Math 110, Spring 2016
HWK06 due Mar 2

1. Let's use Taylor polynomials to get upper and lower bounds on $\cos(\pi/5)$.

   (a) Compute the quadratic Taylor polynomial $P_2(x)$ for $f(x) = \cos(x)$ at $x = \pi/4$.

   (b) What is $P_2(\pi/5)$?

   Compute some EASY bounds on f''' on the interval $[\pi/5, \pi/4]$ and use these to bound the error $\cos(\pi/5) - P_2(\pi/5)$.

   (c) Write this as upper and lower bounds on $\cos(\pi/5)$.

2. Now we'll use Taylor polynomials to approximate a difficult integral such as $\int_0^{1/2} \cos(x^2)\,dx$.

   (a) What is the quartic Taylor polynomial $P_4(x)$ at $x = 0$?

   (b) What is $\displaystyle\int_0^{1/2} P_4(x)\,dx$?

3. Compute the Taylor polynomials as indicated.

   (a) $f(x) = \tan x$, $n = 3$, $a = 0$

   (b) $f(x) = \tan x$, $n = 2$, $a = \pi/4$

   (c) $f(x) = \ln\cos x$, $n = 3$, $a = 0$

   (d) $f(x) = (x + x^2)^{2/3}$, $n = 2$, $a = 1$

4. Compute these Taylor polynomials by making use of substitution, addition and/or multiplication.

   (a) $f(x) = \cos(x^3)$, $n = 8$, $a = 0$

   (b) $f(x) = (1 - 3x + 2x^2)\sin(x)$, $n = 3$, $a = 0$

   (c) $f(x) = (e^x - 1 - x)^3$, $n = 7$, $a = 0$

5. Use Taylor polynomials to approximate the following values without using a calculator. In each case, indicate your choice of f, n and a, choosing n large enough that you expect to be within 2% of the true value (you don't need a rigorous bound - it's OK if you're in the ballpark).

   (a) $\sqrt[3]{10}$

   (b) $\arctan(0.95)$

   (c) $\log_2 65$

   (d) $\left(\frac{17}{20}\right)^{2.1}$

6. A company uses a continuous time model to forecast its revenue, where $f(t)$ is the instantaneous rate of revenue in millions of dollars per year. Based on their trend data over the last year, they are able to estimate the present revenue rate and its first few derivatives:

$$\begin{aligned} f(0) &\approx 45 \\ f'(0) &\approx 15 \\ f''(0) &\approx 6 \\ f'''(0) &\approx 3 \end{aligned}$$

In order to assess a buyout offer, the company would like to forecast the revenue they will receive over the next year (NOT the instantaneous rate after one year but the total over the coming year). Please provide an estimate of this, clearly stating your methods and assumptions.