APPENDIX A
TO: Joseph M. Powers
FROM: James Gallagher
DATE: 3 April 1997
RE: AE360 Project: Part I

The purpose of this project is to solve the governing equations of a shock tube by using the Lax-Wendroff technique. In a duct of length 10 m and cross sectional area of .01 m^2 there is a diaphragm at the center of the tube. The diaphragm separates the tube into halves, where one half has a pressure of 2000 kPa and the other half has a pressure of 100 kPa. Initially the air is steady and it has a constant temerature of 300 K, then at time, $t = 0+$ the diaphragm is burst. The problem is to use the governing equations, initial conditions, and the Lax-Wendroff to model the ensuing flow.

The governing equations that were used were conservation of mass, momentum, and energy in the forms:

$$\frac{\partial}{\partial t}\rho + \frac{\partial}{\partial x}(\rho u) = 0 \tag{1}$$

$$\frac{\partial}{\partial t}(\rho u) + \frac{\partial}{\partial x}(\rho u^2 + p) = 0 \tag{2}$$

$$\frac{\partial}{\partial t}\left(\rho\left(e + \frac{u^2}{2}\right)\right) + \frac{\partial}{\partial x}\left(\rho u\left(e + \frac{u^2}{2} + \frac{p}{\rho}\right)\right) = 0 \tag{3}$$

$$\text{where } e = \frac{1}{\gamma - 1}\frac{p}{\rho} \tag{4}$$

Where t is time, x is position, p is pressure, u is velocity, ρ is density, e is energy, and γ is the ratio of specific heats.

In order to solve these equations for the pressure, density, and velocity of the flow at specific times, the Lax-Wendroff technique, which is a descretization technique, was used. The Lax-Wendroff technique is a two step technique that divides the tube into n points and then finds the initial values at those points (q_i^n) and the values at the half way point between each point ($q_{i+1/2}^n$). In the first step the values for the half points are found at a half time using Equation (5). Then the values at each point for the next time step are found using Equation (6) which are then put back into the first step. This process is continued until the pressure, density, and velociy of the flow are found at each point at the desired time. The Fortran code used for this process can be found on the final pages of this memorandum.

$$q_{i+1/2}^{n+1/2} = q_{i+1/2}^n - \frac{\Delta t/2}{\Delta x}[f(q_{i+1}^n) - f(q_i^n)] \tag{5}$$

$$q_i^{n+1} = q_i^n - \frac{\Delta t}{\Delta x}[f(q_{i+1/2}^{n+1/2}) - f(q_{i-1/2}^{n+1/2})] \tag{6}$$

The results from the simulation are found in Figures 1 - 3. In Figure 1 it is shown how the pressure is increasing and decreasing in the flow as the shock wave moves to the right and the rarefaction wave moves to the left. In Figure 2 the changing density can be seen as the shock and rarefaction waves move across the tube. Finally, in Figure 3 the rise in tempurature due to the shock wave and the decrease in the tempurature due to the rarefaction wave are seen.

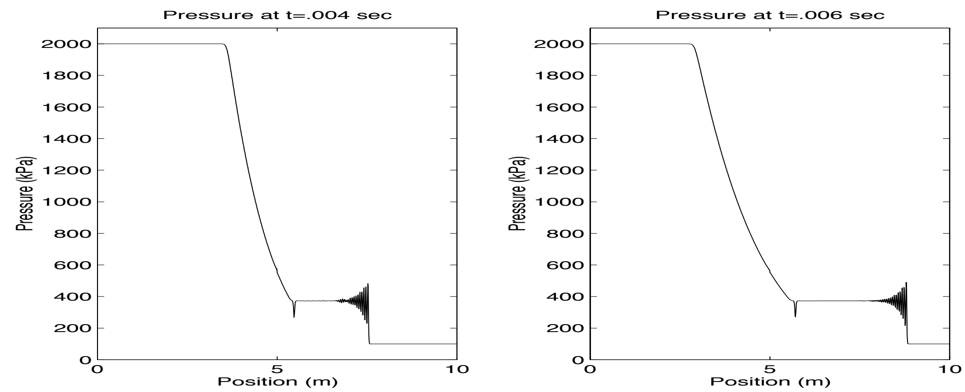

Figure 1: Plots of the pressure distribution at different times

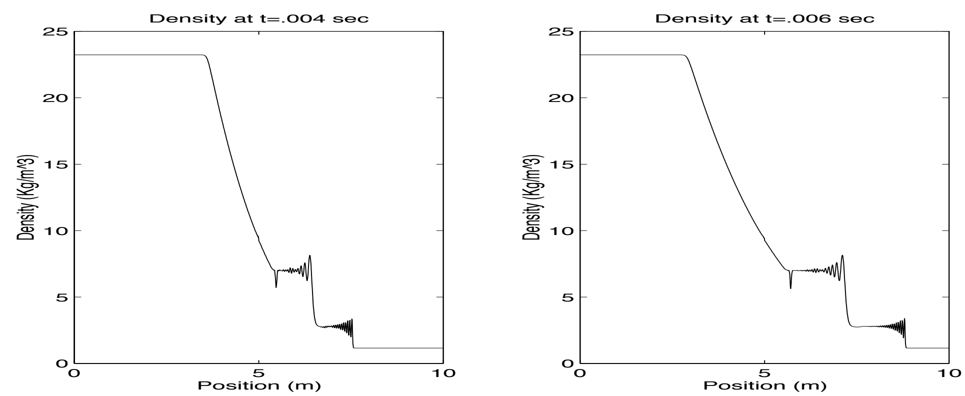

Figure 2: Plots of the density distribution at different times

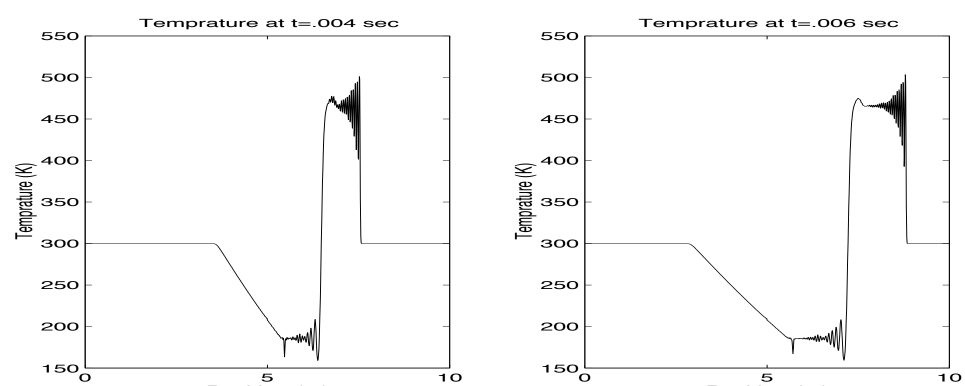

Figure 3: Plots of the temperature distribution at different times