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Five Questions – The Philosophy of Logic

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1. Why were you initially drawn to the philosophy of logic?

In retrospect, I realize that I was drawn to the philosophy of logic long before I knew that there was such a field. When I was an undergraduate, before I had read any philosophy, I took a set theory course, and found myself intrigued by questions not covered in the class – questions like "why *these* axioms rather than some plausible alternatives?" and "what exactly are we trying to model, or to do, with a theory of sets?" I later took some courses in logic, and was again struck by questions not dealt with in the syllabus, questions like "Is there some reason to prefer a logic that satisfies the Löwenheim-Skolem theorems to one that doesn't?", "Is logic just a matter of 'relations of ideas'?" (I'd read some Hume by then), and "What exactly does it mean to say that number theory is 'part of' or 'reducible to' logic, and what does that claim imply with respect to the nature of mathematics?"

Though these questions (and others like them) intrigued me from early on, I was initially inclined to think that they were insufficiently precise to admit of clear answers, or even of clear proposals for answers, and so it didn't initially occur to me to try to do serious work on them. In graduate school, I thought at first that I'd work on the philosophy of language, and leave questions like the above for moments of idle musing.

Two things changed my attitude towards the reasonableness of pursuing serious work in the philosophy of logic. One of these was that I read John Etchemendy's *The Concept of Logical Consequence*, in which I found a beautiful example of an attempt to take seriously the question of how a particular, central pretheoretic notion (that of logical consequence) matches up with, or fails to match up with, a particular kind of modern account of that notion.[1] Etchemendy's generally-negative conclusion, to the effect that the relation of what we might call "model-theoretic consequence" does not in general match up very well with the relation that underlies the correctness of ordinary inference, is in my view (which is apparently the minority one) essentially the correct one to draw. In any case, the book made it clear to me that the kinds of questions that had always fascinated me were, at least on occasion, susceptible to careful philosophical treatment.

The second thing that made it clear to me that the philosophy of logic could be done well was reading some history of logic, and discovering that the titans of modern logic were themselves philosophers at heart. I was intrigued to discover that Frege, Hilbert, Cantor, Zermelo, Brouwer, and Gödel, just to name a few, were deeply invested in purely-philosophical questions about the nature of logic,

[1] Etchemendy [1990]

and about the role of formal systems in studying logic and mathematics. The debate between Frege and Hilbert over the nature of independence-proofs, to choose my own favorite example, was for me an eye-opening exchange. Here the subject-matter is that of the relationship between a purely-formal technique (essentially, the construction of models) and a handful of pre-formal questions (e.g. the independence of the parallels postulate, or the consistency of a theory). The debate involves not just the central issue of whether or not Hilbert's model-construction technique is a reliable method for establishing independence and consistency, but also a series of important related questions, having to do with the nature of mathematical axioms, the connection between formal deduction and intuitive provability, the importance of semantics to logic, and so forth. For me, debates of this kind between those involved in laying the foundations of logic were a vivid demonstration both of the importance of the debates themselves, and of the possibility of treating them with rigor and care.

The first issue that I thought I might make some headway on had to do with the connection between logic and mathematics, specifically the logicist attempts to "reduce" mathematics to logic. I soon discovered that there are as many different logicisms as there are logicists, with very different criteria of success, and different implications for the nature of the reduced and the reducing theories. Thinking that I might write a dissertation exploring those differences, the first chapter of which would have been on Frege, I soon found that Frege's logicism itself was sufficiently rich to keep me busy for quite a long time. The first chapter became the whole dissertation, as I have since learned is not an unusual pattern in dissertation-writing. My interests have since branched out to the history and philosophy of logic more broadly, but the issues raised in Frege's writings – see below – continue to play a central role in my work.

2. What are your main contributions to the philosophy of logic?

My work has mainly focused on Frege, and on a handful of philosophical issues that arise in his work. The technical details of Frege's work are relatively well known, but it has seemed to me that there's a lot that isn't yet well understood regarding the philosophical lessons to be learned from the successes and the failures of various parts of that work. Two of the issues I've been especially interested in are these: (i) Frege's conception of the connection between formal systems of logic and ordinary, non-formalized relations of entailment, consistency, independence, and other pretheoretic logical notions, and (ii) Frege's conception of the relation of "reducibility," that relation that he attempted to establish between logic and a large part of mathematics.

