

AME 50531 Homework Solutions¹

Fall 2011

Homework 1

1. CPIG air enters an isentropic nozzle at 1.30 atm and 24 °C with a velocity of 2.5 m/s. The nozzle entrance diameter is 120 mm. The air exits the nozzle at 1.24 atm with a velocity of 90 m/s. Determine the temperature of the exiting air and the nozzle exit diameter.

Given: CPIG air, ① $T_1 = 24$ °C, $P_1 = 1.30$ atm, $v_1 = 2.5$ m/s, $d_1 = 120$ mm ② $P_2 = 1.24$ atm, $v_2 = 90$ m/s

Assumptions:

Find: T_2 , d_2

For T_2 , use energy conservation:

$$h_1 + \frac{v_1^2}{2} = h_2 + \frac{v_2^2}{2}$$

$$(h_1 - h_2) + \frac{v_1^2}{2} = \frac{v_2^2}{2}$$

$$c_p(T_1 - T_2) + \frac{v_1^2}{2} = \frac{v_2^2}{2}$$

$$c_p(T_1 - T_2) = 1/2(v_2^2 - v_1^2)$$

$$1004.5 \frac{\text{J}}{\text{kg K}}(T_1 - T_2) = 1/2((90 \text{ m/s})^2 - (2.5 \text{ m/s})^2)$$

$$T_1 - T_2 = 4.029 \text{ °C}$$

$$\rightarrow \boxed{T_2 = 19.97 \text{ °C.}}$$

For d_2 , use mass conservation:

$$\dot{m}_1 = \dot{m}_2$$

$$\rho_1 A_1 v_1 = \rho_2 A_2 v_2$$

$$P = \rho RT \rightarrow \rho = P/RT$$

$$\frac{P_1 A_1 v_1}{T_1} = \frac{P_2 A_2 v_2}{T_2}$$

$$A_2 = \frac{T_2 P_1 v_1}{T_1 P_2 v_2} A_1$$

$$\frac{\pi d_2^2}{4} = \frac{T_2 P_1 v_1}{T_1 P_2 v_2} \frac{\pi d_1^2}{4}$$

$$d_2^2 = \frac{(293.12 \text{ K})(1.30 \text{ atm})(2.5 \text{ m/s})}{(297.15 \text{ K})(1.24 \text{ atm})(90 \text{ m/s})} (120 \text{ mm})^2$$

$$\rightarrow \boxed{d_2 = 20.34 \text{ mm.}}$$

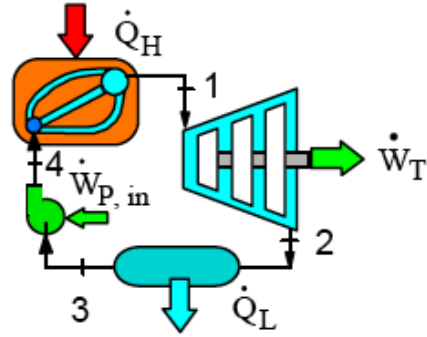
2. 9.47 Consider a steam turbine power plant operating near critical pressure. As a first approximation, it may be assumed that the turbine and the pump processes are reversible and adiabatic. Neglect any changes in kinetic and potential energies.

¹Solutions adapted from Borgnakke, Sonntag (2008) "Solutions Manual," *Fundamentals of Thermodynamics, 7th Edition* and previous AME 50531 Homework Solutions documents.

Given: CPIG air, ① $T_1 = 750 \text{ }^\circ\text{C}$,
 $P_1 = 20 \text{ MPa}$ ② $P_2 = 15 \text{ kPa}$ ③
 $T_3 = 40 \text{ }^\circ\text{C}$, $P_3 = 15 \text{ kPa}$ ④ $P_4 = 20 \text{ MPa}$

Assumptions: $\Delta v = 0$, $\Delta z = 0$, pump water is incompressible

Find: (a) w_T , turbine exit state (b) w_P , h_4
(c) η_{TH}



(a)

$$h_1 = 3939.45 \text{ kJ/kg}, s_1 = 6.9269 \frac{\text{kJ}}{\text{kg K}}$$

$$s_2 = s_1 = 6.9269 \frac{\text{kJ}}{\text{kg K}}$$

$$s_2 = 0.7548 \frac{\text{kJ}}{\text{kg K}} + x_2 \left(7.2536 \frac{\text{kJ}}{\text{kg K}} - 0.7548 \frac{\text{kJ}}{\text{kg K}} \right) = 6.9269 \frac{\text{kJ}}{\text{kg K}}$$

$$x_2 = 0.9497$$

Thus

$$h_2 = 53.97 \text{ kJ/kg} + x_2 (2373.14 \text{ kJ/kg}) = 2307.74 \text{ kJ/kg}$$

$$w_T = h_1 - h_2 = 3939.45 \text{ kJ/kg} - 2307.74 \text{ kJ/kg} = \boxed{1631.71 \text{ kJ/kg} = w_T}$$

Turbine exit state: liquid-vapor mixture

(b) $h_3 = 167.54 \text{ kJ/kg}$, $v_3 = 0.001018 \text{ m}^3/\text{kg}$

$$w_P = v_3(P_4 - P_3)$$

$$w_P = (0.001018 \text{ m}^3/\text{kg})(20000 \text{ kPa} - 15 \text{ kPa})$$

$$\rightarrow \boxed{w_P = 20.14 \text{ kJ/kg}}$$

$$h_4 = h_3 + w_P = \boxed{187.68 \text{ kJ/kg} = h_4}$$

(c)

$$\eta_{TH} = \frac{w_{NET}}{q_H} = \frac{(h_1 - h_2) - (h_4 - h_3)}{h_1 - h_4} = \frac{w_T - w_P}{q_H}$$

$$\eta_{TH} = \frac{1631.71 \text{ kJ/kg} - 20.14 \text{ kJ/kg}}{3939.45 \text{ kJ/kg} - 187.68 \text{ kJ/kg}}$$

$$\rightarrow \boxed{\eta_{TH} = 0.430}$$

Homework 2

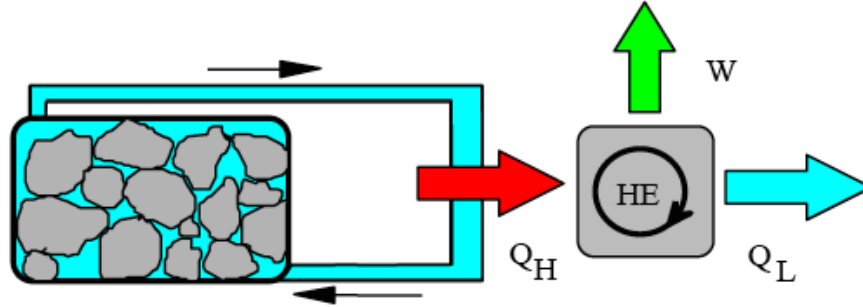
1. 7.65 A thermal storage is made with a rock (granite) bed of 2 m^3 which is heated to 390 K using solar energy. A heat engine receives a Q_h from the bed and rejects heat to the ambient at 290 K . The rock bed therefore cools down and as it reaches 290 K the process

stops. Find the energy the rock bed can give out. What is the heat engine efficiency at the beginning of the process and what is it at the end of the process?

Given: $T_i = 390 \text{ K}$, $T_f = 290 \text{ K}$, $V = 2 \text{ m}^3$

Assumptions: Reversible, heat engine operates in a Carnot cycle

Find: ${}_1Q_2$, η_i , η_f



$${}_1q_2 = u_2 - u_1 = C\Delta T = (0.89 \text{ kJ/kg K})(390 \text{ K} - 290 \text{ K}) = 89.0 \text{ kJ/kg}$$

$$m = \rho V = (2750 \text{ kg/m}^3)(2 \text{ m}^3) = 5500 \text{ kg}$$

$$Q = m{}_1q_2 = (5500 \text{ kg})(89.0 \text{ kJ/kg}) = \boxed{489,500 \text{ kJ} = {}_1Q_2} \leftarrow$$

To get the efficiencies, use the Carnot cycle:

$$\eta_i = 1 - T_0/T_i = 1 - 290/390 = \boxed{0.256 = \eta_i} \leftarrow$$

$$\eta_f = 1 - T_0/T_i = 1 - 290/290 = \boxed{0 = \eta_f} \leftarrow$$

2. 8.95 An insulated piston/cylinder setup contains carbon dioxide gas at 400 kPa, 300 K which is then compressed to 4 MPa in a reversible adiabatic process. Calculate the final temperature and the specific work using (a) ideal gas Table A.8 and (b) using constant specific heats Table A.5.

Given: ① $T_1 = 300 \text{ K}$, $P_1 = 400 \text{ kPa}$, $\rightarrow u_1 = 157.70 \text{ kJ/kg K}$ (from Table A.8) ② $P_2 = 4 \text{ MPa}$

Assumptions: CV CO_2 , a control mass undergoing a reversible, adiabatic (isentropic) process

Find: T_2 , ${}_1q_2$

(a) From Table A.8 for CO_2 and Eq. (8.19),

$$s_2 - s_1 = 0 = s_{T_2}^o - s_{T_1}^o - R \ln P_2/P_1$$

$$s_{T_2}^o = s_{T_1}^o + R \ln (P_2/P_1) = 4.8631 + 0.1889 \ln (4000/400) = 5.2981 \frac{\text{kJ}}{\text{kg K}}$$

Now interpolate in Table A.8: $u_2 = 291.91 \text{ kJ/kg K}$, $\boxed{T_2 = 481.2 \text{ K}} \leftarrow$

$${}_1w_2 = -(u_2 - u_1) = -(291.91 - 157.70) = \boxed{-134.21 \text{ kJ/kg} = {}_1q_2} \leftarrow$$

(b) Table A.5: $k = 1.289$, $C_{v0} = 0.653$ kJ/kg K. From Eq. (8.23):

$$T_2 = T_1 \left(\frac{P_2}{P_1} \right)^{\frac{k-1}{k}} = 300 \left(\frac{4000}{400} \right)^{\frac{0.289}{1.289}} = \boxed{502.72 \text{ K} = T_2} \leftarrow$$

$${}_1w_2 = -C_{v0}(T_2 - T_1) = -0.653(502.72 - 300) = \boxed{-132.38 \text{ kJ/kg} = {}_1q_2} \leftarrow$$

3. 8.121 Ammonia is contained in a rigid sealed tank of unknown quality at 0°C . When heated in boiling water to 100°C its pressure reaches 1200 kPa. Find the initial quality, the heat transfer to the ammonia and the total entropy generation.

