NAME: SOLUTION
AME 20231, Thermodynamics
Examination 2
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1. (20) A tank with a fixed volume of 2 m^3 contains a calorically imperfect ideal gas whose specific heat is

$$c_v(T) = c_{vo} + \alpha T + \beta T^2.$$

Here, T is the absolute temperature, and $c_{vo} = 1.05\ kJ/(kg\ K)$, $\alpha = -3.65\times 10^{-4}\ kJ/(kg\ K^2)$, and $\beta = 8.5 \times 10^{-7}\ kJ/(kg\ K^3)$. The specific volume of the gas is 0.8 m^3/kg. Calculate how much heat needs to be added to the gas to bring it from $T_1 = 300\ K$ to $T_2 = 400\ K$.

Solution

$$m = \frac{V}{v} = 2.5\ kg.$$

$$_1Q_2 = m \int_{T_1}^{T_2} c_v(T) dT.$$

$$_1Q_2 = m \int_{T_1}^{T_2} \left(c_{vo} + \alpha T + \beta T^2\right) dT.$$

$$_1Q_2 = m \left(c_{vo}(T_2 - T_1) + \frac{\alpha}{2}\left(T_2^2 - T_1^2\right) + \frac{\beta}{3}\left(T_2^3 - T_1^3\right)\right).$$

$$\boxed{_1Q_2 = 256.771\ kJ.}$$

2. (30) A tank with a volume of 0.85 m^3 initially contains water as a two-phase liquid-vapor mixture at 260°C and a quality of 0.7. Saturated water vapor at 260°C is slowly withdrawn through a valve at the top of the tank. At the same time, heat is added to the water to maintain the temperature inside the tank to be constant. This continues until the tank is filled with saturated vapor at 260°C. Assume that we can ignore changes in kinetic energy and potential energy.

   (a) Determine the initial specific volume, mass and specific internal energy of the water in the tank. (7)

   (b) Determine the specific enthalpy of the water vapor withdrawn through the valve. (7)

   (c) Determine the final pressure, specific internal energy and mass of the water in the tank. (7)

   (d) Determine the amount of heat transferred to the tank. (9)

Solution

$$v_1 = v_{f1} + x_1 v_{fg1} = 0.001276 + (0.7)(0.04093) = \boxed{0.029927\ \frac{m^3}{kg}.}$$

$$m_1 = V/v_1 = 0.85/0.029927 = \boxed{28.4024\ kg.}$$

$$u_1 = u_{f1} + x_1 u_{fg1} = 1128.37 + (0.7)(1470.64) = \boxed{2157.82\ \frac{kJ}{kg}.}$$

$$h_e = \boxed{2796.89\ \frac{kJ}{kg}.}$$

$$P_2 = P_1 = \boxed{4688.6\ kPa.}$$

$$v_2 = 0.04220\ \frac{m^3}{kg}.$$

$$m_2 = V/v_2 = 0.85/0.04220 = \boxed{20.1422\ kg.}$$

$$u_2 = \boxed{2599.01\ \frac{kJ}{kg}.}$$

$$\begin{aligned} \frac{dm}{dt} &= -\dot{m}_e, \\ m_2 - m_1 &= -\dot{m}_e \Delta t. \end{aligned}$$

$$\begin{aligned} \frac{d}{dt}(mu) &= -\dot{m}_e h_e + \dot{Q}, \\ m_2 u_2 - m_1 u_1 &= -\dot{m}_e \Delta t h_e + \dot{Q}\Delta t, \\ m_2 u_2 - m_1 u_1 &= (m_2 - m_1) h_e + {}_1Q_2, \\ {}_1Q_2 &= m_2 u_2 - m_1 u_1 - m_2 h_e + m_1 h_e \end{aligned}$$

$$\boxed{{}_1Q_2 = 14165.5\ kJ.}$$

3. (50) In a solar tower system, mirrors are used to focus sunlight to a tower in which the working fluid is heated to high temperatures (see Fig. 1). The tower system functions as the boiler used in conventional Rankine cycles. This solar tower is incorporated into a Rankine cycle to generate electricity as shown in Fig. 1. The Rankine cycle utilizes water, with a mass flow rate of 25 kg/s. Some properties are listed in Table 1. Assume steady-state and that the changes in kinetic energy and potential energy are negligible.

