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MAGNETOHYDRODYNAMIC NATURAL CONVECTION HEAT TRANSFER FROM HORIZONTAL CYLINDERS

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NOMENCLATURE

b	constant
B	magnetic flux density
C_1–C_6	constants
C_p	specific heat at constant volume
d	cylinder diameter
Gr	Grashof number, $g\beta\Delta Td^3/\nu^2$
g	gravitational acceleration
h	film heat transfer coefficient
k	thermal conductivity
l	cylinder length
Ly	Lykoudis number, $\sigma B^2(d/g\beta\Delta T)^{1/2}/\rho$
M	Hartmann number, $Bd(\sigma/\mu)^{1/2}$
Nu	Nusselt number, hd/k
Nu_c	conduction Nusselt number
Nu_0	Nusselt number for zero Hartmann number
Pr	Prandtl number, $C_p\mu/k$
t	test cell's fluid height
T_c	cylinder surface temperature
T_e	environmental temperature
u	free convection velocity
x, y, z	coordinates

Greek symbols

α	thermal diffusivity
β	volume coefficient of thermal expansion
δ	velocity boundary layer thickness
δ_T	thermal boundary layer thickness
ΔT	temperature difference, $T_c - T_e$
γ	angle between normal to cylinder's surface and vertical direction
θ	angle from lower stagnation point of cylinder
λ	$(Nu - Nu_c)/(Nu_0 - Nu_c)$
μ	absolute viscosity
ν	kinematic viscosity, μ/ρ
ρ	density
σ	electrical conductivity

1. INTRODUCTION

THIS note presents an approximate 2-dim. solution of magnetohydrodynamic (MHD) natural convection heat transfer from a finite cylinder at various orientations with respect to an applied magnetic field. The work is an extension of that of Lykoudis and Dunn [1], in which they measured and successfully predicted the MHD natural convection heat transfer from hot-film probes aligned axially with a horizontal magnetic field.

The prediction of MHD heat transfer from a finite cylinder at various orientations with respect to an applied magnetic field is particularly useful in the determination of the heat transfer from a hot-film probe used in a single-phase liquid metal (e.g. [1–3]) and two-phase liquid-metal inert-gas MHD studies (e.g. [4, 5]). It also is relevant to the prediction of heat transfer in various fusion reactor blankets, such as those in which lithium flows through tubes immersed in pools of stagnant lithium [6] or those involving natural circulation of a liquid metal [7].

Detailed local measurements of the OHD (ordinary-hydrodynamic) and MHD natural convection heat transfer from horizontal finite cylinders to mercury for two magnetic field orientations have been presented by Michiyoshi *et al.* [8]. Average measurements of MHD natural convection heat transfer to mercury for the third mutually orthogonal magnetic field orientation have been presented by Dunn [9]. These experimental studies and the analytical work of Lykoudis and Dunn [1] are the bases of the present work.

2. EXPERIMENTS

The experimental cases considered here are the three mutually orthogonal orientations of an applied magnetic field with respect to a finite cylinder's axis, as shown in Fig. 1. Cases 1 and 2 were examined by Michiyoshi *et al.* [8] using a cylindrical heater (effective heating length-to-diameter ratios of 6.0 and 10.4) immersed in mercury. Case 3 was studied by Dunn [9] employing TSI quartz-coated hot-film probes (sensing area length-to-diameter ratios of 13.1 and 19.7) immersed in mercury. The operating parameters of all three experiments are presented in Table 1. For all three of these cases, MHD interaction was observed to reduce the natural convection heat transfer from the cylinder to the liquid metal.

This observed reduction in natural convection heat transfer from a finite cylinder in the presence of a magnetic field results from the interaction of the magnetic field with the flow field adjacent to the cylinder. This interaction occurs not only around the circumference of the cylinder but also around its ends and in the convective wake above the horizontal cylinder. Because the cylinder is finite, some degree of interaction and thereby some reduction in heat transfer will occur at any magnetic field orientation. The amount of reduction in heat transfer, however, will vary for each magnetic field orientation. As noted by Malcolm [10], the reduction in heat transfer that results from an applied magnetic field oriented as in case 3 will be the least when compared to the other two cases. This is because in case 3, when the cylinder's length-to-diameter ratio becomes large, the flow approaches a 2-dim. MHD configuration in which no MHD interaction with the flow is predicted to occur, except at its ends [11]. Reductions in heat

