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CONVECTIVE HEAT TRANSFER WITH ELECTRIC AND MAGNETIC FIELDS

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HANDBOOK OF SINGLE-PHASE CONVECTIVE HEAT TRANSFER

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dimensionless parameter

$$N\rho_c = \left(\frac{2I\epsilon}{A_w dK} \right)^{1/2} \frac{d^2}{\mu K} \tag{9.50}$$

As reported for tube flows, friction factors were found to increase at low Reynolds numbers with increasing corona current. No appreciable EHD effects were noted at Reynolds numbers above 1000.

9.5 BASIC CONCEPTS OF MAGNETOHYDRODYNAMICS (MHD)

9.5.1 Governing Equations

In classical MHD theory, the fluid is considered to be a continuum. The transport coefficients, e.g., electrical conductivity, are assumed to be isotropic and the fluid to be electrically neutral. It is assumed further that the dielectric constant ϵ and the permeability μ_e are scalars, the net space-charge density ρ_c is neglected, and the convection (displacement) and polarization currents are ignored.

When an electromagnetic field is applied to an electrically conducting fluid at rest, four forces can arise [85]. These are electrostatic (forces applied on particles with free electric charges), ponderomotive (the macroscopic summation of the elementary Lorentz forces applied on charged particles), electrostrictive (forces resulting from variations in the dielectric constant with the mass density of the fluid), and magnetostrictive (forces arising from variations in the magnetic permeability with the mass density of the fluid). Typically, in MHD the ponderomotive force is the only one of the above forces that is comparable to other hydrodynamic forces. (There are exceptions–for example, in electrostrictive natural convection, in which ϵ is a function of the mass density of the fluid [47, 67].) Maxwell's equations for the fixed (laboratory) reference frame [14], written for the rationalized MKS (m-kg-s) system or SI units [39], subject to the aforementioned idealizations, are

$$\nabla \cdot \mathbf{B} = 0 \tag{9.51}$$

$$\nabla \cdot \mathbf{D} = 0 \tag{9.52}$$

$$\nabla \times \mathbf{H} = \mathbf{J} \tag{9.53}$$

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \tag{9.54}$$

where

$$\mathbf{B} = \mu_e \mathbf{H} \quad \text{and} \quad \mathbf{D} = \epsilon \mathbf{E} \tag{9.55}$$

Ohm's law for this case is

$$\mathbf{J} = \sigma(\mathbf{E} + \mathbf{u} \times \mathbf{B}) \tag{9.56}$$

in which **E** is measured in the laboratory reference frame. As shown by this equation, the motion of a conducting fluid through an applied magnetic field contributes to the current density **J**. This, by virtue of Eqs. (9.53) and (9.55), implies that the applied field, in turn, will be altered. The resultant, or total, field is described by the magnetic induction equation,

$$\frac{\partial \mathbf{B}}{\partial t} = \frac{1}{\sigma \mu_e} \nabla^2 \mathbf{B} + \nabla \times (\mathbf{u} \times \mathbf{B}) \tag{9.57}$$

This equation is derived from Eqs. (9.56), (9.53), and (9.55) and illustrates the coupling of the electromagnetic and hydrodynamic fields. The electrical conductivity and density are assumed constant.

The governing hydrodynamic equations are the equations of conservation of mass, momentum, and energy. The momentum equation is

$$\rho \left[\frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u} \cdot \nabla)\mathbf{u} \right] = -\nabla p + \mu \nabla^2 \mathbf{u} + \rho \mathbf{g} + \mathbf{J} \times \mathbf{B} \tag{9.58}$$

Magnetic field interaction with the flow occurs through the ponderomotive force, $\mathbf{J} \times \mathbf{B}$.

The energy equation is

$$\rho c_p \frac{DT}{Dt} = -\nabla \cdot \mathbf{q}'' + \Phi + q''' + \frac{J^2}{\sigma} \tag{9.59}$$

The term J^2/σ represents the dissipative energy resulting from Joule heating of the conducting fluid. Thus, the effect of an applied magnetic field enters the energy equation explicitly through Joule heating and implicitly through the viscous-dissipation and convective terms.

9.5.2 Dimensionless Groups

The dimensionless momentum equation is

$$\rho \frac{D\mathbf{u}}{Dt} = -\nabla p + \frac{\mathrm{Gr}}{\mathrm{Re}^2}\theta + \frac{1}{\mathrm{Re}}\nabla^2 \mathbf{u}$$

$$+ \frac{M^2}{\mathrm{Re}}(\kappa \mathbf{E} \times \mathbf{B}) + \frac{M^2}{Re}(\mathbf{u} \times \mathbf{B} \times \mathbf{B}) \tag{9.60}$$

where all vector operators are dimensionless. Also,

$$M^2 = \frac{B_0^2 L^2 \sigma}{\mu} \propto \frac{\text{ponderomotive force}}{\text{viscous force}} \tag{9.61}$$

and

$$\kappa = \frac{E_0}{U_0 B_0} \propto \frac{\text{applied electric field}}{\text{induced electric field}} \tag{9.62}$$

The parameter M^2/Re in Eq. (9.60) represents the ratio of ponderomotive to inertia forces. This ratio is referrred to as the magnetic interaction parameter N. The direction of the applied electric field is specified by the sign of κ.

The dimensionless energy equation is

$$\rho \frac{D\theta}{Dt} = -\frac{1}{\mathrm{Pr\,Re}} \nabla \cdot \mathbf{q}'' + \frac{\mathrm{Ec}}{\mathrm{Re}} \Phi + \mathrm{Ec}\, q''' + \frac{\mathrm{Ec}\, M^2}{\mathrm{Re}} (\mathbf{E} + \mathbf{u} \times \mathbf{B})^2 \qquad (9.63)$$

in which the scaling for the energy source is $L/\rho u_0^3$.

