

## Adaptive Control System Optimization for Vibration Control of Flexible Structures

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### Summary

This paper summarizes recent results in the development of optimal design of direct model reference adaptive controllers. Two methods have been developed: one employs an analytic averaging technique for solving a constrained nonlinear optimization problem yielding a close-form solution; the other method uses a numerical optimization approach with high-level learning capability. These two approaches are outlined below. A mathematical model of a flexible structure experiment facility was employed for testing the two design approaches. Numerical results are discussed and comparative analysis is performed to show the merit of these methods.

#### An Averaging Approach to Optimal Adaptive Control of Large Space Structures

Averaging methods applied to analyze the transient response associated with the adaptive control of large space structures. Using a dominant mode approximation to the plant, an analytical bound is found for the envelope of the adaptive response, which characterizes many of the features of the response useful for control design (e.g., peak values, and settling times). An optimal adaptive design methodology is then formulated based on these expressions. In particular, the product of the settling time and peak torque requirement is chosen to be minimized, since both are required to be small in practice. This leads to a constrained nonlinear optimization problem which, somewhat surprisingly, can be solved in closed-form to give the optimal adaptive design. Several interesting properties of the optimal adaptive solution are discussed including a threshold adaptation gain (TAG) principle. The TAG principle states that there exists a natural threshold beyond which the adaptation gain weight acts to increase the peak torque requirement without improving the settling time. Interestingly, the optimal adaptive design corresponds to setting the adaptive gain weight precisely to its threshold value. Explicit design formulas are given for these threshold values, and the TAG principle is verified by simulation. Further details of the method are presented below.

To date, adaptive systems have been designed almost exclusively using stability-based criteria. The reason for this is essentially due to the availability of analytical tools, (e.g., Lyapunov stability theory, Hyperstability, etc.) for studying the asymptotic portion of the adaptive response. The main results from such investigations are typically adaptive designs which guarantee stability of the closed-loop system in the presence of some prescribed degree of parameter uncertainty. However, such designs make no guarantee as to the transient performance, even when additional plant knowledge is available to the designer. This leads one to wonder whether such a-priori knowledge can be systematically incorporated into the adaptive design process to assure designs which not only are stable, but which provide optimal performance with respect to some

specified criteria.

In contrast to stability based approaches to adaptive design, optimality based approaches must focus on analyzing the transient portion of the adaptive response. This task is inherently more difficult since it involves characterizing and optimizing the transient solution of a highly nonlinear differential equation. Using the adaptive algorithm for control of large space structures, several results have been obtained along these lines. In [6], averaging methods were developed to characterize the envelope equation which is generally much simpler to work with than the original system dynamics; also its solutions are generally much smoother. Furthermore, the envelope characterizes many of the "features" of the response useful for optimal control design (e.g., peak values, quadratic costs, settling times, etc.). Using the envelope equation, closed-form expressions were developed for the peak torque requirement and settling time in [7].

Using averaging methods, an optimal adaptive design methodology was established based on the solution to a particular constrained nonlinear optimization problem. The optimization criteria is to minimize the product of the settling time and peak torque requirement. The constraints on the optimization problem arise naturally due to stability considerations and to ensure accuracy of the averaged expressions. Using an approximation to the error function and a judicious change of variables, the global optimal solution is found in closed-form. This gives rise to a systematic optimal adaptive design methodology and several insights into properties of the optimal design. (See also [8]). Note that this method has also been extended to the multivariable case [9].

Averaging methods have been around for a long time. These methods were developed in Russia by Kryloff and Bogoliuboff [14] for application to nonlinear oscillator equations and the theory was subsequently extended and enhanced [15-18]. The application of averaging methods to adaptive control is not new. In fact, there is presently an entire book dedicated to the subject [19]. To date, however, averaging methods have been used only to characterize the asymptotic properties of the adaptive response with respect to stability properties. The application of averaging methods to analyze and optimize the transient portion of the response appears to be novel. Furthermore, the approach taken here is somewhat different than the time-scale decomposition methods of [19], and is more in line with earlier treatments of averaging arising in the theory of nonlinear oscillations [14-16].

