

AME 50531: Intermediate Thermodynamics

Homework Solutions

Fall 2010

1 Homework 1 Solutions

1.1 Problem 1:

CPIG air enters and isentropic nozzle at 1.30 atm and 25°C with a velocity of 2.5 m/s. The nozzle entrance diameter is 120 mm. The air exits the nozzle at 1.24 atm with a velocity of 90 m/s. Determine the temperature of the exiting air and the nozzle exit diameter.

At the nozzle entrance (State 1):

$$P_1 = 1.30 \text{ atm} \cdot \frac{101.3 \text{ kPa}}{1 \text{ atm}} = 131.7 \text{ kPa} \quad (1)$$

$$T_1 = 25^\circ\text{C} + 273.15 = 298.15 \text{ K} \quad (2)$$

$$v_1 = 2.5 \text{ m/s} \quad (3)$$

$$A_1 = \pi \cdot (0.120 \text{ m})^2 = 0.045 \text{ m}^2 \quad (4)$$

At the nozzle exit (State 2):

$$P_2 = 1.24 \text{ atm} \cdot \frac{101.3 \text{ kPa}}{1 \text{ atm}} = 125.6 \text{ kPa} \quad (5)$$

$$v_2 = 90 \text{ m/s} \quad (6)$$

T_2 and d_2 are unknown.

Because the nozzle is isentropic, we can use the isentropic relation for temperature and pressure to find the exit temperature. For air, $k = 1.4$.

$$\frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{\frac{k-1}{k}} \quad (7)$$

$$T_2 = T_1 \left(\frac{P_2}{P_1}\right)^{\frac{k-1}{k}} = 298.15 \text{ K} \cdot \left(\frac{125.6 \text{ kPa}}{131.7 \text{ kPa}}\right)^{\frac{1.4-1}{1.4}} \quad (8)$$

$$\boxed{T_2 = 294.14 \text{ K} = 21^\circ\text{C}} \quad (9)$$

To find the exit diameter, recall that $\dot{m} = \rho A v$. Then,

$$\dot{m} = \rho_1 A_1 v_1 = \rho_2 A_2 v_2 \quad (10)$$

Knowing that $\rho_1 = \rho_2$,

$$A_1 v_1 = A_2 v_2 \quad (11)$$

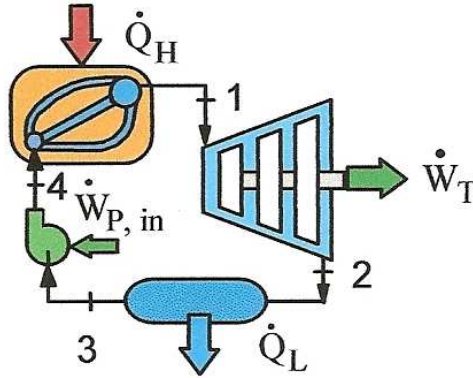
$$A_2 = A_1 \left(\frac{v_1}{v_2}\right) = 0.045 \text{ m}^2 \cdot \left(\frac{2.5 \text{ m/s}}{90 \text{ m/s}}\right) = 0.0012 \text{ m}^2 \quad (12)$$

Therefore, $\boxed{d_2 = 0.020 \text{ m} = 20 \text{ mm}}$.

1.2 Problem 2: 9.47

Consider a steam turbine power plant operation near critical pressure, as shown in Figure 1.2. As a first approximation, it may be assumed that the turbine and pump processes are reversible and adiabatic. Neglecting any changes in kinetic and potential energies, calculate:

- The specific turbine work output and the turbine exit state.
- The pump work input and enthalpy at the pump exit state.
- The thermal efficiency of the cycle.



We are given $P_1 = P_4 = 20 \text{ MPa}$, $T_1 = 800^\circ\text{C}$, $P_2 = P_3 = 10 \text{ kPa}$, and $T_3 = 40^\circ\text{C}$.

State 1: Using Table B.1.3, $h_1 = 4069.80 \text{ kJ/kg}$, $s_1 = 7.0544 \frac{\text{kJ}}{\text{kg}\cdot\text{K}}$

State 2:

$$s_2 = s_1 = 7.0544 \frac{\text{kJ}}{\text{kg}\cdot\text{K}} \quad (13)$$

Using Table B.1.2,

$$s_2 = 0.6492 + 7.5010 \Rightarrow x_2 = 0.8539 \quad (14)$$

$$h_2 = 191.81 + 0.8539 \cdot 2392.82 = 2235.04 \text{ kJ/kg} \quad (15)$$

The specific work output from the turbine,

$$w_t = h_1 - h_2 = 4069.80 \text{ kJ/kg} - 2235.04 \text{ kJ/kg} = \boxed{1834.76 \text{ kJ/kg}} \quad (16)$$

The turbine exit state is a saturated mixture.

State 3: Compressed liquid, using Table B.1.1, $h_3 = 167.54 \text{ kJ/kg}$, $v_3 = 0.001007$

State 4: Using the property relation for constant v ,

$$w_p = -v_3(P_4 - P_3) = -0.001007 \cdot (20000 - 10) = \boxed{-20.13 \text{ kJ/kg}} \quad (17)$$

$$h_4 = h_3 - w_p = 167.54 \text{ kJ/kg} + 20.13 \text{ kJ/kg} = \boxed{187.67 \text{ kJ/kg}} \quad (18)$$

The heat transfer in the boiler is

$$q_{in} = h_1 - h_4 = 4069.80 \text{ kJ/kg} - 187.67 \text{ kJ/kg} = 3882.13 \text{ kJ/kg} \quad (19)$$

$$w_{net} = w_t + w_p = 1834.76 \text{ kJ/kg} - 20.13 \text{ kJ/kg} = 1814.63 \text{ kJ/kg} \quad (20)$$

$$\eta_{TH} = \frac{w_{net}}{q_{in}} = \frac{1814.63}{3882.13} = \boxed{0.4674} \quad (21)$$

2 Homework 2 Solutions:

2.1 Problem 1: 7.64

Helium has the lowest normal boiling point of any of the elements at 4.2 K. At this temperature the enthalpy of evaporation is 83.3 kJ/kmol. A Carnot refrigeration cycle is analyzed for the production of 1 kmol of liquid helium at 4.2 K from saturated vapor at the same temperature. What is the work input to the refrigerator and the coefficient of performance for the cycle with an ambient temperature at 280 K?

For the Carnot cycle the ratio of the heat transfers is the ratio of temperatures:

$$Q_L = n\bar{h}_{fg} = 1 \text{ kmol} \cdot 83.3 \text{ kJ/kmol} = 83.3 \text{ kJ} \quad (22)$$

$$Q_H = Q_L \cdot \frac{T_H}{T_L} = 83.3 \cdot \frac{280}{4.2} = 5553.3 \text{ kJ} \quad (23)$$

$$W_{IN} = Q_H - Q_L = 5553.33 - 83.3 = \boxed{5470 \text{ kJ}} \quad (24)$$

$$\beta = \frac{Q_L}{W_{IN}} = \frac{83.3}{5470} = \boxed{0.0152} \quad \left[= \frac{T_L}{T_H - T_L} \right] \quad (25)$$

2.2 Problem 2: 8.97

A piston/cylinder contains air at 1380 K, 15 MPa, with $V_1 = 9 \text{ cm}^3$, $A_{cyl} = 5 \text{ cm}^2$. The piston is released, and just before the piston exits the end of the cylinder the pressure inside is 200 kPa. If the cylinder is insulated, what is its length? How much work is done by the air inside?

The cylinder is a control volume of air. It is insulated and therefore adiabatic, so $Q = 0$. By the continuity equation,

$$m_2 = m_1 = m \quad (26)$$

From the energy equation (5.11),

$$m(u_2 - u_1) = {}_1Q_2 - {}_1W_2 = -{}_1W_2 \quad (27)$$

From the entropy equation (8.37),

$$m(s_2 - s_1) = \int \frac{dQ}{T} + {}_1S_{2gen} = 0 + {}_1S_{2gen} \quad (28)$$

Pressure and temperature are known at State 1, but only pressure is known at State 2, so T_2 must be obtained. Assume a reversible process.

$${}_1S_{2gen} = 0 \Rightarrow s_2 - s_1 = 0 \quad (29)$$

State 1: Using Table A.7, $u_1 = 1095.2 \text{ kJ/kg}$ and $s_{T1}^\circ = 8.5115 \frac{\text{kJ}}{\text{kg}\cdot\text{K}}$

$$m = \frac{P_1 V_1}{RT_1} = \frac{15000 \cdot 9 \cdot 10^{-6}}{0.287 \cdot 1380} = 0.000341 \text{ kg} \quad (30)$$

State 2: From the entropy equation, $s_2 = s_1$ so from Eq. 8.19,

$$s_{T2}^\circ = s_{T1}^\circ + R \ln \frac{P_2}{P_1} = 8.5115 + 0.287 \ln \left(\frac{200}{15000} \right) = 7.2724 \frac{\text{kJ}}{\text{kg}\cdot\text{K}} \quad (31)$$

Then, use Table A.7 to interpolate and find $T_2 = 447.2 \text{ K}$ and $u_2 = 320.92 \text{ kJ/kg}$.

$$V_2 = V_1 \frac{T_2 P_1}{T_1 P_2} = 9 \cdot \frac{447.2 \cdot 15000}{1380 \cdot 200} = 218.7 \text{ cm}^3 \quad (32)$$

$$\Rightarrow L_2 = \frac{V_2}{A_{cyl}} = \frac{218.7}{5} = \boxed{43.74 \text{ cm}} \quad (33)$$

$${}_1w_2 = u_1 - u_2 = 774.3 \text{ kJ/kg} \quad (34)$$

$${}_1W_2 = m_1 w_2 = \boxed{0.264 \text{ kJ} = 264 \text{ J}} \quad (35)$$

2.3 Problem 3:

CPIG air in a cylinder ($V_1 = 0.03 \text{ m}^3$, $P_1 = 100 \text{ kPa}$, $T_1 = 10^\circ\text{C}$) is compressed reversibly at constant temperature to a pressure of 420 kPa . Determine the entropy change, the heat transferred, and the work done. Also accurately plot this process on T - S and P - V diagrams.

State 1:

$$T_1 = 10^\circ\text{C} = 283.15 \text{ K}$$

$$P_1 = 100 \text{ kPa}$$

$$V_1 = 0.03 \text{ m}^3$$

State 2:

$$T_2 = T_1 = 283.15 \text{ K}$$

$$P_2 = 420 \text{ kPa}$$

$$V_2 = ?$$

We know that $m_1 = m_2$, so,

$$m_1 = m_2 = \frac{P_1 V_1}{RT} = \frac{100 \cdot 0.03}{0.287 \cdot 283.15} = 0.0369 \text{ kg} \quad (36)$$

$$V_2 = \frac{mRT}{P_2} = \frac{0.0369 \cdot 0.287 \cdot 283.15}{420} = 0.00714 \text{ m}^3 \quad (37)$$

$$v_1 = \frac{V_1}{m} = \frac{0.03}{0.0369} = 0.813 \text{ m}^3/\text{kg} \quad (38)$$

$$v_2 = \frac{V_2}{m} = \frac{0.00714}{0.0369} = 0.193 \text{ m}^3/\text{kg} \quad (39)$$

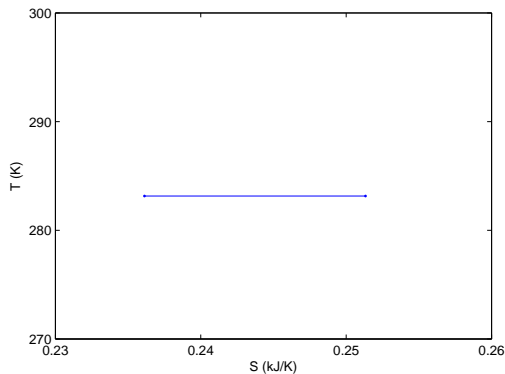
$$S_2 - S_1 = m \left(c_p \ln \frac{T_2}{T_1} - R \ln \frac{P_2}{P_1} \right) = 0.0369 \cdot \left(1.004 \ln 1 - 0.287 \ln \frac{420}{100} \right) \quad (40)$$

$$\boxed{S_2 - S_1 = -0.0152 \text{ kJ/K}} \quad (41)$$

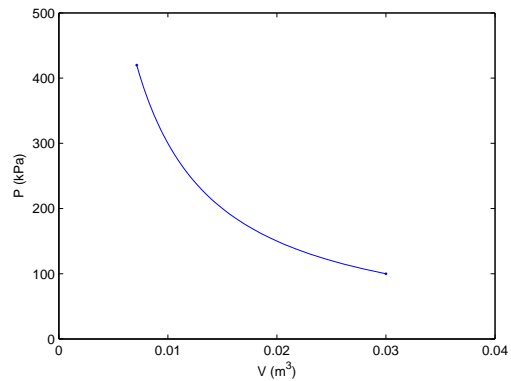
Now, to find the heat transfer and work done,

$${}_1Q_2 = {}_1W_2 = mRT \ln \frac{v_2}{v_1} = 0.0369 \cdot 0.287 \cdot 283.15 \ln \frac{0.193}{0.813} \quad (42)$$

$$\boxed{{}_1Q_2 = {}_1W_2 = -4.312 \text{ kJ}} \quad (43)$$



(a) T-S Diagram



(b) P-V Diagram

2.4 Problem 4: 12.97

Consider an ideal air-standard Stirling cycle with an ideal regenerator. The minimum pressure and temperature in the cycle are 100 kPa , 25°C , the compression ratio is 11, and the maximum temperature in the cycle is 1000°C . Analyze each of the four processes in this cycle for work and heat transfer, and determine the overall performance of the engine.

