Aerosol Science and Technology
Publication details, including instructions for authors and subscription information:
http://www.informaworld.com/smpp/title~content=t713656376

Models of Rebound and Capture for Oblique Microparticle Impacts

Raymond M. Brach [a];Patrick F. Dunn [a]
[a] DEPARTMENT OF AEROSPACE AND MECHANICAL ENGINEERING, UNIVERSITY OF NOTRE DAME, NOTRE DAME, IN, USA

First published on: 01 January 1998

ELSEVIER

Models of Rebound and Capture for Oblique Microparticle Impacts

Raymond M. Brach and Patrick F. Dunn*
DEPARTMENT OF AEROSPACE AND MECHANICAL ENGINEERING, UNIVERSITY OF NOTRE DAME, NOTRE DAME, IN 46556, USA

ABSTRACT. Simple algebraic, rigid body impact models using coefficients of restitution and coefficient of friction have been used extensively in the study of the impact of microparticles with surfaces. Recent work by the authors has shown that rigid body impact theory can be extended to include an adhesion coefficient to model oblique impacts of microparticles with surfaces in the presence of molecular level forces. In this paper, the model is fully described and exploited for engineering applications. The behavior of coefficients is investigated both analytically and experimentally as initial impact velocity and angel of incidence vary. As the initial velocity of microparticles grows smaller and smaller, the significance of adhesion forces increases, eventually reaching a point where no rebound occurs and the particle is captured. Equations for this region of the growth of the influence of adhesion, designed for empirically fitting, are presented. They are capable of representing the behavior of the material restitution coefficient and the adhesion coefficient for a wide variety of materials and conditions. Experimental results of several investigators are examined using these empirical equations. They not only do a remarkable job of representing adhesion in the transition from rebound to capture but are capable of explicitly determing the capture velocity. The paper also contains a study of the sensitivity of the impact coefficients to small changes in the angle of incidence. The dependence of the capture velocity on microsphere radius also is examined. AEROSOL SCIENCE AND TECHNOLOGY 29:379–388 (1998) © **1998 American Association for Aerosol Research**

INTRODUCTION

When a microparticle impacts a surface, it either rebounds from or remains attached to the surface. Capture (also referred to as attachment or deposition) occurs provided the combination of material/adhesion dissipation and molecular-level adhesion forces are high enough. Dahneke (1975), Wall et al. (1990), Dunn et al. (1995), and others have measured microparticle impacts and observed that the coefficient of restitution typically shows a sudden drop as the initial normal velocity decreases. The detailed behavior depends on the size of the particle, the material properties of the surface and particle, including the Dupré surface energy, the surface roughness, and other things. For velocities below a certain value, called the capture (or critical) velocity, the particle remains attached to the surface. One of the problems encountered when analyzing this phenomenon is that the exact value of the

*Corresponding author.

Aerosol Science and Technology 29:379–388 (1998)

Published by Elsevier Science Inc.

0278-6826/98/$19.00
PII S0278-6826(98)00048-8

capture velocity is difficult if not impossible to measure directly. Ordinarily, the experimentally determined coefficient of restitution curve is projected or extrapolated to its zero crossing to determine its value. Another problem is that reasonably accurate and general theoretical models that predict the critical velocity do not exist, partly because little is known about dynamic dissipation both in the materials and of the adhesion process itself. In fact, until recently (see Chen et al., 1991) almost all of the dissipation associated with particle capture was attributed to material dissipation alone, such as plastic deformation. Other problems exist when trying to model microparticle capture, such as the influence of tangential effects (friction) and rigid body rotation when the collision is oblique. Despite the lack of detailed knowledge of most aspects of the mechanics of microsphere impact, a need exists for application models. A goal here is to develop *engineering* models for the incident velocity region of oblique microparticle impact at or near capture.

The approach followed here is to develop a set of equations whose constants are empirical. The resulting equations can be used individually as well as in combination with an existing rigid body impact model. The model uses separate coefficients of restitution related to material dissipation and adhesion dissipation; the empirical equations likewise are set up to determine the material and adhesion restitution. In addition, the empirical equations are configured in a way such that an explicit value of the capture velocity can be determined. The constants that appear in the empirical equations do not have to be found using experimental data; other theories and models of microparticle impact can be used as well to establish their value. The combination of the rigid body impact model and the empirical equations can be used in various ways. One application included in this paper is to carry out a parameter variation analysis to explore the sensitivity of surface roughness on the process of capture.

