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RELATIONSHIPS BETWEEN EVENT RATES AND AGGREGATION IN HIERARCHICAL DISCRETE EVENT SYSTEMS

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ABSTRACT

A discrete event system (DES) is a dynamical system whose evolution in time develops as the result of the occurrence of physical events at possibly irregular time intervals. Although many DES's operation is asynchronous, others have dynamics which depend on a clock or some other complex timing schedule. Here we provide a formal representation of the advancement of time for logical DES via *interpretations of time*. These interpretations of time provide a common framework to characterize deadlock, simultaneous events, and the advancement of time in various DES in the literature. We show that the interpretations of time along with a *timing structure* provide a framework to study principles of the advancement of time for hierarchical DES (HDES). In particular, it is shown that for a wide class of HDES the *event rate* is higher for DES at the lower levels of the hierarchy than at the higher levels of the hierarchy. Relationships between event rate and *event aggregation* are shown. We define a measure for event aggregation and show that there exists an inverse relationship between the amount of event aggregation and the event rate at any two successive levels in a class of HDES. These results for HDES constitute the main results of this paper since they provide the first mathematical characterization of several fundamental timing characteristics of HDES. A manufacturing system example will be provided to illustrate the results.

1.0 INTRODUCTION

We focus on timing characteristics of single DES or HDES which have as components DES that can be accurately modelled with

$$P=(X,U,Y,\delta,\lambda,X_0) \tag{1}$$

where if $\mathbb{P}(X)$ denotes the power set of X,

(i) X is the set of plant states x,

(ii) U is the set of plant inputs u,

(iii) Y is the set of plant outputs y,

(iv) $\delta: U \times X \rightarrow \mathbb{P}(X)$ is the plant state transition function,

(v) $\lambda: U \times X \rightarrow Y$ is the plant output function, and

(vi) $X_0 \subset X$ is the set of possible initial plant states.

We introduce "interpretations of time" as a convenient characterization of the timing characteristics of discrete event systems (DES). In our main results, we show that the interpretations of time along with a *timing structure* provide a framework to study principles of the advancement of time for hierarchical DES (HDES). It is shown that for a wide class of HDES the *event rate* is higher for DES at the lower levels of the hierarchy than at the higher levels of the hierarchy. Relationships between event rate and *event aggregation* are shown. We define a measure for event aggregation and show that a high amount of event aggregation will result in a much lower event rate at higher levels in the HDES while a low amount of event aggregation will result in higher event rates. A manufacturing system example will be provided in the full paper to illustrate the results.

2.0 CHARACTERIZING THE ADVANCEMENT OF TIME IN DES

When a physical plant is modelled via (1), the meaning of the advancement of time must be defined. If Z is an arbitrary set, then Z^* denotes the set of all finite strings of elements from Z. If Z and Z' are arbitrary sets then $Z^{Z'}$ denotes the set of all functions mapping Z' to Z. In order to discuss timing issues for P, an *index set* J and *index sequences*

$$\alpha \in J^* \cup J^{\mathbb{N}}$$

are utilized similar to the approach in [Sain 1981]. The index set J is thought of as a set of times. Let $\mathbb{R}^+$ denote the set of strictly positive real numbers and $\mathbb{R}_+ = \mathbb{R}^+ \cup \{0\}$, the set of non-negative reals and let $\mathbb{N}$ denote the natural numbers. Note that $\mathbb{N}$, $\mathbb{R}_+$, or $\mathbb{R}$ could be candidates for the set J. The index sequences $\alpha \in J^* \cup J^{\mathbb{N}}$ are sequences of time instants that can be of finite or infinite length. If $\alpha \in J^* \cup J^{\mathbb{N}}$ let $|\alpha|$ denote the number of elements in the string α. Note that $\alpha: \mathbb{N} \rightarrow J$ or $\alpha: [0,a] \rightarrow J$ where $[0,a] \subset \mathbb{N}$, and $\alpha(k)$ simply denotes an element in J. An index sequence (function) $\alpha \in J^* \cup J^{\mathbb{N}}$ is said to be *admissible* if