#### Parameter Learning for Performance Adaptation

The goal of the machine learning method discussed here, as applied to the flexible space antenna, is to broaden the region of operability of the adaptive control system by allowing the controller parameters to better adapt to different plant and

environmental conditions. These operating conditions may cause the nominal adaptive system to exceed the tolerances of its design. The parameter learning system determines parameter values for optimal performance for given operating conditions and then stores them in memory. In this way, the controller is able to operate effectively over a wider region. It should be noted that, at the local level, the parameter learning system performs parameter auto-tuning; the overall system, however, does more than this as it learns and it uses the results of auto-tuning to expand the region of operability of the control system.

It is very difficult to control the transient response and the performance of an adaptive control system. This is so because of the mathematical complexity of the nonlinear relationship between the design parameters in the adaptive controller and the output of the compensated system, even for quite simple plants and only recently progress has been made [6,7,8,9]. The parameter learning approach introduced here is general and it offers a viable alternative to analytical approaches and can be used when either no such methods exist or existing methods are too cumbersome. Details of the method are given below.

This learning approach is applicable to any control system where performance depends on a number of adjustable parameters. The mathematical relation between the performance and the parameters does not need to be known. Given particular values for the parameters, a performance index is evaluated via computer simulation or physical experiment; if the mathematical relation is known, the performance evaluation can be done directly. The learning system determines the next set of parameters in a process leading to an optimum performance. In effect, the learning system guides the selection of parameters for optimization; this procedure seems to be a form of learning by observation and discovery rather than learning by example since the experiments are generated by the system itself. After the best parameter values have been discovered, they are stored in memory along with the corresponding operating conditions. This provides the memory which is a necessary element for learning. When the same operating conditions recur, (here it is assumed that the information about the type of operating conditions is provided to the learning system) the system selects the appropriate controller parameter values it has learned. Interpolation is used to select values when similar, but not the same, conditions occur. A more detailed description of the method is now given.

The role of the parameter learning system described here is first to determine the best parameter values given certain system operating conditions, and then to store these values in memory for future use. This approach incorporating learning has been discussed in [1,3,10,11,12,13]. This parameter learning method is applicable to any system where performance depends on a number of adjustable parameters; furthermore, a mathematical model is not necessary as the learning system can be used whenever the performance can be measured via simulation or experiment. A particular application of this learning system to the control of a space antenna is described in [13] and will be discussed in the presentation.

The functional diagram of the parameter learning system is given in Figure 1. First, initial parameter values are assigned; these values can be assigned randomly or using information about the system's behavior, which can be accomplished by utilizing data stored in memory or by some other method such as a systematic procedure over a grid to determine an approximate map of the performance surface; this grid search is not necessary but it helps determine global instead of local minimum. This current set of parameter values,  $X_k$ , is sent to the system and the performance of the system is evaluated by computer simulation or physical experiment. Here the performance is measured via a performance index  $J$  and it is assumed that the parameters  $X_k$  and the performance index  $J_k$  are related by

$$J_k = f(X_k) \quad (1)$$

where the function  $f(\cdot)$  is typically unknown. The performance of the system is actually evaluated using measurable quantities  $Y_k$  via

$$J_k = g(Y_k) \quad (2)$$

where  $g(\cdot)$  is a known function. This is accomplished as follows: as  $X_k$  vary, the measurable quantities  $Y_k$  reflect the changes in system performance and  $J_k$  is then evaluated via (2). To illustrate, in the antenna parameter learning system,  $Y_k$  are measurable quantities such as settling time and maximum output error, while  $g(\cdot)$  of (2) is chosen to be a weighted sum of these quantities. It should be stressed that in a particular problem, given the adjustable parameters  $X_k$ , typically there are many appropriate choices for  $Y_k$  and  $J_k$ ; it is up to the designer to select  $Y_k$  so that they are good measures of the changes in performance and at the same time easy to determine. In the parameter learning system of Figure 1 the performance of the system is then judged to be adequate or inadequate. If inadequate, a new set of parameter values  $X_{k+1}$  is generated to improve the performance. Since the function  $f(\cdot)$  in (1) is not known, an optimization method that does not require a mathematical model is used to generate  $X_{k+1}$ . Here a modified version of the Hooke-Jeeves algorithm was used. If of course the function  $f(\cdot)$  is known, other optimization algorithms may be used and (2) may not be necessary. This process continues until the performance is judged adequate. At that time, the best parameter values found are stored in memory, which here is taken to be a dictionary containing, in each entry, the given system operating conditions and the corresponding best parameter values.