$$P_1 = 100 \text{ kPa}, T_1 = T_2 = 25^\circ\text{C}, \frac{v_1}{v_2} = 11, T_3 = T_4 = 1000^\circ\text{C}$$

From 1-2 at constant temperature, ${}_1w_2 = {}_1q_2 = T_1(s_2 - s_1)$

$${}_1w_2 = -RT_1 \ln \frac{v_1}{v_2} = -0.287 \cdot 298.15 \ln 11 = \boxed{-205.2 \text{ kJ/kg}} \quad (44)$$

From 2-3 at constant volume, ${}_2w_3 = 0$

$$q_{23} = C_{V0}(T_3 - T_2) = 0.717(1000 - 25) = \boxed{699 \text{ kJ/kg}} \quad (45)$$

From 3-4 at constant temperature, ${}_3w_4 = {}_3q_4 = T_1(s_4 - s_3)$

$${}_3w_4 = +RT_3 \ln \frac{v_4}{v_3} = 0.287 \cdot 1273.15 \ln 11 = \boxed{876.2 \text{ kJ/kg}} \quad (46)$$

From 4-1 at constant volume, ${}_4w_1 = 0$

$$q_{41} = C_{V0}(T_1 - T_4) = 0.717(25 - 1000) = \boxed{-699 \text{ kJ/kg}} \quad (47)$$

$$w_{NET} = -205.2 + 0 + 876.2 + 0 = 671 \text{ kJ/kg} \quad (48)$$

Since q_{23} is supplied by $-q_{41}$ (regenerator),

$$q_H = q_{34} = 876.2 \text{ kJ/kg} \quad (49)$$

$$\eta_{TH} = \frac{w_{NET}}{q_H} = \frac{671}{876.2} = \boxed{0.766} \quad (50)$$

NOTE:

$$q_H = q_{34} = RT_3 \ln 11 \quad (51)$$

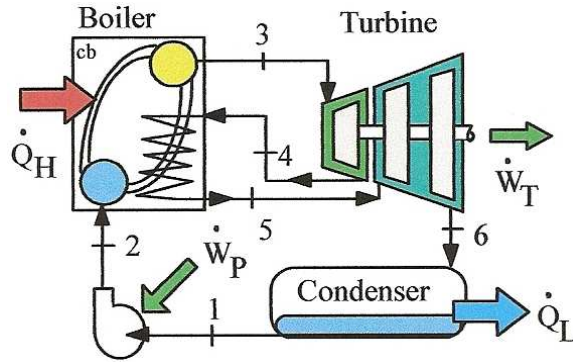
$$q_L = -{}_1q_2 = RT_1 \ln 11 \quad (52)$$

$$\eta_{TH} = \frac{q_H - q_L}{q_H} = \frac{T_3 - T_1}{T_3} = \frac{975}{1273.15} = 0.766 = \text{Carnot efficiency} \quad (53)$$

3 Homework 3 Solutions:

3.1 Problem 1: 11:36

Consider an ideal steam reheat cycle where steam enters the high-pressure turbine at 4.0 MPa, 400°C, and then expands to 0.8 MPa. It is then reheated to 400°C and expands to 10 kPa in the low-pressure turbine. Calculate the cycle thermal efficiency and the moisture content of the steam leaving the low-pressure turbine.



State 3: High-pressure turbine entrance.

$$P_3 = 4 \text{ MPa}, T_3 = 400^\circ\text{C} \Rightarrow h_3 = 3213.51 \text{ kJ/kg}, s_3 = 6.7689 \frac{\text{kJ}}{\text{kg} \cdot \text{K}} \quad (54)$$

State 4: High-pressure turbine exit. $s_4 = s_3 \Rightarrow h_4 = 2817.77 \text{ kJ/kg}$

State 5: Low-pressure turbine entrance.

$$P_5 = 0.8 \text{ MPa}, T_5 = 400^\circ\text{C} \Rightarrow h_5 = 3267.07 \text{ kJ/kg}, s_5 = 7.5715 \frac{\text{kJ}}{\text{kg} \cdot \text{K}} \quad (55)$$

State 6: Low-pressure turbine exit. Use entropy to find the moisture content x_6 :

$$s_6 = s_5 = 7.5715 \frac{\text{kJ}}{\text{kg} \cdot \text{K}} \Rightarrow \text{two-phase state} \quad (56)$$

$$x_6 = \frac{s_6 - s_f}{s_{fg}} = \frac{7.5715 - 0.6492}{7.501} = \boxed{0.92285} \quad (57)$$

$$h_6 = h_f + x_6 h_{fg} = 191.81 + 0.92285 \cdot 2392.82 = 2400.02 \text{ kJ/kg} \quad (58)$$

State 1: Pump entrance. $P_1 = 10 \text{ kPa}, v_1 = 0.00101 \text{ m}^3/\text{kg}, h_1 = 191.81 \text{ kJ/kg}$

State 2: Pump exit. Pump is reversible and adiabatic. Assume incompressible flow.

$$w_P = v_1(P_2 - P_1) = 0.00101(4000 - 10) = 3.94 \text{ kJ/kg} \quad (59)$$

$$h_2 = h_1 + w_P = 191.81 + 3.94 = 195.75 \text{ kJ/kg} \quad (60)$$

$$w_{T,\text{tot}} = h_3 - h_4 + h_5 - h_6 = 3213.51 - 2817.77 + 3267.07 - 2400.02 = 1262.79 \text{ kJ/kg} \quad (61)$$

$$q_{H1} = h_3 - h_2 = 3213.51 - 195.75 = 3017.76 \text{ kJ/kg} \quad (62)$$

$$q_H = q_{H1} + h_5 - h_4 = 3017.76 + 3267.07 - 2817.77 = 3467.06 \text{ kJ/kg} \quad (63)$$

$$\eta_{\text{CYCLE}} = \frac{(w_{T,\text{tot}} - w_P)}{q_H} = \frac{1262.79 - 3.94}{3467.06} = \boxed{0.3631} \quad (64)$$

3.2 Problem 2:

Consider a 600 MW reheat-Rankine cycle steam power plant. The boiler pressure is 1000 psia, the reheater pressure is 200 psia, and the condenser pressure is 1 psia. Both turbine inlet temperatures are 1000°F. The water leaving the condenser is saturated liquid. Determine the thermal efficiency of the power plant and the boiler mass flow rate in lbm/hr.

State 3: HP turbine entrance. $P_3 = 1000 \text{ psia}$, $T_3 = 1000^\circ\text{F}$, $h_3 = 1505.86 \text{ Btu/lb}_m$, $s_3 = 1.6530 \frac{\text{Btu}}{\text{lb}_m \cdot \text{R}}$

State 4: HP turbine exit. $P_4 = 200 \text{ psia}$, $s_4 = s_3 = 1.6530 \frac{\text{Btu}}{\text{lb}_m \cdot \text{R}}$. Interpolating, $h_4 = 1297.58 \text{ Btu/lb}_m$

State 5: LP turbine entrance. $P_5 = 2000 \text{ psia}$, $T_5 = 1000^\circ\text{F}$, $h_5 = 1529.28 \text{ Btu/lb}_m$, $s_5 = 1.8425 \frac{\text{Btu}}{\text{lb}_m \cdot \text{R}}$

State 6: LP turbine exit. $P_6 = 1 \text{ psia}$, $s_6 = s_5 = 1.8425 \frac{\text{Btu}}{\text{lb}_m \cdot \text{R}}$

$$x_6 = \frac{s_6 - s_f}{s_{fg}} = \frac{1.8425 - 0.1323}{1.8461} = 0.9264 \quad (65)$$

$$h_6 = h_f + x_6 h_{fg} = 69.57 + 0.9264 \cdot 1036.11 = 1029.41 \text{ Btu/lb}_m \quad (66)$$

State 1: Pump entrance, saturated liquid. $P_1 = 1 \text{ psia}$, $h_1 = 69.57 \text{ Btu/lb}_m$, $v_1 = 0.016136 \text{ ft}^3/\text{lb}_m$

State 2: Pump exit. $P_2 = 1000 \text{ psia}$

$$w_P = h_1 - h_2 = -v_1(P_2 - P_1) \Rightarrow h_2 = h_1 + v_1(P_2 - P_1) \quad (67)$$

Converting pressures into lb/ft², we get $P_1 = 144 \text{ lb/ft}^2$ and $P_2 = 144000 \text{ lb/ft}^2$.

$$w_P = 0.016136(144000 - 144) \cdot \frac{1.285 \cdot 10^{-3} \text{ Btu}}{1 \text{ lb} \cdot \text{ft}} = 2.983 \text{ Btu/lb}_m \Rightarrow h_2 = 72.55 \text{ Btu/lb}_m \quad (68)$$

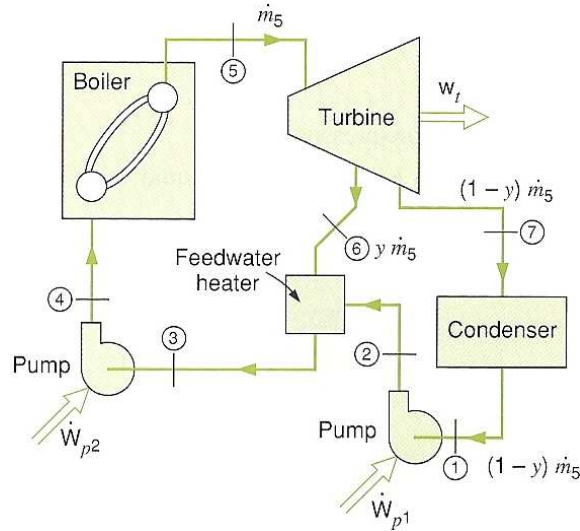
$$\eta_T = \frac{w_{NET}}{Q_{IN}} = \frac{(h_3 - h_4) + (h_5 - h_6) + (h_1 - h_2)}{(h_3 - h_2) + (h_5 - h - 4)} = \boxed{0.4235} \quad (69)$$

$$\dot{W}_{cycle} = 600 \text{ MW} \cdot \frac{1 \text{ Btu/s}}{1.055 \cdot 10^{-3} \text{ MW}} = 5.687 \cdot 10^6 \text{ Btu/s} \quad (70)$$

$$\dot{m} = \frac{\dot{W}_{cycle}}{w_{NET}} = \frac{\dot{W}_{cycle}}{(h_3 - h_4) + (h_5 - h_6) + (h_1 - h_2)} \cdot \frac{3600 \text{ s}}{1 \text{ hr}} = \boxed{2.9033 \cdot 10^7 \text{ lb}_m/\text{hr}} \quad (71)$$

3.3 Problem 3: 11.45

A power plant with one open feedwater heater has a condenser temperature of 45°C , a maximum pressure of 6 MPa , and a boiler exit temperature of 900°C . Extraction steam at 1 MPa to the feedwater heater is mixed with the feedwater line so the exit is saturated liquid into the second pump. Find the fraction of extraction steam flow and the two specific pump work inputs.



State 5: Boiler exit/turbine entrance. $h_5 = 4375.29\text{ kJ/kg}$, $s_5 = 7.8727\frac{\text{kJ}}{\text{kg}\cdot\text{K}}$

State 6: Turbine is reversible and adiabatic: $s_7 = s_6 = s_5$

$$P_6 = 1\text{ MPa}, s_6 = 7.8727\frac{\text{kJ}}{\text{kg}\cdot\text{K}} \Rightarrow h_6 = 3569.39\text{ kJ/kg} \quad (72)$$

State 1: Condenser exit/pump 1 entrance. $h_1 = 188.42\text{ kJ/kg}$, $v_1 = 0.00101\text{ m}^3/\text{kg}$, $P_1 = 9.593\text{ kPa}$

State 2: Pump 1 exit. $P_2 = P_6$

$$w_{p1} = h_2 - h_1 = v_1(P_2 - P_1) = 0.00101(1000 - 9.593) = \boxed{1.0\text{ kJ/kg}} \quad (73)$$

$$\Rightarrow h_2 = h_1 + w_{p1} = 188.42 + 1.0 = 189.42\text{ kJ/kg} \quad (74)$$

State 3: Feedwater exit. $P_3 = P_2$, $h_3 = 762.79$, $v_3 = 0.001127\text{ m}^3/\text{kg}$ The extraction fraction can be expressed as $y = \frac{\dot{m}_6}{\dot{m}_{tot}}$. From the energy equation, $(1-y)h_2 + yh_6 = h_3$

$$y = \frac{h_3 - h_2}{h_6 - h_2} = \frac{762.79 - 189.42}{3569.39 - 189.42} = \boxed{0.1696} \quad (75)$$

State 4: Pump 2 exit. $P_4 = P_5$

$$w_{p2} = h_4 - h_3 = v_3(P_4 - P_3) = 0.001127(6000 - 1000) = \boxed{5.635\text{ kJ/kg}} \quad (76)$$

4 Homework 4 Solutions:

4.1 Problem 1: 12.22

A Brayton cycle produces 14 MW with an inlet state of 17°C, 100 kPa, and a compression ratio of 17 : 1. The heat added in the combustion is 960 kJ. What are the highest temperature and mass flow rate of air, using properties from Table A.7.1?

The specific heat varies; therefore it is necessary to go through the processes individually to find the net work and highest temperature T_3 .