(i) it is order preserving, i.e.,

(a) if $\alpha \in J^{\mathbb{N}}$, then for all $k_1, k_2 \in \mathbb{N}$, $k_1 \leq k_2$ implies that $\alpha(k_1) \leq \alpha(k_2)$,

(b) if $\alpha \in J^*$, then for all $k_1, k_2 \in \mathbb{N}$ with $k_1, k_2 \in [0, |\alpha|-1]$, $k_1 \leq k_2$ implies that $\alpha(k_1) \leq \alpha(k_2)$, and

(ii) it is injective.

Following [Sain 1981], the state of the plant $x \in X$ is associated with the index $\alpha(k)$ for some $\alpha \in J^* \cup J^{\mathbb{N}}$ and is denoted with $x(\alpha(k))$, meaning "the state at time $\alpha(k)$". Similarly, inputs $u \in U$ and outputs $y \in Y$ are associated with that same index and denoted with $u(\alpha(k))$ and $y(\alpha(k))$ respectively. The transition to a state in the set $\delta(u,x)$ can be thought of as leading to the next state, with "next" quantified with the index sequence α as $\alpha(k+1)$. With this, the transition function is given as $x(\alpha(k+1)) \in \delta(u(\alpha(k)), x(\alpha(k)))$ which is often abbreviated as $x_{k+1} \in \delta(u_k, x_k)$. Similarly, the output is often denoted with $y_k = \lambda(u_k, x_k)$ for $k \in \mathbb{N}$. Each run of P $(u_0,x_0,y_0),(u_1,x_1,y_1),\ldots$ has an associated index sequence $\alpha \in J^* \cup J^{\mathbb{N}}$, $\alpha = \alpha(0), \alpha(1), \ldots$ specifying the time instants at which the triples are defined.

A DES often *activates* or *triggers* other DES to act. For instance, in the case where P represents a plant, P may trigger a controller to generate an input to P. In this case, the trigger often represents certain changes that occur in the plant. Here, we consider the case where *events*, to be defined next, are used as the trigger. Similar to [Ramadge and Wonham 1987], we let $E \subset X \times X$ denote the set of *events* e, where

$$E = \{(x,x') \in X \times X : x' \in \delta(u,x)\} \tag{2}$$

An event (x,x') is said to *occur* if the state transition from x to $x' \in \delta(u,x)$ takes place. For convenience, we shall assume that the event occurs (is defined) at the time instant $\alpha(k+1)$ where the next state is defined.

The pair $I=(A,J)$ where J is an index set and $A \subset J^* \cup J^{\mathbb{N}}$ will be referred to as an *interpretation of time* since it specifies the meaning of the advances in time for the occurrence of state transitions, i.e. it specifies the time instants where the variables of the DES P are defined. In general, a system P is said to have a particular interpretation of time $I=(A,J)$ as long as the time instants associated with the elements of the runs of P are elements of J and the index sequences associated with the runs of P are elements of A. The *admissible interpretation of time* will be denoted with $I_{ad}=(A_{ad},J_{ad})$ where J_{ad} is an index set and

$$A_{ad}=\{\alpha\in J_{ad}^{*}\cup J_{ad}^{N}:\alpha \text{ is admissible}\}. \tag{3}$$

Most often we can choose $J_{ad}=\mathbb{R}_+$ and this is what we will assume here.

It is common to discuss the timing characteristics of DES relative to a *clock*. By a "clock" we mean a device which has a fixed interval $T\in\mathbb{R}^+$ between *ticks* and which does not stop ticking (if there is deadlock, the clock keeps ticking but no events occur).

Definition 1: The *asynchronous interpretation of time* is $I_a=(A_a,J_a)$ where $J_a=\mathbb{R}_+$ and
$A_a=\{\alpha\in A_{ad}:\alpha(0)=0\}$.