As to the first: Frege takes it that these pretheoretic relations all obtain between entities that he calls "thoughts," the things that are expressed by meaningful sentences. This includes, in a way that's surprising to a modern audience, the sentences of a formal system: Frege, unlike most of his successors, does not view the sentences of a formal deductive system as partly uninterpreted, but takes them, just like the sentences of natural languages, to express determinate thoughts. This makes it easy to characterize the relationship between *derivability* in a good formal system and *entailment*: a well-designed formal system is one that lets us *derive* a sentence S from a collection P of premise-sentences only if the thought expressed by S really is entailed by the thoughts expressed by the members of P. Frege did not, however, take it that the converse is generally to be expected: he held that the derivability of S from P is a sufficient, but not in general a necessary, condition for the logical entailment of S's thought by P's thoughts. Hence there is an in-principle divide between the important pretheoretic notion and the relation of formal derivability. This divide plays a large role in the debate mentioned above, between Frege and Hilbert over the provability of independence and consistency.[2]

Frege's view of the role of formal derivation is closely connected to the second issue, that of theoretical reduction. Here Frege's story is complicated, but the bottom line is that, as he conceives of it, a reduction of arithmetic to logic would have demonstrated, in a very straightforward way, that the true thoughts expressed by ordinary arithmetical sentences, e.g. the thought expressed by "Every natural number has a successor," are provable from fundamental truths of logic. The latter, the fundamental truths of logic, are themselves thoughts as well, and they are thoughts whose status as "logical" is, as Frege understands it, simply immediately evident.

In my view, Frege's views and arguments about the nature of logic and related areas are generally powerful, and are essential to come to grips with if one wants to have clear views, oneself, about the nature of logic. Part of my contribution to the philosophy of logic, then, has been an extended treatment of Frege's conception of logic, and of his understanding of the connections between logic, language, and mathematics.[3] I argue that Frege's position (which I take to be often badly misunderstood) is an especially cogent one, with powerful and attractive implications for the nature of logic.

Another area in which I've worked concerns the relation of logical consequence, and the role of models in assessing that relation. I have argued for example that a fairly standard practice of treating models of formal theories as in some sense representatives of possible worlds, or of alternative states of affairs, is badly mistaken, and is the source of a good deal of confusion regarding the notion of model-theoretic entailment.[4] This work is closely connected to questions regarding the usefulness of second-order logic in assessing consequence, and to questions about the significance of the completeness of first-order logic.

3. What is the proper role of philosophy of logic in relation to other disciplines, and to other branches of philosophy?

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[2] For my views on the Frege-Hilbert debate, see Blanchette [1996], [2007a], [2007b] and parts of [2012].
[3] See e.g. Blanchette [2012].
[4] Blanchette [2000], [2001]

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Early in the 20th century, philosophical issues had a good deal to do with the design of formal systems and with the particular course followed in the development of modern logic. Questions of realism versus constructivism about the subject-matter of mathematics, questions about the relationship between truth and provability, and other clearly-philosophical issues explicitly motivated the work of the pioneers. In the last half-century, mainstream formal logic has grown into an independent discipline, in most areas a purely-mathematical one, so that to work in pure logic, one doesn't any longer need to be engaged with philosophical issues. Logic is in this way following the pattern of the physical sciences: initially motivated and shaped by philosophical issues and debates, the field has achieved a relatively-stable form, one that can, for the most part, be pursued independently of philosophical engagement.

The role of philosophy in relation to classical logic now, in the more-mature period (i.e. one in which a lot of logicians do no philosophy), is twofold. First of all, philosophers ask and answer questions having to do with the connection between logical systems in general and such pretheoretic notions as that of validity, truth, entailment, definition, analysis, form, number, measure, mathematical truth, and so on. Here the central questions straddle the boundary between the formal and the pretheoretic. We ask, for example, whether the validity, in a pretheoretic sense, of an ordinary argument can be reduced to, or analyzed in terms of, the formal validity of some logical system's representation of that argument; whether the representation of a theory's content via a categorical axiom-system is a particularly revealing method of representing it (and if so, why); whether consistency and independence in the formal logician's sense are reliable indicators of their pretheoretic namesakes; and so on. The hope is that we gain, through this kind of work, a better understanding both of what's going on with formal systems, and of the deeper, pre-formal notions themselves.