State 1: $T_1 = 290.15 \text{ K}$, $P_1 = 100 \text{ kPa}$. From A.7.1, $h_1 = 290.43 \text{ kJ/kg}$, $s_{T1}^\circ = 6.83521 \frac{\text{kJ}}{\text{kg}\cdot\text{K}}$

State 2: The compression is reversible and adiabatic, so from Eq. 8.19,

$$s_2 = s_1 \Rightarrow s_{T2}^\circ = s_{T1}^\circ + R \ln \frac{P_2}{P_1} = 6.83521 + 0.287 \ln 17 = 7.64834 \frac{\text{kJ}}{\text{kg}\cdot\text{K}} \quad (77)$$

$$\Rightarrow T_2 = 642.36 \text{ K}, h_2 = 652.04 \text{ kJ/kg} \quad (78)$$

From the energy equation with compressor work in,

$$w_C = {}_1w_2 = h_2 - h_1 = 652.04 - 290.43 = 361.61 \text{ kJ/kg} \quad (79)$$

State 3: Energy equation for the combustor,

$$h_3 = h_2 + q_H = 652.04 + 960 = 1612.04 \text{ kJ/kg} \quad (80)$$

$$\Rightarrow \boxed{T_3 = 1480.33 \text{ K}}, s_{T3}^\circ = 8.59596 \frac{\text{kJ}}{\text{kg}\cdot\text{K}} \quad (81)$$

State 4: The expansion is reversible and adiabatic so from Eq. 8.19,

$$s_4 = s_3 \Rightarrow s_{T4}^\circ = s_{T3}^\circ + R \ln \frac{P_4}{P_3} = 8.59596 + 0.287 \ln \frac{1}{17} = 7.78283 \frac{\text{kJ}}{\text{kg}\cdot\text{K}} \quad (82)$$

$$\Rightarrow T_4 = 728.36 \text{ K}, h_4 = 744.14 \text{ kJ/kg} \quad (83)$$

From the energy equation with turbine work out,

$$w_T = h_3 - h_4 = 1612.04 - 744.14 = 867.9 \text{ kJ/kg} \quad (84)$$

The net work is

$$w_{net} = w_T - w_C = 867.9 - 361.61 = 506.29 \text{ kJ/kg} \quad (85)$$

The total power requires a mass flow rate of

$$\dot{m} = \frac{\dot{W}_{net}}{w_{net}} = \frac{14000 \text{ KW}}{506.29 \text{ kJ/kg}} = \boxed{27.65 \text{ kg/s}} \quad (86)$$

4.2 Problem 2: 12.36

A large stationary Brayton cycle gas-turbine power plant delivers a power output of 100 MW to an electric generator. The minimum temperature in the cycle is 290 K, and the maximum temperature is 1600 K. The minimum pressure in the cycle is 100 kPa, and the compressor compression ratio is 14 : 1. The compressor has an isentropic efficiency of 85% and the turbine has an isentropic efficiency of 88%. Calculate the power output of the turbine. What fraction of the turbine output is required to drive the compressor? What is the thermal efficiency of the cycle?

Solve using constant C_{P0} . For an ideal compressor, $s_2 = s_1 \Rightarrow$ Implemented by Eq. 8.23

$$T_{2s} = T_1 \left(\frac{P_2}{P_1} \right)^{\frac{k-1}{k}} = 290(14)^{0.286} = 616.4 \text{ K} \quad (87)$$

$$w_{Cs} = h_2 - h_1 = C_{P0}(T_2 - T_1) = 1.004(616.4 - 290) = 327.71 \text{ kJ/kg} \quad (88)$$

For the actual compressor,

$$\Rightarrow w_C = \frac{w_{sC}}{\eta_{sC}} = \frac{327.71}{0.85} = 385.54 \text{ kJ/kg} = C_{P0}(T_2 - T_1) \quad (89)$$

$$\Rightarrow T_2 = T_1 + \frac{w_C}{C_{P0}} = 290 + \frac{385.54}{1.004} = 674 \text{ K} \quad (90)$$

For an ideal turbine, $s_4 = s_3 \Rightarrow$ Implemented by Eq. 8.23

$$T_{4s} = T_3 \left(\frac{P_4}{P_3} \right)^{\frac{k-1}{k}} = 1600 \left(\frac{1}{14} \right)^{0.286} = 752.8 \text{ K} \quad (91)$$

$$w_{Ts} = h_3 - h_4 = C_{P0}(T_3 - T_4) = 1.004(1600 - 752.8) = 850.63 \text{ kJ/kg} \quad (92)$$

For the actual turbine,

$$\Rightarrow w_T = \eta_{sT} \cdot w_{sT} = 0.88 \cdot 850.63 = 748.56 \text{ kJ/kg} = C_{P0}(T_3 - T_4) \quad (93)$$

$$T_4 = T_3 - \frac{w_T}{C_{P0}} = 1600 - \frac{748.56}{1.004} = 854.4 \text{ K} \quad (94)$$

To calculate the overall net work and the cycle efficiency,

$$w_{NET} = w_T - w_C = 748.56 - 385.54 = 363.02 \text{ kJ/kg} \quad (95)$$

$$\dot{m} = \frac{\dot{W}_{NET}}{w_{NET}} = \frac{100000 \text{ kW}}{363.02 \text{ kJ/kg}} = 275.47 \text{ kg/s} \quad (96)$$

$$\dot{W}_{NET} = \dot{m}w_T = 275.47 \text{ kg/s} \cdot 748.56 \text{ kJ/kg} = \boxed{206.203 \text{ MW}} \quad (97)$$

$$\frac{w_C}{w_T} = \frac{385.54}{748.56} = \boxed{0.515} \quad (98)$$

Energy input is from the combustor,

$$q_H = C_{P0}(T_3 - T_2) = 1.004(1600 - 674) = 929.7 \text{ kJ/kg} \quad (99)$$

$$\eta_{TH} = \frac{w_{NET}}{q_H} = \frac{363.02}{929.7} = \boxed{0.3905} \quad (100)$$

4.3 Problem 3: 12.41

An air-standard Ericsson cycle has an ideal regenerator. Heat is supplied at 950°C and heat is rejected at 80°C . Pressure at the beginning of the isothermal compression process is 70 kPa . The heat added is 700 kJ/kg . Find the compressor work, the turbine work, and the cycle efficiency.

Identify the states:

Heat supplied at high temperature: $T_3 = T_4 = 950^{\circ}\text{C} = 1223.15\text{ K}$

Heat rejected at low temperature: $T_1 = T_2 = 80^{\circ}\text{C} = 353.15\text{ K}$

Beginning of the compression: $P_1 = 70\text{ kPa}$

Ideal regenerator:

$${}_2q_3 = -{}_4q_1 \Rightarrow q_H = {}_3q_4 = 700\text{ kJ/kg} \quad (101)$$

$$\Rightarrow w_T = q_H = \boxed{700\text{ kJ/kg}} \quad (102)$$

$$\eta_{TH} = \eta_{CARNOT} = 1 - \frac{353.15}{1223.15} = \boxed{0.7113} \quad (103)$$

$$w_{NET} = \eta_{TH}q_H = 0.7113 \cdot 700 = 497.895\text{ kJ/kg} \quad (104)$$

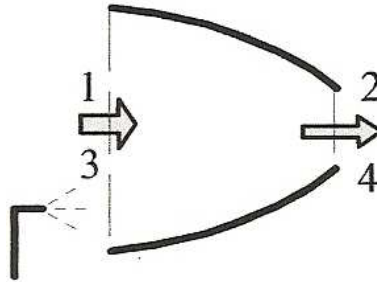
$$w_C = q_L = q_H - w_{NET} = 700 - 497.895 = \boxed{202.105\text{ kJ/kg}} \quad (105)$$

4.4 Problem 4: 12.53

An afterburner in a jet engine adds fuel after the turbine this raising the pressure and temperature due to the energy of combustion. Assume a standard condition of 800 K , 250 kPa after the turbine into the nozzle that exhausts at 95 kPa . Assume the afterburner adds 425 kJ/kg to that state with a rise in pressure for the same specific volume, and neglect any upstream effects on the turbine. Find the nozzle exit velocity before and after the afterburner is turned on.

Without afterburner: $T_1 = 800\text{ K}$, $P_1 = 200\text{ kPa}$, $P_2 = 95\text{ kPa}$

With afterburner: $v_3 = v_1$, $P_4 = 95\text{ kPa}$



Assume reversible adiabatic nozzle flow, then constant s from Eq. 8.23

$$T_2 - T_1 \left(\frac{P_2}{P_1} \right)^{\frac{(k-1)}{k}} = 800\text{ K} \cdot \left(\frac{95}{250} \right)^{0.2857} = 606.8\text{ K} \quad (106)$$

Energy Equation:

$$\frac{1}{2}V_2^2 = C_P(T_1 - T_2) \Rightarrow V_2 = \sqrt{2C_P(T_1 - T_2)} = \sqrt{2 \cdot 1004 \frac{\text{J}}{\text{kg} \cdot \text{K}} (800 - 606.8)\text{ K}} = \boxed{622.8\text{ m/s}} \quad (107)$$

Add the q_{AB} at assumed constant volume then energy equation gives

$$T_3 = T_1 + \frac{q_{AB}}{C_v} = 800 + \frac{425}{0.717} = 1392.63\text{ K} \quad (108)$$

$$v_3 = v_1 \Rightarrow P_3 = P_1 \left(\frac{T_3}{T_1} \right) = 25\text{ kPa} \cdot \frac{1392.63}{800} = 435.2\text{ kPa} \quad (109)$$

Reversible adiabatic expansion, again from Eq. 8.23

$$T_4 = T_3 \left(\frac{P_4}{P_3} \right)^{\frac{(k-1)}{k}} = 1392.63 \cdot \left(\frac{95}{435.2} \right)^{0.2857} = 901.5\text{ K} \quad (110)$$

$$V_2 = \sqrt{2C_P(T_3 - T_4)} = \sqrt{2 \cdot 1004 \frac{\text{J}}{\text{kg} \cdot \text{K}} (1392.63 - 901.5)\text{ K}} = \boxed{993.02\text{ m/s}} \quad (111)$$

Comment: The real process adds some fuel that burns releasing energy so the temperature goes up and due to the confined space then pressure goes up. As the pressure goes up the exit velocity increases altering the mass flow rate through the nozzle. As in most problems the real device is more complicated than we can describe with our simple analysis.

5 Homework 5 Solutions:

5.1 Problem 1: 12.70

A gasoline engine takes air in at 300 K, 90 kPa and then compresses it. The combustion adds 1000 kJ/kg to the air after which the temperature is 2050 K. Use variable heat capacities (Table A.7) and find the compression ratio, the compression specific work and the highest pressure in the cycle.

Standard Otto cycle, solve using Table A.7.1.

State 3: $T_3 = 2050$ K, $u_3 = 1725.71$ kJ/kg

State 2: Combustion Process

$$u_2 = u_3 - q_H = 1725.71 - 1000 = 725.71 \text{ kJ/kg} \quad (112)$$

$$\Rightarrow T_2 = 960.5 \text{ K}, s_{T2}^\circ = 8.0889 \frac{\text{kJ}}{\text{kg} \cdot \text{K}} \quad (113)$$

State 1: Compression from 1 to 2: $u_1 = 214.36$ kJ/kg, $s_1 = 6.86926 \frac{\text{kJ}}{\text{kg} \cdot \text{K}}$, $s_2 = s_1 \Rightarrow$ From Eq. 8.19

$$0 = s_{T2}^\circ - s_{T1}^\circ - R \ln \left(\frac{P_2}{P_1} \right) = s_{T2}^\circ - s_{T1}^\circ - R \ln \left(\frac{T_2 v_1}{T_1 v_2} \right) \quad (114)$$

$$= 8.0089 - 6.86926 - 0.287 \ln \left(\frac{960.5}{300} \right) - 0.287 \ln \left(\frac{v_1}{v_2} \right) \quad (115)$$

$$\text{Solving } \Rightarrow \frac{v_1}{v_2} = \boxed{21.89} \quad (116)$$

Comment: This is much too high for an actual Otto cycle.

$${}_1w_2 = u_2 - u_1 = 725.71 - 214.36 = \boxed{511.35 \text{ kJ/kg}} \quad (117)$$

Highest pressure is after combustion where $v_3 = v_2$ so we get

$$P_3 = P_2 \frac{T_3}{T_2} = P_1 \left(\frac{T_3}{T_1} \right) \left(\frac{v_1}{v_3} \right) = 90 \cdot \left(\frac{2050}{300} \right) \cdot 21.89 = \boxed{13.46 \text{ MPa}} \quad (118)$$

5.2 Problem 2: 12.90

A diesel engine has a state before compression of 95 kPa, 300 K, a peak pressure of 6000 kPa, and a maximum temperature of 2400 K. Find the volumetric compression ratio and the thermal efficiency. Use the properties from Table A.7.

