HW5

1. $\lim_{x\to 0} \frac{\sin x}{x} = 1.$ so $f(0) = 1.$

$f(\frac{\pi}{4}) = \frac{\frac{1}{2}}{(\frac{\pi}{4})^2} = \frac{8}{\pi^2}.$ $f(\frac{\pi}{2}) = \frac{1}{(\frac{\pi}{2})^2} = \frac{4}{\pi^2}$

$f(\frac{3\pi}{4}) = \frac{\frac{1}{2}}{(\frac{3\pi}{4})^2} = \frac{8}{9\pi^2}.$

$f(\pi) = 0$

$\Delta x = \frac{\pi}{4}$

For Trapezoidal rule:

$A_4^{Tra} = \frac{\Delta x}{2} [f(0) + 2f(\frac{\pi}{4}) + 2f(\frac{2\pi}{4}) + 2f(\frac{3\pi}{4}) + f(\pi)]$

$= \frac{\pi}{8} (1 + \frac{16}{\pi^2} + \frac{8\cancel{4}}{\pi^2} + \frac{16}{9\pi^2} + 0)$

$= \frac{\pi}{8} + \frac{\pi}{8} (25 \frac{7}{9})$

For Simpson's rule:

$A_4^{sim} = \frac{\Delta x}{3} (f(0) + 4f(\frac{\pi}{4}) + 2f(\frac{2\pi}{4}) + 4f(\frac{3\pi}{4}) + f(\pi))$

$= \frac{\pi}{12} (1 + \frac{32}{\pi^2} + \frac{8}{\pi^2} + \frac{32}{9\pi^2} + 0)$

$= \frac{\pi}{12} (1 + \frac{392}{9\pi^2})$

2. We want to find

$$\int_0^{\infty} \left(\frac{1}{x+2} - \frac{1}{x+8}\right) dx$$

$$\overset{\text{by def}}{=} \lim_{A\to+\infty} \int_0^{A} \left(\frac{1}{x+2} - \frac{1}{x+8}\right) dx$$

$$= \lim_{A\to+\infty} \left(\ln(x+2) - \ln(x+8) \Big|_0^A \right)$$

$$= \lim_{A\to+\infty} \left(\ln\frac{A+2}{A+8} - \ln\frac{1}{4} \right) \overset{\lim_{A\to\infty}\frac{A+2}{A+8}=1}{=\!=\!=\!=} -\ln\frac{1}{4}$$

$$= \ln 4.$$

3 (a) Not converge. since $\lim_{x\to\infty} x^{-3}e^{4x} = +\infty$

(b) $$\int x e^{-4x} dx = x\cdot\left(-\frac{1}{4}e^{-4x}\right) - \int -\frac{1}{4}e^{-4x} dx$$

$$= -\frac{1}{4}xe^{-4x} - \frac{1}{16}e^{-4x}$$

Thus $$\int_1^{+\infty} x e^{-4x} dx$$

$$= \left(-\frac{1}{4}xe^{-4x} - \frac{1}{16}e^{-4x}\right)\Big|_1^{\infty}$$

$$= \frac{1}{4}e^{-4} + \frac{1}{16}e^{-4} = \frac{5}{16}e^{-4}.$$

(c) $\int (4x-1)^{-\frac{1}{3}} dx$

$= \frac{1}{4} \cdot \frac{3}{2} (4x-1)^{\frac{2}{3}} = \frac{3}{8} (4x-1)^{\frac{2}{3}}.$

Thus $\int_0^2 (4x-1)^{-\frac{1}{3}} dx = \int_0^{\frac{1}{4}} (4x-1)^{-\frac{1}{3}} dx + \int_{\frac{1}{4}}^2 (4x-1)^{-\frac{1}{3}} dx$

$= \lim_{A \to \frac{1}{4}^-} \int_0^A (4x-1)^{-\frac{1}{3}} dx + \lim_{B \to \frac{1}{4}^+} \int_B^2 (4x-1)^{-\frac{1}{3}} dx$

$= \lim_{A \to \frac{1}{4}^-} \frac{3}{8} (4x-1)^{\frac{2}{3}} \Big|_0^A + \lim_{B \to \frac{1}{4}^+} \frac{3}{8} (4x-1)^{\frac{2}{3}}$

$\lim_{A \to \frac{1}{4}^-} \frac{3}{8} (4A-1)^{\frac{2}{3}} = 0$

$= \frac{3}{8} \left(7^{\frac{2}{3}} - \underline{(-1)^{\frac{2}{3}}} \right)$

I guess the function $(4x-1)^{-\frac{1}{3}}$ is not defined at $[0, \frac{1}{4})$.

d) Not converge, since

$$\lim_{x \to +\infty} \frac{\frac{4x+3}{x^2+2}}{\frac{1}{x}} = \lim_{x \to \infty} \frac{4x^2+3x}{x^2+2} = 4.$$

since $\int_0^\infty \frac{1}{x} dx$ diverges.

$\int_0^\infty \frac{4x+3}{x^2+2} dx$ diverges

e) This is not an improper integral

Write $2x = (x+1)^2 - (x^2+1)$

$$\int \frac{2x\,dx}{(x+1)(x^2+1)}\,dx = \int \frac{(x+1)^2 - (x^2+1)}{(x+1)(x^2+1)}\,dx$$

$$= \int \frac{x+1}{x^2+1} - \frac{1}{x+1}\,dx$$

$$= \frac{1}{2}\ln(x^2+1) + \arctan x - \ln(x+1)$$

so $\displaystyle\int_0^2 \frac{2x}{(x+1)(x^2+1)}\,dx = \frac{1}{2}\ln(x^2+1) + \arctan x - \ln(x+1)\Big|_0^2$

$$= \left(\frac{1}{2}\ln 5 + \arctan 2 - \ln 3\right) - \underbrace{\left(\frac{1}{2}\cdot 0 + 0 - 0\right)}_{=0}$$

(f) $$\int \frac{dx}{(x^2-6x+9)} = \int \frac{dx}{(x-3)^2+1}$$

$$= \arctan(x-3).$$

Thus $\displaystyle\int_4^\infty \frac{dx}{x^2-6x+9}\,dx = \arctan(x-3)\Big|_4^\infty$

$$= \frac{\pi}{2} - \frac{\pi}{4} = \frac{\pi}{4}.$$

(g)

$$\int \tan\theta \, d\theta = \int \frac{\sin\theta}{\cos\theta} d\theta$$

$$= \int \frac{-d\cos\theta}{\cos\theta} = -\ln\cos\theta.$$

thus $\displaystyle\int_0^{\frac{\pi}{2}} \tan\theta \, d\theta = -\ln\cos\theta \Big|_0^{\frac{\pi}{2}} = \underbrace{-\ln 0^+ + \ln 1}_{\to\infty}$

thus does not exist.