The dimensionless magnetic induction equation is

$$\frac{\partial \mathbf{B}}{\partial t} = \frac{1}{\mathrm{Re}_m} \nabla^2 \mathbf{B} + \nabla \times (\mathbf{u} \times \mathbf{B}) \qquad (9.64)$$

For small values of Re_m, the applied field is altered solely by diffusion. For large values of Re_m, it is altered solely by convection. When $\mathrm{Re}_m = 0$, the electromagnetic and hydrodynamic equations are decoupled. In most terrestrial applications, $\mathrm{Re}_m \ll 1$ and weak interaction is assumed. Re_m, however, can approach a value of unity in large sodium electromagnetic flowmeters.

9.5.3 Basic Physics of Magnetic Field Effects in Electrically Conducting Liquids

As shown by the governing equations of laminar MHD flow, an applied magnetic field can affect the temperature of a liquid metal directly through Joule heating, and indirectly through altering the liquid metal velocity distribution and thereby convection and viscous dissipation. Joule heat generation is significant primarily in situations in which an electric field is applied externally or in which current flows through an external circuit or conducting duct walls. Usually viscous heat dissipation is negligible except in situations with very high velocity gradients, in which viscous and Joule heatings can be of the same order. In most situations concerning heat transfer between the liquid metal and a physical boundary, the heat transfer is affected by the alternation of the velocity gradient at the wall by the magnetic field. However, because liquid metals have low Prandtl numbers, the heat transfer is governed mostly by conduction. In turbulent MHD flow, heat transfer is affected primarily by the magnetic damping of turbulence.

Noticeable interaction of the magnetic field with the flow field occurs if the magnetic interaction parameter, $N = M^2/\mathrm{Re}$, is on the order of 1 or greater. The resulting ponderomotive force, $\mathbf{J} \times \mathbf{B}$, has two components that interact with the flow. The first component, $\sigma(\mathbf{u} \times \mathbf{B}) \times \mathbf{B}$, always acts to decelerate the flow. The second component, $\sigma \mathbf{E} \times \mathbf{B}$, can act either to accelerate or to decelerate the flow, depending upon the direction of $\mathbf{E}$. It will accelerate the flow if $\mathbf{E}$ is opposite in direction to $\mathbf{u} \times \mathbf{B}$. When $\mathbf{E} = -\mathbf{u} \times \mathbf{B}$, the current density $\mathbf{J}$, and hence Joule dissipation, become zero. In this case, a deceleration of the flow occurs and the heat transfer is decreased. In general, it is not straightforward to determine the change in heat transfer as the result of an applied magnetic field. In some flow situations, the application of a magnetic field can increase the heat transfer rate; in others, it can decrease the rate.

9.6 MHD IN CONFINED FLOWS

9.6.1 Channel Flow

The most basic channel-flow problem is the Hartmann problem [105]. This is a one-dimensional incompressible laminar flow problem in which the spacing between the wall electrodes is large compared to that between the insulating side walls. The flow is fully developed, and no axial currents exist. The magnetic field is applied normal to the side walls. An electric field is established externally between the electrodes.

Romig [105] examined specialized cases of the Hartmann problem in which the mass flow is held constant as M and κ are varied, and in which κ is held constant as M is varied. As M is increased, convection near the wall increases and the temperature becomes more uniform. Internal Joule and viscous-dissipation heating also increase with increasing M. Viscous dissipation is maximum at the wall. The magnitude of Joule heating also depends on κ. When $\kappa = -1$ (the electrically insulated case), internal heating occurs very close to the walls. When $\kappa = 0$ (the open-circuit case) most of the Joule heating occurs near the center of the channel.

Blum et al. [3, 5] theoretically investigated the case of heat transfer for developed Hartmann flow through a channel with electrically insulating walls in a transverse magnetic field. For cases of either constant wall temperature or constant wall heat flux, the application of a magnetic field increases the heat transfer by approximately 30 and 45%, respectively. Most of this initial rise in heat transfer occurs when $M \leq 100$. This is indicative of the Hartmann effect, i.e., the flattening of the velocity profile with a concomitant increase in the velocity gradient at the wall.

Various aspects of heat transfer in a spatially developing laminar flow between parallel conducting walls with various applied magnetic field orientations have been considered theoretically by Rajaram and Yu [97]. Similar theoretical entrance studies with an applied transverse magnetic field have been conducted by Perlmutter and Siegal [94], Hsia [37], and others [97]. The entrance length for flow development was found to depend upon M, Re_m, $\hat{\phi}$, and the direction of the applied magnetic field. The entrance length for velocity decreased with increasing strength of an applied transverse field, whereas with a strong field applied parallel to the flow, it was found to increase even beyond the length for the ordinary hydrodynamic case. The inclination of the magnetic field was determined to have no effect on the heat transfer for low Pr with constant wall temperature. For low Pr with constant wall heat flux, the heat transfer depended weakly on the field inclination, increasing slightly as the inclination increased toward the parallel-field case.

The change in heat transfer for a rectangular channel that occurs in the transition region from laminar to turbulent flow has been measured by Kovner et al. [44]. The data (Fig. 9.21) reveal that the maximum reduction in heat transfer occurs at a Reynolds number equal to approximately twice the critical Reynolds number at a given M. The critical Reynolds number is that at which the flow becomes fully developed turbulent. The values of Re_{cr} are given in [8]. The magnitude of this reduction in heat transfer is proportional to M and decreases as M^2/Re decreases.

The case of turbulent flow between two parallel walls with constant heat flux in a transverse magnetic field was considered analytically by Krasil'nikov [46]. He obtained an expression for the fluid velocity from the semiempirical theory of Kovner [45]. Heat transfer results were approximated by the semiempirical formula [46]

$$\mathrm{Nu} = 10.0 + 0.025\left(\frac{\mathrm{Pe}}{1 + (236M^2/\mathrm{Re})}\right)^{0.8} \tag{9.65}$$

