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ROTATIONAL DISSIPATION DURING MICROSPHERE IMPACT

R. M. Brach*, P. F. Dunn and W. Cheng

Particle Dynamics Laboratory, Department of Aerospace and Mechanical Engineering, University of Notre Dame, Notre Dame, IN 46556, U.S.A.

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Abstract—In this study the dissipative effects due to microsphere rotation in the presence of adhesion during contact were investigated by means of mathematical analysis and numerical simulation. Three sources of rotational moments were considered: a moment about the mass center of the tangential contact force, a moment associated with the material rolling deformation and another with the peeling of the adhesion bond. The latter two are couples proximate to the contact region. A numerical model based on the results of the mathematical analysis was used to simulate the two-dimensional normal and oblique impact of a microsphere. The results show that the magnitude of rolling deformation and adhesion bond peeling moments are proportional to a power of the contact radius. Consequently, because of the small radius of microspheres, the effect of rotational dissipation due to these moments can be neglected. For example, when predicting microsphere rebound during impact, only the moment of the tangential force needs to be considered when considering microsphere rotation.

1. INTRODUCTION

Friction at the contact area through the development of a tangential frictional impulse plays a significant role in the oblique impact of microspheres with surfaces. The tangential impulse and its resulting effect on the translational and rotational velocities of the microsphere not only influence its rebound direction but also contribute significantly to impact energy loss (see Brach and Dunn, 1995). Various studies (see the review by Ziskind *et al.*, 1997) have pointed out that an adhesion moment can play a role in the process of particle detachment from a surface. An adhesion moment often is modeled as the product of an adhesion force, F_A, which is considered as a single-point force acting normal to and at the center of the contact circle. The moment arm is the radius of this circle at equilibrium, r_{ae} (Wang, 1990). In this approach (for detachment studies), it is assumed that due to the action of external lift and drag forces, the particle tips about a forward edge of the (unchanging) contact area and lifts from the surface as the adhesion force and its moment about the tipping axis are overcome. This approach is somewhat naive as it neglects changes in the contact area due to its decrease in size (as a result of the lifting) and due to the sphere's movement along the substrate (as a result of rolling). Rolling is accompanied by an establishment of adhesion contact at the leading edge of the contact area and peeling of the existing adhesion contact at its trailing edge. The peeling of the trailing edge of the contact area and its corresponding dissipation is one of the effects studied in this paper. The analysis is applied to impact but has implications on attachment and detachment as well.

In this paper, expressions of the moments due to rolling contact are developed based upon an analysis of forces distributed over the contact area. Their effects on microsphere motion and energy dissipation are assessed both analytically and numerically for two-dimensional microsphere impacts, based on the equations of the simulation impact model presented by Brach and Dunn (1995).

A few comments are appropriate to explain the approach used to establish the distribution of the rolling moments over the contact surface. Three primary contact forces for

* Author to whom correspondence should be addressed.

microspheres are considered: an adhesion attraction force, the Hertzian repulsive force and the Coulomb friction force. The adhesion force is modeled as an idealized normal tensile ring force around the periphery of the contact circle. The Hertzian force is a surface compressive stress normal to and hemispherically distributed over the contact area of the microsphere. A primary goal of this paper is to assess the rolling resistance of the contact moments, that is, the energy dissipation during rolling contact. The forces of interest are the dissipative forces associated with adhesion and Hertzian compression and not the forces themselves (the adhesion and Hertzian stresses are mechanically conservative, that is, they are nondissipative). Little to nothing is known about the time dependence and distribution of the forces of the dissipation associated with adhesion and rolling compressive contact. Some very primitive assumptions must be made. The dissipation forces associated with adhesion and Hertzian compression are assumed to have the same distribution as the forces themselves (for example, adhesion dissipation is distributed as a ring force around the periphery of the contact area) and they are assumed to have a time dependence proportional to the local relative contact velocity.