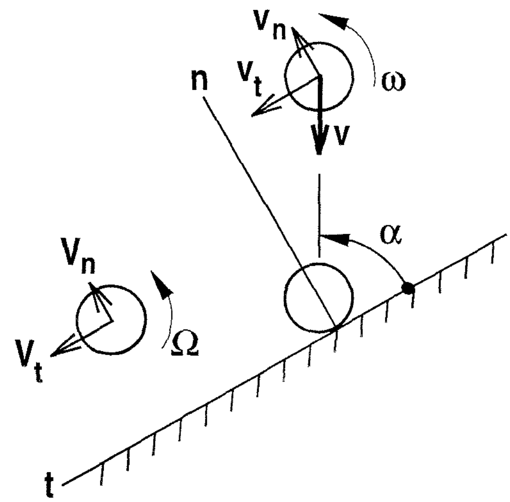

FIGURE 1. Impact configuration, variables, and coordinates; lower case symbols represent initial velocities and upper case symbols represent final values.

SUMMARY OF THE RIGID BODY IMPACT MODEL

The derivation of the equations of the rigid body impact model are presented elsewhere (see Brach et al., 1992, 1995) and is not repeated here. The following is a description of some of the model's features. Consider the planar, oblique impact of a sphere as illustrated in Fig. 1. Newton's equations of motion (in the form of impulse and momentum) provide equations, one for each of the coordinates, n, t, and θ. For the simplest problem formulation (point contact) two other equations come from the definitions of the coefficient of restitution (normal) and an impulse ratio (tangential). These five equations allow for the solution of five unknowns that are three final velocity components and two impulse components. A problem exists here when applying this to microsphere impacts because the usual definition of the coefficient of restitution contains all of the information about the normal contact process, confounding the material restitution/dissipation with the restitution/dissipation of the adhesion process.

To resolve this dilemma a second coefficient is introduced. Consider the normal impact of a particle with an initial normal velocity of v_n ($v_n < 0$; see Fig. 1) in the absence of adhesion and other molecular level effects. The usual approach typically considers a collision to be a rapid static event composed of an approach phase followed by a rebound phase with no oscillations. For the approach phase the normal impulse is P_n^A = $-mv_n$ and for the rebound phase is $P_n^R = mV_n$ where the final normal velocity is V_n. Define a coefficient of restitution as $R = -V_n/v_n$ (Newton's coefficient). Then the rebound impulse is $P_n^R = -Rmv_n$. This can be interpreted as though the particle imparts an impulse of $P_n^A = -mv_n$ to the surface during approach and as a result of inelastic material effects of the particle and surface the restoring impulse is $P_n^R = -Rmv_n$.[1] In fact, this is the basis of the definition of the *kinetic* coefficient of restitution by Poisson, $R = P_n^R/P_n^A$. Note that although the coefficient of restitution can be defined as a velocity ratio (*kinematic* coefficient) and as a ratio of impulses (*kinetic* coefficient), its primary interpretation and utility is as a parameter that represents energy dissipation since for normal (90°) impact, the kinetic energy loss, T_L, can be written as $T_L = \frac{1}{2}(1 - e^2)\, mv_n^2$. Thus for $e = 1$ no energy is lost, a perfectly elastic collision, and for $e = 0$, $T_L = \frac{1}{2}mv_n^2$, a perfectly "plastic" collision.

To return to microparticles, an assumption is made here that the majority of energy loss due to deformation within the particle and surface materials is lost during approach (establishment of contact) and that the majority of energy loss due to molecular level forces, such as adhesion, is irreversible and occurs primarily during rebound (detachment or separation). Then the energy loss due to adhesion can be attributed to the work of an impulse, P_A^R, that acts only during rebound. Let the ratio of this impulse to the elastic restoring impulse be $-\rho$, that is, $P_A^R/P_n^R = -\rho$. As other impact coefficients, ρ is related to energy loss and can be shown to be

$$\rho = 1 - \left(1 - \frac{|W_A|/1/2mv_n^2}{R^2}\right)^{1/2} \quad (1)$$

where W_A is the dynamic work of the adhesion impulse. If $\rho = 1$, then the adhesion impulse completely counteracts the elastic restoring impulse, and the particle remains attached to the surface, that is, capture. If $\rho = 0$, then there is no dissipation of energy by adhesion, essentially equivalent to a macro-impact. Based on these assumptions and this definition, the energy loss for the impact of a microsphere (see Brach et al., 1995) leads to