According to convention $J_a=J_{ad}=\mathbb{R}_+$ with the time instant of zero corresponding to the case where no state transitions have occurred.

Definition 2: The *partially asynchronous interpretation of time* is $I_{pa}=(A_{\gamma\beta},J_{pa})$ with $J_{pa}=\mathbb{R}_+$
and $A_{\gamma\beta}=\{\alpha\in A_a : \alpha(k)+\gamma\le\alpha(k+1)\le\alpha(k)+\beta\}$ for $\gamma,\beta\in\mathbb{R}^+$ where $\beta\ge\gamma$.

Definition 3: The *general synchronous interpretation of time* is $I_s=(A_T,J_s)$ with $J_s=\mathbb{R}_+$
and $A_T=\{\alpha\in A_a : \alpha(k+1)=\alpha(k)+nT \text{ where } n\in\mathbb{N}-\{0\}\}$ with $T\in\mathbb{R}^+$.

When n=1 we shall refer to I_s simply as the *synchronous interpretation of time*.

3.0 TIMING CHARACTERISTICS OF HIERARCHICAL DES

Whereas in Section 2 we represented the timing characteristics of a single DES P, here we shall consider the timing characteristics of many interconnected DES. Our study of the timing characteristics of these HDES was motivated by the work in [Gershwin 1989]. The formation of a control theory for HDES is just beginning [Zhong and Wonham 1988,1989,1990] even though such systems occur quite frequently. Some principles of the evolution of time in hierarchical systems have been postulated but not fully investigated in [Albus, Barbera, and Nagel 1981; Saridis 1983; Valavanis 1986; Mesarovic, Macko, and Takahara 1970; Antsaklis, Passino, and Wang 1989; Passino and Antsaklis 1988]. As with Gershwin what these researchers have recognized is that "systems usually operate at the higher rates at the lower levels in a hierarchical system". We shall verify this intuition for one class of HDES here.

3.1 A Hierarchical DES Model

We shall focus on HDES that have as components two types of DES, G_j, $1\le j\le m$, and P_i, $1\le i\le n$, all defined via (1) except with different timing characteristics. We introduce what we call a *timing structure* which will define how the various components of the hierarchical system influence (are influenced by) the timing characteristics of other components of the HDES. The definition of the timing structure is based on the interpretations of time defined in Section 2.0 and what will be called *input* and *output triggers*. Each P_i, $1\le i\le n$, in the HDES has timing characteristics that are simply specified via their own interpretation of time denoted with $I_{pi}=(A_{pi},J_{pi})$. Roughly speaking, each G_j, $1\le j\le m$, has timing characteristics that depend on P_i, $1\le i\le n$, and G_k for $k\ne j$ via the timing structure as we now discuss in more detail.

Let E_{pi} denote the set of events for P_i, and E_{gj}, the set of events for G_j both defined in a similar manner to the events E for P in (2). In this paper we focus on the case where the *output triggers* for P_i, $1\le i\le n$, and G_j, $1\le j\le m$, are simply defined by the events E_{pi} and E_{gj} respectively. With this choice, the *input triggers* for the G_j are defined by the τ_j maps (or restrictions of the τ_j) for j, $1\le j\le m$, where

$$\tau_j:E_{p1}\times E_{p2}\times \cdots \times E_{pn}\times E_{g1}\times E_{g2}\times \cdots \times E_{gk}\times \cdots \times E_{gm}\rightarrow\{0,1\} \tag{4}$$