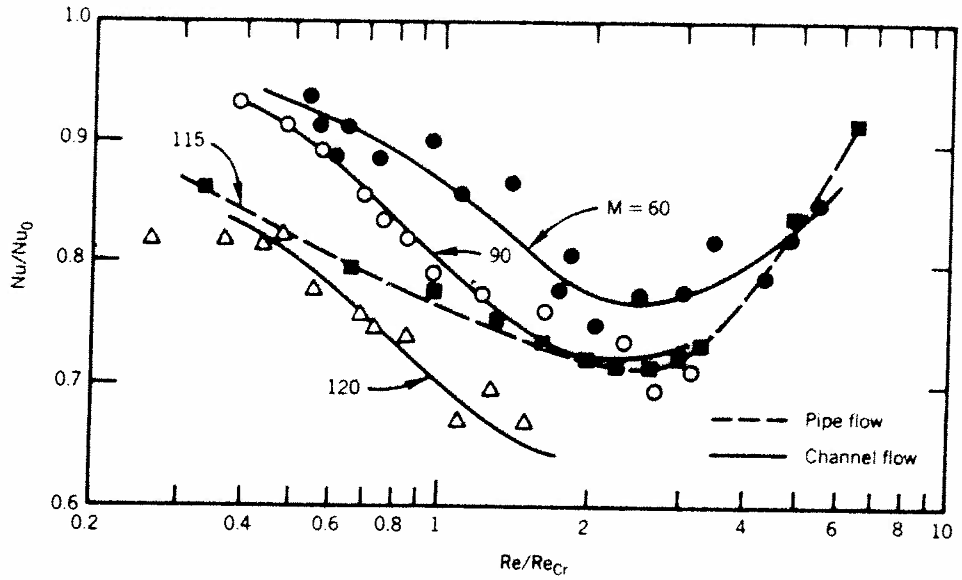

Figure 9.21. Heat Transfer in a transverse magnetic field during laminar-to-turbulent flow transition (adapted from Ref. 8).

in which the characteristic length is the channel width, and the heat transfer coefficient in Nu is based upon the difference between the wall and bulk mean fluid temperatures.

Branover [8] cited experiments performed by Krasil'nikov to study the effect of a longitudinal magnetic field on heat transfer in turbulent rectangular plane-parallel channel flow using gallium (Pr = 0.019). The data obtained for both the ordinary hydrodynamic ($M = 0$) and MHD ($M = 120$) cases are shown in Fig. 9.22. For a constant Pe, application of the longitudinal field was found to reduce the heat transfer. This reduction was caused by the suppression of turbulence and the average velocity gradient near the wall by the magnetic field. Over the range of gathered data

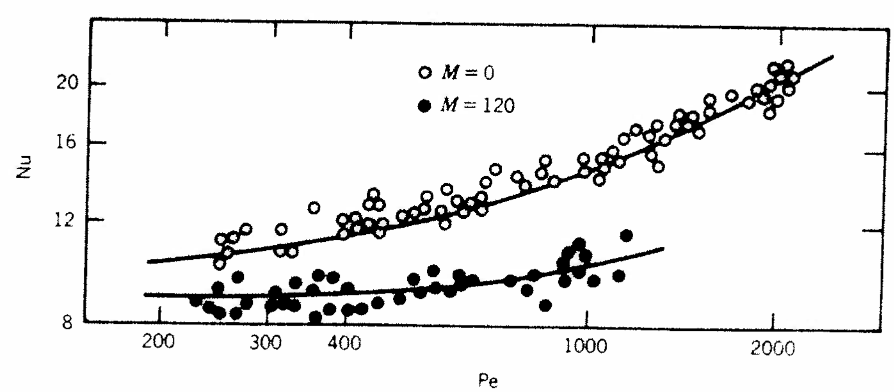

Figure 9.22. Heat transfer data for turbulent flow in a high-aspect ratio rectangular duct in a transverse magnetic field (adapted from Ref. 8).

($8 \leq Nu \leq 12$, $M = 120$, and $200 \leq Pe \leq 1200$), the MHD data were best approximated by

$$Nu = 9.0 + \frac{0.006\,Pe}{1 + (14.8M^2/Re)} \tag{9.66}$$

in which the characteristic length is the channel width. The heat transfer coefficient was determined from the heat flux through the wall, the difference between the wall inside surface temperature and the mean mixed temperature of the liquid. The characteristic temperature for the thermal conductivity was the arithmetic mean of the test-section inlet and outlet liquid temperatures.

Heat transfer experiments on combined free- and forced-convection flow through a vertical channel with conducting walls under the influence of a transverse magnetic field were conducted by Yang and Yu [140]. Application of the field to turbulent flow was found initially to reduce the heat transfer by suppressing free convection and turbulence. At higher field strengths, the flow became laminarized and the heat transfer increased because of the Hartmann effect. The point of minimum heat transfer was found to decrease linearly with increasing Gr/Re^2, i.e., from $Re/M = 217$ at $Gr/Re^2 = 0$ to $Re/M = 80$ at $Gr/Re^2 = 0.4$.

9.6.2. Pipe Flow

As in the case of channel flow, the heat transfer for laminar flow through a pipe with electrically insulating walls increases in the presence of an applied transverse magnetic field. Mittal [76] examined the intermediate Hartmann number cases of $M = 0.8$, 2.0, 2.8, and 4.0, and found that the temperature profile and heat flux at the wall acquire an angular dependence because of the symmetry of the applied transverse field. Increases in the local Nusselt number as much as 100% occurred when $M = 4.0$.

The analytical results of Gardner [27] for a constant wall heat flux show that the average Nusselt number increases approximately 60% as M increases from 0 to 500. (Here the characteristic length for Nu and M is the pipe diameter.) Most of the increase occurs from $1 < M < 100$. The Hartmann number range over which this increase occurs is the same as that predicted by Blum et al. [3, 5] for the analogous channel-flow case.

Experimental heat transfer studies for transition and moderate turbulent flow in an electrically insulated pipe were conducted by Gardner et al. [29] using mercury ($Pr = 0.023$). These results are shown in Fig. 9.23, in which the characteristic length chosen was the pipe diameter. The data for $Re < 10^4$ show no effect on heat transfer, because heat is transferred primarily by conduction. As Re is increased, a decrease in heat transfer then occurs because of the damping of turbulence by the magnetic field. For higher Re, Nu increases because the inertial force becomes much larger than the ponderomotive force and turbulent mixing dominates.