$$e = R(1 - \rho) \quad (2)$$

where, e is the overall coefficient of restitution, $e = -V_n/v_n$, R is the coefficient of restitution in the absence of adhesion, and ρ is the adhesion coefficient defined above. Figure 2 shows a generic relationship between the variables in Eq. (3). The above assumptions and the laws of impulse and momentum lead to the following equations, forming the rigid body impact model:

$$V_n = -R(1 - \rho)v_n \quad (3)$$

$$V_t = v_t - \mu[1 + R(1 - \rho)]v_n \quad (4)$$

$$\Omega = \omega + \frac{5\mu}{2r}[1 + R(1 - \rho)]v_n \quad (5)$$

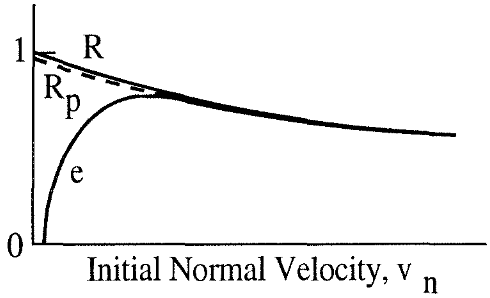

FIGURE 2. Generic restitution with idealized material dissipation alone, R, and combined material and adhesion dissipation, e. R_p, indicates the effect of plastic deformation.

[1]For ductile materials, this implies that any plastic deformation that occurs, happens during approach phase followed by (partial) elastic restoration during rebound. For viscoelastic materials, energy is dissipated throughout the contact duration and the model is not altogether temporally accurate.

The quantity μ is the ratio of the tangential to normal impulse. Under conditions of sliding throughout the contact duration, it has the value equal to the coefficient of friction. If the microsphere is not sliding at the end of contact, it must be rolling and $\mu = \mu_0$, where

$$\mu_0 = \frac{2\eta}{7[1 + R(1 - p)]} \tag{6}$$

where η depends on the initial conditions and is given by

$$\eta = (v_t - r\omega)/v_n \tag{7}$$

Although this model is relatively simple and necessarily approximate it goes a step beyond most other impact models by including a separate coefficient to represent adhesion and having the capability to handle oblique impacts. It is important to recognize that the coefficients R and ρ are not material constants; they depend not only upon the material properties and geometry of the microparticle and surface but the initial conditions of the collision as well.

EMPIRICAL EQUATIONS FOR THE IMPACT COEFFICIENTS, R AND Ρ

Beyond adhesion, the major factor of microparticle impact that influences attachment or capture at low velocities is energy dissipation. In the absence of adhesion, for collisions in general, Goldsmith (1960) shows that dissipation is highly dependent on the initial normal velocity with energy loss related directly to velocity. Dahneke (1975) and others found that adhesion effects for microspheres are also highly dependent on the initial normal velocity. Measurement of the overall coefficient of restitution, e, is straightforward because it is the ratio of final to initial normal velocities. Determination of R and ρ requires a means to separate the observation of these effects. In effect, this is taken care of by the fact that at high initial normal velocities, R is dominant, and at low initial velocities, as capture is approached, ρ is dominant.

Algebraic expressions that do an excellent job for impact coefficients (Brach, 1991) are:

$$R = \frac{k_1^p}{k_2^p + |v_n|^p} \tag{8}$$

$$\rho = \frac{\kappa_1^q}{\kappa_2^q + |v_n - v_c|^q} \tag{9}$$

The constants k, κ, p, q, and v_c are intended to be determined from experimental data. All of these quantities have a functional dependence on the parameters of the process such as particle size, material properties, etc. Note that the k's and κ's have units of velocity.[2] An exponential form for the ρ equation has been proposed (Dunn et al., 1995), but Eq. (10) seems to do a better job. For $k_1 = k_2 = k$ and $\kappa_1 = \kappa_2 = \kappa$ and using Eqs. (8), (9), and (2) the overall coefficient of restitution becomes

$$e = \left[\frac{k^p}{k^p + |v_n|^p}\right]\left[1 - \frac{\kappa^q}{\kappa^q + |v_n - v_c|^q}\right] \tag{10}$$