The main objectives in developing this approach and applying it to the space antenna control problem have been the effective use of all available information and its speed of response. This task appears plausible because the interest is in developing a learning method for a rather specific class of problems where the available information is well defined.

Whether the parameter learning system is invoked will depend upon the time restrictions placed on determining a new parameter set and whether learning is necessary. After the operating conditions have been presented to the learning system, the dictionary containing information about such conditions is consulted to determine if they are known by the system. If these conditions are known, the parameters can be set appropriately, and learning is not required. If the conditions are unknown, a decision to enable learning would be made. If time allows, learning could be enabled. Otherwise, the parameter values could be estimated from known conditions in the dictionary using for example an interpolation method or they may be left unchanged. The decision making mechanism for determining whether the learning system will be invoked is beyond the scope of this work and thus has not been implemented.

The learning method presented here provides performance adaptation for adaptive systems and it appears to be a novel approach to this problem. The method also deals with the question of boundedness of adaptive control systems. While analytical tools, based on stability analysis, do exist to determine whether a system variable will be bounded, typically the analysis does not indicate how large this bound will be. It is possible to exceed the system tolerances and yet be analytically stable. The learning method can be used to determine this bound and use this information in the process of controlling the system.

This method is general and it can be used in any system where performance depends on a number of adjustable parameters. As a matter of fact, the method was also successfully applied for verification purposes to determine the optimum gain in an LQR problem [1,11]. However, specialized methods, when they exist, are obviously more efficient to solve specialized problems. General methods, like the one presented here, are recommended to be used in complicated problems

when traditional methods fail. The method presented is also modular, as both the functional evaluation and the optimization search procedures can be modified to match the particular problem at hand; alternate methods using, for example, the gradient can be used when possible. In addition, functional evaluation can be performed via computer simulation, physical experiment or mathematical calculation.

It is noted that the learning method presented here uses a priori information of what is known about the system. This is in contrast to many machine learning applications where learning is usually accomplished with little a priori information. In engineering applications it is recommended to pay particular attention to utilizing all available information, as the more the system knows, the faster it can learn.

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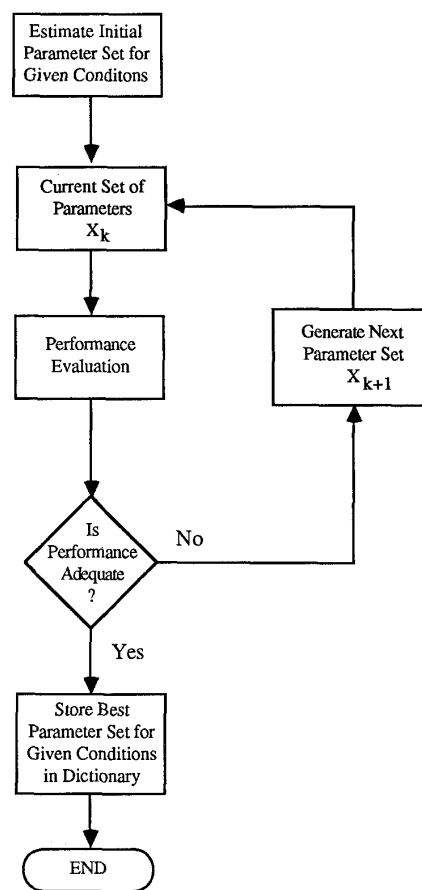


Figure 1: Functional Diagram of the Parameter Learning System