In the turbulent flow regime, experiments were carried out by Gardner and Lykoudis [28]. The effect of a transverse magnetic field on local and average heat transfer was measured for flow through an electrically insulated pipe with constant wall heat flux. For $Re < 50{,}000$, the local heat transfer coefficient depends upon angular orientation with respect to gravity and the applied field. This free-convection effect exists up to $Re = 315{,}000$. As the strength of the magnetic field increased, the centerline temperature of the fluid was lowered, and the temperature near the wall increased. These results demonstrate that the overall influence of the applied field is to inhibit convective heat transfer.

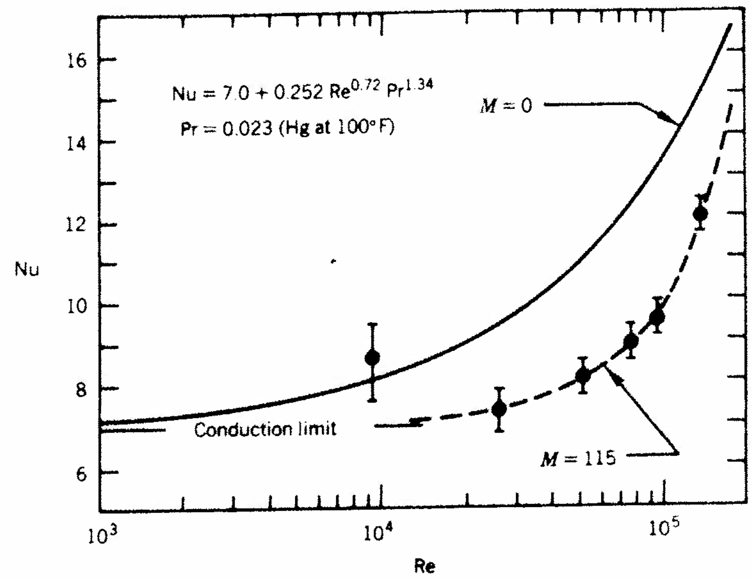

Figure 9.23. Heat transfer during laminar-to-turbulent flow transition for pipe flow in a transverse magnetic field [29].

The effect of a longitudinal magnetic field on heat transfer for turbulent flow of gallium (Pr = 0.019) in an electrically insulated pipe was measured by Kovner et al. [44]. Their results are shown in Fig. 9.24. The overall effect of the longitudinal field is to decrease the heat transfer rate. For low values of Pe ($\leq$ 200), the magnitude of this effect decreases. At Pe $\approx$ 700, suppression of the heat transfer rate is greatest. For high values of Pe ($\geq$ 2000), the applied field has little effect, primarily because the inertial force becomes much greater than the ponderomotive force, i.e., M^2/Re becomes low.

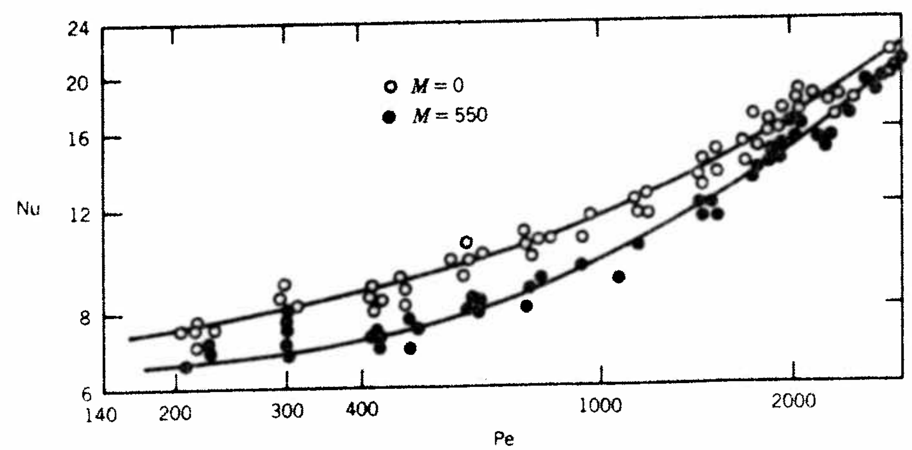

Figure 9.24. Heat transfer data for turbulent pipe flow in a longitudinal magnetic field (adapted from Ref. 8).

The experimental heat transfer results are best described by the expression

$$\text{Nu} = 6.5 + \frac{0.005\,\text{Pe}}{1 + 1890(M/\text{Re})^{1.7}} \tag{9.67}$$

in which the characteristic length is the pipe diameter. (Here the Nusselt number is for fully developed turbulent flow based upon the difference between the inside wall surface temperature fluid bulk mean temperature.) The heat transfer coefficient is determined in the same manner as that for Eq. (9.65) and the thermal conducitivity is based upon the bulk mean temperature. The mean temperature ranged from 22 to 33 K, and the temperature difference from 1.4 to 2.5 K. A theoretical expression similar to the above was developed by Lykoudis [65], based upon his theory of turbulence damping due to the presence of a magnetic field [143, 144]:

$$\text{Nu} = \text{Nu}_c + \frac{\text{Nu}_0 - \text{Nu}_c}{1 + (250M^2/\text{Re}^{1.75})} \tag{9.68}$$

in which the characteristic length is the pipe diameter, Nu_c is the Nusselt number value for pure conduction ($\text{Nu}_c \approx 7.0$), and Nu_0 is that for the ordinary hydrodynamic case. Here, Nu is based upon a heat transfer coefficient defined in terms of the difference between the inside wall temperature and the bulk mean temperature, and the thermal conductivity is based upon the bulk mean temperature. This expression also agrees well with the data of Kovner et al. [44].

9.7 MHD IN EXTERNAL FLOWS AND IN NATURAL CONVECTION

The effect of an applied magnetic field on heat transfer in external flows has been investigated mainly for the cases of flat-plate boundary-layer and blunt-body stagnation-point flows. The works published in these areas are theoretical and appeared in the late 1950s and early 1960s with application to space-vehicle surface heating upon reentry. Because the air in this situation was ionized and therefore conducting, it was envisioned that the application of a transverse magnetic field could be utilized to reduce the local velocity and skin friction drag and thereby the heat transfer to the vehicle's surface.