The quantity, v_c, is the capture velocity or critical velocity. It is important to emphasize that these expressions have some physical basis and are not chosen simply for their convenient algebraic form. For example, for spheres and using Hertzian theory, Stronge (1995) shows that for an initial velocity large enough to initiate plastic deformation, the coefficient of restitution can be expressed as a two-term polynomial in fractional powers of v_n. For low velocities, Hunt and Crossley (1975) show that for hysteretic type dissipation, $R \approx 1 - cv_n$ where c is a constant and the approximation is valid as long as R is near linear. Expansion of Eq. (8) in series form shows that

$$R = 1 - \left(\frac{v_n}{k}\right)^p + \left(\frac{v_n}{k}\right)^{2p} - \left(\frac{v_n}{k}\right)^{3p} + \cdots \tag{11}$$

For certain combinations of parameters differences between successive terms is small, the alternating signs cause cancellations, and all but the first few terms can be neglected so Eq. (11) has the capability of reducing to a

[2] Variations of these equations are possible, and they also can be defined in terms of nondimensional velocities, v_n/k and/or v_n/κ.

TABLE 1. Conditions and Information for Example Particle Impacts

Reference	Particle Dia, μm	Particle Material	Surface Material	k, κ & v_c* (p = 1)	$\frac{\|W_A^c\|}{1/2mv_2^2} = R_c^2$
Dahnke (1975) (Fig. 3)	1.27	polystyrene latex	polished quartz	556.3, 0.273 0.967, q = 1	0.998
Wall et al. (1990) (Fig. 3)	3 to 7	ammonium fluorescein	molybdenum	55.7, 0.586 1.47, q = 1	0.974
		ammonium fluorescein	mica	72.5, 1.333 1.10, q = 1	0.985
Dunn et al. (1995) (Fig. 4)	8.6 [1 to 30]	Ag-coated glass	copper	38.7, 1.610 0.29, q = 1	0.993
		Ag-coated glass	tedlar	51.7, 4.070 −0.44, q = 1	—
		Ag-coated glass	stainless steel	272.0, 1.740 −0.40, q = 1/2	—
Dunn et al. (1996) (Fig. 5)	64 to 76	stainless steel	SiO_2	12.5, 0.065 0.074, q = 1/2	0.994
Simulation (Fig. 7)	35	stainless steel	SiO_2	12.7, 0.197 0.212, q = 1/2	0.966
Simulation (Figs. 5 & 7)	70	stainless steel	SiO_2	12.5, 0.065 0.074, q = 1/2	0.994
Simulation (Fig. 7)	140	stainless steel	SiO_2	11.8, 0.019 0.026, q = 1/2	0.998

Note: All of the values of the constants k, κ, p, and q listed gave a fitting-index value of 0.94 or higher.

finite polynomial form. Likewise, Eq. (9) can be expanded to give:

$$p = 1 - \left(\frac{u_n}{\kappa}\right)^q + \left(\frac{u_n}{\kappa}\right)^{2q} - \left(\frac{u_n}{\kappa}\right)^{3q} + \cdots \quad (12)$$

where $u_n = v_n - v_c$. For arbitrarily-chosen p = 1 and q = 1, Eq. (10) becomes

$$e = \left[\frac{k}{k + |v_n|}\right]\left[\frac{|u_c|}{\kappa + |u_c|}\right] \quad (13)$$

W_A is the work done by adhesion during rebound. If W_A^c is the work of adhesion at capture and R_c is the material coefficient at capture (that is, when ρ = 1 and $v_n = v_c$) then it can be shown from Eq. (1) that

$$\frac{|W_A^c|}{1/2mv_c^2} = R_c^2 \quad (14)$$

Experimentally determined Values of R_c^2 are listed in Table 1. Equation (14) indicates that adhesion must supply just enough additional energy dissipation to the complement the material dissipation, R, to cause capture.