The following study is carried out assuming a linear velocity-dependent dissipation. It is possible, if not likely, that a nonlinear model would be more realistic. An extension of the following analysis that includes nonlinear dissipation was carried out. It was found that nonlinearities in the dissipation terms of the model equations can make differences, which, under most circumstances, are small. The added complexity of the nonlinear model does not make a significant difference. This is even more so considering the fact that rolling dissipation is negligible during microsphere impact (which will be shown). The numerical simulation using linear velocity dependent dissipation presented by Li *et al.* (1998) showed good agreement with the experiment data, so the accuracy of the linear dissipation model likely is acceptable. Furthermore, until more is learned about the actual behavior of dynamic dissipation associated with adhesion, there is no rational way of making a choice about the nature of any nonlinearities.

2. THE DYNAMIC SIMULATION IMPACT MODEL

Figure 1 shows some pertinent features of a microsphere in contact with a surface or substrate. Typically the radius, a, of the contact area is small compared with the radius, r, of the microsphere and the radius of curvature of the contact area (Bowen *et al.*, 1995). Only

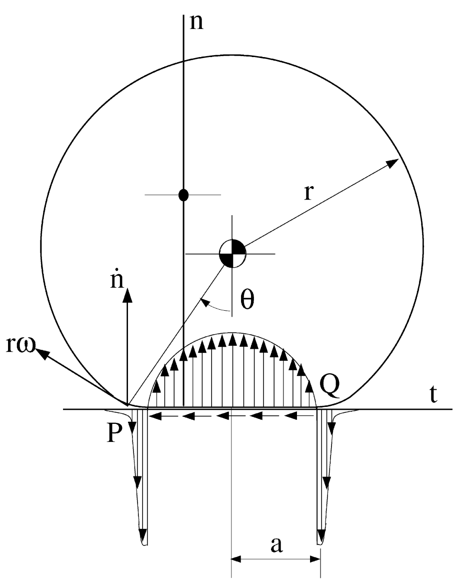

Fig. 1. A rolling microsphere in contact with a flat surface.

elastic deformation is considered (with no permanent deformation). Because the particle is small, gravity is not considered. The mathematical model currently used is based on the two-dimensional impact model established by Brach and Dunn (1995). The equation of motion for the normal direction is

$$m\ddot{n} = F_{\mathrm{H}} + F_{\mathrm{HD}} + F_{\mathrm{A}} + F_{\mathrm{AD}} = F_n(\tau), \tag{1}$$

$$= -\sqrt{r}Kn^{3/2}(1 + C_{\mathrm{H}}\dot{n}) + 2\pi a f_0(1 + C_{\mathrm{A}}\dot{n}), \tag{2}$$

where, τ is time, r is the undeformed particle radius, m is the particle mass and

$$K = \frac{4}{3\pi(k_1 + k_2)} \tag{3}$$

and

$$k_i = \frac{1 - v_i^2}{\pi E_i}, \tag{4}$$

in which v is Poisson's ratio and E is Young's modulus. An effective value of K should be used for greater accuracy. When an applied compressive force exists such as the normal inertial force during impact in combination with adhesion, a reduced stiffness $K = K_{\mathrm{R}}$ should be used (see Brach *et al*., 1998).

The first terms to the right-hand side of equations (1) and (2), F_{H} and $\sqrt{r}Kn^{3/2}$, represent the classical Hertzian restoring force; the second terms, F_{HD} and $\sqrt{r}Kn^{3/2}C_{\mathrm{H}}\dot{n}$, are the Hertzian dissipation force. The third terms represent the idealized adhesion attraction as a circumferential (line) force, and the last terms are the dissipation force due to adhesion. C_{H} and C_{A} are the corresponding damping constants for the Hertzian and adhesion dissipation forces.

The tangential force over the contact surface is modeled using Coulomb friction and is determined by the motion status, consisting of either sliding and rolling or pure rolling. The equation of tangential motion is

$$m\ddot{t} = F_t(\tau), \tag{5}$$

$$F_t = \begin{cases} -fF_n(\tau)sgn(\dot{t} - r\dot{\theta}), & \dot{t} - r\dot{\theta} \neq 0, \\ 0, & \dot{t} - r\dot{\theta} = 0, \end{cases} \tag{6}$$

where f is a friction coefficient, $\dot{t} \neq r\dot{\theta}$ represents the condition of sliding and $\dot{t} = r\dot{\theta}$ represents that of pure rolling.