The classical works in these areas have been reviewed thoroughly by Romig [105]. In particular, the reader is referred to papers by Rossow [145], Bush [146], and Lykoudis [147, 148]. These include the theoretical treatments of MHD heat transfer of flow over a flat plate for the incompressible case, assuming a constant magnetic flux density and either a constant or a variable electrical conductivity [145], and for the compressible case, assuming variable electrical conductivity and variable magnetic flux density [146], constant electrical conductivity, and either constant or variable magnetic flux density [147], or variable electrical conductivity and constant magnetic flux density [148]. These studies show in general that as the boundary layer develops the heat transfer is reduced, and more specifically that the heat transfer is affected by variations of electrical conductivity with temperature, of temperature with velocity, and of magnetic flux density with distance, as well as by the temperature of the surface.

In MHD free convection, the application of a magnetic field reduces the magnitude of heat transfer because ponderomotive forces retard the motion induced by buoyancy. The reader is referred to a recent review article by Lykoudis [142], which covers the

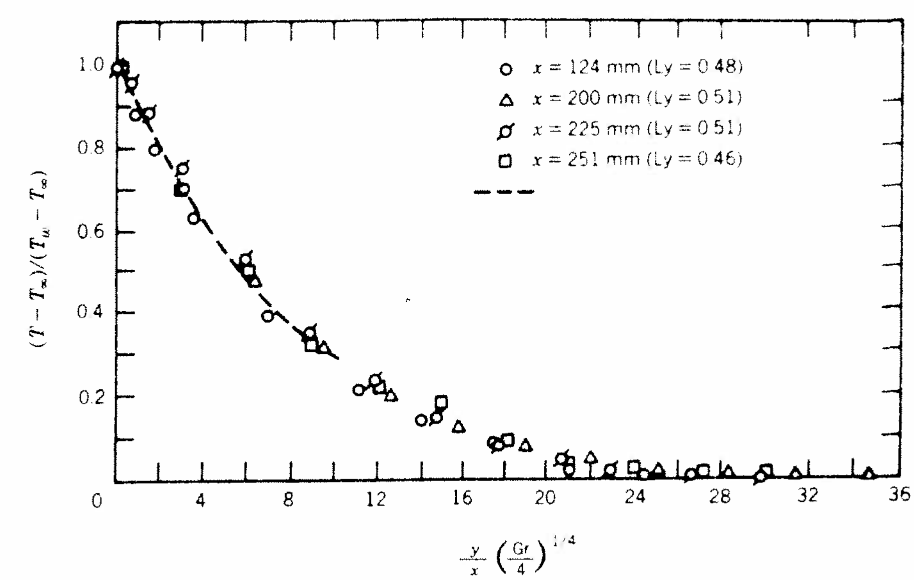

Figure 9.25. Temperature profiles for a heated vertical plate in a spatially varying horizontal magnetic field [87].

same subject material. Free convection heat transfer from a heated vertical plate to a liquid metal has been studied both theoretically and experimentally.

Sparrow and Cess [115] examined the case of laminar free convection from a heated vertical plate with a constant magnetic field normal to the plate. Their results were recast [105] in terms of a parameter equal to the mean Nusselt number divided by $Gr^{1/4}$. This parameter decreases from its hydrodynamic value in direct proportion to the Lykoudis number, which represents the ratio of the ponderomotive force to the square root of the product of the buoyancy and inertia forces. Previously in the literature (e.g., [105, 110, 141]), the Lykoudis number was defined as $2M^2/\sqrt{Gr}$. Recently, however, it has been defined as $Ly = M^2/\sqrt{Gr}$ [133, 142]. The more recent definition is used herein.

Similarity solutions were obtained [30, 64] for conditions like the aforementioned case but with the applied magnetic field varying as $x^{-1/4}$ in the vertical direction, x being the distance from the leading edge. Experimental confirmation of these solutions was obtained by using mercury as the working fluid [87]. Experimental results for the case of $Ly \approx 0.5$ are compared with the theory in Fig. 9.25. At a given value of the similarity coordinate, the dimensionless temperature was found to decrease with increasing magnetic flux density (not shown in Fig. 9.25). For values of Ly up to 1.2, similarity appeared to be maintained, although no exact theoretical solution was available for comparison.

Romig [105] has compared the theoretical predictions of the mean heat transfer parameter for the $x^{-1/4}$ similarity case with that of a constant applied magnetic field. She found that for liquid metals when $Ly < 0.5$, the mean heat transfer is not reduced as effectively as when the field is variable.

Seki et al. [110] conducted both experimental and numerical studies on the heat transfer from a vertical plate with uniform heat flux for the case in which the magnetic field was applied parallel to gravity. The magnetic field increases the surface temperature, and thereby increases Gr and decreases Nu. Data for $M/\mathrm{Gr} < 6 \times 10^{-6}$ ($0 \le M \le 400$, $2 \times 10^7 < \mathrm{Gr} < 5 \times 10^8$) were best approximated by

$$\frac{\mathrm{Nu}}{\mathrm{Nu}_0} \approx 1 - 1.3 \times 10^5 \frac{M}{\mathrm{Gr}} + 7.5 \times 10^9 \left(\frac{M}{\mathrm{Gr}} \right)^2 \qquad (9.69)$$

in which the characteristic length is the heating-surface half height, and the characteristic temperature difference used to determine the heat transfer coefficient is that between the surface temperature at half height and the cold wall, which ranged from 0 to 60 K. These results may be limited to cases having a similar ratio (≈ 0.4) of heated section height to spacing between the hot and cold walls, because of possible thermal interference by the cold wall. Compared to the theoretical predictions of Sparrow and Cess [115] for the case of a field applied normal to the wall, the overall reduction in heat transfer for the parallel field case is less at a given value of Ly.