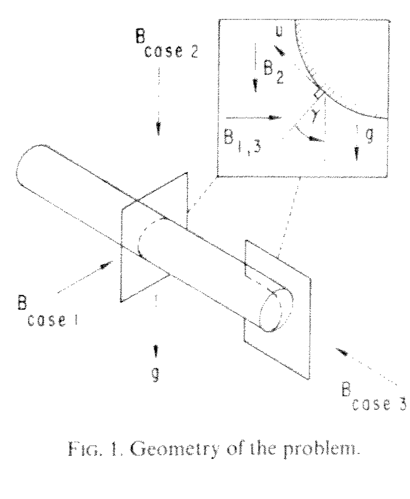

FIG. 1. Geometry of the problem.

Table 1. Parameters for the three cases examined

Case	l (mm)	d (mm)	l/d	t/d	Gr	θ	Nu_0	Nu_c	C_5	C_6	$Ly_{critical}$
1b	50	4.8	10.4	58.3	4.02×10^5	0	3.24	2.42	4.75×10^2	0.152	> 50
1d	50	4.8	10.4	58.3	5.76×10^5	π	2.32	1.66	1.55×10^3	0.235	> 50
2a	39	6.5	6.0	16.2	4.26×10^5	0	2.81	1.69	1.75×10^2	16.7	35
2b	39	6.5	6.0	16.2	1.10×10^6	0	3.83	2.51	1.90×10^2	3.59	30
2c	39	6.5	6.0	16.2	4.84×10^5	π	2.56	1.45	2.04×10^2	8.91	35
2d	39	6.5	6.0	16.2	1.46×10^6	π	2.96	1.92	6.07×10^2	3.49	30
3a	2	0.152	13.1	4.72×10^3	19.3	*	0.674	0.626	1.97×10^3	2.54×10^{-2}	6
3b	2	0.152	13.1	4.72×10^3	9.48	*	0.604	0.566	2.36×10^3	2.46×10^{-2}	6
3c	1	0.051	19.7	1.42×10^4	0.601	*	0.409	0.399	2.52×10^4	1.36×10^{-3}	1
3d	1	0.051	19.7	1.42×10^4	0.281	*	0.426	0.420	7.38×10^4	5.56×10^{-4}	0.3

* Average heat transfer measurements made.

transfer for this case have been measured for cylinders with small length-to-diameter ratios [9]. For the other two cases, reductions in heat transfer also have been measured [8]. In both those cases, local reductions in heat transfer were greatest where the local velocity was perpendicular to the applied magnetic field.

3. ANALYSIS

In the following, an approximate 2-dim. solution is presented that describes the natural convective heat transfer from a horizontal finite cylinder at various orientations with respect to an applied magnetic field. Such a 2-dim. solution can approximate well the measured heat transfer from a cylinder when the product $Gr^{1/2}(l/d)$ is much greater than unity, as shown theoretically by Mahoney [12] for the OHD case and supported experimentally by the work of Lykoudis and Dunn [1] for MHD case 3.

For the cases in which the applied magnetic field is horizontal (cases 1 and 3) and a component of the velocity lies in the B–g plane (around the cylinder's circumference for case 1 and around the cylinder's ends for case 3), the equation of motion in the direction tangent to the cylinder's surface (Fig. 1) can be written as

$$0 = \mu \frac{\partial^2 u}{\partial y^2} - \sigma u B^2 \sin^2 \gamma + g\beta\rho\Delta T \sin \gamma. \tag{1}$$

In this equation, inertial terms involving the square of the velocity are neglected because the magnitude of the free convection velocity is small. Also, because there is no external pressure gradient imposed on the flow, the pressure force in the tangential direction becomes zero. The ponderomotive force ($-\sigma u B^2 \sin^2 \gamma$) and the buoyancy force ($g\beta\rho\Delta T \sin \gamma$) represent body forces per unit volume, where ΔT is the temperature difference between the cylinder's surface and the environment.

For the case in which the applied magnetic field is vertical (case 2) and a component of the velocity lies in the B–g plane (around the cylinder's circumference), the equation of motion in the direction tangent to the cylinder's surface becomes

$$0 = \mu \frac{\partial^2 u}{\partial y^2} - \sigma u B^2 \cos^2 \gamma + g\beta\rho\Delta T \sin \gamma. \tag{2}$$

Equations (1) and (2) differ only by their trigonometric functions in the ponderomotive force term.