The rotational motion equation is

$$mk^2\ddot{\theta} = -rF_t(\tau) + M_{\mathrm{A}}(\tau) + M_{\mathrm{H}}(\tau), \tag{7}$$

where k is the centroidal radius of gyration. The first term on the right-hand side of equation (7) is the moment of the tangential force about the mass center. The second term is a moment due to peeling of the adhesion ring force at the trailing edge of the contact surface during rolling and the last term is the rolling friction moment associated with Hertzian deformation.

3. ANALYSIS OF MOMENTS

Two kinds of moments are considered in equation (7) to simulate the effects of rotation: the tangential force moment, $-rF_t(\tau)$, and the rolling dissipation moments, M_{A} and M_{H}. The latter two depend upon the contact surface area, whereas the former does not. Rolling dissipation is related to the distribution of local relative velocities of the sphere's surface over the contact surface area. Figure 1 is the schematic of the cross section of a rolling sphere in contact with a rigid flat surface over a circular contact area of radius a. Along the periphery of the contact area, the normal microparticle surface velocity due to rotation is not unidirectional. Consider a microsphere that has begun contact. Its mass center is either approaching or receding from the surface with general planar motion. The angular velocity of the sphere is ω and the normal velocity of the mass center is $\dot{n}$. Consider points P and Q. Due to rotation, the relative normal velocity at point P is $\dot{n} + \omega r \cos\theta$ and the relative normal velocity at point Q is $\dot{n} - \omega r \cos\theta$. Hence, the relative normal velocity of the sphere's surface is not uniform. In the current impact model, the Hertzian force and Hertzian dissipation force are related to the relative normal velocity. The normal velocity distribution can result in a distribution of damping forces over the contact area implying the existance of M_A, the moment of the adhesion dissipation force about the rotation center, and M_H, the moment of the Hertzian dissipation force about the rolling center. In the following, each of these two moments will be analyzed in more detail in order to determine the extent of their contributions to the sphere's rotational motion.

3.1. *The analysis of the adhesion rolling dissipation moment*

The dissipation due to M_A is caused by the irreversibility of the adhesion process, that is, the making and breaking of the adhesion bond over the periphery of the contact surface of the microsphere as the microsphere rolls. Figure 2 is used to determine M_A. The normal velocity of the mass center produces only a uniformly distributed adhesion force and does not contribute to the rolling dissipation moment. The presence of $\omega = \dot{\theta}$ during rotation does contribute to M_A and M_H.

The normal component of the relative velocity at point A on S (the edge of the contact area) shown in Figure 2 is