Papailiou and Lykoudis [88] conducted experiments on a free-convection turbulent boundary layer along a vertical wall with constant heat flux subjected to an applied, horizontal magnetic field. The magnetic field reduced convective heat transfer along the plate. The thermal boundary-layer temperature and thickness increased with the strength of the applied field. Heat transfer coefficients were expressed in terms of total Nusselt numbers based upon the length of the heated wall and the average temperature difference between the wall and free stream along the boundary layer. These data are shown in Fig. 9.26. As Ly increases, a reduction in the overall heat transfer coefficient occurs. The change of slope in the curve at Ly $\approx$ 0.33 corresponds to laminarization of the turbulent flow. Based upon measured mean temperature profiles, turbulence intensity distributions, and temperature spectra along the wall, the transition from turbulent to laminar flow occurs at a constant value of the ratio of $\mathrm{Gr}\,\mathrm{Pr}/M$. For six experimental cases, $\mathrm{Gr}\,\mathrm{Pr}/M = 1.2 \times 10^9$. Further analysis [86] showed that below this value a rapid drop in turbulence intensity occurs, as well as marked changes in the turbulence structure.

A relation between the overall heat transfer and Ly has been found also for the case of the natural convection of mercury in a vertical cylindrical container with a heated bottom surface. In experiments by Wagner [133], data were obtained at various saturated pressures in the presence of an applied horizontal magnetic field (Fig. 9.27).

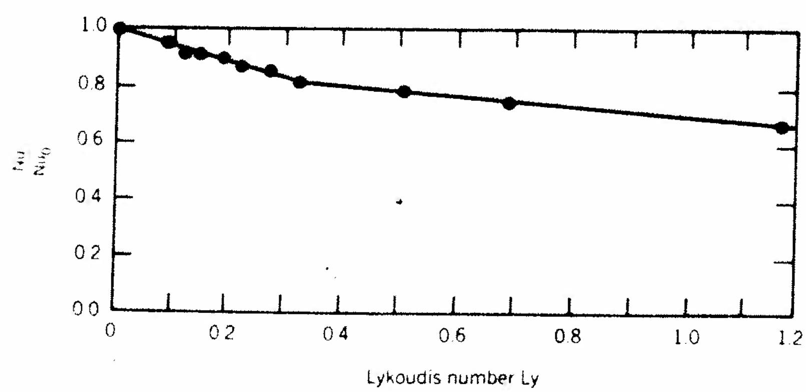

Figure 9.26. Overall heat transfer in laminar and turbulent regimes for a heated vertical plate in a transverse magnetic field [88]

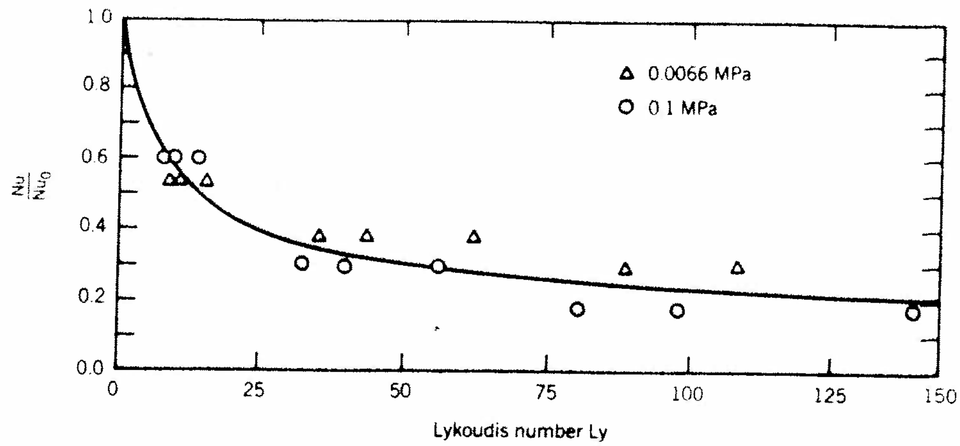

Figure 9.27. Overall heat transfer for natural convection from a heated horizontal surface inside a vertical cylinder in a transverse magnetic field (adapted from Ref. 133).

Reductions of up to 80% in the Nusselt number were measured. The correlation best describing the data is [142]

$$\frac{\mathrm{Nu}}{\mathrm{Nu}_0} = \frac{1}{(1 + 0.15\,\mathrm{Ly})^{0.5}} \tag{9.70}$$

Here, the heat transfer coefficient in Nu is based upon the difference between the temperature of heat transfer surface and liquid bulk mean temperature.

Recent experiments on natural convection heat transfer from finite horizontal cylinders treat magnetic field orientations in all three directions normal to the axis of the cylinder. Measurements in mercury with the magnetic field oriented along the axis were reported by Lykoudis and Dunn [66]. Detailed local heat transfer measurements in mercury for the other two field orientations have been presented by Michiyoshi et al. [72]. Blum and Kronkalns [4] reported data on free-convection heat transfer between a horizontal cylinder and a ferroliquid with the magnetic field normal to the axis of the cylinder. Similar experiments were reported by Kronkalns and Blum [48] for a high-Pr lithium-ammonia solution with the magnetic field normal to the axis and parallel to gravity. For all these cases, the application of the magnetic field reduced natural convection heat transfer from the cylinder.

In the experiments of Lykoudis and Dunn [66], the magnetic field suppressed free convection to the conduction limit. In the experiments of Michiyoshi et al. [72], for a fixed Gr, the Nusselt number (determined from temperature measurements around the cylinder circumference) decreased with increasing M. This decrease gradually levelled off at high M. Dunn [22] derived a semiempirical expression that correlated the data from both these experiments:

$$\frac{\mathrm{Nu} - \mathrm{Nu}_c}{\mathrm{Nu}_0 - \mathrm{Nu}_c} = \left[\frac{\mathrm{Ly}}{\sqrt{C_6}} + \left(1 + \frac{\mathrm{Ly}^2}{C_6}\right)^{1/2}\right]^{1/2} \tag{9.71}$$