The governing energy equation for all three cases is

$$u \frac{\partial T}{\partial x} = \alpha \frac{\partial^2 T}{\partial y^2}. \tag{3}$$

In this expression, the viscous and Joulean dissipation terms are neglected because their magnitudes are comparatively small.

An approximate general solution of the above equations for all three cases can be obtained by using an order-of-magnitude analysis of the type employed by Lykoudis and Yu [13] and by Lykoudis and Dunn [1]. The order of magnitude equations corresponding to equations (1) and (2) and to equation (3) can be written as

$$0 = -\mu \frac{u}{\delta^2} - C_1 \sigma B^2 u + C_2 g\beta\rho\Delta T \tag{4}$$

and

$$C_3 u \frac{\Delta T}{d} = \alpha \frac{\Delta T}{\delta_T^2}. \tag{5}$$

The constants C_1, C_2 and C_3 essentially are empirical. However, C_1 and C_2 are functions of γ.

This approach permits the necessary uncoupling of the momentum and energy equations, which eventually yields [1]

$$Nu = Nu_c + \left[\frac{2Gr\,Pr/C_4}{M^2 + (M^4 + 4Gr\,C_5/C_4^2)^{1/2}}\right]^{1/2} \tag{6}$$

in which Nu_c is the conduction Nusselt number, C_4 equals C_1/C_2C_3 and C_5 equals $1/C_2C_3$.

For the zero Hartmann number case, this expression reduces to

$$Nu_0 = Nu_c + \left(\frac{Gr}{C_5}\right)^{1/4} (Pr)^{1/2}. \tag{7}$$

These equations can be combined into a more convenient form in which λ, the ratio of free convection heat transfer in the presence of a magnetic field to the free convection heat transfer in the absence of a magnetic field, is expressed in terms of C_6 and one nondimensional number, Ly,

$$\lambda = \left[\frac{Ly}{C_6^{1/2}} + \left(1 + \frac{Ly^2}{C_6}\right)^{1/2}\right]^{-1/2} \tag{8}$$

where Ly denotes the Lykoudis number and C_6 equals $4C_2C_3/C_1^2$. The Lykoudis number previously has been shown to characterize natural convection heat transfer in the presence of a transverse magnetic field for the case of a vertical flat plate [14, 15], horizontal pipe flow [16] and a plume above a line heat source [17].

4. RESULTS

Data from all three cases were compared with corresponding values predicted by equation (8). For each set of data in which l/d, Gr and Pr were fixed, the conduction Nusselt number was determined first, and then the values of C_5 and C_6 using equations (7) and (8), respectively.

It was not straightforward to determine the value of Nu_c for each set of data. The value of Nu_c can be determined best by experiments in which the conditions are such that the second term on the RHS of equation (6) becomes negligible. Such