where $k\ne j$ and $\tau_j(\cdot)=1$ (=0) indicates that an event $e_{gj}(\alpha(k+1))\in E_{gj}$ where $e_{gj}(\alpha(k+1))=(x_g(\alpha(k)),x_g(\alpha(k+1)))$ is forced (not) to occur in G_j. Since we require $k\ne j$, event occurrences in G_j cannot directly force other events in G_j to occur via the input trigger τ_j. In fact, we consider here only timing structures that are "tree structured". Let each DES component of the HDES represent a node of a directed graph and let the τ_j define the arcs that connect the P_i and G_j to other G_k, $k\ne j$, in the following manner. If there exists ℓ and $k\ne j$ such that $\tau_j:E_{p1}\times \cdots \times E_{g\ell}\times \cdots E_{pn}\times E_{g1}\times \cdots \times E_{gk}\times \cdots \times E_{gm}\rightarrow\{0,1\}$ then there exists an arc pointing from P_ℓ to G_j and one from G_k to G_j. For an HDES to have a *tree structured* timing structure it must be the case that in this directed graph there does not exist a closed cycle. In this way we eliminate the possibility that some G_j can directly force its own events to occur via the timing structure. Notice that the P_i, $1\le i\le n$, are the "leaves" of the tree structured timing structure.

Whereas the interpretation of time is always specified for the P_i, $1\le i\le n$, the interpretations of time for the G_j are specified in terms of the other G_k, $k\ne j$ and the P_i via the timing structure as we now define. Let $\alpha_{pi}(k+1)$ and $\alpha_{gk}(k+1)$ denote the time instants at which events $e_{pi}\in E_{pi}$ and $e_{gk}\in E_{gk}$ $(k\ne j)$ occur respectively. Suppose that at some time instant $\alpha(k+1)$, $\tau_j(\cdot)=1$ so that $e_{gj}(\alpha(k+1))\in E_{gj}$ occurs. This time instant at which $e_{gj}(\alpha(k+1))$ occurs is given by

$$\alpha(k+1)=\max\{\alpha_{pi}(k+1),\alpha_{gk}(k+1):1\le i\le n,1\le k\le m,k\ne j\} \tag{5}$$

and corresponds to the time instant at which the last event occurred which caused $\tau_j(\cdot)=1$. Each time an event occurs which forces $\tau_j(\cdot)=1$, an event occurs in G_j; hence the "1" represents a pulse sent to G_j via τ_j which forces an event to occur. Hence, if $\tau_j()$ is set equal to 1 at some time instant, an event in G_j must occur at that time instant (unless G_j is deadlocked); if every event in a sequence of events all cause $\tau_j(\cdot)=1$ then there is one event occurrence in G_j for each event in the sequence. The interpretation of time for any G_j is found by executing all possible runs (in all possible orders) of the P_i, $1\le i\le n$, and hence G_k, $1\le k\le m$, where $k \neq j$. Then via equations (4) and (5), the time instants and hence index sequences and interpretations of time for the G_j are specified. We shall study HDES where there is at least one P_i and the interpretations of time for the G_j can be uniquely defined in terms of the P_i.

Note that although we consider only tree structured timing structures we place no restrictions on the manner in which the DES inputs and outputs are connected. This allows our results to apply to a relatively large class of HDES with a wide variety of input/output connecting structures. Tree structured timing structures allow us to study properties of what has been called a "time scale hierarchy". In this hierarchy a DES component is "higher in the hierarchy" than another DES component if its timing characteristics can be influenced by the other DES (i.e., there exists a directed path from one to the other).

3.2 Timing Characteristics of HDES with Multi-Level Timing Structures

To analyze the timing characteristics of HDES we study two fundamental components of interconnected DES. Consider the HDES shown in Figure 3.2 which we will call a HDES with a "multi-level timing structure" (the other one that we study is shown in Figure 3.4). Let the admissible interpretation of time for P_1 be $I_{p1}=(A_{p1},J_{p1})$ with $J_{p1}=\mathbb{R}_+$ and for G_j, $1 \leq j \leq m$, be $I_{gj}=(A_{gj},J_{gj})$.

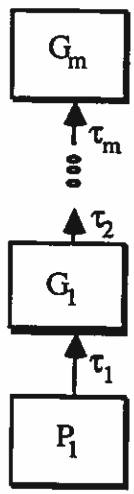

Figure 3.2 Hierarchical DES with m+1 Levels

<u>General Multi-Level Timing Structures</u>

We begin by studying the case where there are no particular restrictions placed on the timing structure so long as it is multi-level. If $\alpha \in J^* \cup J^{\mathbb{N}}$, then $[\alpha]$ will be used to denote the set of elements that make up the sequence α. For any possible run made by P_1 with an index sequence $\alpha_{p1} \in A_{p1}$, the corresponding runs in G_j, $1 \leq j \leq m$, have index sequences denoted by $\alpha_{gj} \in A_{gj}$.