Given: ① $T_1 = 0^\circ\text{C}$, ② $P_2 = 1200$ kPa, $T_2 = 100^\circ\text{C}$

Assumptions: Control volume ammonia, which is a control mass of a control volume

Find: x_1 , ${}_1q_2$, ${}_1s_2$

From the tables: $s_2 = 5.5325 \frac{\text{kJ}}{\text{kg K}}$, $v_2 = 0.14347 \text{ m}^3/\text{kg}$, $u_2 = 1485.8 \text{ kJ/kg}$

The volume is constant, $v_1 = v_2$, so

$$x_1 = \frac{0.14347 - 0.001566}{0.28763} = \boxed{0.493 = x_1} \leftarrow$$

Also because the volume is constant, ${}_1w_2 = 0$, so

$${}_1q_2 = u_2 - u_1 = 1485.8 - 741.28 = \boxed{744.52 \text{ kJ/kg} = {}_1q_2} \leftarrow$$

For the entropy generation,

$${}_1s_2 = s_2 - s_1 - {}_1q_2/T = 5.5325 - 2.9905 - 744.52/373.15$$

$$\rightarrow \boxed{{}_1s_2 = 0.547 \text{ kJ/kg K}}$$

4. 12.98 The air-standard Carnot cycle was not shown in the text; show the Ts diagram for this cycle. In an air-standard Carnot cycle the low temperature is 280 K and the efficiency is 60 %. If the pressure before compression and after heat rejection is 100 kPa, find the high temperature and the pressure just before heat addition.

Given: $T_L = 280$ K, $\eta = 0.6$, $P_1 = 100$ kPa, air-standard Carnot cycle

Assumptions:

Find: the $T - s$ diagram, T_H , P_2

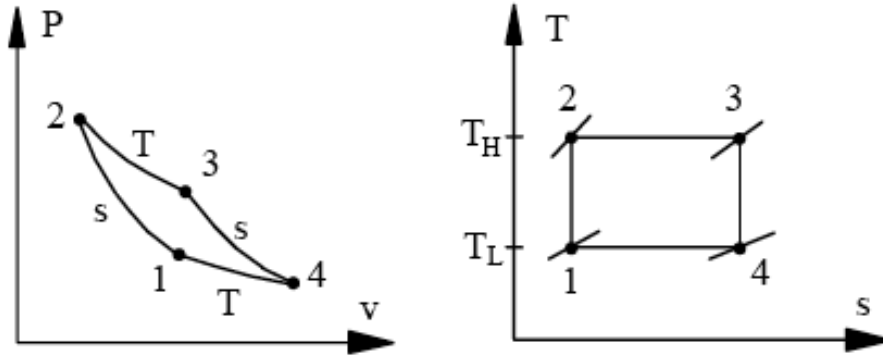
From Eq. (7.5),

$$\eta = 0.6 = 1 - T_H/T_L$$

$$\rightarrow \boxed{T_H = T_L/0.4 = 700 \text{ K.}}$$

State 2 is the state just before heat addition, and we already have the temperature (T_L) and pressure (P_1) for State 1. As seen in the $T - s$ diagram, State 1 to State 2 is an isentropic compression, so using Eq. (8.23),

$$P_2 = P_1(T_H/T_L)^{\frac{k}{k-1}} = 100(700/280)^{\frac{1.4}{0.4}}$$



$$\rightarrow P_2 = 2.47 \text{ MPa.}$$

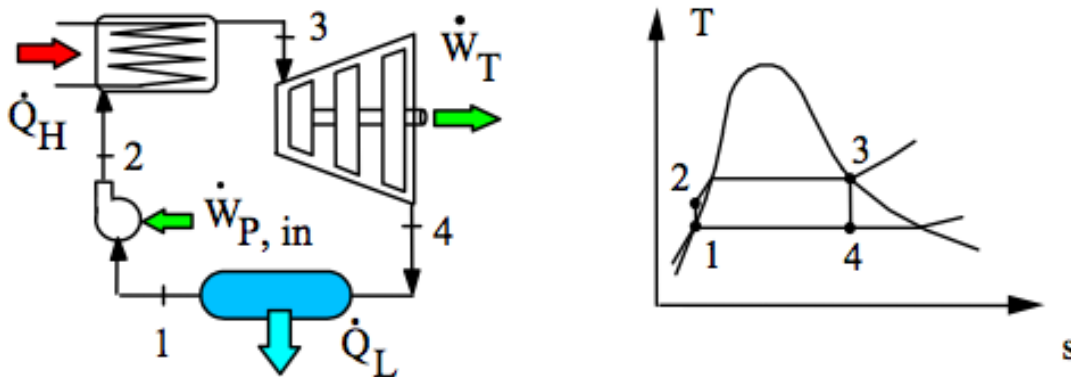
Homework 3

1. 11.21 A supply of geothermal hot water is to be used as the energy source in an ideal Rankine cycle, with R-134a as the cycle working fluid. Saturated vapor R-134a leaves the boiler at a temperature of 85 °C, and the condenser temperature is 35 °C. Calculate the thermal efficiency of this cycle.

Given: R-134a, ideal Rankine Cycle; ① $h_1 = 249.10 \text{ kJ/kg}$, $P_1 = 887.6 \text{ kPa}$ ② $P_2 = \text{kPa}$
 ③ $h_3 = 428.10 \text{ kJ/kg}$ ④

Assumptions:

Find: η_{TH}



$$w_P = h_2 - h_1 = \int_1^2 v \, dP \approx v_1(P_2 - P_1) = 0.000857(2926.2 - 887.6)$$

$$\rightarrow w_P = 1.747 \text{ kJ/kg}$$

$$h_2 = h_1 + w_P = 249.10 + 1.747 = 250.85 \text{ kJ/kg}$$

For the boiler:

$$q_H = h_3 - h_2 = 428.10 - 250.85 = 177.25 \text{ kJ/kg}$$

For the turbine:

$$s_4 = s_3 = 1.6782 = 1.1673 + x_4(0.5465) \rightarrow x_4 = 0.935$$

$$h_4 = 249.10 + x_4(168.42) = 406.57 \text{ kJ/kg}$$

$$w_T = h_3 - h_4 = 428.10 - 406.57 = 21.53 \text{ kJ/kg}$$

$$w_{NET} = w_T - w_P = 21.53 - 1.747 = 19.78 \text{ kJ/kg}$$

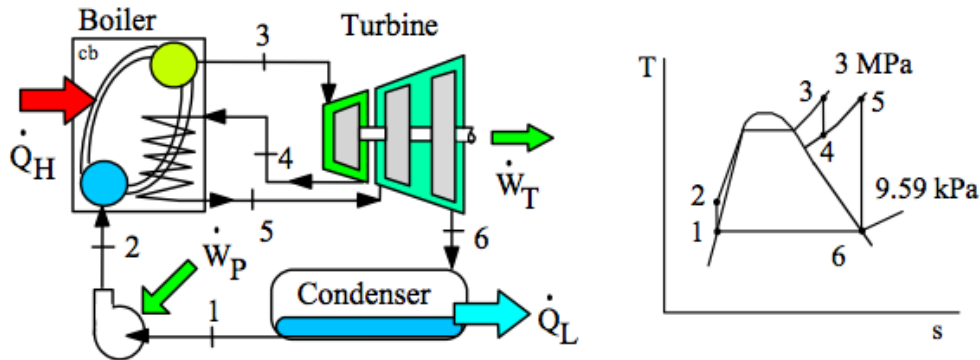
$$\eta_{TH} = w_{NET}/q_H = 19.78/177.25 = \boxed{0.112 = \eta_{TH}} \leftarrow$$

2. 11.37 The reheat pressure affects the operating variables and thus turbine performance. Repeat Problem 11.33 twice, using 0.6 MPa and 1.0 MPa for the reheat pressure.

Given: $\dot{Q}_L = 10,000 \text{ kW}$ ① $x_1 = 0$, $T_1 = 45^\circ\text{C}$ ② $P_2 = 3 \text{ MPa}$ ③ $P_3 = P_2$, $T_3 = 600^\circ\text{C} \rightarrow h_3 = 3682.34 \text{ kJ/kg}$, $s_3 = 7.5084 \text{ kJ/kg K}$ ⑥ $x_6 = 1$, $T_6 = T_1 \rightarrow h_6 = 2583.19 \text{ kJ/kg}$, $s_6 = 8.1647 \text{ kJ/kg K}$

Assumptions: Pump: reversible and adiabatic and incompressible flow, ③ \rightarrow ④ isentropic, ⑤ \rightarrow ⑥ isentropic

Find: T_5 , $\dot{W}_{T,tot}$, \dot{Q}_H



$$w_P = v_1(P_2 - P_1) = 0.00101(3000 - 10) = 3.02 \text{ kJ/kg}$$

$$h_2 = h_1 + w_P = 191.81 + 3.02 = 194.38 \text{ kJ/kg}$$

For $P_4 = 1 \text{ MPa} = P_5$:

$s_3 = s_4 \rightarrow$ state 4 is a superheated vapor, $h_4 = 3295.22 \text{ kJ/kg}$

State 5: $T_5 = 655.6^\circ\text{C}$, $P_4 = P_5 \rightarrow h_5 = 3823.80 \text{ kJ/kg}$, $s_5 = s_6 = 8.1647 \text{ kJ/kg K}$

For $P_4 = 0.6 \text{ MPa} = P_5$:

$s_3 = s_4 \rightarrow$ state 4 is a superheated vapor, $h_4 = 3143.00 \text{ kJ/kg}$

State 5: $T_5 = 561.3^\circ\text{C}$, $P_4 = P_5 \rightarrow h_5 = 3616.54 \text{ kJ/kg}$, $s_5 = s_6 = 8.1647 \text{ kJ/kg K}$

Total condenser heat transfer:

$$q_L = h_6 - h_1 = 2394.77 \text{ kJ/kg}$$

Mass flow rate:

$$\dot{m} = \frac{\dot{Q}_L}{q_L} = \frac{10,000}{2394.77} = 4.176 \text{ kg/s}$$

Total turbine power:

$$\dot{W}_{T,tot} = \dot{m}(w_{T,tot}) = \dot{m}(h_3 - h_4 + h_5 - h_6)$$

Total boiler heat rate:

$$\dot{Q}_H = \dot{m}(h_3 - h_2 + h_5 - h_4)$$

See the results for both iterations of the problem in the table below:

$P_4 = P_5$	T_5	$\dot{W}_{T,tot}$	\dot{Q}_H
1 MPa	655.6 °C	6797 kW	16770 kW
0.6 MPa	561.3 °C	6568 kW	16540 kW

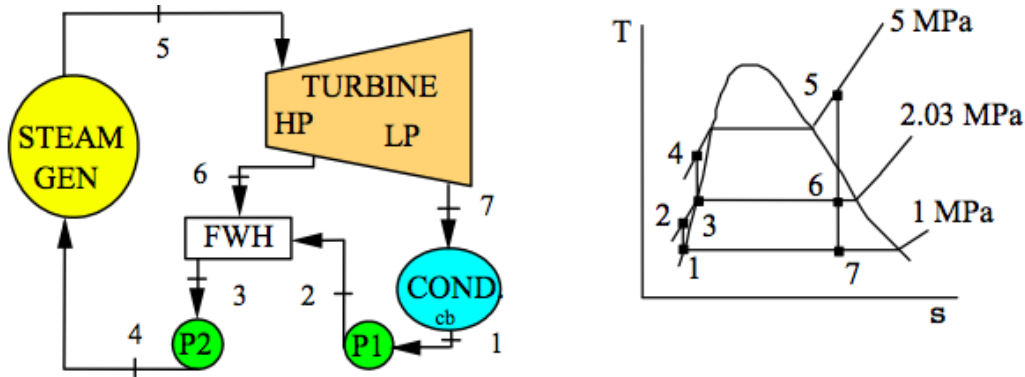
3. 11.42 A Rankine cycle operating with ammonia is heated by some low temperature source so the highest temperature is 130 °C at a pressure of 5000 kPa. Its low pressure is 1003 kPa and it operates with one open feedwater heater at 2033 kPa. The total flow rate is 5 kg/s. Find the extraction flow rate to the feedwater heater assuming its outlet state is saturated liquid at 2033 kPa. Find the total power to the two pumps.