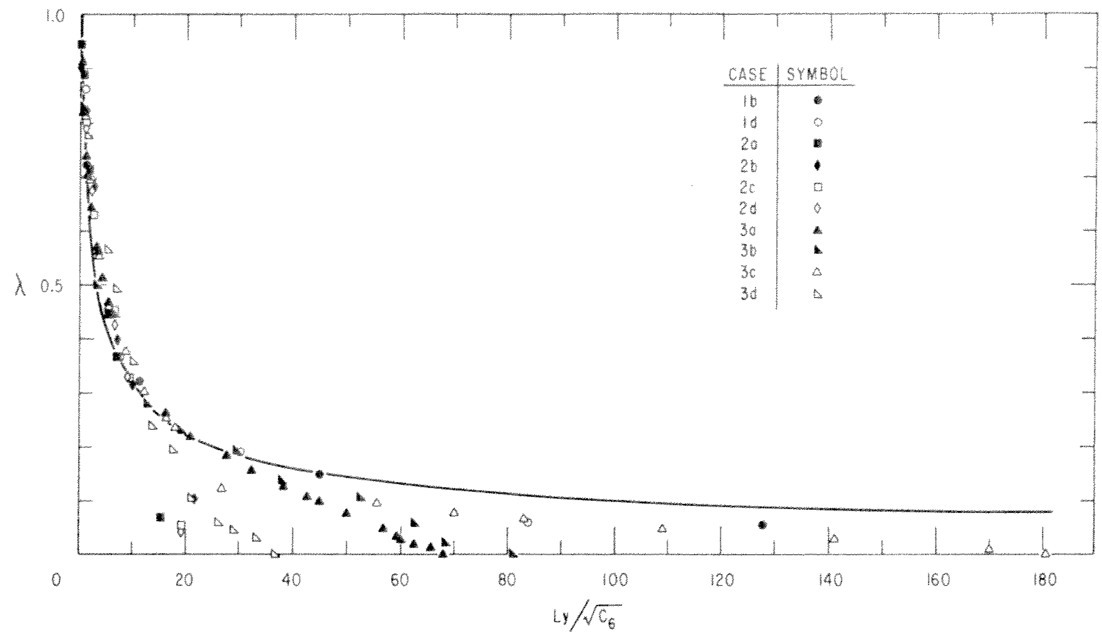

FIG. 2. λ vs $Ly/C_6^{1/2}$ for cases 1, 2 and 3.

conditions were achieved in the experiments of Dunn [1], but not in those of Michiyoshi *et al.* [8]. For cases 1 and 2, in which a conduction Nusselt number was not measured, it was determined by fitting the data, using linear regression analysis, to the expression

$$Nu = Nu_c + b/M, \quad (9)$$

in which b is a constant. This expression becomes the limiting case of equation (6) when $M^4 \gg C_6 Gr$. As a check, for case 3 in which conduction Nusselt numbers were measured, this method was found to predict the measured values to within 1%. The values of Nu_0 and Nu_c for each case are listed in Table 1.

As shown in the table for cases 1 and 2, the values of Nu_0 and Nu_c vary with θ, as well as with Gr, Pr, and l/d. For a fixed Gr, Pr, and l/d, Nu_0 decreases from its maximum at $\theta = 0$ to its minimum at $\theta = \pi$, as measured in all the cases of Michiyoshi *et al.* [8] and as predicted by Hermann [18]. This is because the thermal boundary layer increases in thickness from $\theta = 0$ to $\theta = \pi$. When a magnetic field is applied, the thermal boundary layer increases in thickness further for all θ, i.e. the local Nusselt number decreases with increasing magnetic flux density [8]. This decrease in local Nu gradually levels off at high M, asymptotically approaching its final value, Nu_c. The values of local Nu_c consequently vary for each Gr, Pr, l/d and θ case. For cases in which Gr, Pr and l/d are similar (cases 1b and 1d, 2a and 2c, and 2b and 2d), the local Nu_c decreases from $\theta = 0$ to $\theta = \pi$. For cases in which Pr, l/d and θ are similar (cases 2a and 2b, and 2c and 2d), both the local Nu_0 and the local Nu_c increase with increasing Gr.

Once Nu_c was determined for each case, the value of C_5 using equation (7) was found. The value of C_6 was calculated using equation (8) and the data corresponding to a value of λ between 0.3 and 0.4. This corresponded to the point at which the applied magnetic field was sufficient to suppress a majority of the natural convection heat transfer. The computed values of C_5 and C_6 for each case are listed in Table 1.

The data gathered for all three cases are compared with their predictions in Fig. 2.

5. DISCUSSION AND CONCLUSIONS

As shown in the preceding figure, the 2-dim. solution given by equation (8) predicts well both the reduction in local heat transfer (cases 1 and 2) and in overall heat transfer (case 3) that results from an applied magnetic field. Close agreement between theory and experiment is obtained in all three cases for values of λ between 1 and approximately 0.2. This range of λ is where natural convection is the predominant mode of heat transfer. For lower values of λ, theory and experiment do not compare well. This is anticipated because conduction has become the dominant mode of heat transfer and, therefore, equation (8) is no longer applicable.

The Lykoudis number at which the divergence between theory and experiment occurs will be referred to as the critical Lykoudis number. Its value (listed in Table 1) increases from cases 3 to 2 to 1. That is, the value of the critical Lykoudis number increases from orientations of increasing MHD interaction with the flow field around the cylinder. For a given magnetic field orientation, Gr, Pr and l/d, the critical Lykoudis number can be used to compute the flux density of the magnetic field that is required to suppress natural convection around the heated cylinder. In experiments with a fixed magnetic field orientation, if Gr is increased, e.g. by increasing the temperature difference between the cylinder and the fluid, the Hartmann number required to reach the critical Lykoudis number increases also.