<u>Lemma 1</u>: It is the case that $[\alpha_{gm}] \subset [\alpha_{gm-1}] \subset \cdots \subset [\alpha_{g1}] \subset [\alpha_{p1}]$.

Lemma 1 states the clear fact that the multi-level timing structure can *mask* events and hence remove time instants at which events occur in the higher levels of the hierarchy.

Definition 4: The *event occurrence rate* (*event rate*) in P_i or G_j is the number of events that occur in the time interval $T_u=(r_1,r_2]$ where $(r_1,r_2]\subset \mathbb{R}^+$ and it will be denoted with $\#(P_i,T_u)$ and $\#(G_j,T_u)$ respectively.

Notice that if P_i has a synchronous interpretation of time with $T\in\mathbb{R}^+$ and we choose T_u such that $|r_2-r_1|=T$ then $\#(P_i,T_u)=1$, i.e., there is 1 event occurrence in the time interval T_u no matter what the particular values of r_1 and r_2 are. If P_i has an asynchronous interpretation of time then no matter how T_u is chosen (so long as $|r_2-r_1|$ is bounded) it is possible that $\#(P_i,T_u)=0$, since we cannot guarantee that an event will occur in the given time interval T_u. In fact, we do not know how many events will occur in T_u. It would appear that our definition of event rate is too restrictive. This is, however, not the case since the focus here is on *comparing* the event rates of different DES components in the HDES and this comparison is made relative to T_u, an interval of the real time line. We begin by comparing the event rates of the DES in the multi-level HDES of Figure 3.2.

Theorem 1: $\#(P_1,T_u)\geq\#(G_1,T_u)\geq\#(G_2,T_u)\geq\cdots\geq\#(G_m,T_u)\geq 0$ for all T_u.

In the case where I_{p1} is synchronous the above results support the studies in [Gershwin 1989] where the author assumes that the event rates can be split into "spectra" according to the level in the hierarchy. A similar split can be made for the HDES of Figure 3.2 for the synchronous case. In the more general case, for an asynchronous P_1 for instance, the event rates in DES at the higher levels are also greater than or equal to the event rates at the lower levels.

Multi-Level Timing Structures for Event Aggregation

In this Section we focus on the case where the τ_j perform *event aggregation* and study how this affects the timing characteristics of the components of the multi-level DES. For convenience we shall first consider the case where m=1. Let $E_{a1}\subset E_{p1}$ and $\tau'_1:E_{a1}\rightarrow\{0,1\}$ denote a restriction of τ_1. In this Section the maps τ'_j for j, $1\leq j\leq m$, will be used for event aggregation maps in the multi-level timing structure of Figure 3.2 rather than the τ_j. If $e\in E_{p1}$, $e\notin E_{a1}$ then τ'_1 is said to *ignore* (rather that *mask*) the occurrence of e. Let $B\subset\mathbb{N}-\{0\}$.

Definition 5: $P_1=(X,U,Y,\delta,\lambda,X_0)$ and τ'_1 satisfy the π_1*-event aggregation property* if

(i) There exists a family of sets $X_i\subset X$, $i\in B$ such that

(a) $X_i\cap X_k=\emptyset$ for all $i\neq k$, and $X_0\cap X_i=\emptyset$ for $i\in B$,

(b) If P_1 first enters a state $x \in X_i$ for some $i \in B$, it will take (for all possible runs) at least $\pi_1 > 0$ state transitions before the state of P_1, say x', is such that $x' \notin X_i$, and

(ii) $\tau'_1 : E_{a1} \rightarrow \{0,1\}$ where $E_{a1} = \{e \in E_{p1} : \text{if } e=(x,x'), \text{ for some } i \in B.\ x \in X_i \text{ and } x' \notin X_i\}$.