Given: ① $x_1 = 0$, $h_1 = 298.25$ kJ/kg, $v_1 = 0.001658$ m³/kg ③ $x_3 = 0$, $h_3 = 421.48$ kJ/kg, $v_3 = 0.001777$ m³/kg ⑤ $h_5 = 1621.8$, $s_5 = 4.8187$ kJ/kg K ⑥ $s_6 = s_5 \rightarrow x_6 = (s_6 - s_f)/s_{fg} = (4.8187 - 1.5121)/3.2493 = 0.9827$, $h_6 = 1453.32$ kJ/kg

Assumptions: Incompressible fluid in both pumps

Find: \dot{W}_P

For the pump:



$$w_{P1} = h_2 - h_1 \approx v_1(P_2 - P_1) = 0.001658(2033 - 1003) = 1.708 \text{ kJ/kg}$$

$$\rightarrow h_2 = h_1 + w_{P1} = 298.25 + 1.708 = 299.97 \text{ kJ/kg}$$

For the feedwater heater, call $\dot{m}_6/\dot{m}_{tot} = y$ (the extraction fraction)

Energy equation:

$$(1 - y)h_2 + (y)h_6 = h_3$$

$$y = \frac{h_3 - h_2}{h_6 - h_2} = \frac{421.48 - 299.97}{1453.32 - 299.97} = 0.1054$$

$$\dot{m}_{extr} = y(\dot{m}_{tot}) = 0.1054(5) = 0.527 \text{ kg/s}$$

$$\dot{m}_1 = (1 - y)\dot{m}_{tot} = (1 - 0.1054)5 = 4.473 \text{ kg/s}$$

For pump 2:

$$w_{P2} = h_4 - h_3 = v_3(P_4 - P_3) = 0.001771(5000 - 2033) = 5.272 \text{ kJ/kg}$$

Total pump work:

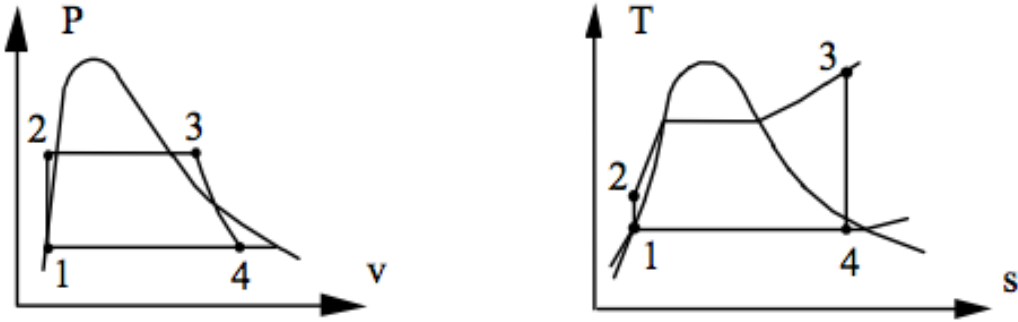
$$\dot{W}_P = \dot{m}_1 w_{P1} + \dot{m}_{tot} w_{P2} = (4.473)(1.708) + (5)(5.272) = \boxed{34.0 \text{ kW} = \dot{W}_P} \leftarrow$$

4. 11.170 From problem 11.30: The power plant in Problem 11.13 is modified to have a super heater section following the boiler so the steam leaves the super heater at 3.0 MPa, 400 °C. Calculate the thermal efficiency of the cycle and the moisture content of the steam leaving the turbine for turbine exhaust pressures of 5, 10, 50, and 100 kPa. Plot the thermal efficiency versus turbine exhaust pressure.

Given: $h_3 = 3230.82 \text{ kJ/kg}$, $s_3 = 6.9211 \text{ kJ/kg K}$

Assumptions:

Find:



Pump:

$$w_P = \int_2^1 v dP \approx v_1(P_2 - P_1)$$

$$h_2 = h_1 + w_P$$

Boiler:

$$q_H = h_3 - h_2$$

Isentropic turbine ($s_4 = s_3$):

$$x_4 = \frac{s_3 - s_f}{s_{fg}}$$

$$h_4 = h_f + (x_4)h_{fg}$$

$$w_{T,s} = h_3 - h_4$$

Finally, the cycle efficiency:

$$\eta_{CYCLE} = \frac{w_{NET}}{q_H} = \frac{w_{T,s} - w_P}{q_H}$$

See the results for all iterations of the problem in the table below:

P_4	v_1	w_P	h_2	q_H	x_4	h_4	$w_{T,s}$	η_{CYCLE}
5	0.001015	3.01	140.80	3090.02	0.814	2110.65	1120.17	0.362
10	0.001001	3.02	194.83	3035.99	0.836	2192.21	1038.61	0.341
50	0.001030	3.04	373.51	2857.31	0.900	2415.33	815.49	0.284
100	0.001043	3.02	420.46	2810.36	0.928	2512.86	717.96	0.254

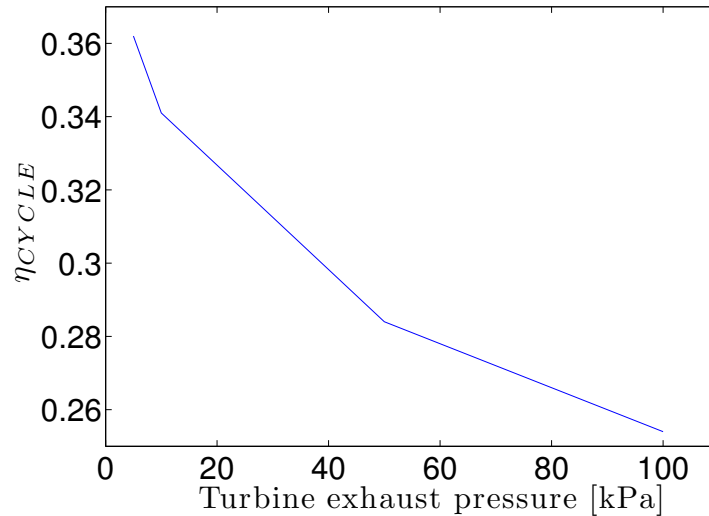


Figure 1: Plot for problem 11.170.

Homework 4

1. 12.18 Repeat Problem 12.17, but assume variable specific heat for the air, table A.7. Consider an ideal air-standard Brayton cycle in which the air into the compressor is at 100 kPa, 20 ° C, and the pressure ratio across the compressor is 12:1. The maximum temperature in the cycle is 1100 °C, and the air flow rate is 10 kg/s. Assume constant specific heat for the air, value from Table A.5. Determine the compressor work, the turbine work, and the thermal efficiency of the cycle.

Given: $h_1 = 293.6$ kJ/kg, $s_{T_1}^\circ = 6.84597$ kJ/kg K

Assumptions: Constant $c_{p_{air}}$

Find: w_C , w_T , η_{TH}

The compression is isentropic:

$$s_{T_2}^\circ = s_{T_1}^\circ + R \ln \frac{P_2}{P_1} = 6.84597 + (0.287) \ln(12) = 7.55914$$

$$\rightarrow T_2 = 590 \text{ K}, h_2 = 597.2 \text{ kJ/kg}$$

Energy equation with compressor work in:

$$w_C = -{}_1w_2 = h_2 - h_1 = 597.2 - 392.6 = 303.6 \text{ kJ/kg}$$

Like the compression, the expansion is isentropic:

$$s_4 = s_3 \rightarrow s_{T_4}^\circ = s_{T_3}^\circ + R \ln \frac{P_4}{P_3} = 8.50554 + (0.287) \ln \frac{1}{12} = 7.79237$$

$$\rightarrow T_4 = 734.8 \text{ K}, h_4 = 751.1 \text{ kJ/kg}$$

Energy equation with turbine work out:

$$w_T = h_3 - h_4 = 1483.1 - 751.1 = 732 \text{ kJ/kg}$$

Find energy rates by multiplying by mass flow rate:

$$\dot{W}_C = \dot{m}w_C = \boxed{3036 \text{ kW} = \dot{W}_C}, \quad \dot{W}_T = \dot{m}w_T = \boxed{7320 \text{ kW} = \dot{W}_T}$$

Energy added by the combustion process:

$$q_H = h_3 - h_2 = 1483.1 - 597.2 = 885.9 \text{ kJ/kg}$$

$$w_{NET} = w_T - w_C = 732 - 303.6 = 428.4 \text{ kJ/kg}$$

$$\eta_{TH} = w_{NET}/q_H = 428.4/885.9 = \boxed{0.484 = \eta_{TH}}$$

2. 12.31 The gas-turbine cycle shown in Fig. P12.31 is used as an automotive engine. In the first turbine, the gas expands to pressure P_5 , just low enough for this turbine to drive the compressor. The gas is then expanded through the second turbine connected to the drive wheels. The data for the engine are shown in the figure and assume that all processes are ideal. Determine the intermediate pressure P_5 , the net specific work output of the engine, and the mass flow rate through the engine. Find also the air temperature entering the burner T_3 , and the thermal efficiency of the engine.

Given:

Assumptions: Ideal generator

Find: P_5 , w_{NET} , \dot{m} , T_3 , η_{TH}

Consider the compressor:

$$s_2 = s_1 \rightarrow T_2 = T_1 \left(\frac{P_2}{P_1} \right)^{\frac{k-1}{k}} = 300(6)^{0.286} = 500.8 \text{ K}$$

$$-w_C = -{}_1w_2 = c_{p0}(T_2 - T_1) = 1.004(500.8 - 300) = 201.6 \text{ kJ/kg}$$

Now consider the turbine:

$$w_{T1} = -w_C = 201.6 = c_{p0}(T_4 - T_5) = 1.004(1600 - T_5) \rightarrow T_5 = 1399.2 \text{ K}$$

$$s_5 = s_4 \rightarrow P_5 = P_4 \left(\frac{T_5}{T_4} \right)^{\frac{k}{k-1}} = 600 \left(\frac{1399.2}{1600} \right)^{3.5} = \boxed{375 \text{ kPa} = P_5}$$

$$s_6 = s_5 \rightarrow T_6 = T_5 \left(\frac{P_6}{P_5} \right)^{\frac{k-1}{k}} = 1399.2 \left(\frac{100}{375} \right)^{0.286} = 958.8 \text{ K}$$

The second turbine gives the net work out:

$$w_{T2} = c_{p0}(T_5 - T_6) = 1.004(1399.2 - 958.8) = 442.2 \text{ kJ/kg}$$

$$\dot{m} = \dot{W}_{NET}/w_{T2} = 150/442.2 = \boxed{0.339 \text{ kg/s} = \dot{m}}$$

Ideal generator: $T_6 = \boxed{T_3 = 958.8 \text{ K}}$

$$q_H = c_{p0}(T_5 - T_6) = 1.004(1600 - 958.8) = 643.8 \text{ kJ/kg}$$

$$\eta_{TH} = w_{NET}/q_H = 442.2/643.8 = \boxed{0.687 = \eta_{TH}}$$

3. 12.40 Consider an ideal air-standard Ericsson cycle that has an ideal regenerator as shown in Fig. P12.40. The high pressure is 1 MPa and the cycle efficiency is 70 %. Heat is rejected in the cycle at a temperature of 350 K, and the cycle pressure at the beginning of the isothermal compression process is 150 kPa. Determine the high temperature, the compressor work, and the turbine work per kilogram of air.

