Math 104, Fall 2017

Practice Problems for Midterm 2

1. Use the Trapezoidal Rule and Simpson's rule to approximate:

   (a) $\int_1^5 \frac{1}{x^2}\,\mathrm{d}x$, using 4 subintervals

   (b) $\int_0^4 x^3\,\mathrm{d}x$, using 4 subintervals

2. Compute the following improper integrals:

   (a) $\int_0^1 \frac{\mathrm{d}x}{x^2}$

   (b) $\int_1^\infty \frac{\mathrm{d}x}{x^2}$

   (c) $\int_0^\infty \frac{\mathrm{d}x}{x^2}$

   (d) $\int_1^\infty \frac{\mathrm{d}x}{\sqrt{x}}$

   (e) $\int_0^1 \frac{\mathrm{d}x}{\sqrt{x}}$

3. Decide if the following integrals converge or diverge. If they converge, you **do not** need to compute their value.

   (a) $\int_1^\infty \frac{\sin^2 x}{x^2}\mathrm{d}x$

   (b) $\int_2^\infty \frac{\mathrm{d}x}{\sqrt{x^2-1}}$

   (c) $\int_2^\infty \frac{\mathrm{d}x}{\sqrt{x^4-1}}$

   (d) $\int_1^\infty \frac{e^x}{x}\,\mathrm{d}x$

   (e) $\int_1^\infty \frac{1-e^{-x}}{x}\,\mathrm{d}x$

4. Consider the function $f(x) = \begin{cases} 0, & x < 0 \\ Cx^2e^{-4x}, & x \geq 0. \end{cases}$

   (a) What value of C makes the function $f(x)$ a probability density?

   (b) What is the mean of this probability distribution?

5. Decide if the sequence $\{a_n\}$ converges or diverges. If it converges, find its limit.

a) $a_n = \dfrac{n+1}{n^3}$

b) $a_n = 3^n$

c) $a_n = \dfrac{n^3}{3^n}$

d) $a_n = 3^{1/n}$

e) $a_n = \left(\dfrac{1}{3}\right)^n$

f) $a_n = \left(\dfrac{1}{3}\right)^{1/n}$

g) $a_n = \sqrt[n]{3n}$

h) $a_n = \sqrt[3n]{n}$

6. Compute $\lim_{n\to\infty} \left(1+\dfrac{a}{n}\right)^{bn}$, where a and b are any nonzero real numbers.

7. Compute $\lim_{n\to\infty} \sqrt[n]{n!}$ or justify (with inequalities) if it diverges.

8. Compute $\lim_{n\to\infty} \dfrac{n!}{2^n}$ or justify (with inequalities) if it diverges.

9. Given any $a > 0$, compute $\lim_{n\to\infty} \dfrac{n!}{a^n}$ or justify (with inequalities) if it diverges.

10. Decide if the following series converge or diverge. If they converge, find their limit.

(a) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{4^n}$

(b) $\displaystyle\sum_{n=1}^{\infty} \frac{7}{2^n}$

(c) $\displaystyle\sum_{n=1}^{\infty} \frac{6^n}{n^2+4}$

(d) $\displaystyle\sum_{n=1}^{\infty} \tan(n) - \tan(n+1)$

(e) $\displaystyle\sum_{n=1}^{\infty} \arccos\left(\frac{1}{n+1}\right) - \arccos\left(\frac{1}{n+2}\right)$

11. Use a convergence test to determine if the following series converge or not:

(a) $\sum_{n=1}^{\infty} \frac{n^2}{n^4+1}$

(b) $\sum_{n=1}^{\infty} \frac{n^5}{n^6+2n^3+1}$

(c) $\sum_{n=2}^{\infty} \frac{3^{n+2}}{\ln n}$

(d) $\sum_{n=1}^{\infty} \left(\frac{4n+1}{2n-5}\right)^n$

(e) $\sum_{n=1}^{\infty} \frac{4^n}{(3n)^n}$

(f) $\sum_{n=1}^{\infty} \frac{n+2}{4^n}$

12. A dosage of 100mg of a certain drug is given to a patient at 8:00am each day. Suppose 10% of the drug remains in the body after one full day period (8:00am next day).

a) What is the amount of drug in the body three days after the treatment started before the next dose is given at 8:00am?

b) Use a geometric series to estimate the amount of drug in the body after a very long time **before** a new dose is given.

13. Suppose that a basketball rebounds 2/3 of its previous height after each bounce. If you drop this basketball from a height of 3m, how far does it travel up and down until it stops moving?