EXPERIMENTAL VALUES, R AND ρ

Nonlinear least-square regression fits of Eq. (10), with $p = q = 1$, using data of Dahneke (1975) and data of Wall et al. (1990) are shown in Fig. 3. Figure 4 shows nonlinear least-square regression fits of the data of

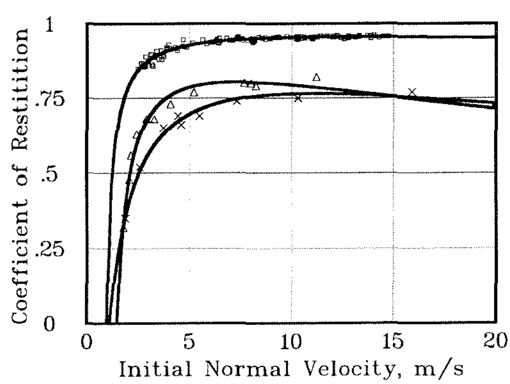

FIGURE 3. Experimental data from normal impacts fit to Eq. (10) Dahneke (1975): □ - quartz surface; Wall et al., Δ - moly and X - mica surfaces; see Table 1 for specific conditions.

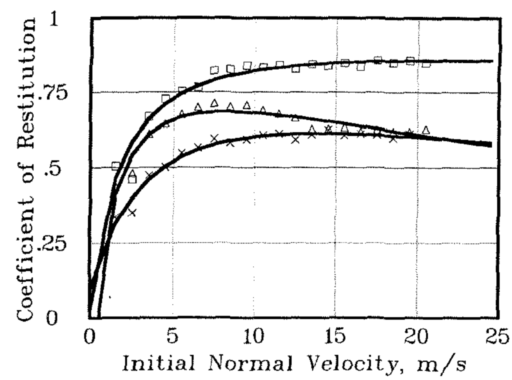

FIGURE 4. Experimental data from normal impacts fit with Eq. (10) Dunn et al., (1995): □ - stainless steel; Δ - copper and X - tedlar surfaces; see Table 1 for specific conditions.

Dunn et al. (1995).[3] The values of the constants k and κ, the capture velocities v_c and the adhesion work at capture for each target material are presented in Table 1. Fitting of the equations was done using commercially available software. Initially, the values of p and q were set. Then the values of k, κ, and v_c were varied iteratively by the software to maximize a fitting index. Individual values of the fitting index are not shown, but in all cases they range from 0.94 to 0.99. The capture velocities indicated by the data in Fig. 3 are fairly close to each other, despite the fact that the value of the constant k of one case is an order of magnitude greater than the others. High values of k represent high restitution (low dissipation) for a broad range of initial velocities. Small values of κ indicate that adhesion is significant over only a relatively small range of initial velocities. Figure 4 shows greater experimental disparity in the capture velocity. In fact, except for the copper particles, two of the three fitted curves have a negative capture velocity, indicating that conditions for capture likely did not exist. A negative capture velocity can result from insufficient low-velocity data or could be from variations due to uncontrolled factors. Stainless steel in Fig. 4 shows a significantly higher level of material restitution (lower dissipation) than the others, particularly at higher initial velocities. All of the results show that the transition from rebound to capture is relatively sudden as the initial velocity is decreased. The effects of adhesion are almost totally absent above about 10 to 15 m/s and become quite significant below about 5 m/s.

Fitting of Eq. (10) (or, alternatively, Eqs. [8] and [9]) for a range of impact conditions (particle and surface materials, surface conditions, particle sizes, etc.) allows Eqs. (3), (4), and (5) to serve as a simple but effective model for predicting the impact response of microparticles. The third impact coefficient, the impulse ratio μ, is controlled by the friction coefficient and the critical impulse ratio. "Fitting" μ amounts to observing the experimental value of the friction coefficient such as discussed in Dunn et al. (1995). The process of fitting the coefficients e, R, and ρ is illustrated more fully in the following.

NORMAL AND OBLIQUE COLLISIONS

All of the data illustrated in Figs. 3 and 4 are for normal collisions. Highly controlled experiments such as these are necessary to learn the basic characteristics of the adhesion contact process under conditions of impact. However, most naturally occurring collisions of microparticles are not normal but oblique. The analysis of oblique collisions, of course, must take into account the effects of rolling, sliding, and rotation. Do any tangential effects influence capture? In the absence of microforces tangential effects are uncoupled from the normal motion. Whether or not this remains true in the presence of adhesion remains to be addressed. When normal velocity experiments are run, the incidence angle is held fixed (at 90°) and the particle's initial velocities are varied. Typically in oblique experiments the velocity is held fixed, and the angles of incidence are varied. Both types of experiments have been conducted by Li et al. (1997) using stainless steel particles and ultraflat SiO_2 surfaces. Figure 5 shows the coefficients of restitution from oblique collisions taken over a range of

[3] In the data of Dunn et al (1995) and Li et al. (1997) each point represents the average of from 30 to 60 collisions.