Compression:

$$\text{State 1: } T_1 = 300 \text{ K}, u_1 = 214.36, s_1 = 6.869 \frac{\text{kJ}}{\text{kg}\cdot\text{K}}$$

$$\text{State 2: } s_2 = s_1 \Rightarrow \text{From Eq. 8.19}$$

$$s_{T_2}^\circ = s_{T_1}^\circ + R \ln \left(\frac{P_2}{P_1} \right) = 6.869 + 0.287 \ln \left(\frac{6000}{95} \right) = 8.0588 \frac{\text{kJ}}{\text{kg}\cdot\text{K}} \quad (119)$$

$$\text{A.7.1} \Rightarrow T_2 = 935.32 \text{ K}, h_2 = 972.90 \text{ kJ/kg} \quad (120)$$

$$\text{State 3: } h_3 = 2755.78 \text{ kJ/kg}, s_{T_3}^\circ = 9.19586 \frac{\text{kJ}}{\text{kg}\cdot\text{K}}$$

$$q_H = h_3 - h_2 = 2755.78 - 972.90 = 1782.88 \text{ kJ/kg} \quad (121)$$

$$CR = \frac{v_1}{v_2} = \left(\frac{T_1}{T_2} \right) \left(\frac{P_2}{P_1} \right) = \left(\frac{300}{935.32} \right) \left(\frac{6000}{95} \right) = \boxed{20.26} \quad (122)$$

Expansion process

$$s_{T_4}^\circ = s_{T_3}^\circ + R \ln \left(\frac{P_4}{P_3} \right) = s_{T_3}^\circ + R \ln \left(\frac{T_4}{T_3} \right) + R \ln \left(\frac{v_3}{v_4} \right) \quad (123)$$

$$\frac{v_3}{v_4} = \frac{v_3}{v_1} = \left(\frac{v_2}{v_1} \right) \cdot \left(\frac{T_3}{T_2} \right) = \left(\frac{T_3}{T_2} \right) \left(\frac{1}{CR} \right) = \left(\frac{2400}{935.32} \right) \left(\frac{1}{20.26} \right) = 0.1267 \quad (124)$$

$$s_{T_4}^\circ - R \ln \left(\frac{v_3}{v_4} \right) = s_{T_3}^\circ + R \ln \left(\frac{T_4}{T_3} \right) = 9.1958 + 0.287 \ln 0.1267 = 8.6029 \quad (125)$$

Trial and error on T_4 since it appears both in $s_{T_4}^\circ$ and the ln function

$$T_4 = 1300 \text{ K}, LHS = 8.4405 - 0.287 \ln \left(\frac{1300}{2400} \right) = 8.6160 \quad (126)$$

$$T_4 = 1250 \text{ K}, LHS = 8.3940 - 0.287 \ln \left(\frac{1250}{2400} \right) = 8.5812 \quad (127)$$

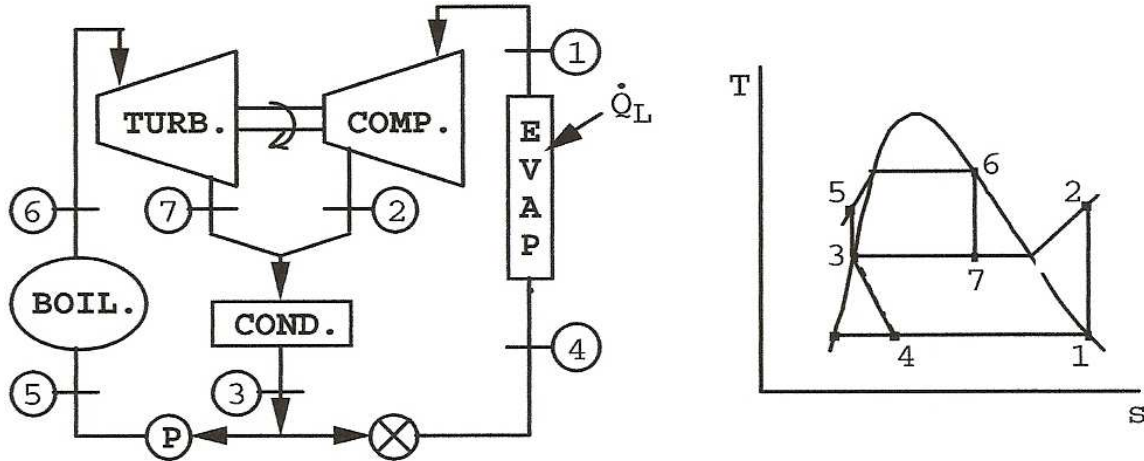
Now linearly interpolate $\Rightarrow T_4 = 1280.7 \text{ K}, u_4 = 1004.81 \text{ kJ/kg}$

$$q_L = u_4 - u_1 = 1004.81 - 214.36 = 790.45 \text{ kJ/kg} \quad (128)$$

$$\eta = 1 - \left(\frac{q_L}{q_H} \right) = 1 - \left(\frac{790.45}{1782.88} \right) = \boxed{0.5566} \quad (129)$$

5.3 Problem 3: 11.122

Consider an ideal dual-loop heat-powered refrigeration cycle using R-134a as the working fluid, as shown in Figure 5.3. Saturated vapor at 90°C leaves the boiler and expands in the turbine to the condenser pressure. Saturated vapor at -15°C leaves the evaporator and is compressed to the condenser pressure. The ratio of the flows through the two loops is such that the turbine produces just enough power to drive the compressor. The two exiting streams mix together and enter the condenser. Saturated liquid leaving the compressor at 45°C is then separated into two streams in the necessary proportions. Determine the ratio of mass flow rate through the power loop to the refrigeration loop. Find also the performance of the cycle, in terms of the ratio $\frac{\dot{Q}_L}{\dot{Q}_H}$.



From Table B.5.1, $T_6 = 90^\circ\text{C}$ sat. vapor $\Rightarrow P_5 = P_6 = 3.2445 \text{ MPa}$

From Table B.5.1, $T_3 = 45^\circ\text{C}$ sat. liquid $\Rightarrow P_2 = P_3 = P_7 = 1.1602 \text{ MPa}$

$$T_1 = -15^\circ\text{C}, h_1 = 389.20 \text{ kJ/kg}, h_3 = h_4 = 264.11 \text{ kJ/kg}, h_6 = 425.70 \text{ kJ/kg} \quad (130)$$

C.V. Turbine

$$s_7 = s_6 = 1.6671 \frac{\text{kJ}}{\text{kg} \cdot \text{K}} = 1.2145 + x_7 \cdot 0.4962, x_7 = 0.912 \quad (131)$$

$$h_7 = 264.11 + 0.912 \cdot 157.85 = 408.09 \text{ kJ/kg} \quad (132)$$

C.V. Compressor

$$s_2 = s_1 = 0.9258 \Rightarrow h_2 = 429.89 \text{ kJ/kg} \quad (133)$$

CV: turbine + compressor

$$\text{Continuity Eq.: } \dot{m}_1 = \dot{m}_2, \dot{m}_6 = \dot{m}_7 \quad (134)$$

$$\text{Energy Eq.: } \dot{m}_1 h_1 + \dot{m}_6 h_6 = \dot{m}_2 h_2 + \dot{m}_7 h_7 \quad (135)$$

$$\frac{\dot{m}_6}{\dot{m}_1} = \frac{(h_2 - h_1)}{(h_6 - h_7)} = \frac{429.89 - 389.20}{425.70 - 408.09} = \boxed{2.31} \quad (136)$$

CV: pump

$$w_P = v_3(P_5 - P_3) = 0.000890(3244.5 - 1160.2) = 1.855 \text{ kJ/kg} \quad (137)$$

$$h_5 = h_3 + w_P = 265.97 \text{ kJ/kg} \quad (138)$$

CV: evaporator $\Rightarrow \dot{Q}_L = \dot{m}_1(h_1 - h_4)$

CV: boiler $\Rightarrow \dot{Q}_H = \dot{m}_6(h_6 - h_5)$

$$\beta = \frac{\dot{Q}_L}{\dot{Q}_H} = \frac{\dot{m}_1(h_1 - h_4)}{\dot{m}_6(h_6 - h_5)} = \frac{389.20 - 264.11}{2.31(425.70 - 265.97)} = \boxed{0.339} \quad (139)$$

5.4 Problem 4: 12.112

A small utility gasoline engine of 250 cc runs at 1500 RPM with a compression ratio of 7 : 1. The inlet state is 75 kPa, 17°C and the combustion adds 1500 kJ/kg to the charge. This engine runs a heat pump using R-410a with a high pressure of 4 MPa and an evaporator operating at 0°C. Find the rate of heating the pump can deliver. Overall cycle

efficiency is from Eq. 12.12, $r_v = \frac{v_1}{v_2} = 7$

$$\eta_{TH} = 1 - r^{1-k} = 1 - 7^{-0.4} = 0.5408 \quad (140)$$

$$w_{NET} = \eta_{TH} \cdot q_H = 0.5408 \cdot 1500 = 811.27 \text{ kJ/kg} \quad (141)$$

We also need specific volume to evaluate Eqs. 12.9 to 12.11

$$v_1 = \frac{RT_1}{P_1} = \frac{0.287 \frac{\text{kJ}}{\text{kg}\cdot\text{K}} \cdot 290 \text{ K}}{75 \text{ kPa}} = 1.1097 \text{ m}^3/\text{kg} \quad (142)$$

$$v_2 = \frac{v_1}{CR} = 0.15853 \text{ m}^3/\text{kg} \quad (143)$$

$$P_{meff} = \frac{w_{NET}}{v_1 - v_2} = \frac{811.27 \text{ kJ/kg}}{1.097 - 0.15853 \text{ m}^3/\text{kg}} = 852.9 \text{ kPa} \quad (144)$$

Now we can find the power from Eq. 12.11 (assume 4-stroke engine)

$$\dot{W} = P_{meff} V_{displ} \frac{\text{RPM}}{60} \frac{1}{2} = 852.9 \cdot 2.5 \cdot 10^{-4} \cdot \frac{1500}{60} \cdot \frac{1}{2} = 2.665 \text{ kW} \quad (145)$$

For the refrigeration cycle we have:

$$\text{State 1: } h_1 = 279.12 \text{ kJ/kg}, s_1 = 1.0368 \frac{\text{kJ}}{\text{kg}\cdot\text{K}}$$

$$\text{State 2: } P_2 = 4 \text{ MPa}, s_2 = s_1, \text{ interpolate } \Rightarrow h_2 = 323.81 \text{ kJ/kg}$$

$$\text{State 3: } P_3 = 4 \text{ MPa}, \text{ interpolate } \Rightarrow h_3 = 171.62 \text{ kJ/kg}$$

$$\beta_{HP} = \frac{q_H}{w_C} = \frac{h_2 - h_3}{h_2 - h_1} = \frac{323.81 - 171.62}{323.81 - 279.12} = 3.405 \quad (146)$$

The work out of the heat engine equals the input to the heat pump

$$\dot{Q}_H = \beta_{HP} \dot{W} = 3.405 \cdot 2.665 \text{ kW} = \boxed{9.07 \text{ kW}} \quad (147)$$

6 Homework 6 Solutions:

6.1 Problem 1: 13.20

A 100 m^3 storage tank with fuel gases is at 300 K , 100 kPa containing a mixture of acetylene C_2H_2 , propane C_3H_8 and butane C_4H_{10} . A test shows the partial pressure of C_2H_2 is 15 kPa and that of C_3H_8 is 65 kPa . How much mass is there of each component?

Assume ideal gases, then the ratio of partial to total pressure is the mole fraction, $y = \frac{P}{P_{tot}}$

$$y_{\text{C}_2\text{H}_2} = \frac{15}{100} = 0.15, y_{\text{C}_3\text{H}_8} = \frac{65}{100} = 0.65, y_{\text{C}_4\text{H}_{10}} = \frac{20}{100} = 0.20 \quad (148)$$

$$n_{tot} = \frac{PV}{\bar{R}T} = \frac{100 \text{ kPa} \cdot 100 \text{ m}^3}{8.31451 \frac{\text{kJ}}{\text{kmol}\cdot\text{K}} \cdot 300 \text{ K}} = 4.0091 \text{ kmol} \quad (149)$$

$$m_{\text{C}_2\text{H}_2} = (nM)_{\text{C}_2\text{H}_2} = y_{\text{C}_2\text{H}_2} n_{tot} M_{\text{C}_2\text{H}_2} = 0.15 \cdot 4.0091 \text{ kmol} \cdot 26.038 \text{ kg/kmol} = \boxed{15.659 \text{ kg}} \quad (150)$$

$$m_{\text{C}_3\text{H}_8} = (nM)_{\text{C}_3\text{H}_8} = y_{\text{C}_3\text{H}_8} n_{tot} M_{\text{C}_3\text{H}_8} = 0.65 \cdot 4.0091 \text{ kmol} \cdot 44.097 \text{ kg/kmol} = \boxed{114.905 \text{ kg}} \quad (151)$$

$$m_{\text{C}_4\text{H}_{10}} = (nM)_{\text{C}_4\text{H}_{10}} = y_{\text{C}_4\text{H}_{10}} n_{tot} M_{\text{C}_4\text{H}_{10}} = 0.20 \cdot 4.0091 \text{ kmol} \cdot 58.124 \text{ kg/kmol} = \boxed{46.604 \text{ kg}} \quad (152)$$

6.2 Problem 2: 13.29

A flow of 1 kg/s argon at 300 K and another flow of 1 kg/s CO₂ at 1600 K both at 200 kPa are mixed without any heat transfer. Find the exit T, P using variable specific heats.

No work implies no pressure change for a simple flow.

$$P_e = \boxed{200 \text{ kPa}} \quad (153)$$

The energy equation becomes

$$\dot{m}h_i = \dot{m}h_e = (\dot{m}h_i)_{Ar} + (\dot{m}h_i)_{CO_2} = (\dot{m}h_e)_{Ar} + (\dot{m}h_e)_{CO_2} \quad (154)$$

$$\Rightarrow \dot{m}_{CO_2}(h_e - h_i)_{CO_2} + \dot{m}_{Ar}C_{PAr}(T_e - T_i)_{Ar} = 0 \quad (155)$$

$$\Rightarrow 1 \text{ kg/s} \cdot (h_e - 1748.12 \text{ kJ/kg}) + (1 \cdot 0.52) \text{ kW/K} \cdot (T_e - 300 \text{ K}) = 0 \quad (156)$$

$$h_{eCO_2} + 0.52T_e = 1748.12 + 0.52 \cdot 300 = 1904.12 \text{ kJ/kg} \quad (157)$$

Trial and error on T_e using Table A.8 for h_{eCO_2}

$$T_e = 1200 \text{ K} : LHS = 1223.34 + 0.52 \cdot 1200 = 1847.34 \quad (158)$$

$$T_e = 1300 \text{ K} : LHS = 1352.28 + 0.52 \cdot 1300 = 2028.28 \quad (159)$$

Interpolating,

$$T_e = 1200 + 100 \frac{1904.12 - 1847.34}{2028.28 - 1847.34} = \boxed{1231.4 \text{ K}} \quad (160)$$

6.3 Problem 3: 13.46

A mixture of 2 kg oxygen and 2 kg argon is in an insulated piston cylinder arrangement at 150 kPa, 300 K. The piston now compresses the mixture to half its initial volume. Find the final pressure, temperature, and the piston work.