Given: $P_2 = P_3 = 1 \text{ MPa}$, $T_1 = T_2 = 350 \text{ K}$, $P_1 = 150 \text{ kPa}$, $r_P = P_2/P_1 = 10$

Assumptions: Ideal regenerator

Find: T_H , w_C , w_T

Ideal regenerator:

$$2q_3 = -4q_1 \rightarrow q_H = 3q_4, \quad w_T = q_H$$

$$\eta_{TH} = \eta_{CARNOT} = 1 - T_L/T_H = 0.7 \rightarrow T_3 = T_4 = \boxed{T_H = 1167 \text{ K}}$$

$$q_L = w_C = \int v dP = RT_1 \ln \left(\frac{P_2}{P_1} \right) = (0.287)(350) \ln \frac{1000}{150} = \boxed{190.56 \text{ kJ/kg} = q_L}$$

$$w_T = q_H = - \int v dP = -RT_3 \ln \left(\frac{P_4}{P_3} \right) = \boxed{635.2 \text{ kJ/kg} = w_T}$$

4. 12.53 An afterburner in a jet engine adds fuel after the turbine thus raising the pressure and temperature due to the energy of combustion. Assume a standard condition of 800 K, 250 kPa after the turbine into the nozzle that exhausts at 95 kPa. Assume the afterburner adds 475 kJ/kg to that state with a rise in pressure for same specific volume, and neglect any upstream effects on the turbine. Find the nozzle exit velocity before and after the afterburner is turned on.

Given: ① $T_1 = 800 \text{ K}$, $P_1 = 250 \text{ kPa}$ ② $P_2 = 95 \text{ kPa}$; After afterburner is turned on: ③ $v_3 = v_1$, ④ $P_4 = 95 \text{ kPa}$

Assumptions: Isentropic nozzle flow

Find:

For the nozzle:

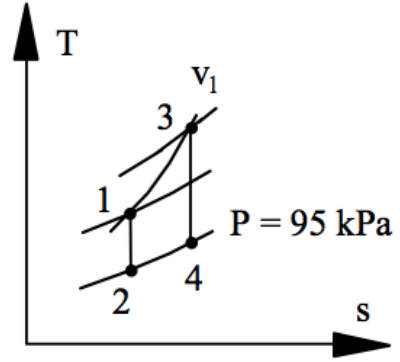
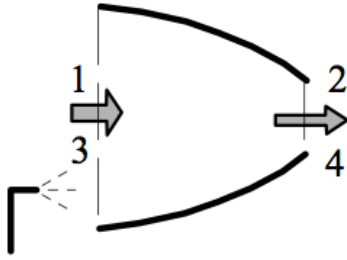
$$T_2 = T_1(P_2/P_1)^{\frac{k-1}{k}} = (800)(95/250)^{0.2857} = 606.8 \text{ K}$$

Energy equation: $(1/2)v_2^2 = c_p(T_1 - T_2)$

$$v_2 = \sqrt{2c_p(T_1 - T_2)} = \sqrt{(2)(1004)(800 - 606.8)} = \boxed{622.8 \text{ m/s} = v_2} \leftarrow$$

Now with the q_{AB} at assumed constant volume gives the energy equation as

$$T_3 = T_1 + q_{AB}/c_v = 800 + 475/0.717 = 1462.5 \text{ K}$$



$$v_3 = v_1 \rightarrow P_3 = P_1(T_3/T_1) = 250(1462.5/800) = 457.0 \text{ kPa}$$

The expansion is isentropic, so from Eq. 8.23:

$$T_4 = T_3(P_4/P_3)^{(k-1)/k} = 1462.5(95/457)^{0.2857} = 933.7 \text{ K}$$

$$v_2 = \sqrt{2c_p(T_1 - T_2)} = \sqrt{(2)(1004)(1462.5 - 933.7)} = \boxed{1030.5 \text{ m/s} = v_2} \leftarrow$$

Homework 5

12.45

The turbine section in a jet engine receives gas (assume air) at 1200 K, 800 kPa with an ambient atmosphere at 80 kPa. The turbine is followed by a nozzle open to the atmosphere and all the turbine work drives a compressor. Find the turbine exit pressure so the nozzle has an exit velocity of 800 m/s. Hint: take the CV around both turbine and nozzle.

Solution:

C.V. Reversible and adiabatic turbine and nozzle. This gives constant s , from Eq.8.23 we can relate the T's and P's

State 3: 1200 K, 800 kPa State 5: 80 kPa; $s_5 = s_3$

$$\text{Eq.8.23: } T_5 = T_3 (P_5/P_3)^{(k-1)/k} = 1200 \text{ K } (80/800)^{0.2857} = 621.56 \text{ K}$$

$$\text{Energy: } h_3 + 0 = h_5 + (1/2)V_5^2 + w_T = h_4 + w_T$$

$$w_T = h_3 - h_5 - (1/2)V_5^2 \cong C_p(T_3 - T_5) - (1/2)V_5^2$$

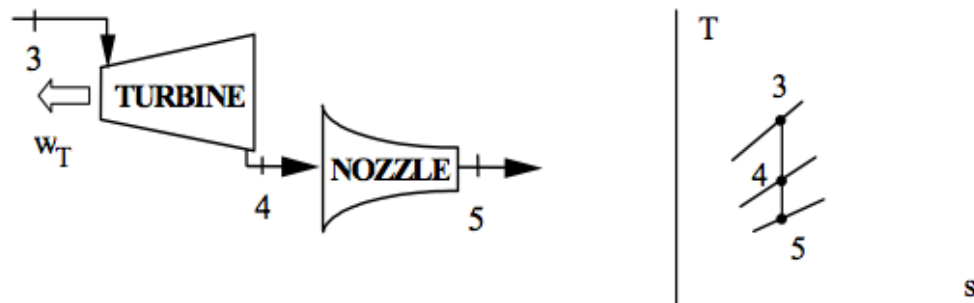
$$\begin{aligned} &= 1.004 \text{ kJ/kg-K } (1200 - 621.56) \text{ K} - (1/2) \times 800^2 (\text{J/kg}) / 1000 \text{ J/kJ} \\ &= 580.75 - 320 = 260.75 \text{ kJ/kg} \end{aligned}$$

C.V. Nozzle alone to establish state 4 (same s as state 5 and 3).

$$h_4 = h_5 + (1/2)V_5^2 = h_3 - w_T$$

$$T_4 = T_5 + (1/2)V_5^2/C_p = 621.56 + 320/1.004 = 940.29 \text{ K}$$

$$P_4 = P_3 (T_4/T_3)^{k/(k-1)} = 800 \text{ kPa} \times (940.29/1200)^{3.5} = \mathbf{340.7 \text{ kPa}}$$



12.74

Repeat Problem 12.67, but assume variable specific heat. The ideal gas air tables, Table A.7, are recommended for this calculation (or the specific heat from Fig. 5.10 at high temperature).

Solution:

Table A.7 is used with interpolation.

$$T_1 = 283.2 \text{ K}, \quad u_1 = 202.3 \text{ kJ/kg}, \quad s_{T1}^{\circ} = 6.8113 \text{ kJ/kg K}$$

Compression 1 to 2: $s_2 = s_1 \Rightarrow$ From Eq.8.19

$$0 = s_{T2}^{\circ} - s_{T1}^{\circ} - R \ln(P_2/P_1) = s_{T2}^{\circ} - s_{T1}^{\circ} - R \ln(T_2 v_1 / T_1 v_2)$$

$$s_{T2}^{\circ} - R \ln(T_2/T_1) = s_{T1}^{\circ} + R \ln(v_1/v_2) = 6.8113 + 0.287 \ln 7 = 7.3698$$

This becomes trial and error so estimate first at 600 K and use A.7.1.

$$\text{LHS}_{600} = 7.5764 - 0.287 \ln(600/283.2) = 7.3609 \text{ (too low)}$$

$$\text{LHS}_{620} = 7.6109 - 0.287 \ln(620/283.2) = 7.3860 \text{ (too high)}$$

$$\text{Interpolate to get: } T_2 = 607.1 \text{ K}, \quad u_2 = 440.5 \text{ kJ/kg}$$

$$\Rightarrow {}_1w_2 = u_2 - u_1 = 238.2 \text{ kJ/kg},$$

$$u_3 = 440.5 + 1800 = 2240.5 \Rightarrow T_3 = \mathbf{2575.8 \text{ K}}, \quad s_{T3}^{\circ} = 9.2859 \text{ kJ/kgK}$$

$$P_3 = P_1 (v_1/v_3) T_3/T_1 = 90 \text{ kPa} \times 7 \times 2575.8 / 283.2 = \mathbf{5730 \text{ kPa}}$$

Expansion 3 to 4: $s_4 = s_3 \Rightarrow$ From Eq.8.19 as before

$$s_{T4}^{\circ} - R \ln(T_4/T_3) = s_{T3}^{\circ} + R \ln(v_3/v_4) = 9.2859 + 0.287 \ln(1/7) = 8.7274$$

This becomes trial and error so estimate first at 1400 K and use A.7.1.

$$\text{LHS}_{1400} = 8.5289 - 0.287 \ln(1400/2575.8) = 8.7039 \text{ (too low)}$$

$$\text{LHS}_{1450} = 8.5711 - 0.287 \ln(1450/2575.8) = 8.7360 \text{ (too high)}$$

$$\text{Interpolation } \Rightarrow T_4 = 1436.6 \text{ K}, \quad u_4 = 1146.9 \text{ kJ/kg}$$

$${}_3w_4 = u_3 - u_4 = 2240.5 - 1146.9 = 1093.6 \text{ kJ/kg}$$

Net work, efficiency and mep

$$\rightarrow w_{\text{net}} = {}_3w_4 + {}_1w_2 = 1093.6 - 238.2 = 855.4 \text{ kJ/kg}$$

$$\eta_{\text{TH}} = w_{\text{net}} / q_{\text{H}} = 855.4 / 1800 = \mathbf{0.475}$$

$$v_1 = RT_1/P_1 = (0.287 \times 283.2)/90 = 0.9029 \text{ m}^3/\text{kg}$$

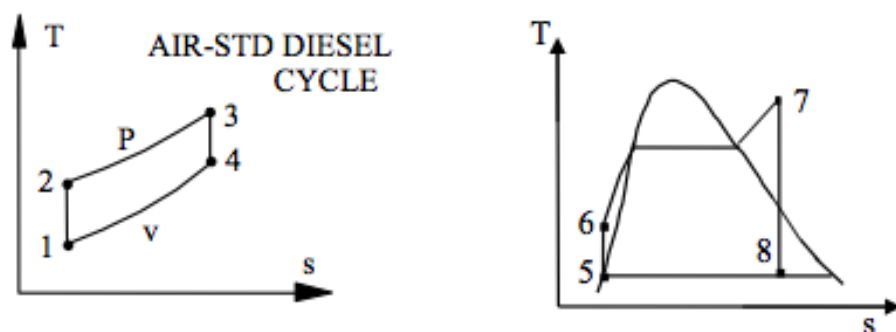
$$v_2 = (1/7) v_1 = 0.1290 \text{ m}^3/\text{kg}$$

$$P_{\text{meff}} = \frac{w_{\text{net}}}{v_1 - v_2} = 855.4 / (0.9029 - 0.129) = \mathbf{1105 \text{ kPa}}$$

12.111

A simple Rankine cycle with R-410a as the working fluid is to be used as a bottoming cycle for an electrical generating facility driven by the exhaust gas from a Diesel engine as the high temperature energy source in the R-410a boiler. Diesel inlet conditions are 100 kPa, 20°C, the compression ratio is 20, and the maximum temperature in the cycle is 2800°C. The R-410a leaves the bottoming cycle boiler at 80°C, 4 MPa and the condenser pressure is 1800 kPa. The power output of the Diesel engine is 1 MW. Assuming ideal cycles throughout, determine

- The flow rate required in the diesel engine.
- The power output of the bottoming cycle, assuming that the diesel exhaust is cooled to 200°C in the R-410a boiler.