Such a definition for τ'_1 results in a type of "event aggregation" between P_1 and G_1 since some sequences of events in P_1 can be *ignored* by the higher level G_1 and others can be *masked*. The number π_1 provides a measure for the amount of aggregation.

Theorem 2: If P_1 and τ'_1 satisfy the π_1-event aggregation property and $T_u=(r_1,r_2]$ where $|r_2-r_1|$ is sufficiently large, then

$$\frac{\#(P_1,T_u)}{\pi_1} + 1 \geq \#(G_1,T_u). \tag{6}$$

Notice that if P_1 and τ'_1 satisfy the π_1-event aggregation property and $T_u=(r_1,r_2]$ where $r_1=0$ then for all $r_2>0$,

$$\frac{\#(P_1,T_u)}{\pi_1} \geq \#(G_1,T_u). \tag{7}$$

Hence if T_u is chosen appropriately then we get a tighter bound on the number of events that occur at the higher level G_1 than in (6).

Next, we state the result analogous to Theorem 2 for the higher levels of the HDES. For m>1 we shall use the π_j-event aggregation property of G_j and τ'_{j+1} where π_j and τ'_{j+1}, the families of states, and initial states at each level are defined in a similar manner to that for Definition 5 above.

Corollary 1: If G_j and τ'_{j+1} satisfy the π_j-event aggregation property and $T_u=(r_1,r_2]$ where $|r_2-r_1|$ is sufficiently large, then for j, $1<j\leq m$,

$$\frac{\#(G_j,T_u)}{\pi_j} + 1 \geq \#(G_{j+1},T_u). \tag{8}$$

A result similar to (7) holds for m>1 if T_u is chosen as in (7). The τ'_j can be viewed as maps that cause event aggregation; consequently, Theorem 2 and Corollary 1 provide a relationship between event aggregation and event rates for one class of HDES: *As events are aggregated to the higher levels in the hierarchy, fewer events occur.* If there is a high measure of aggregation at level j (large π_j) then there will be a lot fewer events occurring at level j+1 ($\#(G_{j+1},T_u) \leq \#(G_j,T_u)/\pi_j + 1$). This illustrates that there is an inverse relationship between event aggregation and event rate between two levels of a HDES. In general, hierarchical systems researchers have observed a similar inverse relationship between "time scale density" ("time granularity") and "model abstractness" [Antsaklis, Passino, and Wang 1989; Saridis 1983]. The above results provide the first mathematical validation of these researcher's intuition about relationships between event aggregation and event rates for a class of HDES.

3.3 Timing Characteristics of HDES with Single-Branch Timing Structures

The other fundamental component of a HDES with a tree structured timing structure is what we will call a "single-branch timing structure" and it is shown in Figure 3.4. Results analogous to Theorems 1 and 2 will be developed for this HDES.

General Single-Branch Timing Structures

In this case $\tau_1:E_{p1}\times \cdots \times E_{pn}\rightarrow\{0,1\}$. Notice that even though we consider only P_i at the lower level, it requires only a simple modification to consider a mix of P_i and G_j at the lower level and our results that follow still hold. For any possible runs made by P_i, $1\leq i\leq n$, with index sequences $\alpha_{pi}\in A_{pi}$, the corresponding run in G_1 has index sequence denoted by $\alpha_{g1}\in A_{g1}$.

Lemma 2: It is the case that $[\alpha_{g1}] \subset \bigcup_{i=1}^{n}[\alpha_{pi}]$.

Lemma 2 states the clear fact that τ_1 can mask events in any P_i, $1\leq i\leq n$. This basic result can be used to compare the interpretations of time and event rates of the DES in the HDES of Figure 3.4.

Theorem 3: $\sum_{i=1}^{n}\#(P_i,T_u)\geq\#(G_1,T_u)$ for all T_u.