Secondly, philosophers ask, and answer, questions about the philosophical significance of various results in modern logic. What do we learn about the representative capacity of first-order logic when we discover that no first-order axiom-system with infinite models is categorical? (Does this provide pressure to move to a stronger language, or is categoricity not what we should be aiming for?) What do we learn about the deductive power of first-order logic when we learn that it's complete? (And what does the incompleteness of the stronger second-order logic tell us about its adequacy?) Does the fact that arithmetical truth is not recursively enumerable tell us something important about the nature of arithmetic? Does the independence of e.g. the continuum hypothesis from the typical axioms of set theory show us that there's something incoherent about some forms of set-theoretic realism? (Or that we should be searching for stronger axioms?) In general, logic itself provides a rich trove of precise formal results to which one needs to be sensitive in investigating a host of topics that have always been of central concern to philosophy; and it is the goal of the philosopher of logic to investigate exactly what those results imply with respect to those philosophical issues.

In addition to this engagement with classical logic, philosophers of logic currently play a fundamental role in the development and the discussion of alternative systems of logic. Modal logics, many-valued logics, free logics, relevance logics, paraconsistent logics, nonmonotonic logics, and so on are formal logics motivated by purely philosophical concerns. In general, the fundamental idea is that in some areas of discourse (e.g. discourse about necessity and possibility, or about empty domains, or about paradoxical situations), there are particular patterns, or rules, of inference that can be codified and studied via a formal logic specific to that kind of discourse. The role of the philosopher of logic here is to present reasons for adding to or departing from classical logic in these domains, to devise well-behaved formal systems reflecting the patterns in question, and to show that we can learn significant things about the kind of discourse in question, and about the knowledge involved in that discourse, by pursuing the formal project.

As to the relationship between the philosophy of logic and the rest of philosophy: here the connections are numerous. Modern philosophy of science is closely connected with the philosophy of logic via a fabric of interconnected questions and issues, and a shared history. The question of whether, and how, the axioms of a theory can provide implicit definitions of its central concepts has at times been a central question in modern philosophy of science, taking its defining concepts (those of modern axiom, implicit definition, and satisfaction) from 20^{th}-century logic. The closely-related question of how we are to understand a scientific theory (whether for example as the deductive closure of its axioms, or as its class of models, or in some other way altogether) is informed by the logician's characterization of these notions. The questions of whether a scientific theory ought to be understood in terms of its "structure," and of how a theory applies to the observable world, are questions motivated in large part by the logician's characterization of theories in terms of structure. And so on. In general, the shared concern with the nature of axioms and theories, with the role of mathematics within theories, and with the application of theories to the real world, make the philosophy of logic and the philosophy of science natural partners.

Also extremely closely linked with the philosophy of logic is the philosophy of mathematics. This is partly because some of the central issues in each field are the same: in each case, knowledge in the field would seem to be obtained *a priori*, while the truths so obtained are essential to empirical research. This has made both logic and mathematics significant in their apparent challenge to various forms of empiricism, and has presented in each case an intriguing set of questions regarding the applicability of the discipline. The connection is also very close historically: modern formal logic was devised by those attempting to provide a foundation for mathematics, and by those who viewed it as a tool to be used within mathematical research. The question of how, exactly, logic and mathematics are connected – and in particular in what sense parts of mathematics are "reducible" to logic – was a driving question in a good deal of early logical research. That the reducibility question is of less-central importance today (though still alive in some quarters) is due largely to two factors. First, we have some negative results – e.g. the discovery that the most natural axiomatizations (Frege's, Russell's) that are rich enough to generate mathematics are either inconsistent or contain axioms whose logical status is questionable, and the lesson of Gödel's incompleteness theorem that no axiomatization will do the trick – that undermine the most straightforward reductionist projects. Secondly, via the development of set theory, the line between logic and mathematics has blurred: it is clear that set theory is sufficient for the "reduction" of much of mathematics, but set theory itself has enough arguably-mathematical content that this form of reduction is by no means the same project as the early logicist attempts to reduce mathematics to logic. The questions of what kinds of reduction are possible, and what we are to learn from them, both about the nature of mathematics and about the nature of logic, are on-going concerns in the philosophy of logic and the philosophy of mathematics.