C.V. Mixture. Control mass, boundary work, adiabatic and assume reversible.

$$\text{Energy Eq. 5.11: } u_2 - u_1 = {}_1q_2 - {}_1w_2 = -{}_1w_2 \quad (161)$$

$$\text{Entropy Eq. 8.37: } s_2 - s_1 = 0 + 0 = 0 \quad (162)$$

Process: constant $s \Rightarrow P v^k = \text{constant}$, $v_2 = \frac{v_1}{2}$,

Assume ideal gases ($T_1 \gg T_c$) and use k_{mix} and $C_{v,mix}$ for properties.

$$\text{Eq. 13.15: } R_{mix} = \sum c_i R_i = 0.5 \cdot 0.25983 + 0.5 \cdot 0.20813 = 0.234 \frac{\text{kJ}}{\text{kg} \cdot \text{K}} \quad (163)$$

$$\text{Eq. 13.23: } C_{P,mix} = \sum c_i C_{P,i} = 0.5 \cdot 0.9216 + 0.5 \cdot 0.5203 = 0.721 \frac{\text{kJ}}{\text{kg} \cdot \text{K}} \quad (164)$$

$$C_{v,mix} = C_{P,mix} - R_{mix} = 0.487 \frac{\text{kJ}}{\text{kg} \cdot \text{K}} \quad (165)$$

Ratio of specific heats: $k_{mix} = \frac{C_{P,mix}}{C_{v,mix}} = 1.4805$ The relations for the polytropic process

$$\text{Eq. 8.25: } P_2 = P_1 \left(\frac{v_1}{v_2} \right)^k = P_1 (2)^k = 150(2)^{1.4805} = \boxed{418.5 \text{ kPa}} \quad (166)$$

$$\text{Eq. 8.24: } T_2 = T_1 \left(\frac{v_1}{v_2} \right)^{k-1} = T_1 (2)^{k-1} = 300(2)^{0.4805} = \boxed{418.6 \text{ K}} \quad (167)$$

Work from the energy equation

$${}_1W_2 = m_{tot}(u_1 - u_2) = m_{tot}C_v(T_1 - T_2) = 4 \text{ kg} \cdot 0.487 \frac{\text{kJ}}{\text{kg} \cdot \text{K}}(300 - 418.6) \text{ K} = \boxed{-231 \text{ kJ}} \quad (168)$$

6.4 Problem 4: 13.54

Carbon dioxide gas at 320 K is mixed with nitrogen at 280 K in an insulated mixing chamber. Both flows are at 100 kPa and the mass ratio of carbon dioxide to nitrogen is 2 : 1. Find the exit temperature and total entropy generation per kg of the exit mixture.

CV mixing chamber. The inlet ratio is so $\dot{m}_{CO_2} = 2\dot{m}_{N_2}$ and assume no external heat transfer, no work involved.

$$\text{Continuity Eq. 6.9: } \dot{m}_{N_2} + 2\dot{m}_{N_2} = \dot{m}_{ex} = 3\dot{m}_{N_2} \quad (169)$$

$$\text{Energy Eq. 6.10: } \dot{m}_{N_2}(h_{N_2} - 2h_{CO_2}) = 3\dot{m}_{N_2}h_{mixex} \quad (170)$$

Take 300 K as reference and write $h = h_{300} + C_{Pmix}(T_{mixex} - 300)$

$$C_{PN_2}(T_{iN_2} - 300) + 2C_{PCO_2}(T_{iCO_2} - 300) = 3C_{Pmix}(T_{mixex} - 300) \quad (171)$$

$$C_{Pmix} = \sum c_i C_{Pi} = \frac{2}{3} \cdot 0.842 + \frac{1}{3} \cdot 1.042 = 0.9087 \frac{kJ}{kg \cdot K} \quad (172)$$

$$3C_{Pmix}T_{mixex} = C_{PN_2}T_{iN_2} + 2C_{PCO_2}T_{iCO_2} = 830.64 \text{ kJ/kg} \quad (173)$$

$$T_{mixex} = \boxed{304.7 \text{ K}} \quad (174)$$

To find the entropies we need the partial pressures, which assuming ideal gas are equal to the mole fractions times the total pressure:

$$y_i = \frac{\left[\frac{c_i}{M_i} \right]}{\sum \frac{c_j}{M_j}} \quad (175)$$

$$y_{N_2} = \frac{\left[\frac{0.3333}{28.013} \right]}{\frac{0.3333}{28.013} + \frac{0.6666}{44.01}} = 0.44 \quad (176)$$

$$y_{CO_2} = 1 - y_{N_2} = 0.56 \quad (177)$$

$$\dot{S}_{gen} = \dot{m}_{ex}s_{ex} - (\dot{m}s)_{iCO_2} - (\dot{m}s)_{iN_2} = \dot{m}_{N_2}(s_e - s_i)_{N_2} + 2\dot{m}_{N_2}(s_e - s_i)_{CO_2} \quad (178)$$

$$\frac{\dot{S}_{gen}}{3\dot{m}_{N_2}} = \frac{1}{3} \left[C_{PN_2} \ln \frac{T_{ex}}{T_{iN_2}} - R_{N_2} \ln y_{N_2} \right] + \frac{2}{3} \left[C_{PCO_2} \ln \frac{T_{ex}}{T_{iCO_2}} - R_{CO_2} \ln y_{CO_2} \right] \quad (179)$$

$$= \frac{1}{3} \left[1.042 \ln \left(\frac{304.7}{280} \right) - 0.2968 \ln 0.44 \right] + \frac{2}{3} \left[0.842 \ln \left(\frac{304.7}{320} \right) - 0.1889 \ln 0.56 \right] \quad (180)$$

$$\dot{S}_{gen} = \boxed{0.1561 \frac{kJ}{kg_{mix} K}} \quad (181)$$

7 Homework 7 Solutions:

7.1 Problem 1: 13.73

Ambient moist air enters a steady-flow air-conditioning unit at 105 kPa, 30°C, with a 60% relative humidity. The volume flow rate entering the unit is 100 L/s. The moist air leaves the unit at 95 kPa, 15°C, with a relative humidity of 100%. Liquid condensate also leaves the unit at 15°C. Determine the rate of heat transfer for this process.

State 1:

$$P_{V1} = \phi_1 P_{G1} = 0.60 \cdot 4.246 \text{ kPa} = 2.5476 \text{ kPa} \quad (182)$$

$$w_1 = 0.622 \cdot \frac{2.5476}{105 - 2.5476} = 0.0154668 \quad (183)$$

$$\dot{m}_A = \frac{P_{A1} \dot{V}_1}{R_A T_1} = \frac{102.45 \cdot 0.1}{0.287 \cdot 303.15} = 0.1178 \text{ kg/s} \quad (184)$$

State 2:

$$P_{V2} = P_{G2} = 1.705 \text{ kPa} \quad (185)$$

$$w_2 = 0.622 \cdot \frac{1.705}{95 - 1.705} = 0.1137 \quad (186)$$

Energy Eq. 6.10:

$$\dot{Q}_{CV} + \dot{m}_A h_{A1} + \dot{m}_{V1} h_{V1} = \dot{m}_A h_{A2} + \dot{m}_{V2} h_{V2} + \dot{m}_3 h_{L3} \quad (187)$$

$$\frac{\dot{Q}_{CV}}{\dot{m}_A} = C_{p0A}(T_2 - T_1) + w_2 h_{V2} - w_1 h_{V1} + (w_1 - w_2) h_{L3} \quad (188)$$

$$= 1.004(15 - 30) + 0.1137 \cdot 2528.91 - 0.0154668 \cdot 2556.25 + 0.00456 \cdot 63.0 = -26.732 \text{ kJ/kg air} \quad (189)$$

$$\dot{Q}_{CV} = 0.1178(-26.732) = \boxed{-3.088 \text{ kW}} \quad (190)$$

7.2 Problem 2: 13.103

Use a psychrometric chart to find the missing property of: ϕ , ω , T_{wet} , T_{dry}

- a. $T_{dry} = 25^\circ\text{C}$, $\phi = 80\%$
- b. $T_{dry} = 15^\circ\text{C}$, $\phi = 100\%$

- c. $T_{dry} = 20^\circ\text{C}$, $\omega = 0.008$
- d. $T_{dry} = 25^\circ\text{C}$, $T_{wet} = 23^\circ\text{C}$

a. 25°C , $\phi = 80\% \Rightarrow \boxed{\omega = 0.016, T_{wet} = 22.3^\circ\text{C}}$ (191)

b. 15°C , $\phi = 100\% \Rightarrow \boxed{\omega = 0.0106, T_{wet} = 15^\circ\text{C}}$ (192)

c. 20°C , $\omega = 0.008 \Rightarrow \boxed{\phi = 57\%, T_{wet} = 14.4^\circ\text{C}}$ (193)

d. 25°C , $T_{wet} = 23^\circ\text{C} \Rightarrow \boxed{\omega = 0.017, \phi = 86\%}$ (194)

7.3 Problem 3: 14.35

The Joule-Thomson coefficient μ_J is a measure of the direction and magnitude of the temperature change with pressure in a throttling process. For any three properties x, y, z use the mathematical relation

$$\left(\frac{\partial x}{\partial y}\right)_z \left(\frac{\partial y}{\partial z}\right)_x \left(\frac{\partial z}{\partial x}\right)_y = -1 \quad (195)$$

to show the following relations for the Joule-Thomson coefficient:

$$\mu_J = \left(\frac{\partial T}{\partial P}\right)_h = \frac{T \left(\frac{\partial v}{\partial T}\right)_P^{-v}}{C_P} = \frac{RT^2}{PC_p} \left(\frac{\partial Z}{\partial T}\right)_P \quad (196)$$

Let $x = T$, $y = P$ and $z = h$ and substitute into the relations as

$$\left(\frac{\partial T}{\partial P}\right)_h \left(\frac{\partial P}{\partial h}\right)_T \left(\frac{\partial h}{\partial T}\right)_P = -1 \quad (197)$$

Then we have the definition of specific heat as $C_P = \left(\frac{\partial h}{\partial T}\right)_P$ so solve for the first term

$$\mu_J = \left(\frac{\partial T}{\partial P}\right)_h = -\frac{\frac{1}{C_P}}{\left(\frac{\partial P}{\partial h}\right)_T} = -\frac{1}{C_P} \left(\frac{\partial h}{\partial P}\right)_T \quad (198)$$

The last derivative is substituted with Eq. 14.25 so we get

$$\mu_J = \left(\frac{\partial T}{\partial P}\right)_h = \frac{T \left(\frac{\partial v}{\partial T}\right)_P^{-v}}{C_P} \quad (199)$$

If we use the compressibility factor then we get

$$Pv = ZRT \Rightarrow \left(\frac{\partial v}{\partial T}\right)_P = \frac{ZR}{P} + \frac{RT}{P} \left(\frac{\partial Z}{\partial T}\right)_P = \frac{v}{T} + \frac{RT}{P} \left(\frac{\partial Z}{\partial T}\right)_P \quad (200)$$

so then

$$T \left(\frac{\partial v}{\partial T}\right)_P^{-v} = v + \frac{RT^2}{P} \left(\frac{\partial Z}{\partial T}\right)_P^{-v} = \frac{RT^2}{P} \left(\frac{\partial Z}{\partial T}\right)_P \quad (201)$$

and we have shown the last expression also.

$$\boxed{\mu_J = \left(\frac{\partial T}{\partial P}\right)_h = \frac{T \left(\frac{\partial v}{\partial T}\right)_P^{-v}}{C_P} = \frac{RT^2}{PC_p} \left(\frac{\partial Z}{\partial T}\right)_P} \quad (202)$$

7.4 Problem 4: 14.77

Develop *general* expressions for changes in internal energy, enthalpy, and entropy for a gas obeying the Redlich-Kwong equation of state.

$$\text{Redlich-Kwong equation of state: } P = \frac{RT}{v-b} - \frac{a}{v(v+b)T^{1/2}} \quad (203)$$

$$\left(\frac{\partial P}{\partial T}\right)_v = \frac{R}{v-b} + \frac{a}{2v(v+b)T^{3/2}} \quad (204)$$

From Eq. 14.30

$$u_2 - u_1 = \int_1^2 C_v(T, v) dT + \int_1^2 \frac{3a}{2v(v+b)T^{1/2}} dv = \boxed{\int_1^2 C_v(\hat{T}, v) d\hat{T} - \frac{3a}{2bT^{1/2}} \ln \left[\left(\frac{v_2+b}{v_2} \right) \left(\frac{v_1}{v_1+b} \right) \right]} \quad (205)$$

We find the change in h from change in u , so we do not do the derivative in Eq. 14.27. This is due to the form of the EOS.

$$\boxed{h_2 - h_1 = \int_1^2 C_p(\hat{T}, v) d\hat{T} + P_2 v_2 - P_1 v_1 - \frac{3a}{2bT^{1/2}} \ln \left[\left(\frac{v_2+b}{v_2} \right) \left(\frac{v_1}{v_1+b} \right) \right]} \quad (206)$$

Entropy follows from Eq. 14.35

$$s_2 - s_1 = \int_1^2 C_v(T, v) \frac{dT}{T} + \int_1^2 \left[\frac{R}{v-b} + \frac{a/2}{v(v+b)T^{3/2}} \right] dv \quad (207)$$

$$\boxed{s_2 - s_1 = \int_1^2 C_v(\hat{T}, v) \frac{d\hat{T}}{\hat{T}} + R \ln \left(\frac{v_2-b}{v_1-b} \right) - \frac{a}{2bT^{3/2}} \ln \left[\left(\frac{v_2+b}{v_2} \right) \left(\frac{v_1}{v_1+b} \right) \right]} \quad (208)$$

7.5 Problem 5: 14.81

A flow of oxygen at 235 K, 5 MPa is throttled to 100 kPa in a steady flow process. Find the exit temperature and the specific entropy generation using the Redlich-Kwong equation of state and ideal gas heat capacity. Notice this becomes iterative due to the non-linearity coupling h , P , v and T .