$$v_n = v\cos\alpha = \omega L. \tag{8}$$

From Fig. 2, the geometric relation, $L = a\cos\alpha$, can be used to give

$$v_n = \omega L = \omega a \cos\alpha. \tag{9}$$

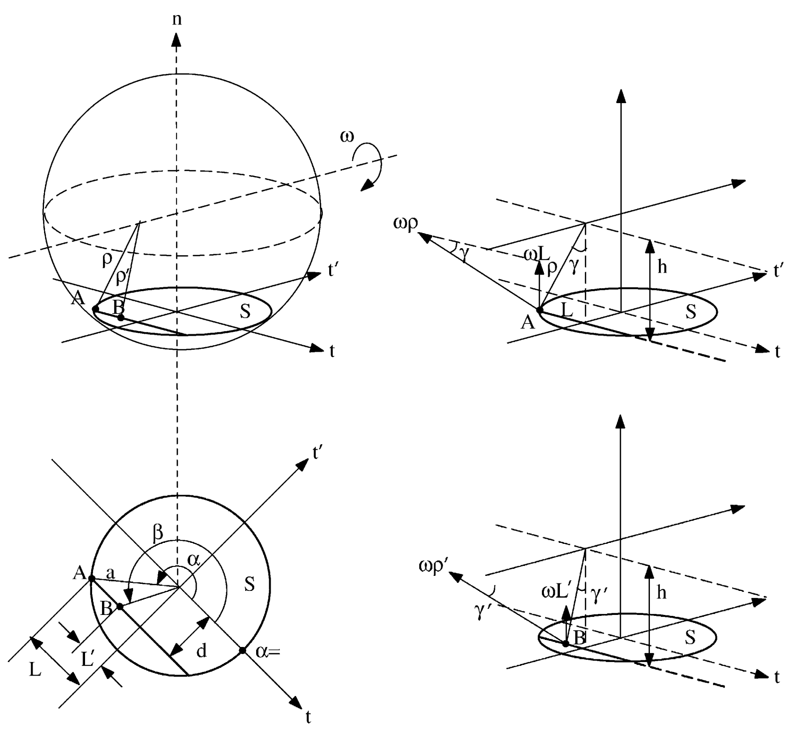

Fig. 2. A schematic illustration of the parameters involved in calculating M_A and M_H, where $\cos\alpha = L/a$, $\sin\gamma = L/\rho$, $h = r - n$, $\rho^2 = L^2 + h^2$, $a^2 = L^2 + d^2$.

According to the impact model, the intensity of the adhesion dissipation force at point A is proportional to the relative normal velocity of the particle at this point. That is

$$\mathrm{d}F_{\mathrm{A}} = f_0 C_{\mathrm{A}} v_n \,\mathrm{d}s = f_0 C_{\mathrm{A}} \omega a^2 \cos\alpha \,\mathrm{d}\alpha. \tag{10}$$

Thus, the local moment of $\mathrm{d}F_{\mathrm{A}}$ about the rotation center is

$$\mathrm{d}m_{\mathrm{A}} = L\,\mathrm{d}F_{\mathrm{A}} = f_0 C_{\mathrm{A}} \omega a^3 \cos^2 \mathrm{d}\alpha. \tag{11}$$

Integrating the local moment over the edge of the contact area S gives

$$M_{\mathrm{A}}(\tau) = \int_0^{2\pi} f_0 C_{\mathrm{A}} \omega a^3 \cos^2\alpha \,\mathrm{d}\alpha = \pi f_0 C_{\mathrm{A}} \omega a^3. \tag{12}$$

The impact model assumes that the adhesion force dissipation is significant only when surfaces are separating. This assumption is made here also and leads to:

- if $\dot{n} < -a\omega$, the normal velocities of all the points of the contact area S on the microsphere surface are negative (leaving the substrate); thus all the points are in the rebound stage and $M_{\mathrm{A}} = \pi f_0 C_{\mathrm{A}} \omega a^3$,
- if $\dot{n} \geqslant -a\omega$, the velocities of all the points of the contact area S are positive (approaching the substrate); thus all points are in the approaching stage and $M_{\mathrm{A}} = 0$,
- if $-a\omega < \dot{n} < a\omega$, only the points within the angle $0 < \alpha < \alpha^*$ on the contact area are in the rebound stage and $M_{\mathrm{A}} = (\alpha^* + \frac{1}{4}\sin 2\alpha) f_0 C_{\mathrm{A}} \omega a^3$, where α^* is the coordinate angle of the point which has the zero relative normal velocity, $\dot{n} - a\omega = 0$.

3.2. *The analysis of the Hertzian rolling dissipation moment*

The Hertzian dissipation force is related to the normal stresses distributed over the contact area. The dissipation moment due to the Hertzian dissipation force, M_{H}, is attributed to the normal velocity distribution over the contact area in this analysis.