There are limitations in the subject experiments that could affect the comparison between theory and experiment. As shown theoretically by Mahoney [12], the 2-dim. solution for natural convective heat transfer in the absence of a magnetic field can predict the heat transfer well provided the experimental conditions are such that the product $Gr^{1/2}(l/d)$ is much greater than unity. In the subject experiments, the values of that product ranged from approximately 10 to 8000. Some distortion of the velocity and temperature fields around the cylinder can result in experiments in which the ratio of the test cell's fluid height, t, to the cylinder's diameter, d, is less than 100 [19]. For cases 1 and 2 of the subject experiments this ratio was less than 100, as listed in Table 1. For horizontal cylinders with l/d less than approximately 1000, there will be some conductive heat loss to the cylinder's end supports that can lead to an overestimate of the amount of natural convective heat transfer [19].

Primarily because of the relatively low length-to-diameter ratios of the cylinders in the subject experiments, a more 'universal' expression governing MHD natural convective heat transfer from horizontal cylinders cannot be established

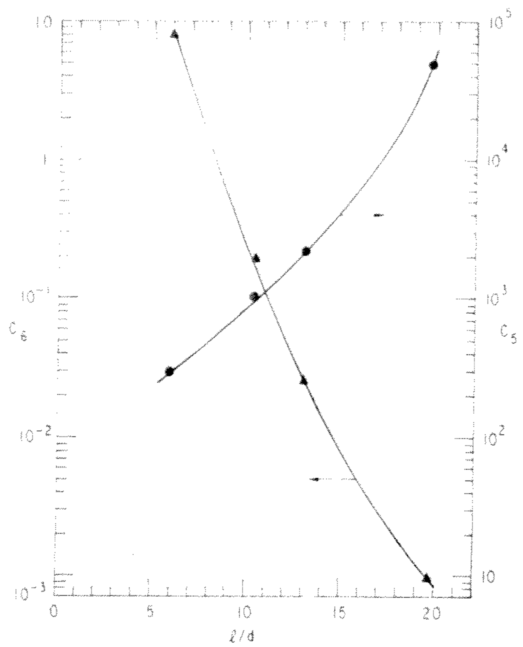

FIG. 3. Constants C_5 and C_6 vs l/d.

at present. This expression would be one in which the constants C_5 and C_6 are related explicitly to parameters such as Gr, Pr, l/d and γ. In the absence of a magnetic field, the natural convective heat transfer from a horizontal cylinder with a very large l/d is related to Gr, Pr and a constant, in a functional form similar to equation (7). For a cylinder with a lower l/d, however, the heat transfer also is related to l/d [19]. In this work, the l/d dependency is implicitly contained in the value of C_5 determined for each experiment. The values of C_5 are listed in Table 1. The relationship between the average value of C_5 for each l/d case and l/d is shown in Fig. 3.

For a horizontal cylinder with a very large l/d in the presence of a magnetic field, the natural convective heat transfer is related not only to Gr, Pr and a constant, but also to M and the orientation of the applied magnetic field. For a cylinder with a lower l/d, the heat transfer probably is related to l/d as well. In this work, both l/d and magnetic field orientation dependencies are implicitly contained in the value of C_6 determined for each experiment. The relationship between the average value of C_6 for each l/d case and l/d is shown in Fig. 3.

Based upon the findings of the present work and those of Mahoney [12] and Morgan [19], a 'universal' value of the constant C_5 could be obtained through experiments in which the conditions $l/d \gg 1$ and $Gr^{1/2}(l/d) \gg 1$ are met. To determine a 'universal' value of C_6, these two conditions must be met in the experiments as well as a third condition that $\lambda \gtrsim 0.5$, to assure that the mode of heat transfer is solely natural convection. The condition $\lambda \gtrsim 0.5$, in most cases, forces Ly to be small. Therefore, large values of M will be required in such experiments because Gr must be high, also, to satisfy the second condition.

It is concluded from the present study that the 2-dim. solution represented by equation (8) can predict the natural convective heat transfer from a horizontal cylinder in the presence of a magnetic field applied at various orientations with respect to the cylinder's axis.

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