a0},X_{af}) \tag{5}$$

be another DES model which we think of as specifying the "allowable" behavior for the plant P. Allowable plant behavior must also be valid plant behavior. Formally, we say that the allowable plant behavior described by A *is contained in* P, denoted with A[P], if the following conditions on A are met: (i) $X_a\subset X$, $Q_a\subset Q$, (ii) $\delta_a:Q_a\times X_a\rightarrow X_a$, $\delta_a(q,x)=\delta(q,x)$ if $\delta(q,x)\in X_a$ and $\delta_a(q,x)$ is undefined otherwise, (iii) $\chi_a:X_a\times X_a\rightarrow\mathbb{R}^+$ is a restriction of $\chi:X\times X\rightarrow\mathbb{R}^+$, (iv) $x_{a0}=x_0$, $X_{af}\subset X_f$. Also, let $E(A)\subset E(P)$ denote the set of *allowable events* defined as in (2), χ_a is defined for all $(x,x')\in E(A)$, and $E_s(A)$ is defined as above. It may be that entering some state, using some input, or going through some sequence of events is undesirable. Such design objectives relating to what is "permissible" or "desirable" plant behavior are captured with A. This formulation is similar to that used for the "supervisor synthesis problems" in the language-theoretic Ramadge-Wonham framework [16].

To specify optimal control problems let the *performance index* be

$$J:X_a^*\rightarrow\mathbb{R}_+ \tag{6}$$

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where the cost of a state trajectory is defined by

$$J(s) = \sum_{(x,x')\in E_s(A)} \chi(x,x') \tag{7}$$

for all $x\in X_a$ and $s\in \mathcal{X}(A,x,X_a)$. By definition, $J(s)=0$ if $s=x$ where $x\in X_a$. Let A describe the allowable behavior for a plant P such that A[P] then we have:

The Optimal Stabilization Problem (OSP): Assume that X_{af} is an invariant set for A and that A is (x_0,X'_{af})-stabilizable where $X'_{af}\subset X_{af}$. Find $u\in Q_a^*$ that drives A along an *optimal* state trajectory s^*, i.e., $s^*\in \mathcal{X}_s(A,x_0,X'_{af})$ such that $J(s^*)=\inf\{J(s):s\in \mathcal{X}_s(A,x_0,X'_{af})\}$.

Here, we focus on the case where X'_{af} is composed of states that form a single cycle. It is not required that X'_{af} be an invariant set. Since we require that $u\in Q_a^*$ and $s\in X_a^*$, the solutions to the OSP will achieve not only optimal but also allowable DES behavior. There may, in general, be more than one optimal state trajectory, i.e., the solution to the OSP is not necessarily unique. The set of optimal state trajectories for A, beginning at state $x\in X_a$, and ending in a cycle in X_z, where $X_z\subset X_a$, is denoted by

$$\mathcal{X}^*_s(A,x,X_z)\subset \mathcal{X}(A,x,X_z). \tag{8}$$

In this paper we are concerned with finding only one optimal state trajectory for the OSP and finding it in a computationally efficient manner.

3. SOLUTIONS TO THE OPTIMAL STABILIZATION PROBLEM

First, it is important to examine the constraints on the OSP to determine what type of solution is sought. Consider the case where $X'_{af}=X_{af}$, $s^*\in \mathcal{X}_s(A,x_0,X_{af})$ is a solution to the OSP, and $s^*=s's''$ where s'' is a cycle in X_{af}. Ultimately, the state of P will cycle on s'' at a cost of J(s'') per cycle; hence it is of primary importance to minimize J(s''). The minimization of J(s's'') or even J(s') is of secondary importance (this treatment of the definition of optimality is similar to that in [6] where the authors consider systems with an infinite time horizon). It is for these reasons that we choose to split the OSP into two problems: (i) finding an optimal cost cycle of all those in X_{af}, and (ii) finding a optimal cost path from x_0 to that cycle. We use the following approach:

(1) Pick some optimal cost cycle s_c^* in X_{af}.

(2) Let $X'_{af}=\{x:x\in s_c^*\}$.

(3) Assume that A is (x_0,X'_{af}) stabilizable and use the adapted A^* algorithm in [14] to find an optimal path from x_0 to X'_{af} and name it s_a^*.

(4) Combine s_c^* and s_a^* to obtain a solution to the OSP.

Note also that some desired cycle may be known for some DES applications then Steps (1) and (2) can be avoided. In this paper we briefly describe two solutions to the OSP which both rely on the results in [14].

kth Shortest Paths Methods: First, notice that several kth shortest paths methods [3] can be adapted to find the cycle s_c^* in X_{af}. For instance, Dantzig's algorithm will not escape X_{af} since it is invariant and hence will result in a shortest cycle s_c^* in X_{af} (one that minimizes the sum of the costs along the arcs in the cycle traversing it just once). Dantzig's algorithm has complexity $O(|X_{af}|^3)$ where X_{af} is normally relatively small. From the results in [14] Step (3) will have quadratic complexity in the worst case so via kth shortest path methods the overall complexity is polynomial. It is for this reason that using kth shortest path methods in conjunction with the A^* approach developed in [14] provides a practical solution procedure for the OSP.

Minimum Ratio Methods: Next we show how a minimum ratio method [3] can be used in conjunction with A^* to find a solution to an OSP. Assume that for each $x\in X_{af}$ there exists $s_{xx'}\in \mathcal{X}(A,x,\{x'\})$ for all $x'\in X_{af}$. Let $m=|\{(x,x'):x'=\delta(q,x) \text{ and } x\in X_{af}\}|$. Use Karp's algorithm (complexity $O(m|X_{af}|)$) [3,5] to find the *minimum average cost cycle* in X_{af} and name it s_c^*. (Karp's algorithm is also used in [2] to compute eigenvalues and critical circuits.) Let $X'_{af}=\{x:x\in s_c^*\}$ and use A^* to find an optimal state trajectory to X'_{af}. The result is a solution to an OSP with polynomial complexity.

Note that the meaning of "optimal" with our two solution approaches is different (i.e., X'_{af} may be different depending on which approach is used). For both cases there are many application areas. For instance, for the kth shortest path method the resulting s_c^* could represent a processing cycle in a manufacturing system that takes a minimum number of steps while s_a^* could represent the steps that must be taken to reach the processing cycle so that resource consumption is minimized. For the minimum ratio method the resulting s_c^* could represent the processing cycle that is completed in minimum average time and s_a^* could represent the steps taken to reach s_c^* in a minimum amount of time.

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