C.V. Throttle. Steady single flow, no heat transfer and no work.

$$\text{Energy Eq.: } h_1 + 0 = h_2 + 2 \text{ so constant } h \quad (209)$$

$$\text{Entropy Eq.: } s_1 + s_{gen} = s_2 \text{ so entropy generation} \quad (210)$$

Find the change in h from Eq. 14.26 assuming C_p is constant.

$$\text{Redlich-Kwong equation of state: } P = \frac{RT}{v-b} - \frac{a}{v(v+b)T^{1/2}} \quad (211)$$

$$\left(\frac{\partial P}{\partial T}\right)_v = \frac{R}{v-b} + \frac{a}{2v(v+b)T^{3/2}} \quad (212)$$

From Eq. 14.31

$$(u_2 - u_1)_T = \int_1^2 \frac{3a}{2v(v+b)T^{1/2}} dv = \frac{-3a}{2bT^{1/2}} \ln \left[\left(\frac{v_2+b}{v_2}\right) \left(\frac{v_1}{v_1+b}\right) \right] \quad (213)$$

We find change in h from change in u , so we do not do the derivative in Eq. 14.27. This is due to the form of the EOS.

$$(h_2 - h_1)_T = P_2 v_2 - P_1 v_1 - \frac{3a}{2bT^{1/2}} \ln \left[\left(\frac{v_2+b}{v_2}\right) \left(\frac{v_1}{v_1+b}\right) \right] \quad (214)$$

Entropy follows from Eq. 14.35

$$(s_2 - s_1)_T = \int_1^2 \left[\frac{R}{v-b} + \frac{a/2}{v(v+b)T^{3/2}} \right] dv = R \ln \left(\frac{v_2-b}{v_1-b} \right) - \frac{a}{2bT^{3/2}} \ln \left[\left(\frac{v_2+b}{v_2}\right) \left(\frac{v_1}{v_1+b}\right) \right] \quad (215)$$

$$P_c = 5040 \text{ kPa}, T_c = 154.6 \text{ K}, R = 0.2598 \frac{\text{kJ}}{\text{kg} \cdot \text{K}} \quad (216)$$

$$b = 0.08664 \frac{RT_c}{P_c} = 0.08664 \cdot \frac{0.2598 \cdot 154.6}{5040} = 0.0006905 \text{ m}^3/\text{kg} \quad (217)$$

$$a = 0.42748 \frac{R^2 T_c^{5/2}}{P_c} = 0.42748 \cdot \frac{(0.2598)^2 \cdot (154.6)^{5/2}}{5040} = 1.7013 \quad (218)$$

We need to find T_2 so the energy equation is satisfied

$$h_2 - h_1 = h_2 - h_x + h_x - h_1 = C_p(T_2 - T_1) + (h_2 - h_1)_T = 0 \quad (219)$$

and we will evaluate it similar to figure 13.4, where the first term is done from state x to 2 and the second term is done from state 1 to state x (at $T_1 = 235 \text{ K}$). We do this as we assume state 2 is close to ideal gas, but we do not know T_2 .

We first need to find v_1 from the EOS, so guess v and find P

$$v_1 = 0.011 \text{ m}^3/\text{kg} \Rightarrow P = 5059 \text{ too high} \quad (220)$$

$$v_1 = 0.01114 \text{ m}^3/\text{kg} \Rightarrow P = 5000.6 \text{ OK} \quad (221)$$

Now evaluate the change in h along the 235 K from state 1 to state x, that requires a value for v_x . Guess ideal gas at $T_x = 235 \text{ K}$,

$$v_x = \frac{RT_x}{P_2} = 0.2598 \cdot \frac{235}{100} = 0.6105 \text{ m}^3/\text{kg} \quad (222)$$

From the EOS: $P_2 = 99.821 \text{ kPa}$ (close) A few more guesses and adjustments gives

$$v_x = 0.6094 \text{ m}^3/\text{kg}, P_2 = 100.0006 \text{ kPa OK} \quad (223)$$

$$(h_x - h_1)_T = P_x v_x - P_1 v_1 - \frac{3a}{2bT^{1/2}} \ln \left[\left(\frac{v_x + b}{v_x} \right) \left(\frac{v_1}{v_1 + b} \right) \right] = 19.466 \text{ kJ/kg} \quad (224)$$

$$\text{From energy eq.: } T_2 = T_1 = \frac{(h_x - h_1)_T}{C_p} = 235 - \frac{19.466}{0.9055} = \boxed{213.51 \text{ K}} \quad (225)$$

Now the change in s is done in a similar fashion,

$$s_{gen} = s_2 - s_1 = (s_x - s_1)_T + s_2 - s_x = R \ln \left(\frac{v_x - b}{v_1 - b} \right) - \frac{a}{2bT^{3/2}} \ln \left[\left(\frac{v_x + b}{v_x} \right) \left(\frac{v_1}{v_1 + b} \right) \right] + C_p \ln \frac{T_2}{T_x} \quad (226)$$

$$= 1.0576 + 0.0202 - 0.0868 = \boxed{0.9910 \frac{\text{kJ}}{\text{kg} \cdot \text{K}}} \quad (227)$$

7.6 Problem 6:

Consider a thermodynamic system in which there are *two* reversible work modes: compression and electrical. So take the version of the text's Eq. 4.16 giving dW to be

$$\delta W = PdV - \mathcal{E}dZ, \quad (228)$$

where \mathcal{E} is the electrical potential difference and dZ is the amount of charge that flows into the system.

- Extend the Gibbs equation to account for electrical work.
- Find the Legendre transformation which renders the independent variables to be P , \mathcal{E} and T and show how the other variables can be determined as functions of these independent variables.
- Find all Maxwell equations associated with this Legendre transformation.

Gibbs equation:

$$dU = \delta Q - \delta W, \quad \delta Q = Tds \quad (229)$$

$$\boxed{dU = TdS - PdV + \mathcal{E}dZ} \quad (230)$$

Legendre transformation:

$$\psi_1 = T, \quad \psi_2 = -P, \quad \psi_3 = \mathcal{E}, \quad x_1 = S, \quad x_2 = V, \quad x_3 = Z \quad (231)$$

$$F_1 = U - \psi_1 x_1 = U - TS \quad (232)$$

$$F_1 = U - \psi_2 x_2 = U - PV \quad (233)$$

$$F_1 = U - \psi_3 x_3 = U - \mathcal{E}Z \quad (234)$$

$$F_{1,2,3} = U - TS + PV - \mathcal{E}Z \quad (235)$$

$$\boxed{F(P, \mathcal{E}, T) = -SdT + VdP - Zd\mathcal{E}} \quad (236)$$

Maxwell equations:

$$-S = \left(\frac{\partial U}{\partial T} \right)_{P, \mathcal{E}}, \quad -V = \left(\frac{\partial U}{\partial P} \right)_{T, \mathcal{E}}, \quad -Z = \left(\frac{\partial U}{\partial \mathcal{E}} \right)_{T, P} \quad (237)$$

$$\boxed{\left(\frac{\partial S}{\partial P} \right)_{T, \mathcal{E}} = - \left(\frac{\partial V}{\partial T} \right)_{P, \mathcal{E}}} \quad (238)$$

$$\boxed{\left(\frac{\partial S}{\partial P \mathcal{E}} \right)_{T, P} = \left(\frac{\partial Z}{\partial T} \right)_{P, \mathcal{E}}} \quad (239)$$

$$\boxed{\left(\frac{\partial V}{\partial \mathcal{E}} \right)_{T, P} = - \left(\frac{\partial Z}{\partial P} \right)_{T, \mathcal{E}}} \quad (240)$$

7.7 Problem 7: 14.113

A 2 kg mixture of 50% argon and 50% nitrogen by mass is in a tank at 2 MPa, 180 K. How large is the volume using a model of (a) ideal gas and (b) Redlich-Kwong equation of state with a, b for a mixture.

a) Ideal gas mixture:

$$\text{Eq. 13.15: } R_{mix} = \sum c_i R_i = 0.5 \cdot 0.2081 + 0.5 \cdot 0.2968 = 0.25245 \frac{kJ}{kg \cdot K} \quad (241)$$

$$V = \frac{m R_{mix} T}{P} = \frac{2 \cdot 0.25245 \cdot 180}{2000} = \boxed{0.0454 \text{ m}^3} \quad (242)$$

b) Redlich-Kwong equation of state:

Before we can do the parameters a, b for the mixture we need the individual component parameters, Eq. 14.54, 13.55.

$$a_{Ar} = 0.42748 \frac{R^2 T_C^{5/2}}{P_c} = 0.42748 \frac{(0.2081)^2 \cdot (150.8)^{2.5}}{4870} = 1.06154 \quad (243)$$

$$a_{N_2} = 0.42748 \frac{R^2 T_C^{5/2}}{P_c} = 0.42748 \frac{(0.2081)^2 \cdot (126.2)^{2.5}}{3390} = 1.98743 \quad (244)$$

$$b_{Ar} = 0.08664 \frac{R T_C}{P_c} = 0.08664 \frac{0.2081 \cdot 150.8}{4870} = 0.000558 \quad (245)$$

$$b_{N_2} = 0.08664 \frac{R T_C}{P_c} = 0.08664 \frac{0.2081 \cdot 126.2}{3390} = 0.000957 \quad (246)$$

Now the mixture parameters are from Eq. 14.84

$$a_{mix} = \left(\sum c_i a_i^{1/2} \right)^2 = \left(0.5 \cdot \sqrt{1.06154} + 0.5 \cdot \sqrt{1.98743} \right)^2 = 1.4885 \quad (247)$$

$$b_{mix} = \sum c_i b_i = 0.5 \cdot 0.000558 + 0.5 \cdot 0.000957 = 0.000758 \quad (248)$$

Using now Eq. 14.53:

$$P = \frac{RT}{v-b} - \frac{a}{v(v+b)T^{1/2}} \quad (249)$$

$$2000 = \frac{0.25245 \cdot 180}{v = 0.000758} - \frac{1.4885}{v(v+0.000758)180^{1/2}} \quad (250)$$

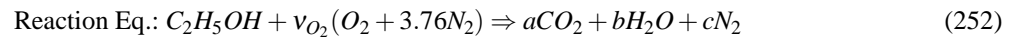
By trial and error we find the specific volume, $v = 0.02102 \text{ m}^3/\text{kg}$

$$V = mv = \boxed{0.04204 \text{ m}^3} \quad (251)$$

8 Homework 8 Solutions:

8.1 Problem 1: 15.20

Calculate the theoretical air-fuel ratio on a mass and mole basis for the combustion of ethanol, C_2H_5OH .



Do the atom balance

$$\text{Balance C: } 2 = a \quad (253)$$

$$\text{Balance H: } 6 = 2b \Rightarrow b = 3 \quad (254)$$

$$\text{Balance O: } 1 + 2v_{O_2}2a + b = 4 + 3 = 7 \rightarrow v_{O_2} = 3 \quad (255)$$

$$\left(\frac{A}{F}\right)_{mol} = \frac{v_{O_2}(1 + 3.76)}{1} = 3 \cdot 4.76 = \boxed{14.28} \quad (256)$$

$$\left(\frac{A}{F}\right)_{mass} = \frac{v_{O_2}M_{O_2} + v_{N_2}M_{N_2}}{M_{Fuel}} = \frac{3 \cdot 31.999 + 11.28 \cdot 28.013}{46.069} = \boxed{8.943} \quad (257)$$

8.2 Problem 2: 16.20

A container has liquid water at 15°C , 100 kPa in equilibrium with a mixture of water vapor and dry air also at 15°C , 100 kPa . How much is the water vapor pressure and what is the saturated water vapor pressure?

From the steam tables we have for a saturated liquid:

$$P_g = \boxed{1.705\text{ kPa}}, \quad v_f = 0.001001\text{ m}^3/\text{kg} \quad (258)$$

The liquid is at 100 kPa so it is compressed liquid still at 15°C so from Eq. 14.15 at constant T

$$g_{liq} - g_f = \int v dP = v_f(P - P_g) \quad (259)$$

The vapor in the moist air is at the partial pressure P_v also at 20°C so we assume ideal gas for the vapor

$$g_{vap} - g_g = \int v dP = RT \ln \frac{P_v}{P_g} \quad (260)$$

We have two saturated phases so $g_f - g_g$ ($q = h_{fg} = Ts_{fg}$) and now for equilibrium the two Gibbs functions must be the same as

$$g_{vap} = g_{liq} = RT \ln \frac{P_v}{P_g} + g_g = v_f(P - P_g) + g_f \quad (261)$$

leaving us with

$$\ln \frac{P_v}{P_g} = \frac{v_f(P - P_g)}{RT} = \frac{0.001001(100 - 1.705)}{0.4615 \cdot 288.15} = 0.0007466 \quad (262)$$

$$P_v = P_g e^{0.0007466} = \boxed{1.7063\text{ kPa}} \quad (263)$$

This is only a minute amount above the saturation pressure. For moist air applications such differences were neglected and assumed the partial water vapor pressure at equilibrium (100% relative humidity) is P_g . The pressure has to be much higher for this to be a significant difference.