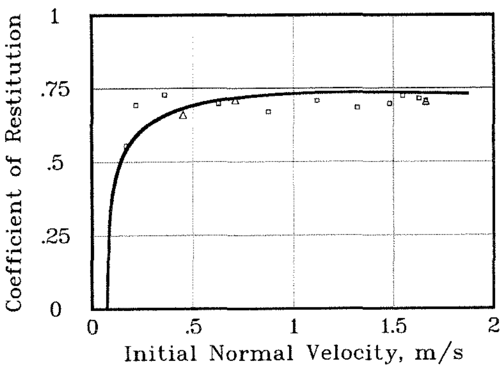

FIGURE 5. The coefficient of restitution, *e*, from normal and oblique impacts of stainless steel particles with a flat SiO2 surface from Li et al. (1997). Solid line is a fit to Eq (10).

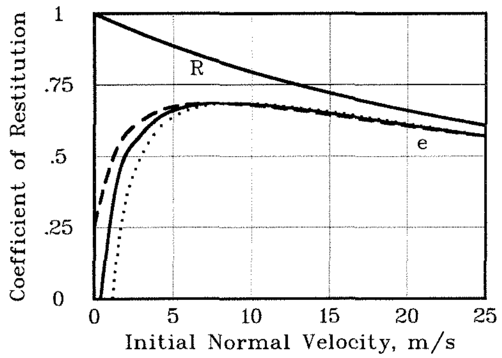

FIGURE 6. Sensitivity to angle of incidence. These curves show the effects of a target surface that is misaligned by an angle ϕ from perpendicular. The dashed curve is for $\phi = +2°$ and dots for $\phi = -2°$. The curve for $\phi = 0°$ corresponds to the copper surface data shown in Fig. 4.

about 5° to 90° along with normal impacts in the same range of initial velocities. A fit of Eq. (10) is also illustrated. The experimental uncertainty of Li's values of the coefficient of restitution has been estimated to be less than 10%, with 95% confidence. Beyond experimental scatter there seems to be no observable significant differences in restitution between the purely normal collisions and the oblique ones. But the question of any fundamental differences remains open and must be answered by additional fundamental studies and experiments.

Although tangential motion may not couple with or significantly influence the normal impact behavior of microparticles, the question of differences with respect to capture and deposition is entirely separate and needs investigation. A value of $e = 0$, for a normal collision against a surface means that all of the kinetic energy has been dissipated and the particle remains on the surface. But for an oblique collision with $e = 0$, a particle can still have significant kinetic energy associated with tangential and rotational motion, and the particle will migrate on the surface. It may or may not eventually leave the surface (depending on other factors such as local surface geometry). The question of capture for oblique impacts is not directly analyzed here, but the fitting of the data from oblique experiments is developed in anticipation of further investigation.

SENSITIVITY TO SURFACE ROUGHNESS

Visual examination of the close agreement of the curves and the data and the high correlation values from the fitting procedure reveal that Eqs. (8) and (9) or (10) have the capability to do an excellent job of matching microparticle impact data in the incident velocity transition region from rebound to attachment. However, because no experiments directly measure capture velocities, the equations and fitting procedure amount to an extrapolation when determining v_c. Extrapolation procedures require special scrutiny. Many factors are known to affect the capture velocity. These include particle size, particle velocity, particle and substrate mechanical properties, the surface chemical conditions (including the presence of oxides, contaminants, etc.), and the surface geometrical conditions such as roughness or flatness. Some of these will be investigated later, but the sensitivity of the capture velocity to the last factor, the surface flatness, is examined here using the above equations.

Except for specially prepared ones, even "smooth" surfaces have local variations in slope to one degree or another. In normal impact experiments there can be a bias, say $\pm\phi$, in the relative direction of particle approach. Figure 6 shows the coefficients of restitution, e and R, for silver-coated, glass spheres colliding with a copper surface (Fig. 4) for the condition of a perfectly flat surface (solid lines). It also shows how the fitted curves for the restitution, e, appear for each of the deviations of $\phi = \pm 2°$. Although in a sense, the three curves for e are relatively close, they predict radically different values of capture velocity. In fact, a 2° bias in the surface angle can cause the capture velocity to change from positive to negative. Obviously, this indicates that extreme care must be used to measure and control angles of incidence in experiments and to treat the values of v_c from the fitting process with care for potential variations due to this sensitivity.

