AE 360
Homework 8
Due: Thursday, 20 March 1997, in class

1. Consider the inviscid Burger's equation in conservative form:

$$\frac{\partial u}{\partial t} + \frac{\partial}{\partial x}\left(\frac{u^2}{2}\right) = 0.$$

   Also take the following initial conditions

$$u(x,0) = U_H, \qquad 0 < x < x_d;$$

$$u(x,0) = U_L, \qquad x_d < x < x_{max}.$$

   It can be shown that the exact solution in this case is a propagating discontinuity:

$$u(x,t) = U_H, \qquad x < x_d + \frac{U_H + U_L}{2}\ t;$$

$$u(x,t) = U_L, \qquad x > x_d + \frac{U_H + U_L}{2}\ t.$$

   For $U_H = 2.0, U_L = 1.0, x_d = \frac{1}{2}, x_{max} = 1$, develop a numerical code based on MacCormack's technique to predict $u(x,t)$ for $x \in [0,1]$, $t \in [0,1]$ Give plots of $u(x,0.1)$, $u(x,0.2)$, $u(x,0.3)$ for both the exact solution and your numerical predictions. Take $\Delta x = 0.01$. Take $\Delta t = \frac{\Delta x}{\frac{U_H+U_L}{2} CFL}$, where the Courant-Friedrichs-Levy number, $CFL = 3$. Attach your code (be it Fortran, matlab, mathematica, etc.) to your homework.

2. For the same model as the previous problem, plot $u(x,0.2)$ for $\Delta x = 0.1, 0.01, 0.001, 0.0001$.

3. Now consider the non-conservative Burger's equation:

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} = 0.$$

   Again use MacCormack's method to discretize this equation, and with $\Delta x = 0.01$, determine and plot $u(x,0.2)$ on the same plot with the appropriate solution for the conservative version of Burger's equation.

4. Anderson, 7.4, p. 240.

5. Anderson, 7.10, p. 241.

The smaller number of problems this time is so you can begin on the project.