8.3 Problem 3: 16.24:

Calculate the equilibrium constant for the reaction $H_2 \Rightarrow 2H$ at a temperature of 2000 K, using properties from Table A.9. Compare the result with the value listed in Table A.11.

From Table A.9 at 2000 K we find:

$$\Delta \bar{h}_{H_2} = 52942 \text{ kJ/kmol}, \bar{s}_{H_2} = 188.419 \frac{\text{kJ}}{\text{kmol} \cdot \text{K}}, \bar{h}_f^\circ = 0 \quad (264)$$

$$\Delta \bar{h}_{H_2} = 35375 \text{ kJ/kmol}, \bar{s}_{H_2} = 154.279 \frac{\text{kJ}}{\text{kmol} \cdot \text{K}}, \bar{h}_f^\circ = 217999 \quad (265)$$

$$\Delta G^\circ = \Delta H - T\Delta S = H_{RHS} - H_{LHS} - T(S_{RHS}^\circ - S_{LHS}^\circ) \quad (266)$$

$$= 2 \cdot (35375 - 217999) - 52943 - 2000(2 \cdot 154.279 - 182.419) = 213528 \text{ kJ/kmol} \quad (267)$$

$$\ln K = \frac{-\Delta G^\circ}{\bar{R}T} = \frac{-213528}{8.3145 \cdot 2000} = -12.8407 \quad (268)$$

$$\boxed{K = 2.6507 \cdot 10^{-6}} \quad (269)$$

Table A.11: $\ln K = -12.841$ OK

8.4 Problem 4: 16.34

Pure oxygen is heated from 25°C to 3200 K in a steady flow process at a constant pressure of 300 kPa. Find the exit composition and the heat transfer.

The only reaction will be the dissociation of oxygen

$$O_2 \Leftrightarrow 2O, \text{ From A.11: } K(3200) = e^{-3.069} = 0.046467 \quad (270)$$

Look at initially 1 mol Oxygen and shift reaction with x

$$n_{O_2} = 1 - x, n_O = 2x, n_{tot} = 1 + x, y_i = \frac{n_i}{n_{tot}} \quad (271)$$

$$K = \frac{y_O^2}{y_{O_2}^2} \left(\frac{P}{P_0} \right)^{2-1} = \frac{4x^2}{(1+x)^2} \frac{1+x}{1-x} \cdot 3 = \frac{12x^2}{1-x^2} \quad (272)$$

$$x^2 = \frac{K/12}{1 + K/12} \Rightarrow x = 0.06211 \quad (273)$$

$$y_{O_2} = \frac{1-x}{1+x} = \boxed{0.883}, y_O = 1 - y_{O_2} = \boxed{0.117} \quad (274)$$

$$\bar{q} = n_{O_2ex} \bar{h}_{O_2ex} + n_{Oex} \bar{h}_{Oex} - \bar{h}_{O_2in} = (1+x)(y_{O_2} \bar{h}_{O_2} + y_O \bar{h}_O) - 0 \quad (275)$$

$$\bar{h}_{O_2} = 106022 \text{ kJ/kmol}, \bar{h}_O = 249170 + 60767 = 309937 \text{ kJ/kmol} \Rightarrow \bar{q} = 137947 \text{ kJ/kmol } O_2 \quad (276)$$

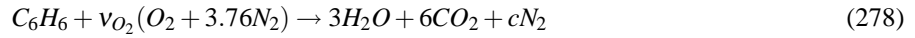
$$q = \frac{\bar{q}}{32} = \boxed{4311 \text{ kJ/kg}} \quad (277)$$

9 Homework 9 Solutions:

9.1 Problem 1: 15:70

An isobaric combustion process receives gaseous benzene C_6H_6 and air in a stoichiometric ratio at P_0, T_0 . To limit the product temperature to 2200 K , liquid water is sprayed in after the combustion. Find the kmol of liquid water added per kmol of fuel and the dew point of the combined products.

The reaction for stoichiometric mixture with C and H balance done is:



$$O \text{ balance: } 2\nu_{O_2} = 3 + 6 \cdot 2 = 15 \Rightarrow \nu_{O_2} = 7.5 \quad (279)$$

$$N \text{ balance: } c = 3.76\nu_{O_2} = 3.76 \cdot 7.5 = 28.2 \quad (280)$$

With $x \text{ kmol}$ of water added per kmol fuel the products are

$$\text{Products: } (3+x)H_2O + 6CO_2 + 28.2N_2 \quad (281)$$

$$\text{Energy Eq.: } H_R = H_R^\circ + x\bar{h}_{f,H_2O,liq}^\circ = H_P^\circ + \Delta H_P = H_P^\circ + x\bar{h}_{f,H_2O,vap}^\circ + (3+x)\Delta\bar{h}_{H_2O} + 6\bar{h}_{CO_2} + 28.2\bar{h}_{N_2} \quad (282)$$

Where the extra water is shown explicitly. Rearrange to get

$$H_R^\circ - H_P^\circ - 6\Delta\bar{h}_{CO_2} - 28.2\Delta\bar{h}_{N_2} - 3\Delta\bar{h}_{H_2O} = x(\bar{h}_{f,H_2O,vap}^\circ - \bar{h}_{f,H_2O,liq}^\circ + \Delta\bar{h}_{H_2O}) \quad (283)$$

$$40756 \cdot 78.114 - 6 \cdot 103562 - 28.2 \cdot 63362 - 3 \cdot 83153 = x[-241826 - (-285830) + 83153] \quad (284)$$

$$525975 = x(127157) \Rightarrow \boxed{x = 4.1364 \text{ kmol/kmol fuel}} \quad (285)$$

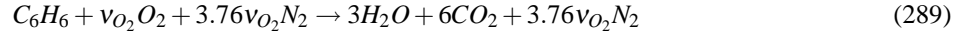
$$\text{Dew point: } y_V = \frac{3+x}{6+28.2+x} = 0.1862 \quad (286)$$

$$\Rightarrow P_V = y_V P = 0.1862 \cdot 101.325 = 18.86 \text{ kPa} \quad (287)$$

$$B.1.2: \boxed{T_{dew} = 58.7^\circ C} \quad (288)$$

9.2 Problem 2: 15.81

A stoichiometric mixture of benzene C_6H_6 and air is mixed from the reactants flowing at $25^\circ C$, 100 kPa . Find the adiabatic flame temperature. What is the error if constant specific heat at T_0 for the products from Table A.5 are used?



$$\nu_{O_2} = 6 + \frac{3}{2} = 7.5 \Rightarrow \nu_{N_2} = 28.2 \quad (290)$$

$$H_P = H_P^\circ + \Delta H_P = H_R = H_R^\circ \Rightarrow \quad (291)$$

$$\Delta H_P = -H_{RP}^\circ = 40576 \cdot 78.114 = 3169554\text{ kJ/kmol} \quad (292)$$

$$\Delta H_P = 6\bar{h}_{CO_2} + 3\bar{h}_{H_2O} + 28.2\bar{h}_{N_2} \quad (293)$$

$$\Delta H_{P\ 2600K} = 6(128074) + 3(104520) + 28.2(77963) = 3280600 \quad (294)$$

$$\Delta H_{P\ 2400K} = 6(115779) + 3(93741) + 28.2(70640) = 2968000 \quad (295)$$

Linear interpolation $\Rightarrow T_{AD} = 2529\text{ K}$

$$\sum \nu_i \bar{C}_{Pi} = 6 \cdot 0.842 \cdot 44.01 + 3 \cdot 1.872 \cdot 18.015 + 28.2 \cdot 1.042 \cdot 28.013 = 1146.66 \frac{\text{kJ}}{\text{kmol} \cdot K} \quad (296)$$

$$\Delta T = \frac{\Delta H_P}{\sum \nu_i \bar{C}_{Pi}} = \frac{3169554}{1146.66} = 2764 \quad (297)$$

$$\Rightarrow T_{AD} = 3062\text{ K}, \quad \boxed{21\% \text{ high}} \quad (298)$$

9.3 Problem 3: 16.49

Water from the combustion of hydrogen and pure oxygen is at 3800 K and 50 kPa. Assuming we only have H_2O , O_2 and H_2 as gases, find the equilibrium composition.

With only the given components we have the reaction



which at 3800 K has an equilibrium constant from A.11 of $\ln K = -1.906$.

Assume we start with 2 mol water and let it dissociate x to the left then

Species	H_2O	H_2	O_2	
Initial	2	0	0	Total: $2 + x$
Change	$-2x$	$2x$	x	
Final	$2 - 2x$	$2x$	x	

Then we have

$$K = e^{-1.906} = \frac{y_{H_2}^2 y_{O_2}}{y_{H_2O}^2} \left(\frac{P}{P_0} \right)^{2+1-2} = \frac{\left(\frac{2x}{2+x} \right)^2 \frac{x}{2+x} \frac{50}{100}}{\left(\frac{2-2x}{2+x} \right)^2} \quad (300)$$

which reduces to

$$0.148674 = \frac{1}{(1-x)^2} \frac{4x^3}{2+x} \frac{1}{4} \frac{1}{2} \text{ or } x^3 = 0.297348(1-x)^2(2+x) \quad (301)$$

Trial and error to solve for $x = 0.54$, then the concentrations are

$y_{H_2O} = \frac{2-2x}{2+x} = 0.362, y_{O_2} = \frac{x}{2+x} = 0.213, y_{H_2} = \frac{2x}{2+x} = 0.425$

(302)

9.4 Problem 4: 16.54

A tank contains 0.1 kmol hydrogen and 0.1 kmol of argon gas at 25°C, 200 kPa and the tank keeps constant volume. To what T should it be heated to have a mole fraction of atomic hydrogen, H , of 8%?

$$\text{For the reaction } H_2 \Leftrightarrow 2H, K = \frac{y_H^2}{y_{H_2}} \left(\frac{P}{P_0} \right)^{2-1} \quad (303)$$

Assume the dissociation shifts right with an amount x , then we get
Assume we start with 2 kmol water and let it dissociate x to the left then

Reaction	H_2	\Leftrightarrow	$2H$	also, Ar	
Initial	0.1		0	0.1	Total: 0.2 + x
Change	$-x$		$2x$	0	
Equilibrium	$0.1 - x$		$2x$	0.1	

$$y_H = \frac{2x}{0.2 + x} = 0.08 \Rightarrow x = 0.008333 \quad (304)$$

We need to find T so K will take on the proper value; since K depends on P we need to evaluate P first.

$$P_1 V = n_1 \bar{R} T_1, P_2 V = n_2 \bar{R} T_2 \Rightarrow P_2 = P_1 \frac{n_2 T_2}{n_1 T_1} \quad (305)$$

where we have $n_1 = 0.2$ and $n_2 = 0.2 + x = 0.2083$

$$K = \frac{y_H^2}{y_{H_2}} \left(\frac{P}{P_0} \right)^{2-1} = \frac{(2x)^2}{(0.1 - x)n_2} \frac{200}{100} \frac{n_2 T_2}{0.2 \cdot 298.15} = 0.0001016 T_2 \quad (306)$$

Now it is trial and error to get T_2 so the above equation is satisfied with K from A.11 at T_2 .

$$3400 \text{ K} : \ln K = -1.519, K = 0.2189, RHS = 0.34544, error = -0.1266 \quad (307)$$

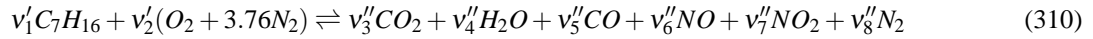
$$3600 \text{ K} : \ln K = -0.611, K = 0.5428, RHS = 0.36576, error = 0.1769 \quad (308)$$

Linear interpolation between the two to make zero error

$$T = 3400 + 200 \cdot \frac{0.1769}{0.1769 + 0.1266} = \boxed{3483.43 \text{ K}} \quad (309)$$

9.5 Problem 5:

Consider the reaction of heptane and air:



Find the most general set of stoichiometric coefficients for the reaction.

$$\begin{matrix} C \\ H \\ O \\ N \end{matrix} \begin{bmatrix} 7 & 0 & 1 & 0 & 1 & 0 & 0 & 0 \\ 16 & 0 & 0 & 2 & 0 & 0 & 0 & 0 \\ 0 & 2 & 2 & 1 & 1 & 1 & 2 & 0 \\ 0 & 7.52 & 0 & 0 & 0 & 1 & 1 & 2 \end{bmatrix} \left\{ \begin{matrix} v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \\ v_6 \\ v_7 \\ v_8 \end{matrix} \right\} = \emptyset \quad (311)$$

Row echelon form:

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 1/22 & -34/1019 & -16/203 & 14/579 \\ 0 & 1 & 0 & 0 & 0 & 25/188 & 25/188 & 25/94 \\ 0 & 0 & 1 & 0 & 15/22 & 103/441 & 1141/2068 & -175/1034 \\ 0 & 0 & 0 & 1 & -4/11 & 67/251 & 326/517 & -53/274 \end{bmatrix} \{ v \} = \emptyset \quad (312)$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \left\{ \begin{matrix} v_1 \\ v_2 \\ v_3 \\ v_4 \end{matrix} \right\} = \begin{bmatrix} -\frac{1}{22}v_5 & \frac{34}{1019}v_6 & \frac{16}{203}v_7 & -\frac{14}{579}v_8 \\ 0 & -\frac{25}{188}v_6 & -\frac{25}{188}v_7 & -\frac{25}{94}v_8 \\ -\frac{15}{22}v_5 & -\frac{103}{441}v_6 & -\frac{1141}{2068}v_7 & \frac{175}{1034}v_8 \\ \frac{4}{11}v_5 & -\frac{67}{251}v_6 & -\frac{326}{517}v_7 & \frac{53}{274}v_8 \end{bmatrix} \quad (313)$$