At the point $B(\rho', \alpha')$ in the contact area (refer to Fig. 2), the relative velocity caused by rotation has a normal component

$$v_n = \omega L' = \omega \rho' \cos\alpha'. \tag{13}$$

The Hertzian dissipation force intensity due to rotation at point $B(\rho', \alpha')$ is

$$\mathrm{d}F_{\mathrm{HD}}(\rho', \alpha') = P(\rho', \alpha') C_{\mathrm{H}} v_n(\rho', \alpha') \rho' \,\mathrm{d}\alpha \,\mathrm{d}\rho' \tag{14}$$

where, $P(\rho', \alpha')$ is the Hertzian force at point B. Integrating equation (14) over the entire area S in Fig. 2 to find the moment about the t' axis gives

$$M_{\mathrm{H}} = \int_0^a \int_0^{2\pi} P(\rho', \alpha') C_{\mathrm{H}} v_n(\rho', \alpha') \rho'^2 \cos\alpha' \,\mathrm{d}\rho' \,\mathrm{d}\alpha \tag{15}$$

$$= \frac{3K}{2\pi} \sqrt{\frac{n}{R}} C_{\mathrm{H}} \omega \int_0^a \int_0^{2\pi} \rho'^3 \left(1 - \frac{\rho'^2}{a^2}\right)^{1/2} \cos^2\alpha' \,\mathrm{d}\rho' \,\mathrm{d}\alpha' \tag{16}$$

$$= \frac{3K}{2} \sqrt{\frac{n}{R}} C_{\mathrm{H}} \omega \int_0^a F(\rho')\, d\rho' \tag{17}$$

Here, $F(\rho') = \rho'^3 (1 - \rho'^2/a^2)^{1/2}$. Thus,

$$M_{\mathrm{H}} = \frac{K}{5} \sqrt{\frac{n}{R}} C_{\mathrm{H}} \omega a^4. \tag{18}$$

3.3. *Summary of the analysis of the rolling dissipation moment*

In the impact simulation presented by Brach and Dunn (1995), the equation of rotation is

$$mk^2 \ddot{\theta} = -rF_t(\tau) + g(\tau), \tag{19}$$

where $g(\tau)$ is a dissipation moment due to adhesion damping during rotation

$$g(\tau) = \lambda\zeta_A\left(\frac{r^3Kv_n^{1/2}}{m}\right)^{-2/5}a\dot{\theta} \tag{20}$$

and ζ_A is a nondimensional dissipation parameter defined as

$$\zeta_A = C_A\left(\frac{r^3Kv_n^{1/2}}{m}\right)^{2/5} \tag{21}$$

Thus,

$$g(\tau) = \lambda C_A a\dot{\theta}. \tag{22}$$

Equating $g(\tau)$ and M_A from equation (14) gives

$$\lambda = \pi a^2 f_0. \tag{23}$$

Equation (23) is an analytical expression for the adhesion rolling dissipation moment parameter λ. It is a dimensional parameter with the units of N m. It is proportional to a^2. In the case of microsphere impact, because the particle radius is on the order of 10^{-6} m and f_0 correlates with the particle radius as: $f_0 \sim r^{1/3}$ (Li *et al.*, 1998), the order of λ is about 10^{-10} N m.

Figure 3 shows the effect on the final rotational velocity, Ω, of the microsphere of varying the value of λ over a wide range. Three cases are considered: the normal impact, $+40^\circ$ oblique impact and -40° oblique impact. In each case, the magnitude of the initial velocity of the mass center is 1.66 m s^{-1} and the initial angular velocity is 10^6 degree s^{-1}. When