Diesel cycle information given means: $\dot{W}_{\text{DIESEL}} = 1 \text{ MW}$

$$P_1 = 100 \text{ kPa}, T_1 = 20^\circ\text{C}, CR = v_1/v_2 = 20, T_3 = 2800^\circ\text{C}$$

Consider the Diesel cycle

$$T_2 = T_1(v_1/v_2)^{k-1} = 293.2 \text{ K} (20)^{0.4} = 971.8 \text{ K}$$

$$P_2 = P_1(v_1/v_2)^k = 100 \text{ kPa} (20)^{1.4} = 6629 \text{ kPa}$$

$$q_H = C_{p0}(T_3 - T_2) = 1.004 \text{ kJ/kg-K}(3073.2 - 971.8) \text{ K} = 2109.8 \text{ kJ/kg}$$

$$v_1 = \frac{0.287 \times 293.2}{100} = 0.8415 \text{ m}^3/\text{kg}, v_2 = \frac{0.8415}{20} = 0.04208 \text{ m}^3/\text{kg}$$

$$v_3 = v_2(T_3/T_2) = 0.04208 \text{ m}^3/\text{kg} (3073.2/971.8) = 0.13307 \text{ m}^3/\text{kg}$$

$$T_4 = T_3 \left(\frac{v_3}{v_4} \right)^{k-1} = 3073.2 \text{ K} \left(\frac{0.13307}{0.8415} \right)^{0.4} = 1469.6 \text{ K}$$

$$q_L = 0.717(293.2 - 1469.6) = -843.5 \text{ kJ/kg}$$

$$w_{\text{NET}} = 2109.8 - 843.5 = 1266.3 \text{ kJ/kg}$$

$$\dot{m}_{\text{AIR}} = \dot{W}_{\text{NET}}/w_{\text{NET}} = 1000 \text{ kW}/1266.3 \text{ kJ/kg} = \mathbf{0.79 \text{ kg/s}}$$

Homework 6

1. 13.21 A 2 kg mixture of 25 % N_2 , 50 % O_2 and 25 % CO_2 by mass is at 150 kPa and 300 K. Find the mixture gas constant and the total volume.

Given: $m = 2$ kg, $c_{N_2} = 0.25$, $c_{O_2} = 0.5$, $c_{CO_2} = 0.25$

Assumptions:

Find: R_{mix} , V

From Eq. 13.15:

$$R_{mix} = \sum c_i R_i = (0.25)(0.2968) + (0.5)(0.2598) + (0.25)(0.1889) = \boxed{0.2513 \text{ kJ/kg K} = R_{mix}}$$

Ideal gas law: $PV = mR_{mix}T$

$$\rightarrow V = \frac{mR_{mix}T}{P} = \frac{(2 \text{ kg})(0.2513 \text{ kJ/kg K})(300 \text{ K})}{150 \text{ kPa}} = \boxed{1.005 \text{ m}^3 = V}$$

13.28

A flow of 1 kg/s argon at 300 K and another flow of 1 kg/s CO_2 at 1600 K both at 150 kPa are mixed without any heat transfer. What is the exit T, P?

No work implies no pressure change for a simple flow.

$$P_e = 150 \text{ kPa}$$

The energy equation becomes

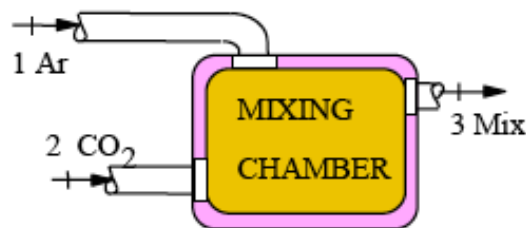
$$\dot{m}h_i = \dot{m}h_e = (\dot{m}h_i)_{Ar} + (\dot{m}h_i)_{CO_2} = (\dot{m}h_e)_{Ar} + (\dot{m}h_e)_{CO_2}$$

$$\Rightarrow \dot{m}_{CO_2} C_{p, CO_2} (T_e - T_i)_{CO_2} + \dot{m}_{Ar} C_{p, Ar} (T_e - T_i)_{Ar} = 0$$

$$\Rightarrow \dot{m}_{Ar} C_{p, Ar} T_i + \dot{m}_{CO_2} C_{p, CO_2} T_i = [\dot{m}_{Ar} C_{p, Ar} + \dot{m}_{CO_2} C_{p, CO_2}] T_e$$

$$(1 \times 0.520 \times 300 + 1 \times 0.842 \times 1600) \text{ kW} = (1 \times 0.520 + 1 \times 0.842) \text{ kW/K} \times T_e$$

$$T_e = 1103.7 \text{ K}$$



13.44

Two insulated tanks A and B are connected by a valve. Tank A has a volume of 1 m^3 and initially contains argon at 300 kPa, 10°C . Tank B has a volume of 2 m^3 and initially contains ethane at 200 kPa, 50°C . The valve is opened and remains open until the resulting gas mixture comes to a uniform state. Determine the final pressure and temperature.

Solution:

C.V. Tanks A + B. Control mass no W, no Q.

$$\text{Energy Eq. 5.11: } U_2 - U_1 = 0 = m_{\text{Ar}} C_{V0}(T_2 - T_{A1}) + m_{\text{C}_2\text{H}_6} C_{V0}(T_2 - T_{B1})$$

$$m_{\text{Ar}} = P_{A1} V_A / R T_{A1} = (300 \times 1) / (0.2081 \times 283.15) = 5.0913 \text{ kg}$$

$$m_{\text{C}_2\text{H}_6} = P_{B1} V_B / R T_{B1} = (200 \times 2) / (0.2765 \times 323.15) = 4.4767 \text{ kg}$$

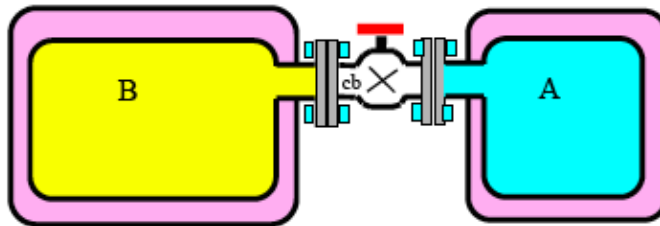
$$\text{Continuity Eq.: } m_2 = m_{\text{Ar}} + m_{\text{C}_2\text{H}_6} = 9.568 \text{ kg}$$

$$\begin{aligned} \text{Energy Eq.: } & 5.0913 \times 0.312 (T_2 - 283.2) \\ & + 4.4767 \times 1.490 (T_2 - 323.2) = 0 \end{aligned}$$

Solving, $T_2 = 315.5 \text{ K}$

$$R_{\text{mix}} = \sum c_i R_i = \frac{5.0913}{9.568} \times 0.2081 + \frac{4.4767}{9.568} \times 0.2765 = 0.2401 \text{ kJ/kg K}$$

$$\begin{aligned} P_2 &= m_2 R T_2 / (V_A + V_B) = 9.568 \text{ kg} \times 0.2401 \text{ kJ/kg-K} \times 315.5 \text{ K} / 3 \text{ m}^3 \\ &= 242 \text{ kPa} \end{aligned}$$



13.55

Carbon dioxide gas at 320 K is mixed with nitrogen at 280 K in an insulated mixing chamber. Both flows are coming in at 100 kPa and the mole ratio of carbon dioxide to nitrogen is 2:1. Find the exit temperature and the total entropy generation per kmole of the exit mixture.

CV mixing chamber, steady flow. The inlet ratio is $\dot{n}_{\text{CO}_2} = 2 \dot{n}_{\text{N}_2}$ and assume no external heat transfer, no work involved.

Continuity: $\dot{n}_{\text{CO}_2} + 2\dot{n}_{\text{N}_2} = \dot{n}_{\text{ex}} = 3\dot{n}_{\text{N}_2}$;

Energy Eq.: $\dot{n}_{\text{N}_2}(\bar{h}_{\text{N}_2} + 2\bar{h}_{\text{CO}_2}) = 3\dot{n}_{\text{N}_2} \bar{h}_{\text{mix ex}}$

Take 300 K as reference and write $\bar{h} = \bar{h}_{300} + \bar{C}_{\text{Pmix}}(T - 300)$.

$$\bar{C}_{\text{P N}_2}(T_{\text{i N}_2} - 300) + 2\bar{C}_{\text{P CO}_2}(T_{\text{i CO}_2} - 300) = 3\bar{C}_{\text{P mix}}(T_{\text{mix ex}} - 300)$$

Find the specific heats in Table A.5 to get

$$\begin{aligned} \bar{C}_{\text{P mix}} &= \sum y_i \bar{C}_{\text{P}i} = (1.042 \times 28.013 + 2 \times 0.842 \times 44.01)/3 \\ &= 34.43 \text{ kJ/kmol K} \end{aligned}$$

$$3\bar{C}_{\text{P mix}} T_{\text{mix ex}} = \bar{C}_{\text{P N}_2} T_{\text{i N}_2} + 2\bar{C}_{\text{P CO}_2} T_{\text{i CO}_2} = 31889 \text{ kJ/kmol}$$

$$T_{\text{mix ex}} = 308.7 \text{ K}$$

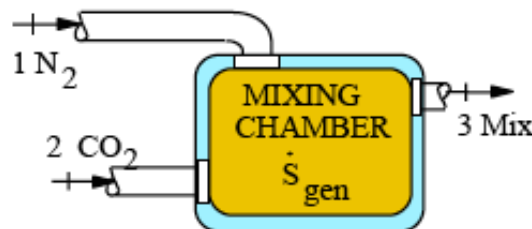
Partial pressures are total pressure times molefraction

$$P_{\text{ex N}_2} = P_{\text{tot}}/3; \quad P_{\text{ex CO}_2} = 2P_{\text{tot}}/3$$

$$\dot{S}_{\text{gen}} = \dot{n}_{\text{ex}} \bar{s}_{\text{ex}} - (\dot{n}\bar{s})_{\text{i CO}_2} - (\dot{n}\bar{s})_{\text{i N}_2} = \dot{n}_{\text{N}_2}(\bar{s}_{\text{e}} - \bar{s}_{\text{i}})_{\text{N}_2} + 2\dot{n}_{\text{N}_2}(\bar{s}_{\text{e}} - \bar{s}_{\text{i}})_{\text{CO}_2}$$

$$\dot{S}_{\text{gen}}/3\dot{n}_{\text{N}_2} = [\bar{C}_{\text{P N}_2} \ln \frac{T_{\text{ex}}}{T_{\text{i N}_2}} - \bar{R} \ln y_{\text{N}_2} + 2\bar{C}_{\text{P CO}_2} \ln \frac{T_{\text{ex}}}{T_{\text{i CO}_2}} - 2\bar{R} \ln y_{\text{CO}_2}]/3$$

$$= [2.8485 + 9.1343 - 2.6607 + 6.742]/3 = 5.35 \text{ kJ/kmol mix K}$$



Homework 7

1. 13.68 A new high-efficiency home heating system includes an air-to-air heat exchanger which uses energy from outgoing stale air to heat the fresh incoming air. If the outside ambient temperature is $-10\text{ }^{\circ}\text{C}$ and the relative humidity is 40 %, how much water will have to be added to the incoming air, if it flows in at the rate of $1\text{ m}^3/\text{s}$ and must eventually be conditioned to $20\text{ }^{\circ}\text{C}$ and 40 % relative humidity?