From there the four stoichiometric equations can be written out

$$v_1 = -\frac{1}{22}v_5 + \frac{34}{1019}v_6 + \frac{16}{203}v_7 - \frac{14}{579}v_8 \quad (314)$$

$$v_2 = -\frac{25}{188}v_6 - \frac{25}{188}v_7 - \frac{25}{94}v_8 \quad (315)$$

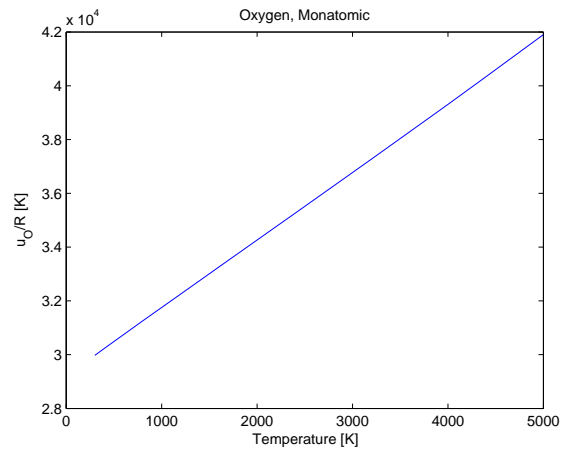
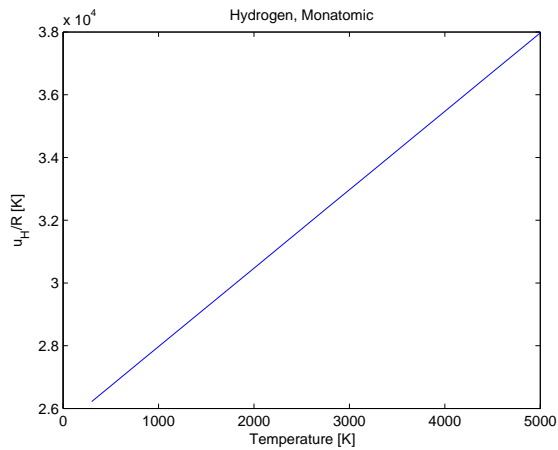
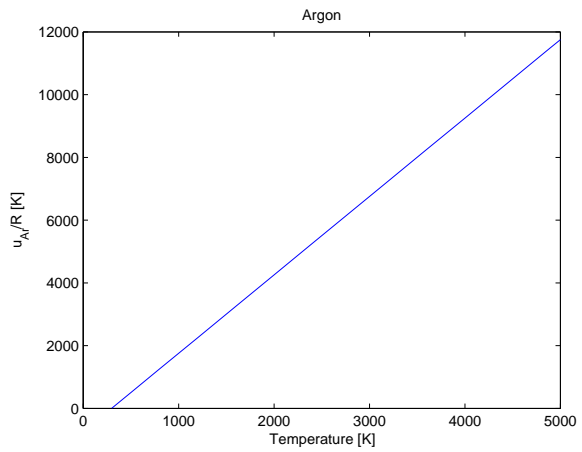
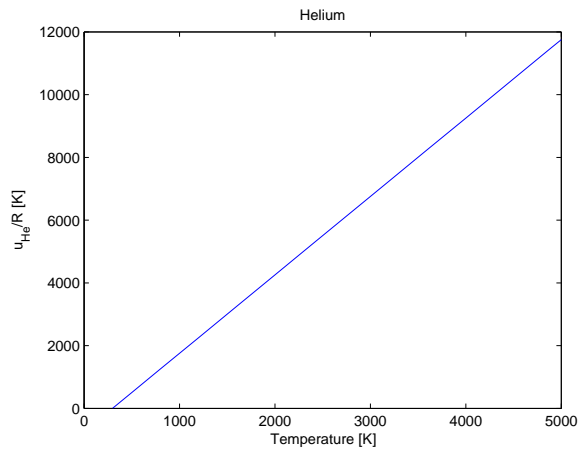
$$v_3 = -\frac{15}{22}v_5 - \frac{103}{441}v_6 - \frac{1141}{2068}v_7 + \frac{175}{1034}v_8 \quad (316)$$

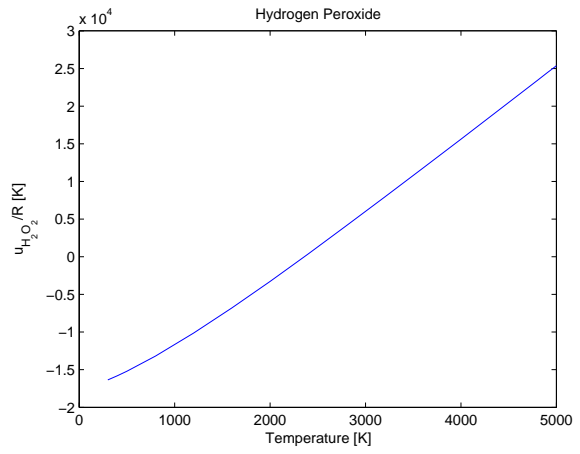
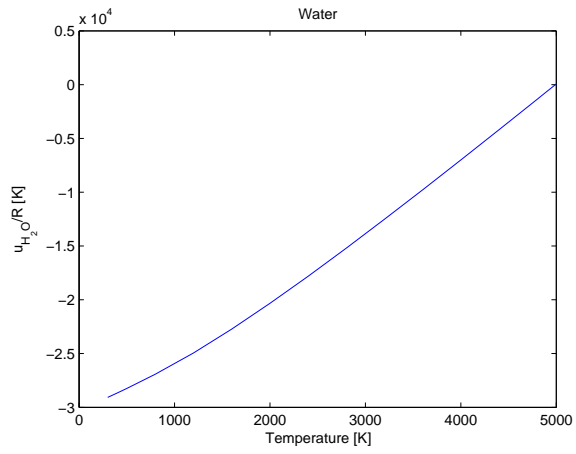
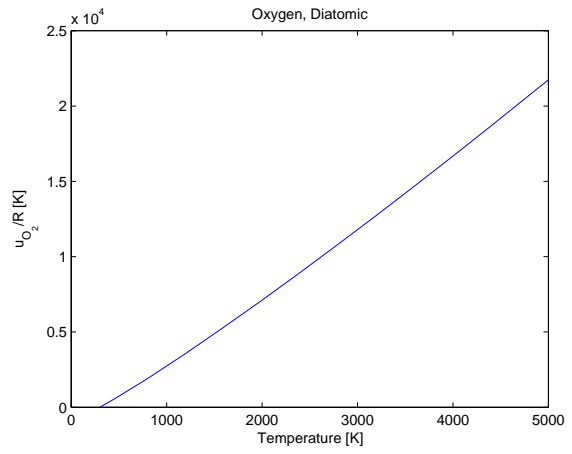
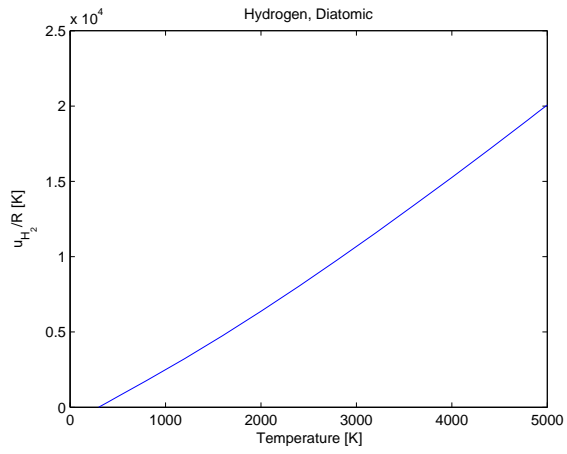
$$v_4 = \frac{4}{11}v_5 - \frac{67}{251}v_6 - \frac{326}{517}v_7 + \frac{53}{274}v_8 \quad (317)$$

10 Homework 10 Solutions:

10.1 Problem 1:

Plot $\frac{\bar{u}}{R}$ versus $T \in [300 \text{ K}, 5000 \text{ K}]$ for He , Ar , H , O , H_2 , O_2 , H_2O , and H_2O_2 . Use the chemkin software package to generate values of $\bar{u}(T)$.





10.2 Problem 2:

Consider a Hydrogen dissociation reaction,



For this reaction

$$a = 2.23 \cdot 10^{12} \frac{cm^3 \cdot K^{-\frac{1}{2}}}{mole \cdot s} \quad (319)$$

$$\beta = 0.5 \quad (320)$$

$$\bar{E} = 92600 \frac{cal}{mole} \quad (321)$$

Consider an isochoric, isothermal reaction in which $n_{H_2} = 1 \text{ kmole}$, $n_H = 0 \text{ kmole}$ at $t = 0$, and for which $T = 5200 \text{ K}$ and $P = 1500 \text{ kPa}$.

a) Formulate the reaction kinetics in the form

$$\frac{d\bar{\rho}_{H_2}}{dt} = f(\bar{\rho}_{H_2}) \quad (322)$$

b) Find all equilibria.

c) Ascertain the stability of each equilibrium point.

d) Use any appropriate numerical method such as Mathematica's NDSolve, MATLAB, or a straightforward Euler method to integrate the governing differential equation from the initial state to the equilibrium state.

e) Repeat this using chemkin to generate the reaction rates and dlsode to integrate the differential equations.

f) Plot the Gibbs free energy, $G = n_{H_2}\bar{g}_{H_2} + n_H\bar{g}_H$, as a function of time.

Formulation of the reaction kinetics:

$$\frac{d\bar{\rho}_{H_2}}{dt} = v_1 a T^\beta e^{-\frac{\bar{E}}{RT}} \left(\prod_{k=1}^N \bar{\rho}_k^{v_k'} \right) \left(1 - \frac{1}{k_c} \prod_{k=1}^N \bar{\rho}_k^{v_k} \right) \quad (323)$$

$$a = 2.23 \cdot 10^9 \frac{m^3 \cdot K^{-\frac{1}{2}}}{kmol \cdot s}, \quad \bar{E} = 387,623.6 \text{ kJ/kmol} \quad (324)$$

$$(\bar{\rho}_{H_2})_{t=0} = \frac{P_0}{\bar{R}T} = \frac{1500 \text{ kPa}}{8.314 \frac{kJ}{kmol \cdot K} \cdot 5200 \text{ K}} = 0.034696 \text{ kmol/m}^3 \quad (325)$$

$$V = \frac{(n_{H_2})_{t=0}}{(\bar{r}\bar{h}o_{H_2})_{t=0}} = 28.82 \text{ m}^3 \quad (326)$$

$$-dn_{H_2} = \frac{1}{2}n_H \rightarrow -(n_{H_2} - (n_{H_2})_{t=0}) = \frac{1}{2}(n_{H_2} - (n_{H_2})_{t=0}) \quad (327)$$

$$n_H = 2(1 \text{ kmol} - n_{H_2}), \quad \bar{r}\bar{h}o_H = 2 \left(\frac{1 \text{ kmol}}{28.82 \text{ m}^3} - \bar{r}\bar{h}o_{H_2} \right) \quad (328)$$

$$\frac{d\bar{\rho}_{H_2}}{dt} = v_{H_2} a T^\beta e^{-\frac{\bar{E}}{RT}} (\bar{\rho}_{H_2})^{v_{H_2}'} (\bar{\rho}_H)^{v_H'} \left(1 - \frac{1}{k_c} (\bar{\rho}_{H_2})^{v_{H_2}} (\bar{\rho}_H)^{v_H} \right) \quad (329)$$

$$v_{H_2}' = 2, \quad v_H' = 0, \quad v_{H_2} = -1, \quad v_H = 2 \quad (330)$$

$$\frac{d\bar{\rho}_{H_2}}{dt} = -a T^\beta e^{-\frac{\bar{E}}{RT}} (\bar{\rho}_{H_2})^2 \left(1 - \frac{1}{k_c} \frac{\bar{\rho}_H^2}{\bar{\rho}_{H_2}} \right) \quad (331)$$

$$k(T) = aT^{\beta} e^{\frac{-E}{RT}} = -2.0532 \cdot 10^7 \quad (332)$$

$$k_c = \left(\frac{P_0}{RT} \right)^{\sum_{i=1}^N \nu_i} e^{\left(\frac{-\Delta G}{RT} \right)}, \Delta G = 179564.2 \text{ kJ/kmol} \Rightarrow k_c = 0.14723 \quad (333)$$

$$\boxed{\frac{d\bar{\rho}_{H_2}}{dt} = -2.0532 \cdot 10^7 \cdot \bar{\rho}_{H_2}^2 \left(1 - \frac{1}{0.14723} \cdot \frac{[2(0.034696 - \bar{\rho}_{H_2})]^2}{\bar{\rho}_{H_2}} \right)} \quad (334)$$

Find all equilibria:

Equilibria are located at the points where $\frac{d\bar{\rho}_{H_2}}{dt} = 0$.

$$\frac{d\bar{\rho}_{H_2}}{dt} = 0 \text{ when } \boxed{\bar{\rho}_{H_2} = \{0, 0.0129, 0.0933\}} \quad (335)$$

Ascertain stability:

For stability, $\frac{d^2 \bar{\rho}_{H_2}}{dt^2}$ must be negative. Therefore,

- $\bar{\rho} = 0$ is unstable
- $\bar{\rho} = 0.0129$ is stable
- $\bar{\rho} = 0.0933$ is unstable and non-physical