The philosophy of language and epistemology too are closely linked with the philosophy of logic. As to the first: Logic is pursued by using particular highly-regimented languages, ones that no person has ever used in ordinary communication. Indeed, much of the time, the languages in question are only "partially interpreted," which is to say that, aside from the symbols for "and," "or," "not," "if-then", "for all", and so on, the symbols of the languages used to do logic have no fixed meanings. This gives rise to two issues having to do with what might roughly be called the "interpretation" of the languages in question. The first question is that of how the languages used in formal logic are related to natural languages. When we discover that a particular formula is provable in a good formal system from a set of formulas, what exactly does this tell us about the validity of an argument that's expressed in English? Here, the rough idea is that the formal argument is a "formalization of" an English-language argument, via the kinds of vague (indeed, they're *very* vague) translation-rules that one learns in a first logic course. What we want to know is, in part, what it is that a good formalization preserves from the original argument (do we intend to preserve just the references of the parts of the sentence? Or something more fine-grained, like a Fregean "sense" or an intension?), and – most importantly – whether what's preserved in this process is sufficiently robust to guarantee that the original argument and the formalized version share important logical properties. This raises the question of what, exactly, the bearers of the logical properties and relations are: can we understand validity and entailment to hold of series of sentences, i.e. bare strings of symbols? Or must we take these properties to hold of something more robust, like nonlinguistic propositions? Or is there some middle ground? The answer to this question, pursued by the philosopher of logic, is critical to the philosophy of language, since, for example, if one can only make sense of validity and entailment in terms of such extra-linguistic things as propositions, then it is incumbent on the philosopher of language to make room for such entities in the explanation of how language works. If on the other hand no sense can be made of the expression of such things in natural language, then (assuming that we can express valid arguments in ordinary language) the philosopher of logic must be able to make sense of the logical properties (validity, entailment) in terms of such things as sentences. This is just one instance of the rich connection between the philosophy of language and the philosophy of logic:

in the end, the two are inseparable largely because we need languages in order to do logic, and we need to recognize logical relations in order to speak a language.

Epistemological questions were a driving force for some of the early pioneers of logic. Frege and Russell were both interested in the question of whether a "reduction" of mathematics to logic would help to explain the nature of mathematical knowledge and its certainty. Frege's student Carnap, along with a number of modern positivists, hoped that the close connections between mathematics and logic could be exploited to show that mathematics poses no problem for empiricism. Brouwer thought that the nature of mathematical knowledge placed strict limits on the reliability of various forms of logical inference. Gödel, holding a radically different picture of mathematical knowledge, held a correspondingly different view of the fundamental rules of logic. And so on. Logic has forever been viewed as a tool of *inference*, i.e. a tool by means of which we obtain new knowledge from old, with the result that views about the nature of knowledge and views about the laws of logic are inextricably connected. As above, epistemological concerns have been a driving force behind the development of a large number of alternative systems of logic, some of which supplement, and some of which depart from, ordinary classical logic. The connection between epistemology and the philosophy of logic is a robust two-way street: while the philosopher of logic must pay attention to the fact that logic is a tool for expanding knowledge, the epistemologist finds in logic a rich body of knowledge that cries out for explanation: is our knowledge of logic simply a species of linguistic or conventional knowledge, or is there something more robust, and harder to explain, behind our knowledge of logical truths?

In short, the philosophy of logic is deeply connected with a large proportion of the rest of philosophy, in no small part because logic itself is crucial to the reasoning that we engage in, and that we reflect upon, when we do philosophy.

4. What have been the most significant advances in the philosophy of logic?

Philosophy of logic, like most of philosophy, doesn't advance in discrete steps (like the proof of new earth-shaking theorems or the discovery of new species), but gains ground by slowly, carefully bringing some illumination to areas and questions that were earlier characterized by confusion.

My own view is that the most significant changes in this area over the last century have all had to do with increasing clarification about (i) the connections between language, mathematics and logic, and (ii) the nature of logical entailment itself.