Given: $\phi_1 = 0.4$, $T_1 = -10\text{ }^{\circ}\text{C}$, $T_2 = 20\text{ }^{\circ}\text{C}$, $P_{g1} = 0.2602\text{ kPa}$ (from Table B.1.5)

Assumptions: $P_1 = P_2 = 100\text{ kPa}$

Find: $\dot{m}_{liq,in}$

Outside ambient air:

$$P_{v1} = (\phi_1)(P_{g1}) = (0.40)(0.2602) = 0.1041\text{ kPa}$$

$P_1 = P_2 = 100\text{ kPa}$, so $P_{a1} = 100 - 0.1041 = 99.896\text{ kPa}$.

$$\dot{m}_a = \frac{P_{a1}v_1}{R_aT_1} = \frac{(99.896)(1)}{(0.287)(263.2)} = 1.3225\text{ kg/s}$$

From Eq. 13.28: $w_1 = 0.622 \left(\frac{0.1041}{99.896} \right) = 0.000648$. On the outside:

$$P_{v2} = (\phi_2)(P_{g2}) = (0.4)(2.339) = 0.9356\text{ kPa}$$

Eq. 13.28: $w_2 = 0.622 \frac{0.9356}{99.064} = 0.0587$ Now applying the continuity equation to the water:

$$\dot{m}_{liq,in} = \dot{m}_a(w_2 - w_1) = 1.3228(0.0587 - 0.000648) = \boxed{0.00691\text{ kg/s} = 24.9\text{ kg/h} = \dot{m}_{liq,in}}$$

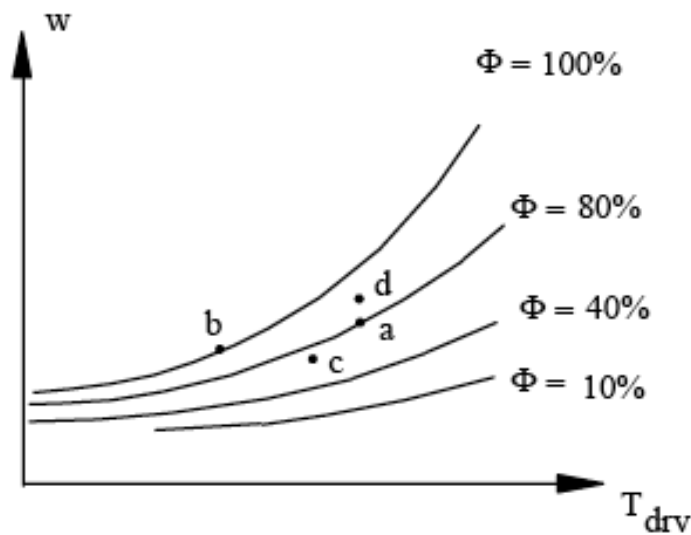
13.103

Use the psychrometric chart to find the missing property of: ϕ , ω , T_{wet} , T_{dry}

- | | |
|---|--|
| a. $T_{\text{dry}} = 25^\circ\text{C}$, $\phi = 80\%$ | b. $T_{\text{dry}} = 15^\circ\text{C}$, $\phi = 100\%$ |
| c. $T_{\text{dry}} = 20^\circ\text{C}$, and $\omega = 0.008$ | d. $T_{\text{dry}} = 25^\circ\text{C}$, $T_{\text{wet}} = 23^\circ\text{C}$ |

Solution:

- | | | |
|---|---------------|--|
| a. 25°C , $\phi = 80\%$ | \Rightarrow | $\omega = 0.016$; $T_{\text{wet}} = 22.3^\circ\text{C}$ |
| b. 15°C , $\phi = 100\%$ | \Rightarrow | $\omega = 0.0106$; $T_{\text{wet}} = 15^\circ\text{C}$ |
| c. 20°C , $\omega = 0.008$ | \Rightarrow | $\phi = 57\%$; $T_{\text{wet}} = 14.4^\circ\text{C}$ |
| d. 25°C , $T_{\text{wet}} = 23^\circ\text{C}$ | \Rightarrow | $\omega = 0.017$; $\phi = 86\%$ |



13.167E

An indoor pool evaporates 3 lbm/h of water, which is removed by a dehumidifier to maintain 70 F, $\Phi = 70\%$ in the room. The dehumidifier is a refrigeration cycle in which air flowing over the evaporator cools such that liquid water drops out, and the air continues flowing over the condenser, as shown in Fig. P12.71. For an air flow rate of 0.2 lbm/s the unit requires 1.2 Btu/s input to a motor driving a fan and the compressor and it has a coefficient of performance, $\beta = Q_L / W_C = 2.0$. Find the state of the air after the evaporator, T_2 , ω_2 , Φ_2 and the heat rejected. Find the state of the air as it returns to the room and the compressor work input.

The unit must remove 3 lbm/h liquid to keep steady state in the room. As water condenses out state 2 is saturated.

$$1: 70 \text{ F}, 70\% \Rightarrow P_{g1} = 0.363 \text{ psia}, h_{g1} = 1092.0 \text{ Btu/lbm},$$

$$P_{v1} = \phi_1 P_{g1} = 0.2541 \text{ psia}, w_1 = 0.622 P_{v1} / (P_{\text{tot}} - P_{v1}) = 0.01094$$

$$\text{CV 1 to 2: } \dot{m}_{\text{liq}} = \dot{m}_a (w_1 - w_2) \Rightarrow w_2 = w_1 - \dot{m}_{\text{liq}} / \dot{m}_a$$

$$q_L = h_1 - h_2 - (w_1 - w_2) h_{f2}$$

$$w_2 = 0.01094 - 3 / (3600 \times 0.2) = \mathbf{0.006774}$$

$$P_{v2} = P_{g2} = P_{\text{tot}} w_2 / (0.622 + w_2) = \frac{14.7 \times 0.006774}{0.628774} = 0.1584 \text{ psia}$$

$$\text{Table F.7.1: } T_2 = \mathbf{46.8 \text{ F}}, h_{f2} = 14.88 \text{ btu/lbm}, h_{g2} = 1081.905 \text{ Btu/lbm}$$

$$q_L = 0.24(70 - 46.8) + 0.01094 \times 1092 - 0.006774 \times 1081.905 - 0.00417 \times 14.88 = 10.12 \text{ Btu/lbm dry air}$$

$$\dot{W}_c = \dot{m}_a q_L / \beta = \mathbf{1 \text{ Btu/s}}$$

CV Total system :

$$\begin{aligned} \tilde{h}_3 - \tilde{h}_1 &= \dot{W}_{el} / \dot{m}_a - (w_1 - w_2) h_f = 1.2 / 0.2 - 0.062 = 5.938 \text{ Btu/lbm dry air} \\ &= C_{p,a} (T_3 - T_1) + w_2 h_{v3} - w_1 h_{v1} \end{aligned}$$

Trial and error on T_3

$$3: w_3 = w_2, h_3 \Rightarrow T_3 = \mathbf{112 \text{ F}}, P_{g3} = 1.36 \text{ psia}, P_{v3} = P_{v2} = 0.1584$$

$$\phi_3 = P_{v3} / P_{g3} = 0.12 \quad \text{or} \quad \phi_3 = \mathbf{12\%}$$

14.35

The Joule-Thomson coefficient μ_J is a measure of the direction and magnitude of the temperature change with pressure in a throttling process. For any three properties x, y, z use the mathematical relation

$$\left(\frac{\partial x}{\partial y}\right)_z \left(\frac{\partial y}{\partial z}\right)_x \left(\frac{\partial z}{\partial x}\right)_y = -1$$

to show the following relations for the Joule-Thomson coefficient:

$$\mu_J = \left(\frac{\partial T}{\partial P}\right)_h = \frac{T \left(\frac{\partial v}{\partial T}\right)_P - v}{C_P} = \frac{RT^2}{PC_P} \left(\frac{\partial Z}{\partial T}\right)_P$$

Let $x = T$, $y = P$ and $z = h$ and substitute into the relations as:

$$\left(\frac{\partial T}{\partial P}\right)_h \left(\frac{\partial P}{\partial h}\right)_T \left(\frac{\partial h}{\partial T}\right)_P = -1$$

Then we have the definition of specific heat as $C_P = \left(\frac{\partial h}{\partial T}\right)_P$ so solve for the first term

$$\mu_J = \left(\frac{\partial T}{\partial P}\right)_h = -\frac{1}{C_P} / \left(\frac{\partial P}{\partial h}\right)_T = -\frac{1}{C_P} \left(\frac{\partial h}{\partial P}\right)_T$$

The last derivative is substituted with Eq.14.25 so we get

$$\mu_J = \left(\frac{\partial T}{\partial P}\right)_h = \frac{T \left(\frac{\partial v}{\partial T}\right)_P - v}{C_P}$$

If we use the compressibility factor then we get

$$Pv = ZRT \quad \Rightarrow \quad \left(\frac{\partial v}{\partial T}\right)_P = \frac{ZR}{P} + \frac{RT}{P} \left(\frac{\partial Z}{\partial T}\right)_P = \frac{v}{T} + \frac{RT}{P} \left(\frac{\partial Z}{\partial T}\right)_P$$

so then

$$T \left(\frac{\partial v}{\partial T}\right)_P - v = v + \frac{RT^2}{P} \left(\frac{\partial Z}{\partial T}\right)_P - v = \frac{RT^2}{P} \left(\frac{\partial Z}{\partial T}\right)_P$$

and we have shown the last expression also.

$$\mu_J = \left(\frac{\partial T}{\partial P}\right)_h = \frac{T \left(\frac{\partial v}{\partial T}\right)_P - v}{C_P} = \frac{RT^2}{PC_P} \left(\frac{\partial Z}{\partial T}\right)_P$$

Homework 8

1. 14.77 Develop general expressions for $u(T, v)$, $h(T, v)$, $s(T, v)$. That is, do not restrict your analysis to isothermal changes. In this problem, you may have to admit that $c_v = c_v(T, v)$.