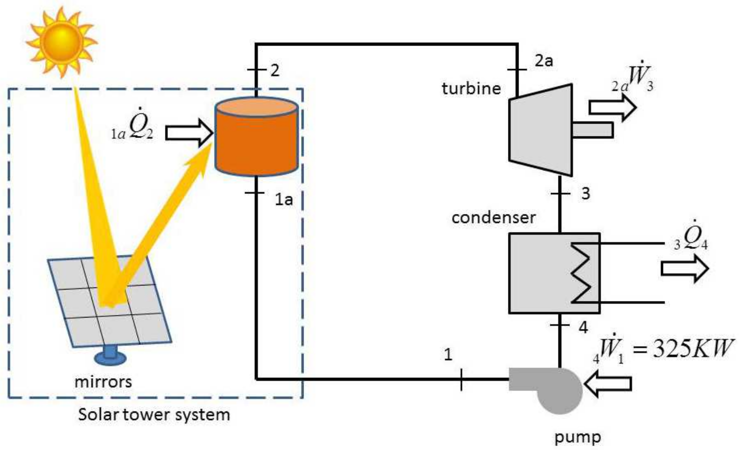

Figure 1: A solar energy-based Rankine cycle.

Table 1: Thermodynamic properties at different states in the cycle

State	1	1a	2	2a	3	4
P (kPa)	6000	5000	5000	5000	10	
T ($°C$)		20	500			40
$h(kJ/kg)$				3422		
x					0.95	0

(a) If the pump is adiabatic, what is the specific enthalpy and temperature at state 1? (10)

(b) In the pipeline from the pump to the tower (1-1a), there is a heat loss. What is the rate of the heat transfer loss along this pipe? (6)

(c) What is the rate of heat transfer to the working fluid inside the solar tower? (7)

(d) What is the power output from the turbine which functions adiabatically? (6)

(e) Assume that the cooling water in the condenser comes from a lake at 12$°C$ and returns at 25$°C$. Determine the rate of heat transfer in the condenser and the mass flow rate of the cooling water from the lake. (8)

(f) What is the overall thermal efficiency? What is theoretical maximum possible efficiency of this cycle? (8)

(g) Assume that the solar radiation energy flux is 1366 W/m^2, and the solar-to-heat conversion efficiency of this solar tower system is 90%. How large an area of mirrors is needed to power this Rankine cycle? (note: efficiency is defined as the ratio of what you want and what you paid for). (5)

Solution

A few other answers than those given below were accepted. Specifically, liquid properties could be estimated via the compressed liquid tables or the saturation tables at the appropriate temperature. Sign conventions were only loosely enforced as the arrows on the chart did not strictly satisfy the sign convention.

$$_4w_1 = {_4\dot{W}_1}/\dot{m} = 325/25 = 13 \ \frac{kJ}{kg}.$$

$$h_1 = h_4 + {_4w_1} = 167.54 + 13 = \boxed{180.54 \ \frac{kJ}{kg}.}$$

Interpolate to get T_1:

$$T_1 = 40 + (180.54 - 167.54)/(188.42 - 167.54)(5) = \boxed{43.11^\circ C}.$$

$$h_{1a} = 88.64 \ \frac{kJ}{kg}.$$

$$_1\dot{Q}_{1a} = \dot{m}(h_{1a} - h_1) = (25)(88.64 - 180.54) = \boxed{-2297.5 \ kW.}$$

$$h_2 = 3433.76 \ \frac{kJ}{kg}.$$

$$_{1a}\dot{Q}_2 = \dot{m}(h_2 - h_{1a}) = (25)(3433.76 - 88.64) = \boxed{83628 \ kW.}$$

$$h_{2a} = 3422 \ \frac{kJ}{kg}.$$

$$x_3 = 0.95.$$

$$h_3 = h_{f3} + x_3 h_{fg3} = 191.81 + (0.95)(2392.82) = 2464.99 \ \frac{kJ}{kg}.$$

$$_{2a}\dot{W}_3 = \dot{m}(h_{2a} - h_3) = (25)(3422 - 2464.99) = \boxed{23925.3 \ kW.}$$

An energy balance for the heat exchanger with the cooling water yields

$$\dot{m}_{cw} c(T_H - T_L) = \dot{m}(h_3 - h_4).$$

$$\dot{m}_{cw} = \dot{m}\frac{h_3 - h_4}{c(T_H - T_L)} = (25)\frac{2464.99 - 167.54}{4.186(25 - 12)} = \boxed{1055.46 \ \frac{kg}{s}.}$$

$$_3\dot{Q}_4 = \dot{m}(h_4 - h_3) = (25)(167.54 - 2464.99) = \boxed{-57436.2 \ kW.}$$

$$\eta = \frac{{_{2a}\dot{W}_3} - {_4\dot{W}_1}}{_{1a}\dot{Q}_2} = \frac{23925.3 - 325}{83628} = \boxed{0.282205.}$$

$$\eta_{max} = 1 - \frac{T_L}{T_H} = 1 - \frac{20 + 273}{500 + 273} = \boxed{0.620957}.$$

Energy balance for solar panels:

$$q''_{solar} A \eta_{solar} = {_{1a}\dot{Q}_2}.$$

$$A = \frac{_{1a}\dot{Q}_2}{q''_{solar}\eta_{solar}} = \frac{83628}{(1366/1000)(0.9)} = \boxed{68023.4 \ m^2}$$

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