IMPACT SIMULATION, CAPTURE DYNAMICS, AND CAPTURE VELOCITIES

In this section the fitting procedure begun for the data in Fig. 5 is expanded, providing an example of one of the many ways in which the empirical equations can be used. Figure 5 shows data from experiments run with (nominally) 70 μm diameter stainless steel microspheres and a corresponding curve from Eq. (10). If experimental data were available over a wider range of conditions, such as for smaller and larger diameters, then a family of fitted curves could be generated and, among other things, the change in capture velocity could be observed. Such data are not available, but a digital computer simulation (Brach et al., 1995) of microsphere impact dynamics can be used to generate additional values of the coefficient of restitution.

To establish the input to the simulation, physical and material constants are used to calculate the stiffness coefficients of the sphere and surface, here corresponding to stainless steel microspheres and a SiO_2 surface. Then experimental data, shown in Fig. 5, are used to establish the damping coefficients. One relatively high-velocity datum value (of the coefficient of restitution) is used to establish the simulation's dissipation constant for the material, and another, relatively low-velocity datum value is used to establish the simulation's dissipation constant for adhesion dissipation. All of these coefficients then are held fixed for the remainder of the simulation as parameters such as size, initial velocity, angle of incidence, etc. are varied. The simulation provides full particle motion and deformation as a function of time. However, only the coefficient of restitution, e, is examined here. The procedure followed was to treat the coefficient of restitution values generated by the simulation as input to the fitting of Eq. (10). Figure 7 shows the corresponding curves for particles with diameters of 35, 70, and 140 μm with the corresponding values of the fitted parameters k, κ, p, q, and v_c given in Table 1. The same experimental data shown in Fig. 5 are shown in Fig. 7.

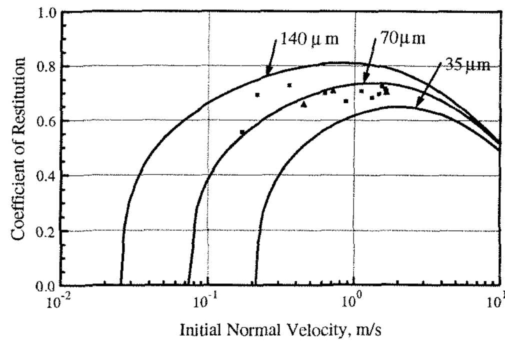

FIGURE 7. The solid curves are simulation results fitted using Eq (10). Experimental points are the same as Fig. 5.

Figure 5 contains data from collisions of microspheres with a distribution of diameters ranging from about 64 μm to 76 μm and a relatively limited range of initial velocities. The simulation results leading to the curves in Fig. 7 can be considered in some sense to be extrapolations both to other particle sizes and other initial velocities and, of course, are only as accurate as the simulation. Improvements of the simulation are continuing, particularly the manner of expressing the adhesion force; reports of progress are planned for the future.

Despite the need for additional verification, it is interesting to examine the depen-

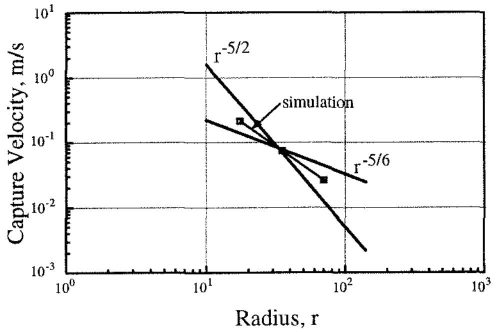

FIGURE 8. Comparison of the capture velocity trends between the simulation, a -5/6 power law (see Johnson and Pollock, 1994) and a line with a -5/2 slope.