Given:

Assumptions:

Find: $u(T, v)$, $h(T, v)$, $s(T, v)$

Redlich-Kwong equation of state:

$$P = \frac{RT}{v-b} - \frac{a}{v(v+b)T^{1/2}}$$
$$\left(\frac{\partial P}{\partial T}\right)_v = \frac{RT}{v-b} - \frac{a}{v(v+b)T^{3/2}}$$

From Eq. (14.30):

$$u_2 - u_1 = \int_1^2 C_v(T, v) dT + \int_1^2 \frac{3a}{2v(v+b)T^{1/2}}$$
$$= \boxed{\int_1^2 C_v(\hat{T}) d\hat{T} - \frac{3a}{2bT^{1/2}} \ln \left[\left(\frac{v_2+b}{v_2} \right) \left(\frac{v_1}{v_1+b} \right) \right] = u_2 - u_1}$$

We find the change in h from the change in u ($dh = du + Pdv$):

$$\boxed{h_2 - h_1 = \int_1^2 C_p(\hat{T}) d\hat{T} - \frac{3a}{2bT^{1/2}} \ln \left[\left(\frac{v_2+b}{v_2} \right) \left(\frac{v_1}{v_1+b} \right) \right] + P_2 v_2 - P_1 v_1}$$

Entropy follows from Eq. (14.35):

$$s_2 - s_1 = \int_1^2 C_v(T, v) \frac{dT}{T} + \int_1^2 \left[\frac{R}{v-b} + \frac{a/2}{v(v+b)T^{3/2}} \right] dv$$
$$\boxed{s_2 - s_1 = \int_1^2 C_v(\hat{T}, v) \frac{d\hat{T}}{\hat{T}} + R \ln \frac{v_2 - b}{v_1 - b} - \frac{a}{2bt^{3/2}} \ln \left[\left(\frac{v_2+b}{v_2} \right) \left(\frac{v_1}{v_1+b} \right) \right]}$$

14.80

Oxygen in a rigid tank with 1 kg is at 160 K, 4 MPa. Find the volume of the tank by iterations using the Redlich-Kwong EOS. Compare the result with the ideal gas law.

For the ideal gas law: $Pv = RT$ so $v = RT/P$

$$v = 0.2598 \times 160 / 4000 = 0.0104 \text{ m}^3/\text{kg}; \quad V = mv = \mathbf{0.0104 \text{ m}^3}$$

For Redlich-Kwong, Eq.14.53 and oxygen

$$P_c = 5040 \text{ kPa}; \quad T_c = 154.6 \text{ K}; \quad R = 0.2598 \text{ kJ/kg K}$$

$$b = 0.08664 \frac{RT_c}{P_c} = 0.08664 \times \frac{0.2598 \times 154.6}{5040} = 0.0006905 \text{ m}^3/\text{kg}$$

$$a = 0.42748 \frac{R^2 T_c^{5/2}}{P_c} = 0.42748 \times \frac{0.2598^2 \times 154.6^{5/2}}{5040} = 1.7013$$

$$P = \frac{RT}{v-b} - \frac{a}{v(v+b)T^{1/2}} \quad \text{trial and error to get } v \text{ due to nonlinearity}$$

$$v = 0.01 \text{ m}^3/\text{kg} \Rightarrow P = 4465.1 - 1279.9 = 3185.2 \text{ kPa} \quad \text{too low}$$

$$v = 0.008 \text{ m}^3/\text{kg} \Rightarrow P = 5686.85 - 1968.1 = 3718.8 \text{ kPa} \quad \text{too low}$$

$$v = 0.0075 \text{ m}^3/\text{kg} \Rightarrow P = 6104.41 - 2227.43 = 3876.98 \text{ kPa}$$

$$v = 0.007 \text{ m}^3/\text{kg} \Rightarrow P = 6588.16 - 2541.70 = 4046.46 \text{ kPa}$$

Now we interpolate between the last two entries and check

$$v = 0.00714 \text{ m}^3/\text{kg} \Rightarrow P = 6445.15 - 2447.3 = 3997.8 \text{ kPa} \quad \text{OK}$$

$$V = mv = \mathbf{0.00714 \text{ m}^3} \quad (69\% \text{ of the ideal gas value})$$

3. Consider a thermodynamic system in which there are two reversible work modes: compression and electrical. So take the version of the texts Eq. (4.16) giving dW to be

$$dW = PdV - EdZ,$$

where E is the electrical potential difference and dZ is the amount of charge that flows into the system.

- Extend the Gibbs equation to account for electrical work.
- Find the Legendre transformation which renders the independent variables to be P , E , and T and show how the other variables can be determined as functions of these independent variables.
- Find all Maxwell relations associated with this Legendre transformation.

Gibbs equation:

$$dU = \delta Q - \delta W$$

Knowing $\delta Q = Tds$,

$$\boxed{dU = TdS - PdV + EdZ} \quad (1)$$

Legendre transformation:

$$\psi_1 = T, \psi_2 = -P, \psi_3 = E, \chi_1 = S, \chi_2 = V, \chi_3 = Z$$

$$F_1 = U - \psi_1\chi_1 = U - TS$$

$$F_2 = U - \psi_2\chi_2 = U - PV$$

$$F_3 = U - \psi_3\chi_3 = U - EZ$$

$$F_{1,2,3} = U - TS + PV - EZ$$

$$dF_{1,2,3} = dU - TdS - SdT + PdV + VdP - EdZ - ZdE$$

Re-arranging this equation and recalling Eq. (1),

$$\begin{aligned} dF_{1,2,3} + TdS + SdT - PdV - VdP + EdZ + ZdE &= dU \\ &= TdS - PdV + EdZ \end{aligned}$$

Canceling like terms and solving for $dF_{1,2,3}$ shows that

$$\boxed{dF_{1,2,3} = -SdT + VdP - ZdE}, \quad (2)$$

and that $F_{1,2,3} = F_{1,2,3}(T, P, E)$. From Eq. (2):

$$-S = \left(\frac{\partial F_{1,2,3}}{\partial T} \right)_{P,E}$$

$$V = \left(\frac{\partial F_{1,2,3}}{\partial P} \right)_{T,E}$$

$$-Z = \left(\frac{\partial F_{1,2,3}}{\partial E} \right)_{T,P}$$

Taking mixed partials of the right-hand sides of the three equations above results in the Maxwell relations:

$$\frac{\partial^2 F_{1,2,3}}{\partial T \partial P} = - \left(\frac{\partial S}{\partial P} \right)_{P,E}, \quad \frac{\partial^2 F_{1,2,3}}{\partial P \partial T} = \left(\frac{\partial V}{\partial T} \right)_{T,E} \rightarrow \boxed{- \left(\frac{\partial S}{\partial P} \right)_{P,E} = \left(\frac{\partial V}{\partial T} \right)_{T,E}}$$

$$\frac{\partial^2 F_{1,2,3}}{\partial T \partial E} = - \left(\frac{\partial S}{\partial E} \right)_{P,E}, \quad \frac{\partial^2 F_{1,2,3}}{\partial E \partial T} = - \left(\frac{\partial Z}{\partial T} \right)_{P,T} \rightarrow \boxed{\left(\frac{\partial S}{\partial E} \right)_{P,E} = \left(\frac{\partial Z}{\partial T} \right)_{P,T}}$$

$$\frac{\partial^2 F_{1,2,3}}{\partial P \partial E} = \left(\frac{\partial V}{\partial E} \right)_{T,E}, \quad \frac{\partial^2 F_{1,2,3}}{\partial E \partial T} = - \left(\frac{\partial Z}{\partial P} \right)_{P,T} \rightarrow \boxed{\left(\frac{\partial V}{\partial E} \right)_{T,E} = - \left(\frac{\partial Z}{\partial P} \right)_{P,T}}$$

14.113

A 2 kg mixture of 50% argon and 50% nitrogen by mass is in a tank at 2 MPa, 180 K. How large is the volume using a model of (a) ideal gas and (b) Redlich Kwong equation of state with a, b for a mixture.

a) Ideal gas mixture

$$\text{Eq.13.15: } R_{\text{mix}} = \sum c_i R_i = 0.5 \times 0.2081 + 0.5 \times 0.2968 = 0.25245 \text{ kJ/kg K}$$

$$V = \frac{mR_{\text{mix}}T}{P} = \frac{2 \times 0.25245 \times 180}{2000} = 0.0454 \text{ m}^3$$

b) Redlich Kwong equation of state. Before we can do the parameters a, b for the mixture we need the individual component parameters, Eq.14.54, 13.55.

$$a_{\text{Ar}} = 0.42748 \frac{R^2 T_c^{5/2}}{P_c} = 0.42748 \frac{0.2081^2 \times 150.8^{2.5}}{4870} = 1.06154$$

$$a_{\text{N}_2} = 0.42748 \frac{R^2 T_c^{5/2}}{P_c} = 0.42748 \frac{0.2968^2 \times 126.2^{2.5}}{3390} = 1.98743$$

$$b_{\text{Ar}} = 0.08664 \frac{RT_c}{P_c} = 0.08664 \frac{0.2081 \times 150.8}{4870} = 0.000558$$

$$b_{\text{N}_2} = 0.08664 \frac{RT_c}{P_c} = 0.08664 \frac{0.2968 \times 126.2}{3390} = 0.000957$$

Now the mixture parameters are from Eq.14.84

$$a_{\text{mix}} = \left(\sum c_i a_i^{1/2} \right)^2 = (0.5 \times \sqrt{1.06154} + 0.5 \times \sqrt{1.98743})^2 = 1.4885$$

$$b_{\text{mix}} = \sum c_i b_i = 0.5 \times 0.000558 + 0.5 \times 0.000957 = 0.000758$$

$$\text{Using now Eq.14.53: } P = \frac{RT}{v-b} - \frac{a}{v(v+b)T^{1/2}}$$

$$2000 = \frac{0.25245 \times 180}{v - 0.000758} - \frac{1.4885}{v(v + 0.000758) 180^{1/2}}$$

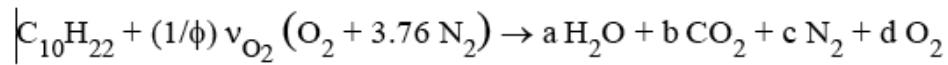
By trial and error we find the specific volume, $v = 0.02102 \text{ m}^3/\text{kg}$

$$V = mv = 0.04204 \text{ m}^3$$

Homework 9

15.21

A certain fuel oil has the composition $C_{10}H_{22}$. If this fuel is burned with 150% theoretical air, what is the composition of the products of combustion?