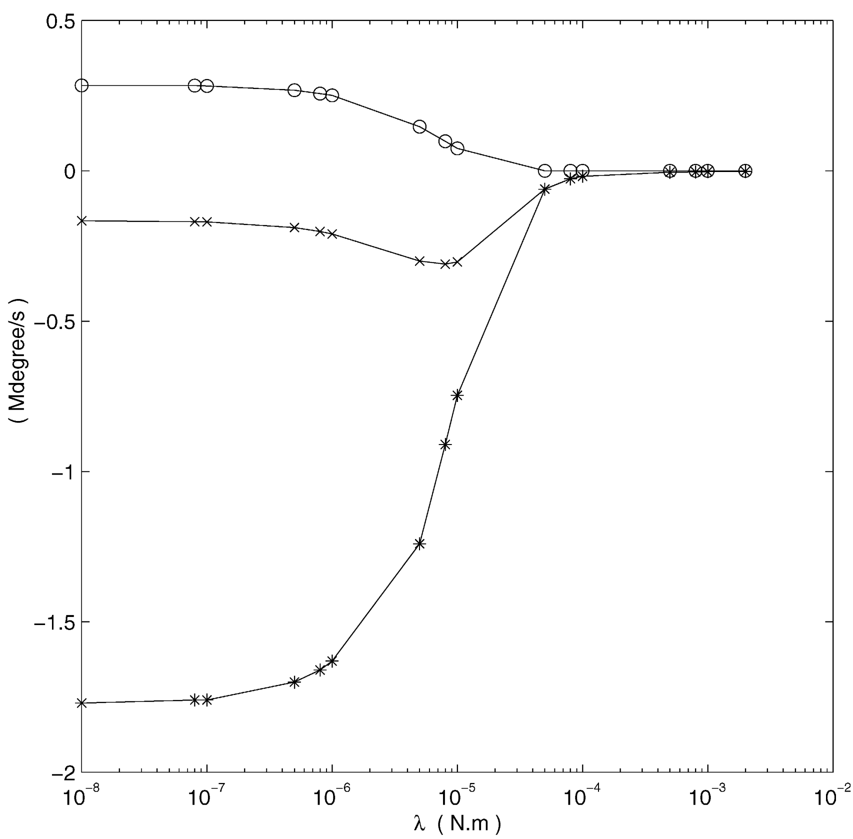

Fig. 3. Numerical results: Final rotational velocity, Ω, as a function of the parameter λ for impact. Three cases: $\theta_0 = 0$, normal impact ("○"), $\theta_0 = +40^\circ$ ("$\times$") and $\theta_0 = -40^\circ$ ("$*$"), all for $\omega_0 = 10^6$ degree s^{-1}.

$\lambda < 10^{-7}$ N m, there is no change in the final rotational velocities. This implies that the effect of rolling dissipation due to adhesion is negligible for microspheres. This is supported by equation (12), in which $M_A \sim a^3$, and by equation (18), in which $M_H \sim a^4$. Thus, it is reasonable to neglect both M_A and M_H in the case of microsphere impact.

4. THE NUMERICAL SIMULATION OF THE ROLLING PARTICLE IMPACT

Equations (1), (5) and (7) are three second-order, nonlinear, ordinary differential equations which can be solved using standard numerical techniques. As part of the simulation of Brach and Dunn, the Runge–Kutta–Gill method was used. The nondimensional dissipation parameter ζ_A is defined in equation (21) and ζ_H is defined as

$$\zeta_H = C_H \left(\frac{r^3 K v_n^{1/2}}{m} \right)^{2/5}. \tag{24}$$

Values of parameters chosen for the numerical simulation are shown in Table 1.

Figure 4 shows the magnitude of the moments about the mass center of the microsphere that arise as functions of time during normal impact. In this figure, the total rolling dissipation, $M_A + M_H$, is compared to the tangential force moment, $-rF_t(\tau)$. The

Table 1. The parameters for numerical simulation

Initial vel. ($\mathrm{m\,s^{-1}}$)	ω_0 (degree $\mathrm{s^{-1}}$)	r (μm)	Density ($\mathrm{kg\,m^{-3}}$)
1.66	10^6	35	8000
$K(\times 10^{10}$ Pa)	f_0(N/m)	ζ_A	ζ_H
11.4	325	400	100