dence of the capture velocity on the particle size and to present the results for future comparisons. Johnson et al. (1994) have developed equations that provide the value of the capture velocity under the JKRS theory. They find that the capture velocity is proportional to the microsphere radius raised to the -5/6 power. Figure 8 shows the capture velocities obtained from the fit of Eq. (10), a line with a slope proportional to the -5/6 power and, for additional comparison, a line showing proportionality to the -5/2 power. A disparity with Johnson and Pollock's results is not surprising because of differences in underlying assumptions from Brach and Dunn's simulation. The JKRS theory is based on an elastic, quasi-static approach where energy loss is attributed to instabilities in the microsphere's motion (snap on and snap off) and to the imposition of irreversibility through a change in the value of the surface energy parameter between attachment and detachment. Whereas in the simulation, dissipation is from two processes, one associated with a nonlinear, velocity-dependent material dissipation and the other with a similar but distinct adhesion dissipation. Some of the assumptions associated with the two approaches are common. Both are based on Hertzian theory and both treat adhesion as an irreversible process. Much more work remains in order to learn more about the dependence of microsphere dynamics and the conditions of capture of microparticles by surfaces during impact.

CONCLUSIONS

A set of empirical equations with constants k, κ, v_c, p, and q have been developed with a form suitable for representing the coefficient of restitution and adhesion coefficient for microparticles colliding with a surface in the presence of adhesion. These constants are found from data (from experiments or theoretical calculations) by using curve-fitting procedures such as least squares. Sophisticated fitting methods (such as with weighted deviations) could be used, but this was not pursued since the degree of fit was found to be excellent without them. The resulting equations can be used as stand-alone expressions for the restitution of normal impacts. Or they can be used in place of the coefficient of restitution in the rigid body impact model providing a true physical model (based on Newton's laws) applicable to oblique collisions.

Two nice features of the fitted equations are that they provide an estimate of the dynamic work of adhesion determine and a value for the capture velocity without direct experimental measurement of the capture velocity. For certain conditions (oblique impact at shallow angles), the fitted value of the capture velocity can be sensitive to small variations in the surface flatness, but ordinarily, the accuracy of the value of the capture velocity is as good as the data used in the fitting of the equations. Another feature of the methods presented here is that with an algebraic form they are quite simple and are efficient components for large-scale computer simulations, such as those that model turbulence and/or the air quality of buildings and other large enclosures.

The financial support by the Center for Indoor Air Research and the Electric Power Research Institute for the research leading to the above results is gratefully acknowledged.

REFERENCES

Brach, R. M. (1991) *Mechanical Impact Dynamics*, John Wiley & Sons, New York.

Brach, R. M., and Dunn, P. F. (1992). A Mathematical Model of Impact and Adhesion of Microspheres, *Aerosol Science & Technology* 16(1):51–64.

Brach, R. M., and Dunn, P. F. (1995). Macrodynamics of Microspheres, *Aerosol Science & Technology* 23(1):51-71.

Chen, Y. L., Helm, C. A., and Israelachvili, J. N. (1991). Molecular Mechanisms Associated with Adhesion and Contact Angle Hysteresis of Monolayer Surfaces, *J Physical Chem.* 95:10,736–10,747.

Dahneke, B. (1975). Further Measurements of the Bouncing of Small Latex Spheres, *J. Colloid & Interface Sci.* 51:58–65.

Dunn, P. F., Brach, R. M., and Caylor, M. J. (1995). Experiments on the Low Velocity Impact of Microspheres with Planar Surfaces, *Aerosol Science & Technology* 23(1):80–95.

Goldsmith, W. (1960). *Impact*, Edw. Arnold Ltd., London.

Hunt, K. H., and Crossley, F. R. E. (1975). Coefficient of Restitution Interpreted as Damping in Vibroimpact, *J Appl Mech.* 97:440–445.

Johnson, K. L., and Pollock, H. M. (1994). The Role of Adhesion in the Impact of Elastic Spheres, *J Adhesion Sci Technol.* 8:(11):1323–1332.

Li, X., Dunn, P. F., and Brach, R. M. (1997). Experiments on the Oblique Impact of Microspheres with Planar Surfaces with Known Patterns, in *Proceedings of the American Association for Aerosol Research 16th Annual Meeting,* Denver, CO, pp. 481.

Stronge, W. J. (1995). Coupling of Friction and Internal Dissipation in Planar Collision of Compliant Bodies, in *Contact Mechanics* (M. Raous et al., eds) Plenum Press, New York.

Wall, S., John, W., and Wang, H.-C. (1990). Measurement of Kinetic Energy Loss for Particles Impact Surfaces, *Aerosol Science & Technology* 12:926–946.

Received October 17, 1997; accepted June 15, 1998.