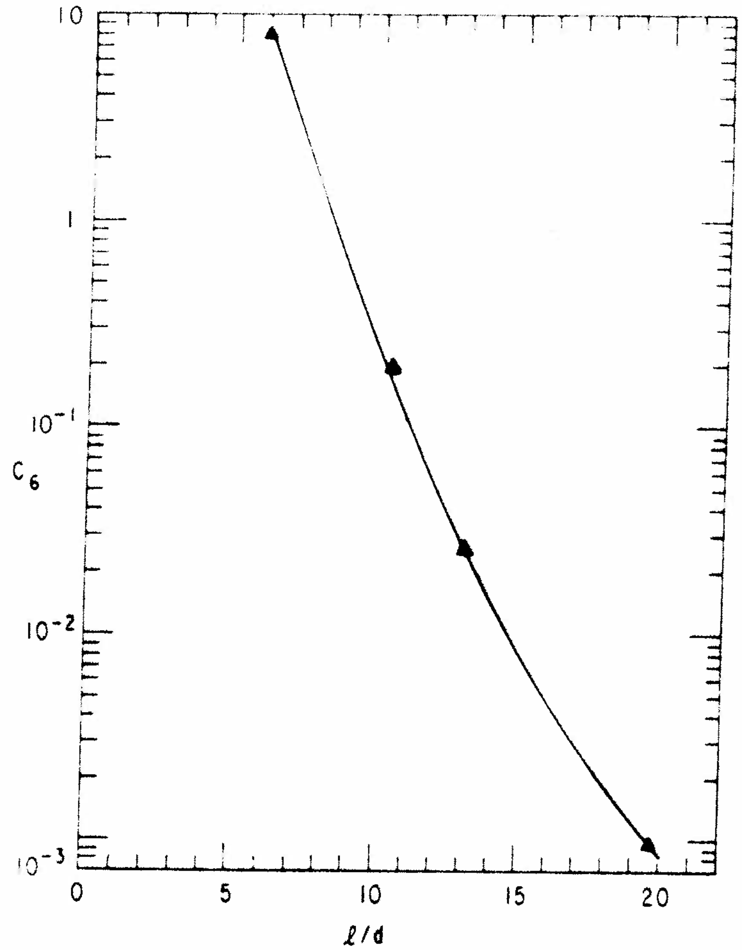

Figure 9.28. Empirical constant C_6 versus cylinder aspect ratio l/d [22].

in which C_6 is a function of the cylinder aspect ratio as shown in Fig. 9.28 [22] and the characteristic length is the cylinder diameter. This expression is compared with the experimental data in Fig. 9.29, in which cases 1 and 2 are from Ref. 72 and case 3 from Ref. 66. For cases 1 and 2, Nu represents a local Nusselt number whose heat transfer coefficient is based upon the temperature difference between the cylinder surface and the ambient mercury. This difference ranged from 0 to 80 K. For case 3, Nu represents an average Nusselt number based upon the temperature difference between the probe's surface (as determined from the probe's overheat ratio) and the ambient mercury. This difference varied from 12 to 27 K. The semiempirical expression predicts the reduction in both local heat transfer and overall heat transfer. Close agreement between theory and experiment is obtained in all three cases for $(Nu - Nu_c)/(Nu_0 - Nu_c) \geq 0.2$. For lower values, theory and experiment do not compare well because conduction has become the dominant mode of heat transfer.

The effect of a magnetic field on forced convection from a horizontal cylinder (a hot-film probe) was measured also in the experiments of Lykoudis and Dunn [66]. In

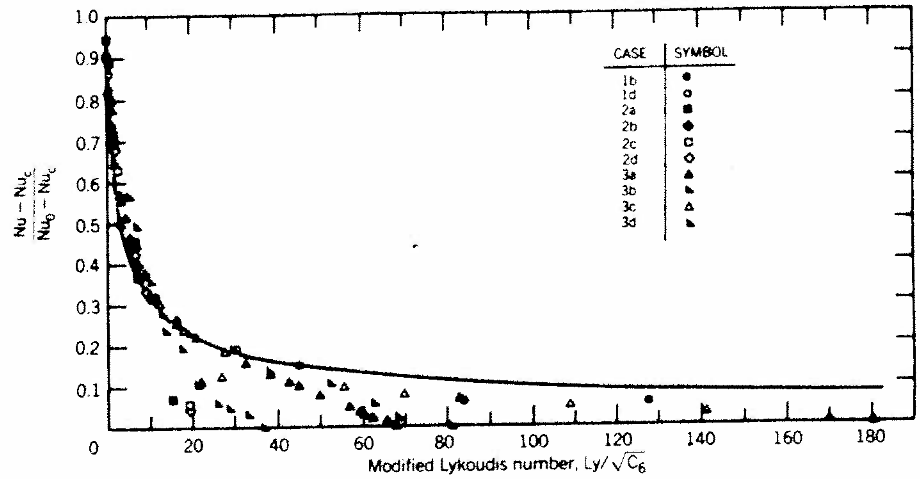

Figure 9.29. Heat transfer for natural convection from heated horizontal cylinders in magnetic fields of different orientations (adapted from Ref. 22).

their experiments, the value of the interaction parameter N was of order 1. Heat transfer data were gathered over the ranges $0 < \mathrm{Re} < 130$ and $0 < M < 4.7$. Their results are shown in Fig. 9.30. For a given value of Re, the heat transfer from the probe decreased with increasing M. Decreases of up to 50% in the Nusselt number were measured at Re = 100. Similar experiments were conducted at low values (< 0.005) of the interaction parameter N by Platnieks [96], and no effect was found.