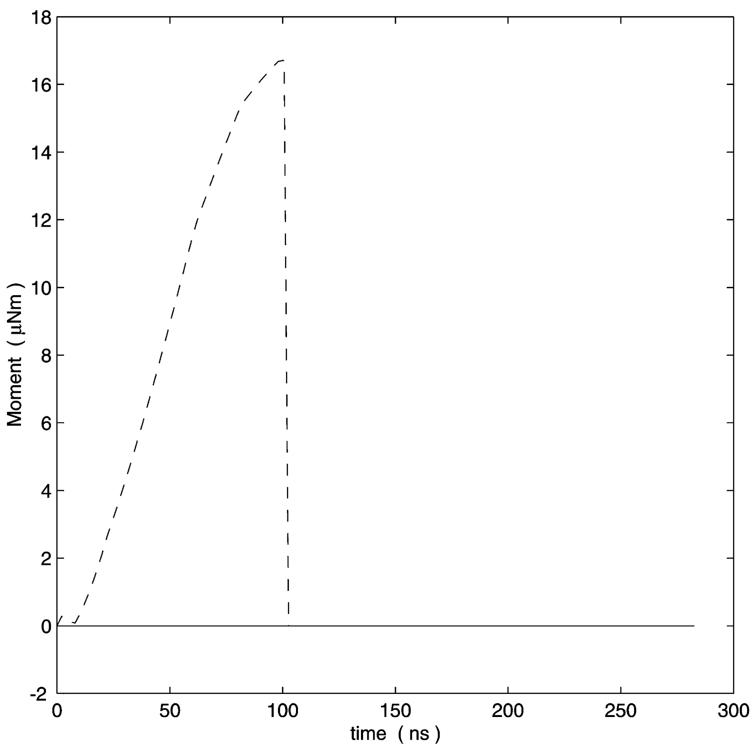

Fig. 4. Numerical results: Magnitude of the tangential force moment (dash line) and the rolling dissipation moments (solid line) during normal impact with $\omega_0 = 10^6$ degree $\mathrm{s^{-1}}$.

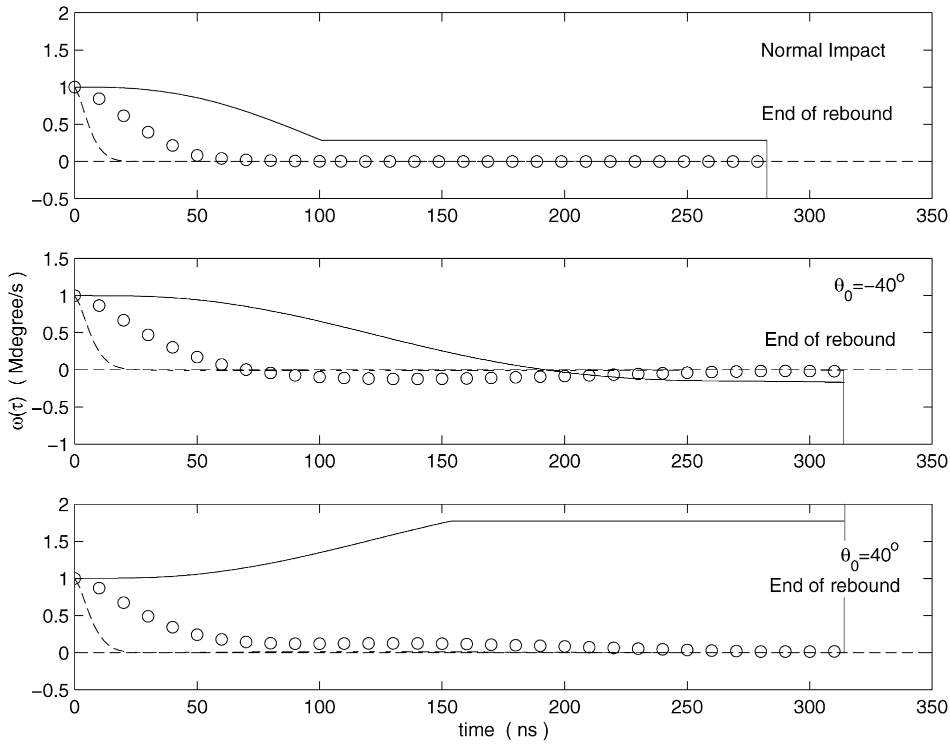

Fig. 5. Numerical results: The particle angular velocity during impact. Three cases: $\theta_0 = 0$, (normal impact), $\theta_0 = +40°$, and $\theta_0 = -40°$ all for $\omega_0 = 10^6$ degree s^{-1}. Dashed line: $\lambda = 10^{-3}$ N m, Circles: $\lambda = 10^{-4}$ N m, solid line: the analytical model.