For this basic hierarchy we see that it is also the case that the event rate for the lower level is greater than or equal to the event rate at the higher level. Next, we highlight the importance of using an appropriate definition of what it means for a DES to be at a certain level in a hierarchy. Assume that $\tau_1(\cdot)=1$ in all cases for the HDES in Figure 3.4 so that no events will be masked. In this case, if any P_i has the asynchronous interpretation of time then G_1 has a asynchronous interpretation of time. For any possible run made by any P_i, $1\leq i\leq n$, with an index sequence $\alpha_{pi}\in A_{pi}$, the corresponding run in G_1 has index sequence $\alpha_{g1}\in A_{g1}$ where each α_{pi} is a subsequence of α_{g1}; hence $\#(G_1,T_u)\geq\#(P_j,T_u)$ for all T_u and each j, $1\leq j\leq n$. The event rate is higher in G_1 than in any single one of the other DES P_i.

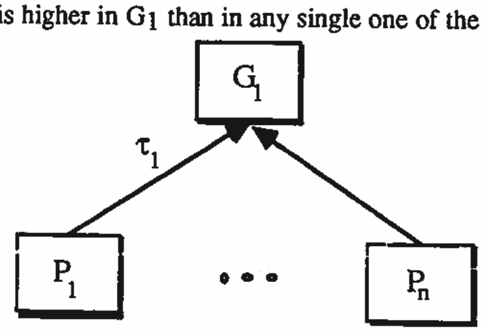

Figure 3.4 Hierarchical DES with Single-Branch

Single-Branch Timing Structures for Event Aggregation

Next we consider the case where τ_1 performs event aggregation for each P_j, $1 \le j \le n$. Let $E_{aj} \subset E_{pj}$ for all j, $1 \le j \le n$, and $\tau'_1 : E_{a1} \times E_{a2} \times \cdots \times E_{an} \rightarrow \{0,1\}$ denote a restriction of the τ_1 used in Lemma 2 and Theorem 3. We use this τ'_1 in place of τ_1 in Figure 3.4 for event aggregation. Hence, as above, τ'_1 *ignores* events $e \in E_{pj}$, $e \notin E_{aj}$ and can *mask* others. Let $B_j \subset N - \{0\}$ for j, $1 \le j \le n$.

Definition 6: $\{P_j = (X^j, U^j, Y^j, \delta^j, \lambda^j, X_{0j}) : 1 \le j \le n\}$ and τ'_1 satisfy the *$(\pi^1, \pi^2, \ldots, \pi^n)$-event aggregation property* if for each j, $1 \le j \le n$,

(i) There exists a family of sets $X_{ij} \subset X^j$, $i \in B_j$ such that

(a) $X_{ij} \cap X_{kj} = \emptyset$ for all $i \ne k$, and $X_{0j} \cap X_{ij} = \emptyset$ for $i \in B_j$,

(b) If P_j first enters a state $x \in X_{ij}$ for some $i \in B_j$, it will take (for all possible runs) at least $\pi^j > 0$ state transitions before the state of P_j, say x', is such that $x' \notin X_{ij}$,

(ii) $\tau'_1 : E_{a1} \times E_{a2} \times \cdots \times E_{an} \rightarrow \{0,1\}$ where $E_{aj} = \{e \in E_{pj} : e = (x, x')$ and for some $i \in B_j$, $x \in X_{ij}$, $x' \notin X_{ij}\}$.

Theorem 4: If $\{P_j = (X^j, U^j, Y^j, \delta^j, \lambda^j, X_{0j}) : 1 \le j \le n\}$ and τ'_1 satisfy the $(\pi^1, \pi^2, \ldots, \pi^n)$-event aggregation property and $T_u = (r_1, r_2]$ and $|r_2 - r_1|$ is sufficiently large, then

$$\sum_{i=1}^{n} \left\{ \frac{\#(P_i, T_u)}{\pi^i} + 1 \right\} \ge \#(G_1, T_u). \tag{9}$$

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