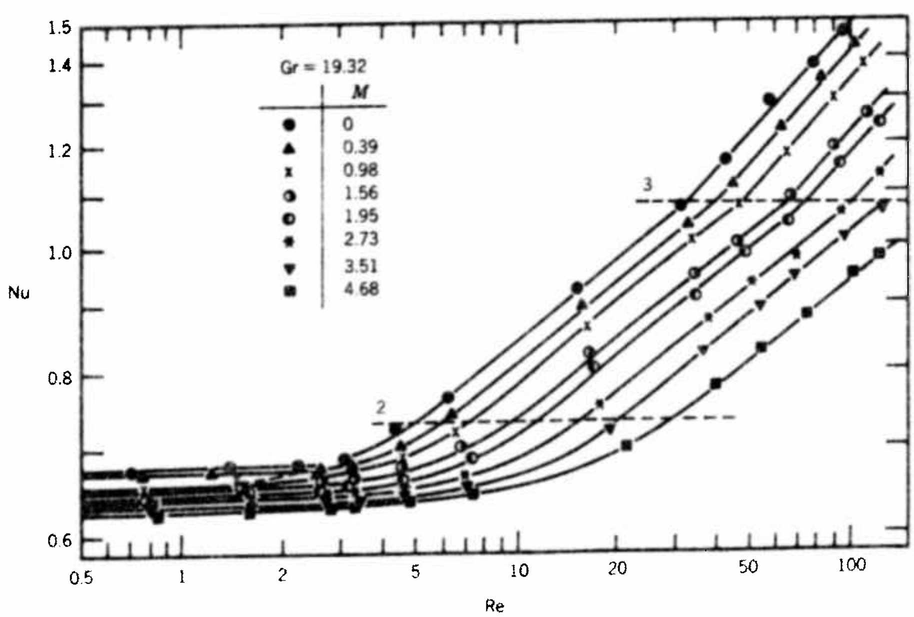

Figure 9.30. Heat transfer for forced convection from a horizontal cylindrical hot-film probe in an axially aligned magnetic field [66].

NOMENCLATURE

A_w	wall area, m^2, ft^2
B	magnetic flux density, T, Wb/m^2, Wb/ft^2
C, C^*	constants depending on geometry
C_6	empirical constant
c_p	specific heat, J/(kg · K), Btu/(lb_m · °F)
D	dielectric displacement field, C/m^2, C/ft^2
d	spacing between electrode and corona discharge source, m, ft
$\mathbf{E}$	electric field vector (E_x, E_y, E_z), V/m, V/ft
E	average electric field, V/m, V/ft
E_0	threshold electric field, V/m, V/ft
Ec	Eckert number = $U^2/c_p\,(T_2 - T_1)$
F	load factor
$\mathbf{F}_e$	electric body force, C/(m^2 · s), C/ft^2 · s)
f	Fanning friction factor = $\tau_\omega/(\rho U^2/2)$
f_0	Fanning friction with no electric or magnetic field
g	gravitational acceleration, m/s^2, ft/s^2
Gr	Grashof number = $\rho^2 g\beta\,\Delta T\,L^3/\mu^2$
H	magnetic field intensity, A/m, A/ft
h	heat transfer coefficient, W/(m^2 · K), Btu/(h · ft^2 · ° F)
h_0	stagnation heat transfer coefficient, W/(m^2 · K), Btu/(h · ft^2 · °F)
h_x	local heat transfer coefficient, W/(m^2 · K), Btu/(h · ft^2 · °F)
I	electric current, A
I_0	electric current scale, A
i	enthalpy per unit mass, J/kg, Btu/lb_m
$\mathbf{J}$	current density (J_x, J_y, J_z), A/m^2, A/ft^2
J	magnitude of current density, A/m^2, A/ft^2
K	ion mobility, m^2/(V · s), ft^2/(V · s)
k	thermal conductivity, W/(m · K), Btu/(h · ft · °F)
L	characteristic length, m, ft
Ly	Lykoudis number = $M^2/\sqrt{\mathrm{Gr}}$
L_0	length scale, m, ft
M	Hartmann number = $BL\sqrt{\sigma/\mu}$
N	Interaction parameter = M^2/Re
Ne	dimensionless electric number, Eq. (9.15)
Ne*	dimensionless number, Eq. (9.48)
Nu	Nusselt number = hd/k
Nu_0	Nusselt number with no electric or magnetic field
N_ρ	dimensionless charge number, Eq. (9.50)
P	modified pressure, Eq. (9.10), Pa, lb_f/ft^2
p	pressure, Pa, lb_f/ft^2
Pe	Péclet number = Pr Re

Pr	Prandtl number = ν/α
q'''	volumetric heat generation, W/m^3, Btu/(h · ft^3)
q''	heat flux, W/m^2, Btu/(h · ft)
q''_w	magnitude of wall heat flux, W/m^2, Btu/(h · ft^2)
Re	Reynolds number = UL/ν
Re_{cr}	critical Reynolds number
Re_{EHD}	electrical Reynolds number, Eq. (9.17)
Re_m	magnetic Reynolds number, $\sigma\mu_e UL$
R_1	inner radius, m, ft
R_2	outer radius, m, ft
r	radius, m, ft
T	temperature, °C, K, °F, °R
T_w	wall temperature, °C, K, °F, °R
T_1, T_2	reference temperature, °C, K, °F, °R
T_∞	free-stream temperature, °C, K, °F, °R
t	time, s
U	bulk velocity, m/s, ft/s
U_c	characteristic corona wind velocity, m/s, ft/s
U_0	velocity scale, m/s, ft/s
u	velocity vector (u, v, w), m/s, ft/s
V	voltage, V
V_i	ion drift velocity, m/s, ft/s
X	dimensionless length, Eq. (9.27)
x	Cartesian coordinate, m, ft
y	Cartesian coordinate, m, ft

Greek Symbols

α	thermal diffusivity, m^2/s, ft^2/s
β	coefficient of thermal expansion, K^{-1}, °R^{-1}
δ	boundary-layer thickness, m, ft
δ_t	thermal boundary-layer thickness, m, ft
ϵ	dielectric constant, gas permittivity, C/(m · V), C/(ft · V)
ϵ_0	permittivity of free space, C/(m · V), C/(ft · V)
θ	dimensionless temperature, Eq. (9.13)
κ	(applied electric field)/(induced electric field)
λ	linear charge density, C/m, C/ft, or point charge density, C/m^2, C/ft^2
μ	dynamic viscosity, Pa · s, lb$_m$/(h · ft)
μ_e	magnetic permeability, H/m, H/ft
ν	kinematic viscosity = μ/ρ
ρ	density, kg/m^3, lb$_m$/ft^3
ρ_c	space-charge density, C/m^3, C/ft^3
ρ_{c0}	reference space-charge density, C/m^3, C/ft^3

σ	electrical conductivity, S/m, S/ft
τ_w	fluid shear stress at wall, Pa, lb_f/ft^2
χ	dimensionless space-charge number, Eq. (9.15)
ϕ	electric potential, V
ϕ_0	reference or threshold electric potential, V
$\hat{\phi}$	conductance ratio

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