tangential force moment increases during the attachment phase of impact and then decreases to zero at ~100 ns when the relative tangential velocity on the contact area becomes zero (the beginning of pure rolling). The numerical results confirm that the magnitude of the tangential force moment is much greater than the sum of the rolling dissipation moments.

Figure 5 shows the temporal evolution of the particle rotational velocity during contact under different incident angles. Three cases are compared: normal impact, positive angle oblique impact ($\theta_0 = 40°$) and negative angle oblique impact ($\theta_0 = -40°$) . Two different models for adhesion rolling dissipation were used in the simulations. One is the constant λ model by solving equations (19) and (20) (the rolling dissipation moment parameter, λ, is treated as constant). The other is the analytical model by solving equation (7). In this model M_A and M_H are given by equations (12) and (18), respectively. The dashed line is for the result of the constant λ model with $\lambda = 10^{-3}$ N m, the line of circles for $\lambda = 10^{-4}$ N m and the solid line for the analytical model. The modeling of rolling dissipation plays a significant role in the impact simulation. By the constant λ model, when λ is large (refer to the results for $\lambda = 10^{-3}$ N m and 10^{-4} N m), the adhesion rolling dissipation is predominant, the final particle angular velocity reduces to zero during contact, which implies a very "sticky" contact surface. Nevertheless, the smaller the value of λ the longer the time for the particle to stop rolling. The results of the analytical model (in which the equivalent λ is not constant but smaller than 10^{-7} N m) show that the microsphere remains at a constant rotational velocity at the end of the contact. Thus, the rolling dissipation is not significant as compared with the case of larger λ. On the other hand, the incident angle also plays a role in particle contact. For the constant λ model ($\lambda = 10^{-3}$ and 10^{-4} N m), the time at which the angular velocity of the particle reduces to zero in the normal impact is shorter than that for the oblique impacts, even though for all the cases the angular velocities reduce to zero during contact (because the rolling dissipation is predominant). By the analytical model, for the case of normal impact and positive angle oblique impact ($\theta_0 = 40°$), the angular velocity of the particle does not change direction during contact, whereas for a negative angle oblique impact ($\theta_0 = -40°$), the angular velocity changes direction in the contact time. This result agrees with what one would intuitively expect.

5. CONCLUSIONS

From the above analysis, the rolling dissipation due to Hertzian and adhesion contact moments are related directly to the size and shape of the contact area. Because of the typically small contact radius of microspheres during impact these rolling dissipation moments are negligible. In other words, among the terms on the right-hand side of the rotational equation of motion (equation (7)) $|M_A + M_H| \ll rF_t$. Therefore, changes in angular velocity are dominated by the moment of the tangential (frictional) force about the mass center of the microsphere. On the other hand, for pure rolling, $F_t = 0$ but M_A and M_H still exist. However, for this case, the effect of these moments is still insignificant because the duration of contact is short and the rotational displacement is very small.

In nonimpact situations, the duration of rolling can be relatively long and the rolling dissipation can play a significant role in the total energy loss. A hint of this is shown in Fig. 5. For the case when $\theta_0 = -40°$ and $\lambda = 10^{-4}$ N m, the angular velocity changes sign and eventually returns to zero ($\dot{\theta} = 0$) and rolling stops. Although the value of $\lambda = 10^{-4}$ N m is unrealistically high, for lower values of λ the rolling dissipation ($M_A + M_H$) for a longer duration of pure rolling could be significant. In particular, this would be the case in the studies of microsphere resuspension.

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