On both of these issues, I think of Frege's work as having been especially significant. Frege understood that we need to have a clear appreciation of how language works if we are to even begin to understand the nature of logic. His idea, for example, that the principles of logic have to do not with expressions but with what they express, makes sense only against the background of a theory of meaning that takes something like sense, or intension, into account. And though he was not unique in this way, his clear and dogged pursuit of the connections between a theory of meaning and an account of the nature of logic served to set the agenda for much of modern philosophy. We now know that there are alternative ways of understanding meaning that will make sense of the connection between meaning and inference, and that there are arguments both for and against the specifically Fregean accounts of meaning and of the logical relations. But one enduring lesson of Frege's work is that theories of logical entailment and of meaning cannot be pursued in isolation from one another: whatever and however it is that our words and sentences mean, it has become clear that semantic properties are determined by, and determine, logical properties: our sentences mean what they mean in large part because of what they logically imply (and are implied by), and these relations of implication are determined in large part by what those sentences mean.

Frege's view that we can come to understand the nature of arithmetic by reducing it to logic, and that this reductionist project must involve both a careful axiomatization of the logic and careful conceptual analyses of arithmetical notions, has borne a great deal of fruit. We have learned that a certain conceptual richness of arithmetic, together with a lack of corresponding richness in pure logic, makes a reduction of the kind Frege envisioned impossible. This is as clear as philosophy ever gets, it seems to me, to establishing a firm and foundational result of a surprising kind. As a result, we have achieved a considerably clearer conception of what is at stake in the attempt to explain mathematical truth and mathematical knowledge, and also a considerably clearer conception of the bounds of the purely logical.

In the post-Fregean era, logic itself has developed dramatically, and some of the most important work in the philosophy of logic has involved the exploration of exactly what we learn from modern results in pure logic. Over the last eighty years or so, we have, for example, achieved a much clearer conception of the bounds of provability. We have learned that axiomatizations even of such apparently-simple theories as that of natural-number arithmetic are very hard to come by, and so we've recognized that the principles of inference needed for such theories are vastly richer than they at first appeared to be. We have learned that the notions of "true sentence" and of "sentence provable-in-system-S" are much less straightforwardly characterized than one might have thought, and that the difficulties in so characterizing them help us establish boundaries to the expressive power of formal systems. We've managed to characterize the notion of "computable function," and to show how surprisingly narrow the bounds of computability are. These and related modern results regarding the expressive and proof-theoretic limitations of formal systems seem to me to be extremely important in coming to understand how logic is related to language, to mathematics, and to reasoning in general.

5. What are the most important open problems in philosophy of logic, and what are the prospects for progress?

I don't have anything like a "top-ten" favorite list of discrete open problems, in part because most of the interesting questions in the philosophy of logic are so closely tied to one another as to form one large mass, rather than a handful of discrete problems. The connected questions here include those of the right way to understand alternative logics (where such logics conflict, is there one "right" logic, or can we make sense of different logics applicable to different projects or domains?), the significance of various connections between deductive and model-theoretic consequence relations, and hence of completeness and incompleteness results, and that of what, exactly, we should infer from the remarkable success of axiomatic set theory as a foundational tool both in logic and in mathematics. We also need to come to grips with the claims of competing foundational theories, especially that of category theory. In these areas, as in all parts of the philosophy of logic, it seems to me that it will be important for us to do a better job of understanding not just current formal developments, but also the history of the discipline.

One of the further issues about which I would like us to gain greater clarity is that of the nature of logical truth and logical entailment. There was enthusiasm in the early 20th century for the idea that logic is a matter, somehow, of convention, or a byproduct of the forms of our linguistic or mental representation. The attractiveness of such a view is relatively clear: if true, then we seem to have the outline of an answer to the questions of how we know the truths of logic, and of why the relation of logical entailment is immediately and universally applicable. But no clear statement of this idea seems plausible: as Quine has pointed out, it's very difficult to cash out a clear sense of "convention" in which logic can reasonably be said to hold "in virtue of convention;" and it's not at all clear that there is a way of understanding "forms of representation" which fares any better.[5] One of the difficulties is that when we say that *p* holds "in virtue of" *q*, we often mean simply that *p* is logically entailed by *q*; and when the *p* in question is logic itself, such an account is a non-starter. Formulating a clear statement of what exactly the principles of logic *are* (are they rules about representation? About truth? And what's a *rule* anyway?), or a clear line of reasoning to the effect that this is a misguided question, seems to me to be an issue that's still quite unsettled, and worth pursuing. As to prospects for progress: I'm optimistic. We have come a long way in understanding the nature of logic over the last century or so, and I see no reason that we can't press on quite a bit further.

[5] Quine [1936]

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