Stoichiometric combustion: $\phi = 1$, $d = 0$,

C balance: $b = 10$

H balance: $a = 22/2 = 11$,

O balance: $2 v_{O_2} = a + 2b = 11 + 20 = 31 \Rightarrow v_{O_2} = 15.5$

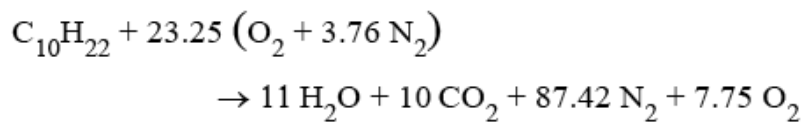
Actual case: $1/\phi = 1.5 \Rightarrow v_{O_2} = 1.5 \times 15.5 = 23.25$

H balance: $a = 11$, C balance: $b = 10$,

N balance: $c = 23.25 \times 3.76 = 87.42$

O₂ balance: $d = 23.25 - 10 - 11/2 = 7.75$ (excess oxygen)

The combustion process is



15.25

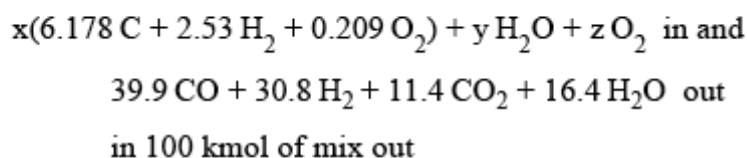
A Pennsylvania coal contains 74.2% C, 5.1% H, 6.7% O, (dry basis, mass percent) plus ash and small percentages of N and S. This coal is fed into a gasifier along with oxygen and steam, as shown in Fig. P14.26. The exiting product gas composition is measured on a mole basis to: 39.9% CO, 30.8% H₂, 11.4% CO₂, 16.4% H₂O plus small percentages of CH₄, N₂, and H₂S. How many kilograms of coal are required to produce 100 kmol of product gas? How much oxygen and steam are required?

Convert the mass concentrations to number of kmol per 100 kg coal:

$$\text{C: } n = 74.2/12.01 = 6.178 \quad \text{H}_2: n = 5.1/2.016 = 2.530$$

$$\text{O}_2: n = 6.7/31.999 = 0.209$$

Now the combustion equation reads



Now we can do the atom balance to find (x, y, z)

$$\text{C balance: } 6.178 x = 39.9 + 11.4 \rightarrow x = 8.304$$

$$\text{H}_2 \text{ balance: } 2.53 \times 8.304 + y = 30.8 + 16.4 \rightarrow y = 26.191$$

$$\text{O}_2 \text{ balance: } 0.209 \times 8.304 + \frac{26.191}{2} + z = \frac{39.9}{2} + 11.4 + \frac{16.4}{2} \rightarrow z = 24.719$$

Therefore, for 100 kmol of mixture out

require: **830.4 kg** of coal
 26.191 kmol of steam
 24.719 kmol of oxygen



16.20

A container has liquid water at 20°C, 100 kPa in equilibrium with a mixture of water vapor and dry air also at 20°C, 100 kPa. How much is the water vapor pressure and what is the saturated water vapor pressure?

From the steam tables we have for saturated liquid:

$$P_g = 2.339 \text{ kPa}, \quad v_f = 0.001002 \text{ m}^3/\text{kg}$$

The liquid is at 100 kPa so it is compressed liquid still at 20°C so from Eq.14.15 at constant T

$$g_{\text{liq}} - g_f = \int v \, dP = v_f (P - P_g)$$

The vapor in the moist air is at the partial pressure P_v also at 20°C so we assume ideal gas for the vapor

$$g_{\text{vap}} - g_g = \int v \, dP = RT \ln \frac{P_v}{P_g}$$

We have the two saturated phases so $g_f = g_g$ ($q = h_{fg} = T s_{fg}$) and now for equilibrium the two Gibbs function must be the same as

$$g_{\text{vap}} = g_{\text{liq}} = RT \ln \frac{P_v}{P_g} + g_g = v_f (P - P_g) + g_f$$

leaving us with

$$\ln \frac{P_v}{P_g} = v_f (P - P_g) / RT = \frac{0.001002 (100 - 2.339)}{0.4615 \times 293.15} = 0.000723$$

$$P_v = P_g \exp(0.000723) = 2.3407 \text{ kPa.}$$

This is only a minute amount above the saturation pressure. For the moist air applications in Chapter 13 we neglected such differences and assumed the partial water vapor pressure at equilibrium (100% relative humidity) is P_g . The pressure has to be much higher for this to be a significant difference.

16.25

For the dissociation of oxygen, $O_2 \rightleftharpoons 2O$, around 2000 K we want a mathematical expression for the equilibrium constant $K(T)$. Assume constant heat capacity, at 2000 K, for O_2 and O from Table A.9 and develop the expression from Eqs. 16.12 and 16.15.

From Eq. 16.15 the equilibrium constant is

$$K = \exp\left(-\frac{\Delta G^0}{RT}\right); \quad \Delta G^0 = \Delta H^0 - T \Delta S^0$$

and the shift is

$$\Delta G^0 = 2 \bar{h}_O - \bar{h}_{O_2} - T(2\bar{s}_O^0 - \bar{s}_{O_2}^0)$$

Substitute the first order approximation to the functions \bar{h} and \bar{s}^0 as

$$\bar{h} = \bar{h}_{2000\text{ K}} + \bar{C}_p (T - 2000); \quad \bar{s}^0 = \bar{s}_{2000\text{ K}}^0 + \bar{C}_p \ln \frac{T}{2000}$$

The properties are from Table A.9 and $\bar{R} = 8.3145 \text{ kJ/kmol K}$

Oxygen O_2 : $\bar{h}_{2000\text{ K}} = 59\,176 \text{ kJ/kmol}$, $\bar{s}_{2000\text{ K}}^0 = 268.748 \text{ kJ/kmol K}$

$$\bar{C}_p = \frac{\bar{h}_{2200\text{ K}} - \bar{h}_{1800\text{ K}}}{2200 - 1800} = \frac{66\,770 - 51\,674}{400} = 37.74 \text{ kJ/kmol K}$$

Oxygen O : $\bar{h}_{2000\text{ K}} = 35\,713 + 249\,170 = 284\,883 \text{ kJ/kmol}$,

$$\bar{s}_{2000\text{ K}}^0 = 201.247 \text{ kJ/kmol K}$$

$$\bar{C}_p = \frac{\bar{h}_{2200\text{ K}} - \bar{h}_{1800\text{ K}}}{2200 - 1800} = \frac{39\,878 - 31\,547}{400} = 20.8275 \text{ kJ/kmol K}$$

Substitute and collect terms

$$\frac{\Delta G^0}{RT} = \frac{\Delta H^0}{RT} - \frac{\Delta S^0}{R} = \frac{\Delta H_{2000}^0}{RT} + \frac{\Delta \bar{C}_p 2000}{R} \left[\frac{T-2000}{T} - \ln \frac{T}{2000} \right] - \frac{\Delta S_{2000}^0}{R}$$

Now we have

$$\Delta H_{2000}^0 / \bar{R} = (2 \times 284\,883 - 59\,176) / 8.3145 = 61\,409.6 \text{ K}$$

$$\Delta \bar{C}_p 2000 / \bar{R} = (2 \times 20.8275 - 37.74) / 8.3145 = 0.470864$$

$$\Delta S_{2000}^0 / \bar{R} = (2 \times 201.247 - 268.748) / 8.3145 = 16.08587$$

so we get

$$\begin{aligned}\frac{\Delta G^0}{RT} &= \frac{61\,409.6}{T} + 0.470864 \left[\frac{T-2000}{T} - \ln \frac{T}{2000} \right] - 16.08587 \\ &= \frac{60\,467.9}{T} - 15.615 - 0.470864 \ln \frac{T}{2000}\end{aligned}$$

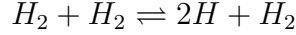
Now the equilibrium constant $K(T)$ is approximated as

$$K(T) = \exp \left[15.615 - \frac{60\,467.9}{T} + 0.470864 \ln \frac{T}{2000} \right]$$

Homework 10

2. Consider an isochoric, isothermal reaction in which $n_{H_2} = 1$ kmole, $n_H = 0$ kmole at $t = 0$, and for which $T = 5200$ K and $P = 1500$ kPa.

Consider a Hydrogen dissociation reaction,



For this reaction $a = 2.23 \times 10^{12} \frac{\text{cm}^3 \text{K}^{-1/2}}{\text{mole s}}$, $\beta = 0.5$, and $\bar{E} = 92600 \frac{\text{cal}}{\text{mole}}$ (a) Formulate the reaction kinetics in the form

$$\frac{d\bar{\rho}_{h_2}}{dt} = f(\bar{\rho}_{H_2})$$

$$\frac{d\bar{\rho}_{h_2}}{dt} = \nu_1 a T^\beta e^{\frac{-\bar{E}}{RT}} \left(\prod_{k=1}^N \bar{\rho}^{\nu'_k} \right) \left(1 - \frac{1}{k_c} \prod_{k=1}^N \bar{\rho}^{\nu_k} \right)$$

$$(\bar{\rho}_{H_2})_{t=0} = \frac{P_0}{RT} = \frac{1500 \text{ kPa}}{8.314 \frac{\text{kJ}}{\text{kmol} \cdot \text{K}} \cdot 5200 \text{ K}} = 0.034696 \frac{\text{kmol}}{\text{m}^3}$$

$$V = \frac{(n_{H_2})_{t=0}}{(\bar{\rho}_{H_2})_{t=0}} = 28.82 \text{ m}^3$$

$$-dn_{H_2} = 1/2 n_H \rightarrow -(n_{H_2} - (n_{H_2})_{t=0}) = 1/2 (n_{H_2} - (n_{H_2})_{t=0})$$

Therefore:

$$\frac{d\bar{\rho}_{h_2}}{dt} = \nu_{H_2} a T^\beta e^{\frac{-\bar{E}}{RT}} (\bar{\rho}_{H_2})^{\nu'_{H_2}} (\bar{\rho}_H)^{\nu'_H} \left(1 - \frac{1}{k_c} (\bar{\rho}_{H_2})^{\nu_{H_2}} (\bar{\rho}_H)^{\nu_H} \right)$$

$$\nu'_{H_2} = 2, \nu'_H = 0, \nu_{H_2} = -1, \nu_H = 2$$

$$\frac{d\bar{\rho}_{h_2}}{dt} = -a T^\beta e^{\frac{-\bar{E}}{RT}} (\bar{\rho}_{H_2})^2 \left(1 - \frac{1}{k_c} \frac{(\bar{\rho}_{H_2})^2}{(\bar{\rho}_H)} \right)$$

$$k(T) = a T^\beta e^{\frac{-\bar{E}}{RT}} = -2.0532 \times 10^7$$

$$k_c = \left(\frac{P_0}{RT} \right)^{\sum_i \nu_i} e^{\frac{-\Delta G}{RT}}, \Delta G = 179564.2 \frac{\text{kJ}}{\text{kmol}} \rightarrow k_c = 0.14723$$

$$\rightarrow \boxed{\frac{d\bar{\rho}_{h_2}}{dt} = -2.0532 \times 10^7 (\bar{\rho}_{H_2})^2 \left(1 - \frac{1}{0.14723} \cdot \frac{[2(0.034696 - \bar{\rho}_{H_2})]^2}{\bar{\rho}_{H_2}} \right)}$$

(b) Find all equilibria. Equilibria are located at the points where $\frac{d\bar{\rho}_{h_2}}{dt} = 0$.

$$\bar{\rho}_{H_2} = \{0, 0.0129, 0.0933\}$$

(c) Ascertain the stability of each equilibrium point. For stability, $\frac{d^2 \bar{\rho}_{h_2}}{dt^2}$ must be negative. Therefore,

$$\begin{array}{ll} \bar{\rho} = 0 & \text{unstable} \\ \bar{\rho} = 0.0129 & \text{stable} \\ \bar{\rho} = 0.0933 & \text{unstable and non-physical} \end{array}$$